

# Overfitting, prediction error and trade-offs

Filippo Biscarini (CNR, Milan, Italy)

[filippo.biscarini@cnr.it](mailto:filippo.biscarini@cnr.it)



# Overfitting



# What is overfitting?

We fitted a linear model on our dataset and made predictions; we then measured the “accuracy” of these predictions: **did we do it right?**



# What is overfitting?

We fitted a linear model on our dataset and made predictions; we then measured the “accuracy” of these predictions: **did we do it right?**

- short answer: **NO!**
- main reason: **overfitting**



# What is overfitting?

Overfitting:

Fitting too well the data:  $R^2$  too large ( $\approx 1$ )



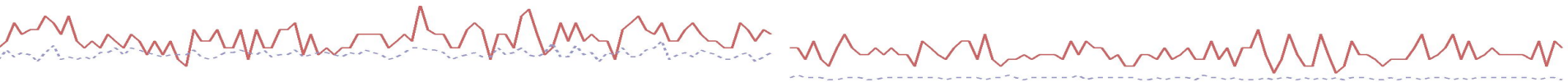
# What is overfitting?

## Overfitting:

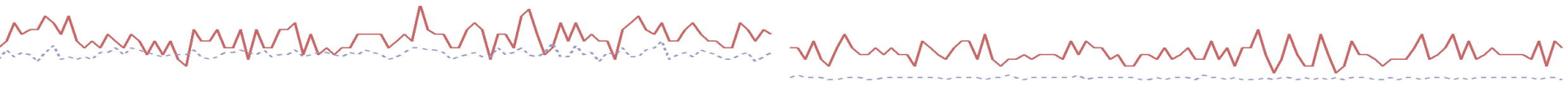
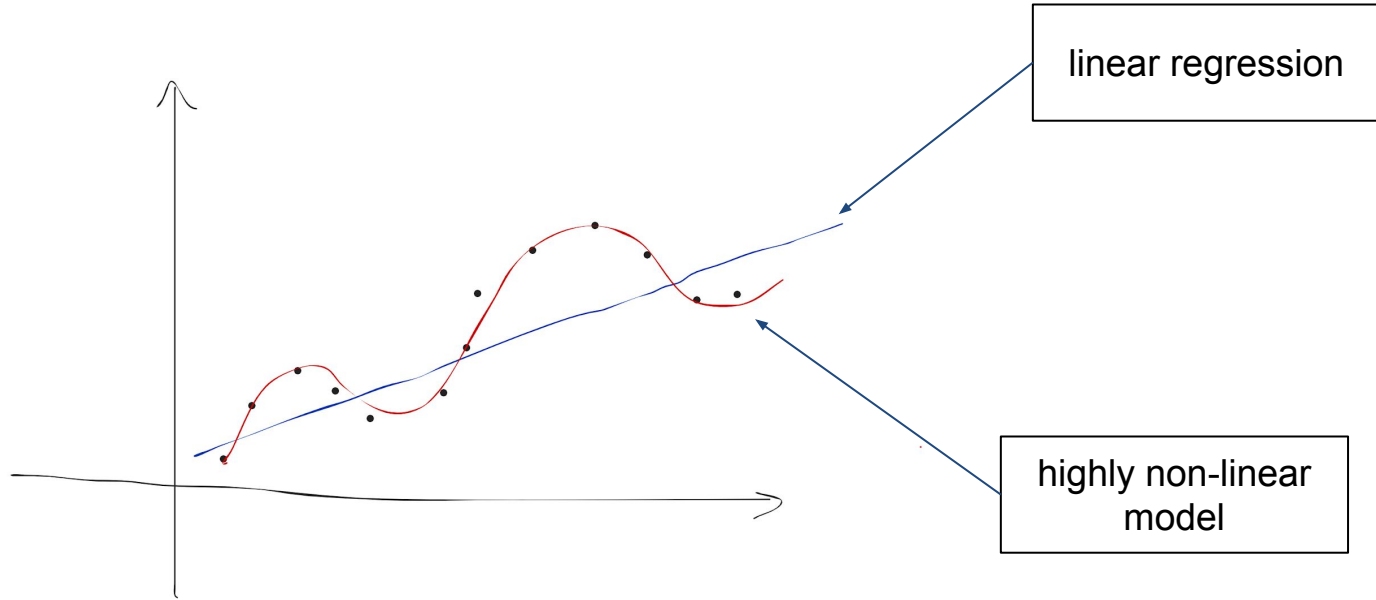
Fitting too well the data:  $R^2$  too large ( $\approx 1$ )

overfitting happens with:

- using the same data to fit the model and make predictions
- overparameterization of the model (e.g. too many effects)
- flexible methods (e.g. polynomial functions, splines, classification trees etc.)

