Probability Final Exam Cheat Sheet

Gamma function

$\Gamma(z) = \int_0^\infty t^{z-1}e^{-t}dt$; for $\mathcal{R}(z) > 0$
$\Gamma(z+1) = z\Gamma(z)$; $\Gamma(n+1) = n!$ for $n \in \mathbb{N}$

Gamma distribution

$f_Y(y) = \frac{\beta^\alpha}{\Gamma(\alpha)} y^{\alpha-1}e^{-\beta y}$; $y, \alpha, \beta > 0$
$E[Y] = \alpha/\beta$; $Var(Y) = \alpha/\beta^2$
$\text{MGF}(t) = \left(1 - \frac{t}{\beta}\right)^{-\alpha}$ for $t < \beta$
$\chi^2(k) \sim \text{Gamma}(k/2, 1/2)$
Y_i indip. $\text{Gamma}(\alpha_i, \beta) \Rightarrow \sum_{i=1}^n Y_i \sim \text{Gamma}(\sum_{i=1}^n \alpha_i, \beta)$
$k * \text{Gamma}(\alpha, \beta) = \text{Gamma}(\alpha, \beta/k)$

Beta distribution

$f_Y(y) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} y^{\alpha-1}(1-y)^{\beta-1}$; $\alpha, \beta > 0$, $y \in [0, 1]$
$E[Y] = \frac{\alpha}{\alpha+\beta}$; $Var(Y) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$

Multivariate Normal distribution

$f_Y(y) = \frac{\det(A)^{1/2}e^{-\frac{1}{2}b^TAb}}{(2\pi)^{d/2}} \exp\{-\frac{1}{2}(y^TAy) + b^Ty\}$
$E[Y] = A^{-1}b$; $\text{MGF}(t) = \exp\{\mu t + \sigma^2t^2/2\}$
$Y_1|Y_2 = y_2 \sim N(\mu_1 + \Sigma_{12}\Sigma_{22}^{-1}(y_2 - \mu_2); \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21})$

t-Student distribution

$f_Y(y) = \frac{\Gamma(\frac{\nu+1}{2})}{\sqrt{\nu\pi}\Gamma(\frac{\nu}{2})}\left(1 + \frac{y^2}{\nu}\right)^{-(\nu+1)/2}$
$\nu > 0$

Uniform distribution

$Y \sim Unif(a, b)$
$f_Y(y) = \frac{1}{b-a}$ for $y \in [a, b]$ otherwise 0
$F_Y(y) = \frac{y-a}{b-a}$ for $y \in [a, b]$
$E[Y] = \frac{1}{2}(a+b)$; $Var(Y) = \frac{1}{12}(b-a)^2$

Hypergeometric distribution

$f_Y(y) = \frac{\binom{K}{y}\binom{N-K}{n-y}}{\binom{N}{n}}$
$E[Y] = n\frac{K}{N}$; $Var(Y) = n\frac{N-K}{N}\frac{K}{N}\frac{N-n}{N-1}$

Negative Binomial distribution

$f_Y(k) = \binom{k+r-1}{k}(1-p)^kp^r$
$E[Y] = \frac{r(1-p)}{p}$; $Var(Y) = \frac{r(1-p)}{p^2}$

Series

$\sum_{i=0}^n r^i = \frac{1-r^{n+1}}{1-r}$ for $|r| < 1$
$e^x = \sum_{n=0}^\infty x^n/n! = \lim_{n\to\infty}\left(1 + \frac{x}{n}\right)^n$
$\sum_{i=0}^n \frac{1}{x^p}$ for $p > 1$

Trigonometric summations

$\sin(x \pm y) = \sin(x)\cos(y) \pm \sin(y)\cos(x)$
$\cos(x \pm y) = \cos(x)\cos(y) \mp \sin(x)\sin(y)$
$z^2 = x^2 + y^2 - 2xy\cos(\text{angle}(x, y))$

Log base change

$\log_a(x) = \frac{\log_b(x)}{\log_b(a)}$

Derivatives

$\frac{d}{dx}\sin(x) = \cos(x)$; $\frac{d}{dx}\cos(x) = -\sin(x)$
$\frac{d}{dx}\arcsin(x) = \frac{1}{\sqrt{1-x^2}}$; $\frac{d}{dx}\arccos(x) = -\frac{1}{\sqrt{1-x^2}}$
$\frac{d}{dx}\tan(x) = \frac{1}{\cos^2(x)}$; $\frac{d}{dx}\arctan(x) = \frac{1}{1+x^2}$

Integrals

$\int x^kdx = \frac{x^{k+1}}{k+1} + C$ $k \neq 1$; $\int \frac{1}{x}dx = \log(|x|) + C$
$\int \tan(x)dx = \log(\frac{1}{\cos(x)}) + C$
$\int \frac{1}{a^2+x^2}dx = \frac{1}{a}\arctan(\frac{x}{a}) + C$
$\int \frac{1}{\sqrt{a^2+x^2}}dx = \arcsin(\frac{x}{a}) + C$

Taylor expansions

$e^x = \sum_{i=0}^k \frac{x^i}{i!} + o(x^k)$ for $x \to 0$
$\log(1+x) = \sum_{i=1}^k (-1)^{i-1}\frac{x^i}{i} + o(x^k)$ for $x \to 0$
$\sin(x) = \sum_{i=0}^n \frac{(-1)^ix^{2i+1}}{(2i+1)!} + o(x^{2n})$
$\cos(x) = \sum_{i=0}^n \frac{(-1)^ix^{2i}}{(2i)!} + o(x^{2n})$

Function of a random variable

$X, Y \in \mathbb{R}^d$; $Y = g(X)$ invertible and with $\det(J_{g^{-1}}(y)) \neq 0$ then $f_Y(y) = f_X(g^{-1}(y))|\det(J_{g^{-1}}(y))|$

Jacobian

$a, b \in \mathbb{R}^d$; $b = f(a)$.
$(J_f(b))_{ij} = \frac{df_i(a)}{da_j}$

Matrix inversion

$A^{-1} = (adjA)/\det(A)$;

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$$m_{11} = \begin{pmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{pmatrix}$$

$$c_{i1} = (-1)^{1+i}\det(m_{i1})$$

$$\det A = \sum_{i=1}^n c_{i1}$$

$$C = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{pmatrix}$$

$$adj(A) = C^T$$

Probability Union

$\mathbb{P}(\overset{n}{\cup}A_i) = \sum_{k=1}^n \sum_{I\subset\{1,..,n\},|I|=k} (-1)^{k-1}\mathbb{P}(\cap A_I)$

Another topic

some stuff