# Abstract

We're going to talk about traffic lights.

# About Notation

## Network Topology

| Symbol | Description |
| --- | --- |
| $i, j \in N$ | nodes (junctions) |
| $a, b \in A \subseteq N \times N$ | directional arcs (lanes or lane groups) |
| $a = (N_a^-, N_a^+)$ | tail node and head node of arc $a$ |
| $A_i^- = \{a \in A \mid N_a^+ = i\}$ | backward star of node $i$ (inbound arcs) |
| $A_i^+ = \{b \in A \mid N_a^- = i\}$ | forward star of node $i$ (outbound arcs) |
| $y, z \in Y$ | manoeuvres |

## Supply and Demand

| Symbol | Unit | Description |
| --- | --- | --- |
| $q_a$ | [veh/s] | demand flow on arc $a$ |
| $\hat{q}_a$ | [veh/s] | saturation flow on arc $a$ |
| $\phi_a = {}^{q_a}/_{\hat{q}_a}$ | | flow ratio of arc $a$ |

## Signalisation

| Symbol | Description |
| --- | --- |
| $p, q \in P_j$ | signal phases at junction $j$ |
| $A^p \subseteq A_j^-$ | arcs gaining the right of way during phase $p$ |

## Signal timing

| Symbol | Unit | Description |
| --- | --- | --- |
| $t_j^C$ | [s] | signal phases at junction $j$ |
| $tp$ | [s] | nominal duration of phase $p$ |
| $ga$ | [s] | effective green time for arc or lane group $a$ |
| $\gamma_a = {}^{g_a}/_{t_j^C}$ | | effective green share for arc or lane group $a$ |
| $\chi_a = {}^{\phi_a}/_{\gamma_a}$ | | saturation of arc or lane group $a$ |
| $t_j^L$ | [s] | time lost at junction $j$ |
| $t_j^O$ | [s] | timing offset of junction $j$ |

## Performance Indices

| Symbol | Unit | Description |
|---|---|---|
| $t_a^Q$ | [s] | queue clearance time on arc $a$ |
| $\omega_a^{stop}$ | | share of vehicles stopped on arc $a$ |
| $\omega_a^d$ | [s] | average user delay $a$ |

# 1. Anatomy Of A Signal Plan

The following chapter briefly illustrates the main features of a signal plan devised for urban traffic regulation. This term encompasses all timings and schedules behind the delicate clockwork of traffic signals, from the elements that constitute a single signal program at one of the many junctions of the network, to the succession of network-wide program changes designed to meet the daily evolution of traffic demand and the propagation of vehicle flows. The features presented in this section fully define what is commonly called a pre-timed plan, and as such do not describe any real-time actuation or decision making logic. They are themselves, however, the decision variables of most optimisation methods and adaptive strategies, and it is crucial to understand their significance in order to appreciate the diversity of setting and control approaches illustrated in more detail throughout this chapter.

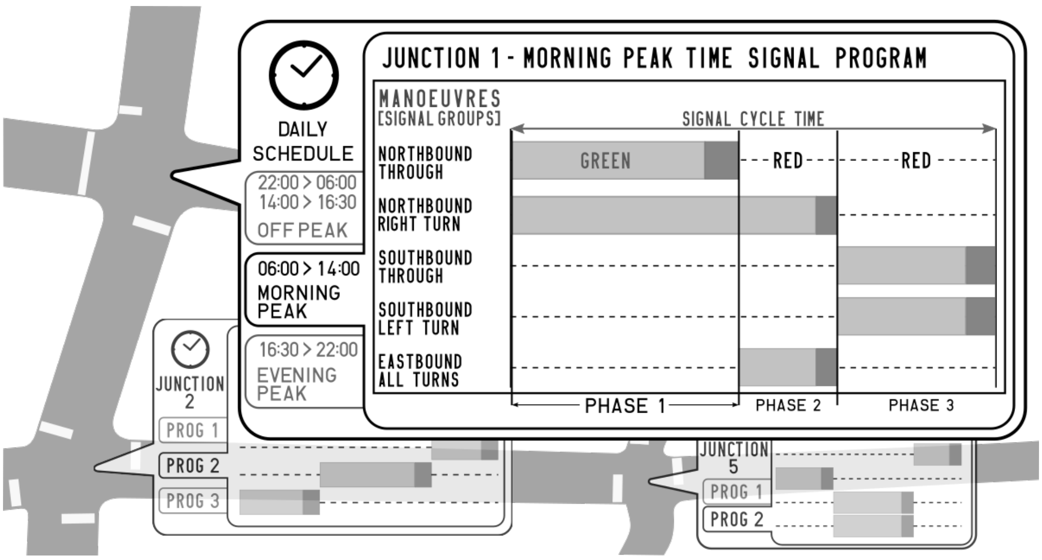


Figure 1.1 – Elements of a network-wide signal plan: a daily schedule specifies the signal programs running at each intersection. The sequence and duration of signal phases repeats over the course of every signal cycle as specified by the different signal programs, administering junction capacity amongst the expected traffic flows. During each phase, a set of compatible manoeuvres is allowed through while the others remain closed.