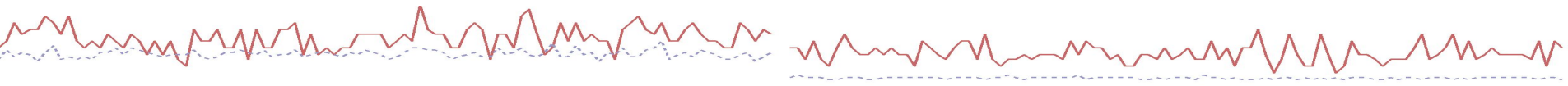
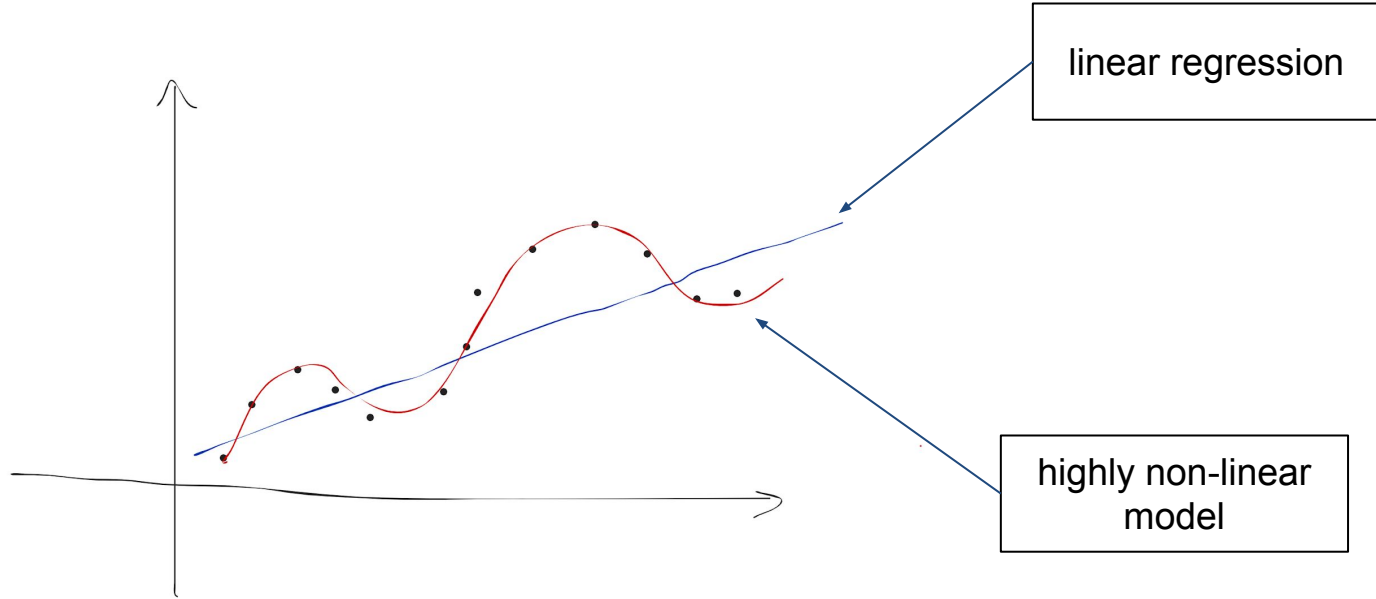


# What is overfitting?



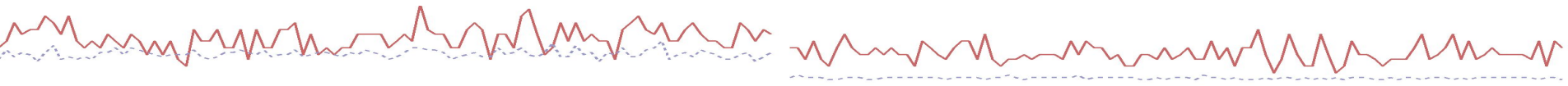
# What is overfitting?

Think of KNN  
with  $k=1$ !





# Prediction error



# Prediction error

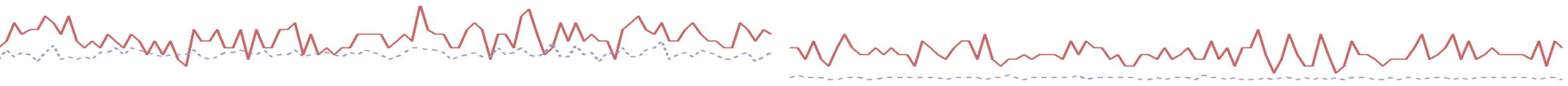
$$E\left(y - \hat{f}(x)\right)^2 = Var\left(\hat{f}(x)\right) + \left[\text{Bias}(\hat{f}(x))\right]^2 + Var(\epsilon)$$



variance



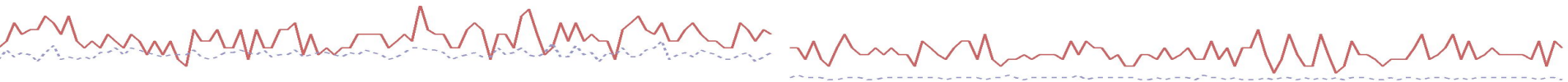
bias<sup>2</sup>



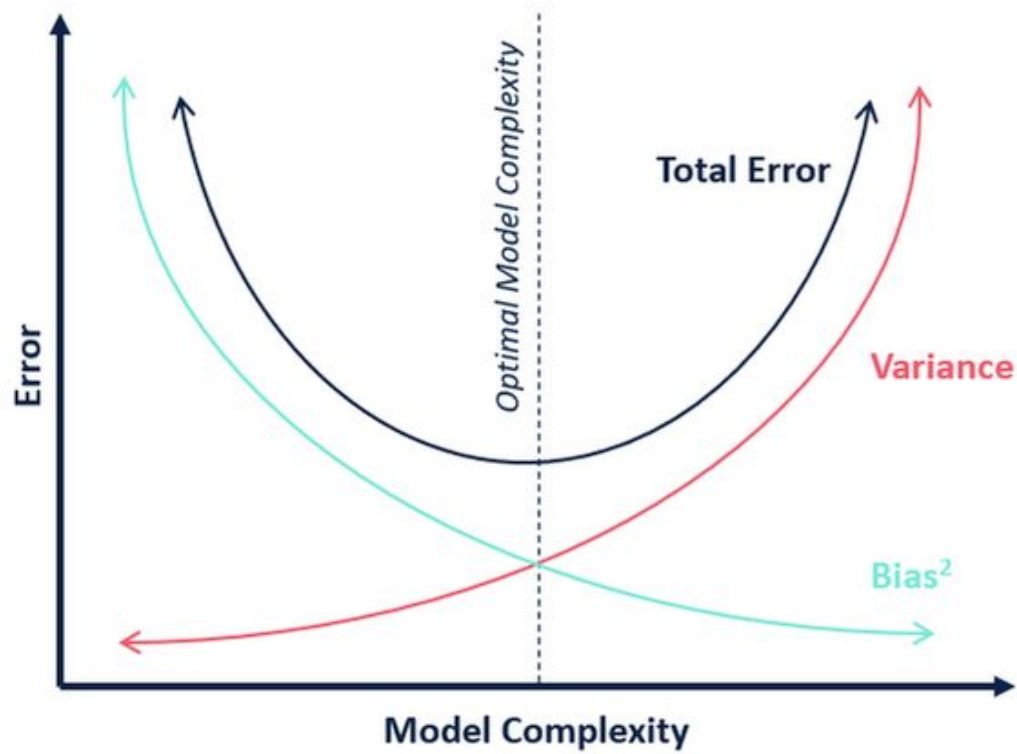
# Prediction error

$$E \left( y - \hat{f}(x) \right) = Var \left( \hat{f}(x) \right) + \left[ \text{Bias} \left( \hat{f}(x) \right) \right]^2 + Var(\epsilon)$$

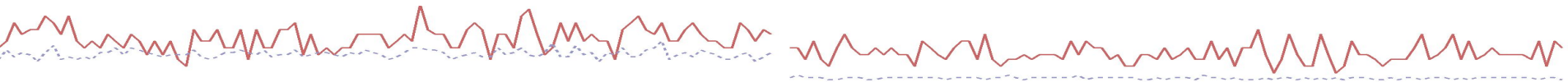
- **variance** refers to the change of the predictor if estimated using different training data
- **bias** refers to the approximation of a real problem by a simpler model



