

Probability Final Exam Cheat Sheet

Gamma function

$$\Gamma(z) = \int_0^\infty t^{z-1} e^{-t} dt; \text{ for } \Re(z) > 0$$
$$\Gamma(z+1) = z\Gamma(z); \Gamma(n+1) = n! \text{ for } n \in \mathbb{N}$$

Gamma distribution

$$f_Y(y) = \frac{\beta^\alpha}{\Gamma(\alpha)} y^{\alpha-1} e^{-\beta y}; y, \alpha, \beta > 0$$
$$E[Y] = \alpha/\beta; \text{Var}(Y) = \alpha/\beta^2$$
$$\text{MGF}(t) = \left(1 - \frac{t}{\beta}\right)^{-\alpha} \text{ for } t < \beta$$
$$\chi^2(k) \sim \text{Gamma}(k/2, 1/2)$$

Beta distribution

$$f_Y(y) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} y^{\alpha-1} (1-y)^{\beta-1}; \alpha, \beta > 0, y \in [0, 1]$$
$$E[Y] = \frac{\alpha}{\alpha+\beta}; \text{Var}(Y) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$$

Multivariate Normal distribution

$$f_Y(y) = \frac{\det(A)^{1/2} e^{-\frac{1}{2} b^T A b}}{(2\pi)^{d/2}} \exp\left\{-\frac{1}{2} (y^T A y) + b^T y\right\}$$
$$E[Y] = A^{-1} b; \quad \text{MGF}(t) = \exp\{\mu t + \sigma^2 t^2 / 2\}$$
$$Y_1 | Y_2 = y_2 \sim N(\mu_1 + \Sigma_{12} \Sigma_{22}^{-1} (y_2 - \mu_2); \Sigma_{11} - \Sigma_{12} \Sigma_{22}^{-1} \Sigma_{21})$$

t-Student distribution

$$f_Y(y) = \frac{\Gamma(\frac{\nu+1}{2})}{\sqrt{\nu\pi} \Gamma(\frac{\nu}{2})} \left(1 + \frac{y^2}{\nu}\right)^{-(\nu+1)/2}$$
$$\nu > 0$$

Hypergeometric distribution

$$f_Y(y) = \frac{\binom{K}{y} \binom{N-K}{n-y}}{\binom{N}{n}}$$
$$E[Y] = n \frac{K}{N}; \text{Var}(Y) = n \frac{N-K}{N} \frac{K}{N} \frac{N-n}{N-1}$$

Negative Binomial distribution

$$f_Y(y) = \binom{k+r-1}{k} (1-p)^k p^r$$
$$E[Y] = \frac{r(1-p)}{p}; \text{Var}(Y) = \frac{r(1-p)}{p^2}$$

Series

$$\sum_{i=0}^n r^i = \frac{1-r^{n+1}}{1-r} \text{ for } |r| < 1$$
$$e^x = \sum_{n=0}^\infty x^n / n! = \lim_{n \rightarrow \infty} \left(1 + \frac{x}{n}\right)^n$$

Trigonometric summations

$$\sin(x \pm y) = \sin(x) \cos(y) \pm \sin(y) \cos(x)$$
$$\cos(x \pm y) = \cos(x) \cos(y) \mp \sin(x) \sin(y)$$
$$z^2 = x^2 + y^2 - 2xy \cos(\text{angle}(x, y))$$

Log base change

$$\log_a(x) = \frac{\log_b(x)}{\log_b(a)}$$

Derivatives

$$\frac{d}{dx} \sin(x) = \cos(x); \frac{d}{dx} \cos(x) = -\sin(x)$$
$$\frac{d}{dx} \arcsin(x) = \frac{1}{\sqrt{1-x^2}}; \frac{d}{dx} \arccos(x) = -\frac{1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx} \tan(x) = \frac{1}{\cos^2(x)}; \frac{d}{dx} \arctan(x) = \frac{1}{1+x^2}$$

Integrals

$$\int x^k dx = \frac{x^{k+1}}{k+1} + C \quad k \neq -1; \quad \int \frac{1}{x} dx = \log(|x|) + C$$
$$\int \tan(x) dx = \log\left(\frac{1}{\cos(x)}\right) + C$$
$$\int \frac{1}{a^2+x^2} dx = \frac{1}{a} \arctan\left(\frac{x}{a}\right) + C$$
$$\int \frac{1}{\sqrt{a^2+x^2}} dx = \arcsin\left(\frac{x}{a}\right) + C$$

Taylor expansions

$$e^x = \sum_{i=0}^k \frac{x^i}{i!} + o\left(\frac{1}{x^k}\right) \text{ for } x \rightarrow 0$$
$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} + o(x^3) \text{ for } x \rightarrow 0$$

Another topic

Some stuff