

Probability Final Exam Cheat Sheet

Gamma function

$$\Gamma(z) = \int_0^\infty t^{z-1} e^{-t} dt; \text{ for } \Re(z) > 0$$

$$\Gamma(z+1) = z\Gamma(z); \Gamma(n+1) = n! \text{ for } n \in \mathbb{N}$$

Gamma distribution

$$f_Y(y) = \frac{\beta^\alpha}{\Gamma(\alpha)} y^{\alpha-1} e^{-\beta y}; y, \alpha, \beta > 0$$

$$E[Y] = \alpha/\beta; \text{Var}(Y) = \alpha/\beta^2$$

$$\text{MGF}(t) = \left(1 - \frac{t}{\beta}\right)^{-\alpha} \text{ for } t < \beta$$

$$\chi^2(k) \sim \text{Gamma}(k/2, 1/2)$$

$$Y_i \text{ indep. } \text{Gamma}(\alpha_i, \beta) \Rightarrow \sum_{i=1}^n Y_i \sim \text{Gamma}(\sum_{i=1}^n \alpha_i, \beta)$$

$$k * \text{Gamma}(\alpha, \beta) = \text{Gamma}(\alpha, \beta/k)$$

Beta distribution

$$f_Y(y) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} y^{\alpha-1} (1-y)^{\beta-1}; \alpha, \beta > 0, y \in [0, 1]$$

$$E[Y] = \frac{\alpha}{\alpha+\beta}; \text{Var}(Y) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$$

Multivariate Normal distribution

$$f_Y(y) = \frac{\det(A)^{1/2} e^{-\frac{1}{2} b^T A b}}{(2\pi)^{d/2}} \exp\left\{-\frac{1}{2}(y^T A y) + b^T y\right\}$$

$$E[Y] = A^{-1}b; \text{MGF}(t) = \exp\{\mu t + \sigma^2 t^2/2\}$$

$$Y_1|Y_2 = y_2 \sim N(\mu_1 + \Sigma_{12}\Sigma_{22}^{-1}(y_2 - \mu_2); \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21})$$

t-Student distribution

$$f_Y(y) = \frac{\Gamma(\frac{\nu+1}{2})}{\sqrt{\nu\pi}\Gamma(\frac{\nu}{2})} \left(1 + \frac{y^2}{\nu}\right)^{-(\nu+1)/2}$$

$$\nu > 0$$

Uniform distribution

$$Y \sim \text{Unif}(a, b)$$

$$f_Y(y) = \frac{1}{b-a} \text{ for } y \in [a, b] \text{ otherwise } 0$$

$$F_Y(y) = \frac{y-a}{b-a} \text{ for } y \in [a, b]$$

$$E[Y] = \frac{1}{2}(a+b); \text{Var}(Y) = \frac{1}{12}(b-a)^2$$

Hypergeometric distribution

$$f_Y(y) = \frac{\binom{K}{y}\binom{N-K}{n-y}}{\binom{N}{n}}$$

$$E[Y] = n \frac{K}{N}; \text{Var}(Y) = n \frac{N-K}{N} \frac{K}{N} \frac{N-n}{N-1}$$

Negative Binomial distribution

$$Y = \text{"n. of failures till r-th success"}$$

$$f_Y(k) = \binom{k+r-1}{k} (1-p)^k p^r$$

$$E[Y] = \frac{r(1-p)}{p}; \text{Var}(Y) = \frac{r(1-p)}{p^2}$$

Series

$$\sum_{i=0}^n r^i = \frac{1-r^{n+1}}{1-r} \text{ for } |r| < 1$$

$$e^x = \sum_{n=0}^\infty x^n/n! = \lim_{n \rightarrow \infty} \left(1 + \frac{x}{n}\right)^n$$

$$\sum_{i=0}^n \frac{1}{x^p} \text{ for } p > 1$$

Trigonometric summations

$$\sin(x \pm y) = \sin(x) \cos(y) \pm \sin(y) \cos(x)$$

$$\cos(x \pm y) = \cos(x) \cos(y) \mp \sin(x) \sin(y)$$

$$z^2 = x^2 + y^2 - 2xy \cos(\text{angle}(x, y))$$

Log base change

$$\log_a(x) = \frac{\log_b(x)}{\log_b(a)}$$

Derivatives

$$\frac{d}{dx} \sin(x) = \cos(x); \frac{d}{dx} \cos(x) = -\sin(x)$$

$$\frac{d}{dx} \arcsin(x) = \frac{1}{\sqrt{1-x^2}}; \frac{d}{dx} \arccos(x) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx} \tan(x) = \frac{1}{\cos^2(x)}; \frac{d}{dx} \arctan(x) = \frac{1}{1+x^2}$$

Integrals

$$\int x^k dx = \frac{x^{k+1}}{k+1} + C \quad k \neq -1; \int \frac{1}{x} dx = \log(|x|) + C$$

$$\int \tan(x) dx = \log\left(\frac{1}{\cos(x)}\right) + C$$

$$\int \frac{1}{a^2+x^2} dx = \frac{1}{a} \arctan\left(\frac{x}{a}\right) + C$$

$$\int \frac{1}{\sqrt{a^2+x^2}} dx = \arcsin\left(\frac{x}{a}\right) + C$$

Taylor expansions

$$e^x = \sum_{i=0}^k \frac{x^i}{i!} + o(x^k) \text{ for } x \rightarrow 0$$

$$\log(1+x) = \sum_{i=1}^k (-1)^{i-1} \frac{x^i}{i} + o(x^k) \text{ for } x \rightarrow 0$$

$$\sin(x) = \sum_{i=0}^n \frac{(-1)^i x^{2i+1}}{(2i+1)!} + o(x^{2n})$$

$$\cos(x) = \sum_{i=0}^n \frac{(-1)^i x^{2i}}{(2i)!} + o(x^{2n})$$

Function of a random vector

$$X, Y \in \mathbb{R}^d; Y = g(X) \text{ invertible and with } \det(J_{g^{-1}}(y)) \neq 0 \text{ then } f_Y(y) = f_X(g^{-1}(y)) |\det(J_{g^{-1}}(y))|$$

Jacobian

$$a, b \in \mathbb{R}^d; b = f(a).$$

$$(J_f(b))_{ij} = \frac{df_i(a)}{da_j}$$

Matrix inversion

$$A^{-1} = (\text{adj} A) / \det(A);$$

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$$m_{11} = \begin{pmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{pmatrix}$$

$$c_{i1} = (-1)^{1+i} \det(m_{i1})$$

$$\det A = \sum_{i=1}^n c_{i1}$$

$$C = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{pmatrix}$$

$$\text{adj}(A) = C^T$$

Probability Union

$$\mathbb{P}(\cup_{i=1}^n A_i) = \sum_{I \subset \{1, \dots, n\}, |I|=k} (-1)^{k-1} \mathbb{P}(\cap_{I} A_I)$$

Another topic

some stuff