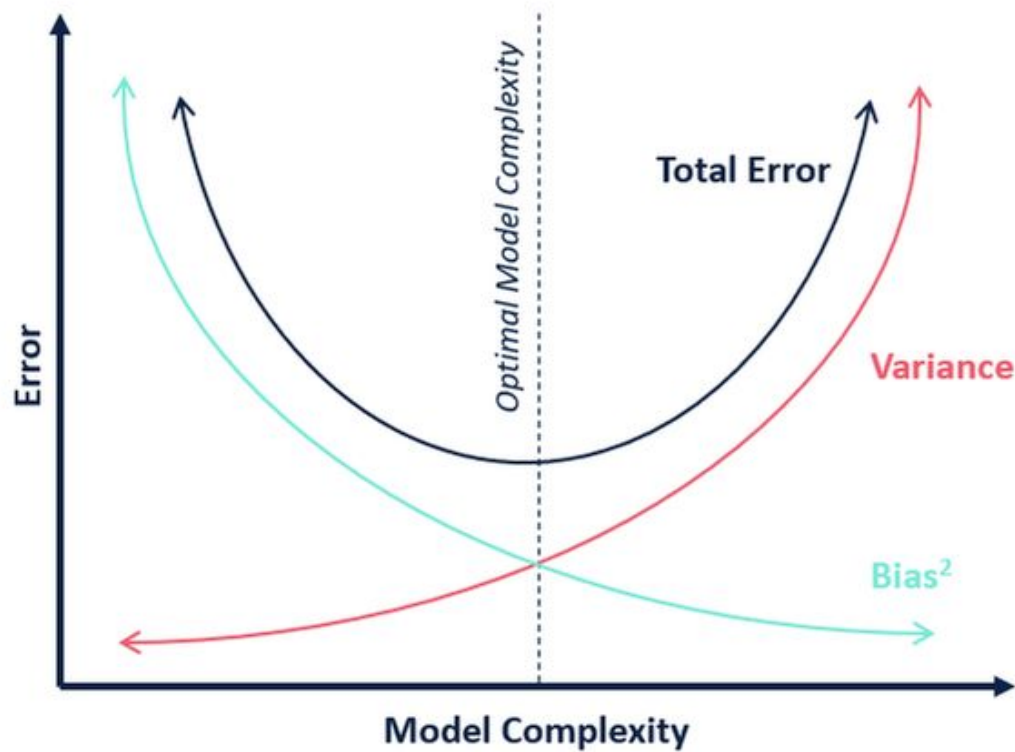
# Bias-variance trade-off



Source: <https://ai-pool.com/a/s/bias-variance-tradeoff-in-machine-learning>

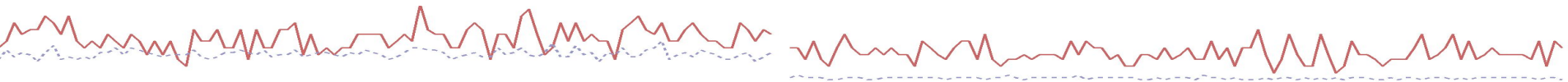


# Bias-variance trade-off



- models with low bias and high variance (e.g. KNN with  $k=1$ )
- models with high bias and low variance (e.g. horizontal line crossing the data)
- → find models/methods with both low variance and low bias

Source: <https://ai-pool.com/a/s/bias-variance-tradeoff-in-machine-learning>



# Bias-variance trade-off

## Related trade-offs

1. Prediction accuracy vs model interpretability:
  - e.g. linear regression is easy to interpret, splines are not
2. Parsimony vs black-box:
  - e.g. variable selection, all-variable models (e.g. RF), Occam's razor



# Bias-variance trade-off

Important for:

1. Correctly estimating the performance of a predictive machine
2. Correctly estimating model parameters
3. Selecting between models



# Bias-variance trade-off

Important for:

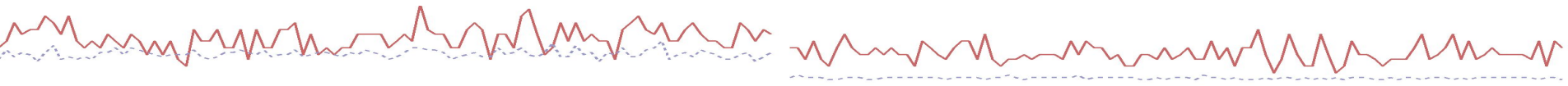
1. Correctly estimating the performance of a predictive machine
2. Correctly estimating model parameters
3. Selecting between models