## 1.1 Signal Phases

Traffic signals exist mainly to separate conflicting traffic flows competing for the right of way at a road intersection. The natural way of doing so is to bundle compatible (e.g. non-secant) manoeuvres which may be safely performed simultaneously into signal phases, so that the corresponding flows may be allowed through the junction in turn. Phases are the fundamental blocks of a signal program, and are usually repeated in the same order at every signal cycle, although some signalisation systems provide phase skipping, usually as part of their public transport prioritisation strategy. Manoeuvres may pertain to different modes of transport, meaning that cars, trams and pedestrians are taken into joint consideration and can be given the right of way during the same signal phase. With reference to Figure 1.2, consider a junction, i.e. a network node $j$ where it is possible to perform a given set of manoeuvres $Y_j$ . The generic manoeuvre $y \in Y_j$ may be a turn, from an arc $a \in A_j^-$ of the node backward star, to a forward star arc $b \in A_j^+$, or a pedestrian crossing affecting one or more arcs either entering or leaving the junction.

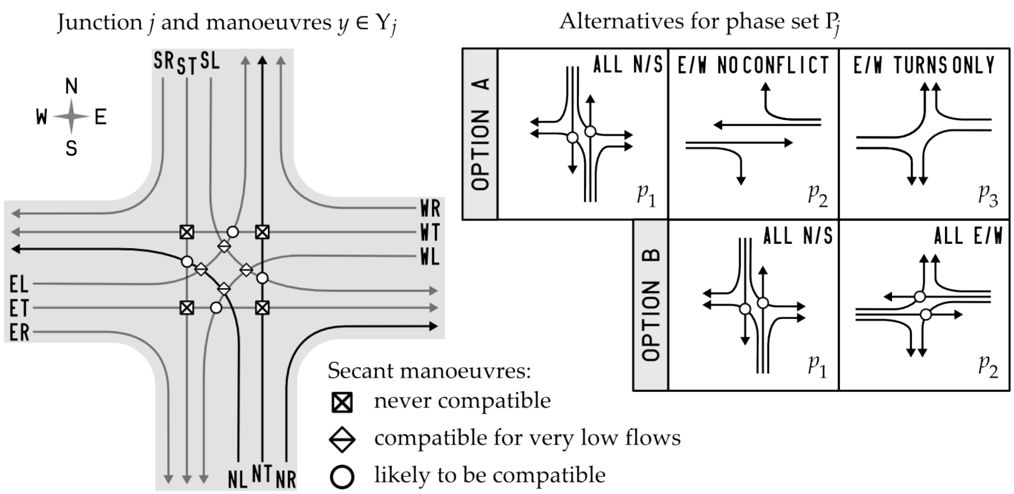


Figure 1.2 – Manoeuvres at an intersection, conflict areas and possible phasing options: option A avoids direct conflicts between Eastbound (E-) and Westbound (W-) manoeuvres, as would be desirable if high volumes were expected along that direction; option B favours a lower number of phase changes (less lost time) assuming flows to be such that left turning vehicles have space to wait at the middle of the intersection, and time to clear after the opposite through flow has decreased sufficiently to let them safely complete the manoeuvre.

From this point on, in order to present a more straightforward correlation between manoeuvres, junction geometrical layout and signalisation, the focus will be on private transport vehicles only. It shall be clear that the principles illustrated may be easily extended to handle more heterogeneous combinations of users. Under given flow conditions and for a given junction layout, manoeuvres may or may not be safe to perform simultaneously: this information is easily represented by a square Boolean matrix where rows and columns correspond to each manoeuvre and elements comply with the following rule:

$$\delta_{yz} = \begin{cases} 1 & \text{if } y \text{ and } z \text{ are compatible} \\ 0 & \text{otherwise} \end{cases} \qquad \forall y, z \in Y_j \tag{1.1}$$

Each signal phase $p \in Y_j$ identifies with a subset of compatible manoeuvres. The set of phases $P_j$ selected for the junction must therefore belong to the space of feasible signal phases, i.e. all possible combinations of manoeuvres contained in the power set $\mathcal{P}(Yj)$ such that no two are incompatible. The union of all phases must also include every available manoeuvre at least once. Formally, $P_j$ must therefore comply with the following properties: