

# Stats Final Exam Cheat Sheet

## Gamma function

$$\Gamma(z) = \int_0^\infty t^{z-1} e^{-t} dt; \text{ for } \Re(z) > 0$$

$$\Gamma(z+1) = z\Gamma(z); \Gamma(n+1) = n! \text{ for } n \in \mathbb{N}$$

## Gamma distribution

$$f_Y(y) = \frac{\beta^\alpha}{\Gamma(\alpha)} y^{\alpha-1} e^{-\beta y}; y, \alpha, \beta > 0$$

$$E[Y] = \alpha/\beta; \text{Var}(Y) = \alpha/\beta^2$$

$$\text{MGF}(t) = \left(1 - \frac{t}{\beta}\right)^{-\alpha} \text{ for } t < \beta$$

$$\chi^2(k) \sim \text{Gamma}(k/2, 1/2)$$

$$Y_i \text{ indep. } \text{Gamma}(\alpha_i, \beta) \Rightarrow \sum_{i=1}^n Y_i \sim \text{Gamma}(\sum_{i=1}^n \alpha_i, \beta)$$

$$k * \text{Gamma}(\alpha, \beta) = \text{Gamma}(\alpha, \beta/k)$$

## Inverse Gamma distribution

$$X \sim \text{Gamma}(\alpha, \beta) \quad Y = 1/X \sim \text{I-Gamma}(\alpha, \beta)$$

$$f_Y(y) = \frac{\beta^\alpha}{\Gamma(\alpha)} y^{-\alpha-1} e^{-\beta/y}; y, \alpha, \beta > 0$$

$$E[Y] = \beta/(\alpha-1); \text{Var}(Y) = \beta^2/[(\alpha-1)^2(\alpha-2)]$$

$$\text{Mode} = \beta/(\alpha+1)$$

## Inverse Gaussian distribution

$$Y \sim \text{I-G}(\phi, \lambda)$$

$$f_Y(y) = \frac{\sqrt{\lambda}}{\sqrt{2\pi}} e^{\sqrt{\lambda\phi} y - 3/2} \exp\left\{-\frac{1}{2}\left(\phi y + \frac{\lambda}{y}\right)\right\};$$

$$y, \phi \geq 0, \lambda > 0$$

$$E[Y] = \left(\frac{\lambda}{\phi}\right)^{1/2}; \text{Var}(Y) = \sqrt{\frac{\lambda}{\phi^3}}$$

## Normal Inverse Gamma distribution

$$\theta \sim \text{N-IGa}(\eta_0, \lambda_0, \alpha_0, \beta_0)$$

$$\sigma^2 \sim \text{IGa}(\alpha_0, \beta_0)$$

$$\mu|\sigma^2 \sim \text{N}(\eta_0, \sigma^2/\lambda_0)$$

$$f(\mu, \sigma^2) \propto \left(\frac{1}{\sigma^2}\right)^{\alpha_0+3/2} \exp\left\{\lambda_0 \eta_0 \frac{\mu}{\sigma^2} - \frac{1}{2\sigma^2}(\lambda_0 \eta_0^2 + 2\beta_0) - \lambda_0 \frac{\mu^2}{2\sigma^2}\right\}$$

Posterior params (from Normal model s.r.s.):

$$\alpha = \alpha_0 + \frac{n}{2}, \lambda = \lambda_0 + n$$

$$\eta = \frac{\lambda_0 \eta_0 + n \bar{y}}{\lambda_0 + n}$$

$$\beta = \beta_0 + \frac{n}{2} v^2 + \frac{n \lambda_0}{2(\lambda_0 + n)} (\bar{y} - \eta_0)^2$$

$$v = n^{-1} \sum_{i=1}^n (y_i - \bar{y})^2$$

## Beta distribution

$$f_Y(y) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} y^{\alpha-1} (1-y)^{\beta-1}; \alpha, \beta > 0, y \in [0, 1]$$

$$E[Y] = \frac{\alpha}{\alpha+\beta}; \text{Var}(Y) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$$

## Multivariate Normal distribution

$$f_Y(y) = \frac{\det(A)^{1/2} e^{-\frac{1}{2} b^T A b}}{(2\pi)^{d/2}} \exp\left\{-\frac{1}{2}(y^T A y) + b^T y\right\}$$

$$E[Y] = A^{-1}b; \text{MGF}(t) = \exp\{\mu t + \sigma^2 t^2/2\}$$

$$Y_1|Y_2 = y_2 \sim N(\mu_1 + \Sigma_{12}\Sigma_{22}^{-1}(y_2 - \mu_2); \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21})$$

## t-Student distribution

$$f_Y(y) = \frac{\Gamma(\frac{\nu+1}{2})}{\sqrt{\nu\pi}\Gamma(\frac{\nu}{2})} \left(1 + \frac{y^2}{\nu}\right)^{-(\nu+1)/2}$$

$$\nu > 0$$

## Uniform distribution

$$Y \sim \text{Unif}(a, b)$$

$$f_Y(y) = \frac{1}{b-a} \text{ for } y \in [a, b] \text{ otherwise } 0$$

$$F_Y(y) = \frac{y-a}{b-a} \text{ for } y \in [a, b]$$

$$E[Y] = \frac{1}{2}(a+b); \text{Var}(Y) = \frac{1}{12}(b-a)^2$$

## Hypergeometric distribution

$$f_Y(y) = \frac{\binom{K}{y} \binom{N-K}{n-y}}{\binom{N}{n}}$$

$$E[Y] = n \frac{K}{N}; \text{Var}(Y) = n \frac{N-K}{N} \frac{K}{N} \frac{N-n}{N-1}$$

## Negative Binomial distribution

$$Y = \text{"n. of failures till r-th success"}$$

$$f_Y(k) = \binom{k+r-1}{k} (1-p)^k p^r$$

$$E[Y] = \frac{r(1-p)}{p}; \text{Var}(Y) = \frac{r(1-p)}{p^2}$$

## Weibull distribution

$$Y \sim \text{Wei}(\gamma, \beta) \quad y, \beta, \gamma > 0$$

$$f(y) = \gamma \beta^{-\gamma} y^{\gamma-1} e^{-(y/\beta)^\gamma}$$

$$E[Y] = \beta \Gamma(1 + 1/\gamma)$$

## Series

$$\sum_{i=0}^n r^i = \frac{1-r^{n+1}}{1-r} \text{ for } |r| < 1$$

$$e^x = \sum_{n=0}^\infty x^n/n! = \lim_{n \rightarrow \infty} \left(1 + \frac{x}{n}\right)^n$$

$$\sum_{i=0}^n \frac{1}{x^i} \text{ for } p > 1$$

## Log base change

$$\log_a(x) = \frac{\log_b(x)}{\log_b(a)}$$

## Derivatives

$$\frac{d}{dx} \sin(x) = \cos(x); \frac{d}{dx} \cos(x) = -\sin(x)$$

$$\frac{d}{dx} \arcsin(x) = \frac{1}{\sqrt{1-x^2}}; \frac{d}{dx} \arccos(x) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx} \tan(x) = \frac{1}{\cos^2(x)}; \frac{d}{dx} \arctan(x) = \frac{1}{1+x^2}$$

## Integrals

$$\int x^k dx = \frac{x^{k+1}}{k+1} + C \quad k \neq -1; \int \frac{1}{x} dx = \log(|x|) + C$$

$$\int \tan(x) dx = \log\left(\frac{1}{\cos(x)}\right) + C$$

$$\int \frac{1}{a^2+x^2} dx = \frac{1}{a} \arctan\left(\frac{x}{a}\right) + C$$

$$\int \frac{1}{\sqrt{a^2+x^2}} dx = \arcsin\left(\frac{x}{a}\right) + C$$

$$\int_0^\infty y^\alpha e^{-ky} dy < \infty \text{ if } \alpha > -1 \quad (k > 0)$$

## Taylor expansions

$$e^x = \sum_{i=0}^k \frac{x^i}{i!} + o(x^k) \text{ for } x \rightarrow 0$$

$$\log(1+x) = \sum_{i=1}^k (-1)^{i-1} \frac{x^i}{i} + o(x^k) \text{ for } x \rightarrow 0$$

$$\sin(x) = \sum_{i=0}^n \frac{(-1)^i x^{2i+1}}{(2i+1)!} + o(x^{2n})$$

$$\cos(x) = \sum_{i=0}^n \frac{(-1)^i x^{2i}}{(2i)!} + o(x^{2n})$$

## Matrix

$$X = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

If  $X$  is a  $2 \times 2$  matrix:

$$X^{-1} = \frac{1}{\det X} \begin{pmatrix} D & -B \\ -C & A \end{pmatrix}$$

$X$  is positive definite if  $A > 0$  and  $\det X > 0$

If  $X$  is a block Matrix  $X^{-1}$  has elements:

$$(A - BD^{-1}C)^{-1}$$

$$-A^{-1}B(D - CA^{-1}B)^{-1}$$

$$-(D - CA^{-1}B)^{-1}CA^{-1}$$

$$(D - CA^{-1}B)^{-1}$$

$$i^{\psi\psi} = (i_{\psi\psi} - i_{\psi\lambda}(i_{\lambda\lambda})^{-1}i_{\lambda\psi})^{-1}$$

$$i^{\psi\lambda} = -i^{\psi\psi}i_{\psi\lambda}i_{\lambda\lambda}^{-1}$$

## Order Statistics

i.i.d continuous case.

$$f_{Y_{(1)}, \dots, Y_{(n)}}(y_1, \dots, y_n) = \frac{n!}{n!} f(y_1) f(y_2) \dots f(y_n) = f_Y(y_r) f_Y(y_r) = \frac{n!}{(n-r)!(r-1)!} [F(y_r)]^{r-1} f(y_r) [1 - F(y_r)]^{n-r}$$

$$f_{Y(r)Y(s)}(y, z) = \frac{n!}{(n-s)!(s-r-1)!(r-1)!} [F(y)]^{r-1} f(y) [F(z) - F(s)]^{s-r-1} f(z) [1 - F(z)]^{n-r}$$

## Profile pivot

Likelihood profile pivot  $\theta = (\psi, \lambda)$

$$W_{P_e}(\psi) = (\hat{\psi} - \psi)^T j_P(\hat{\psi}) (\hat{\psi} - \psi)$$

$$W_{P_u}(\psi) = U_P(\psi)^T i_P^{\psi\psi}(\psi, \hat{\lambda}_\psi) U_P(\psi)$$

we can use  $j^{\psi\psi}(\theta) = (j_{\psi\psi} - j_{\psi\lambda}(j_{\lambda\lambda})^{-1}j_{\lambda\psi})^{-1}$

## Confidence intervals and Test

Binomial: for  $n\theta, n(1-\theta) > 5$

$$q_2(\theta, \hat{\theta}) = \frac{\hat{\theta} - \theta}{\sqrt{\frac{\theta(1-\theta)}{n}}}$$

$$\frac{\hat{\theta} + (z_{1-\alpha/2})^2/2n}{1 + (z_{1-\alpha/2})^2/n} \pm \frac{z_{1-\alpha/2}/\sqrt{n}}{1 + (z_{1-\alpha/2})^2/n} \sqrt{\hat{\theta}(1-\hat{\theta}) + \frac{(z_{1-\alpha/2})^2}{4n}}$$

Clopper Pearson:

$$\{\theta \in (0, 1) : \mathbb{P}_\theta(Y \leq y) \geq \alpha/2 \text{ and } \mathbb{P}_\theta(Y \geq y) \geq \alpha/2\}$$

Exact distribution test:  $n\hat{\theta} \sim \text{Bi}(n, \theta)$

Mid-pvalue:  $\alpha_{mid}^{\text{oss}} = \sup_{\theta \in \Theta_0} \{\mathbb{P}_\theta(t(Y) > t(y)) + \frac{1}{2}\mathbb{P}_\theta(t(Y) = t(y))\}$

## Netwon - Raphson

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

## Total Variance

$$\text{Var}(Y) = E[\text{Var}(Y|X)] + \text{Var}(E[Y|X])$$

## Delta Method

$$\sqrt{n}(Y_n - \theta) \xrightarrow{D} U$$

$$\sqrt{n}(g(Y_n) - g(\theta)) \xrightarrow{D} g'(\theta)U$$

### Posterior Predictive Density

if  $(y^*|y, \theta) = (y^*|\theta)$   
 $p(y^*|y) = \int p(y^*|\theta)p(\theta|y)d\theta$

### Another topic

some stuff