**So, how do we control for overfitting and the bias-variance trade-off?**





# Training and test sets



# Training and testing sets



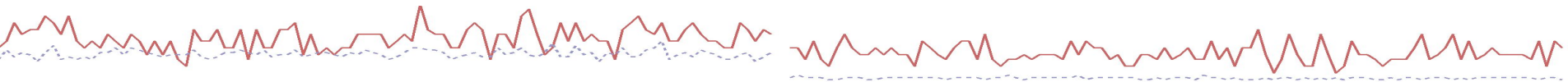
**Training data**

the predictive model is **trained here**



**Test data**

the predictive model is **evaluated here**

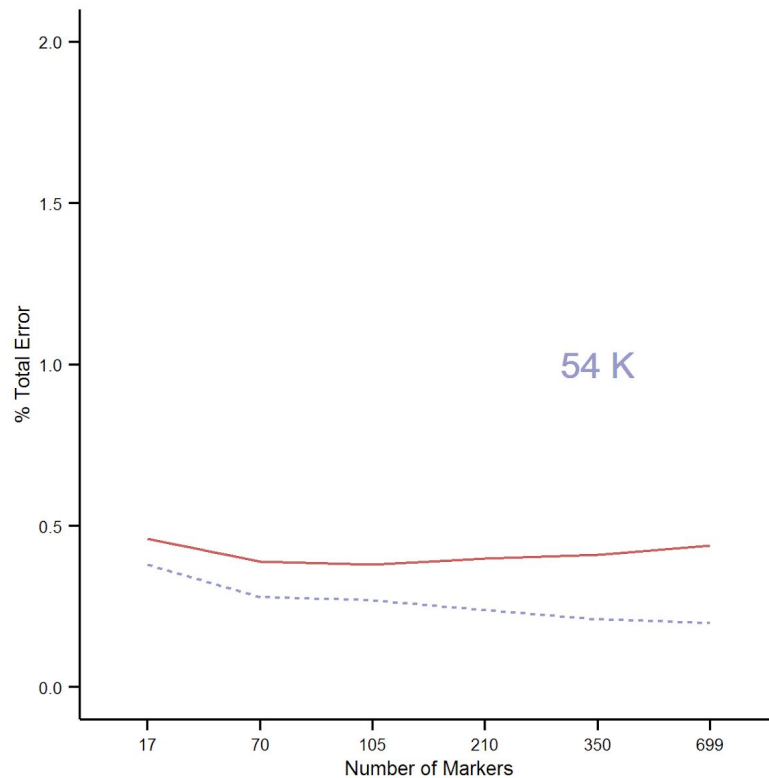
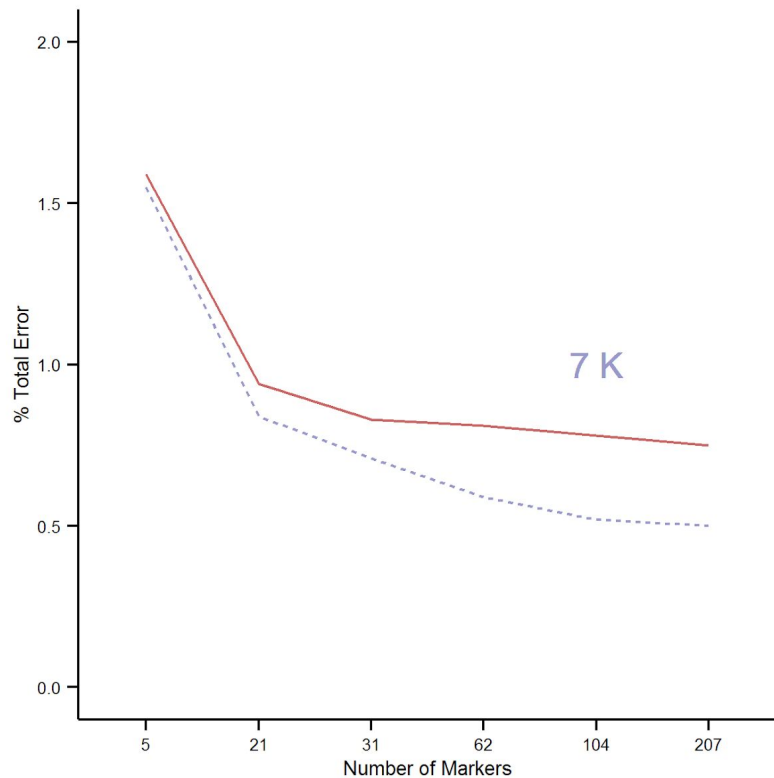


# Training and testing sets

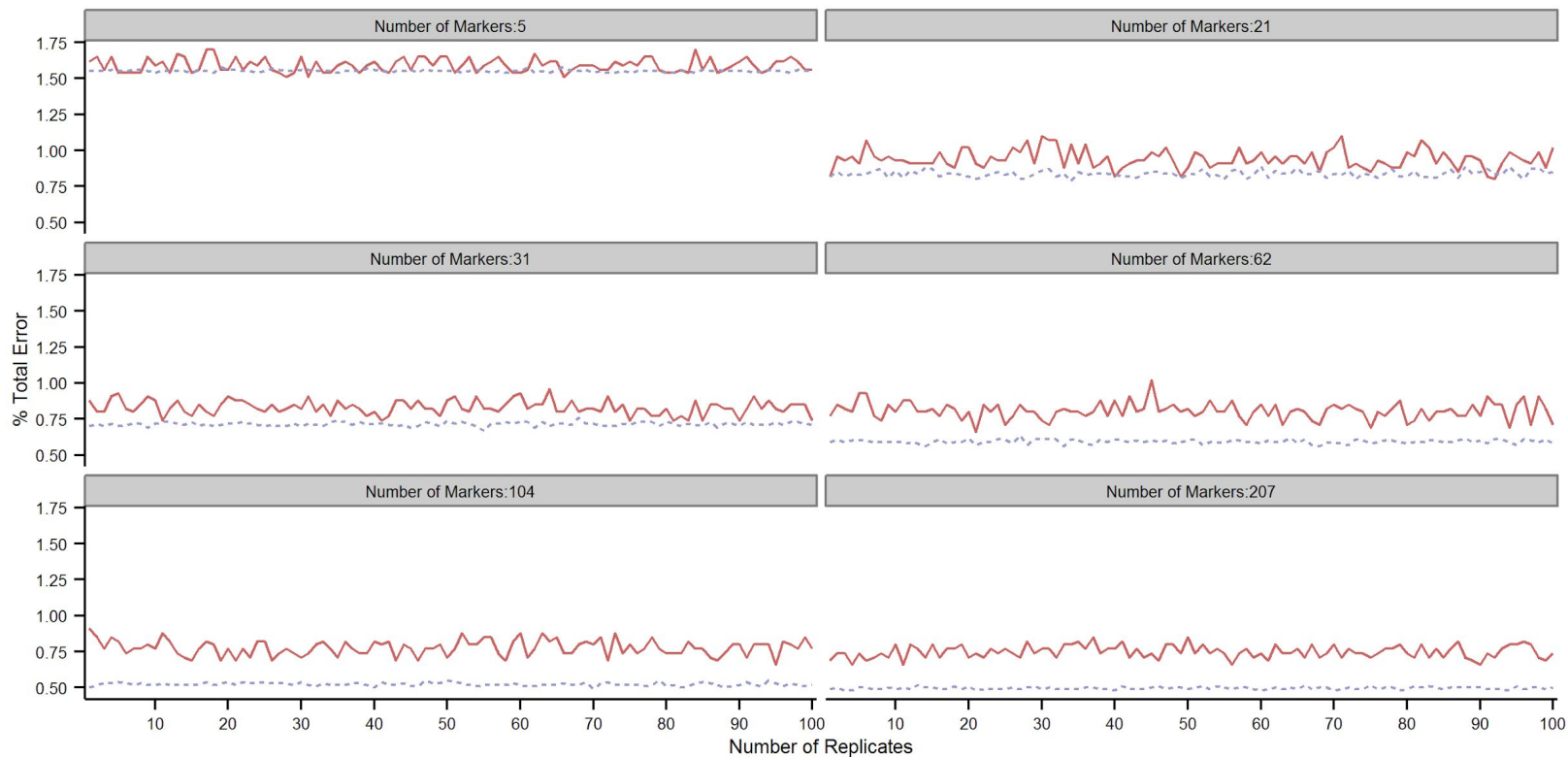
- accuracy (model performance) on the training set is “optimistic” (biased upward ← *overfitting*)
- a better estimate of model performance can be obtained from independent test data
- usually we are interested in the predictive performance on new data
- accuracy in the test set is usually lower than in the training set



# Training and testing sets



# Training and testing sets



# Overfitting - hands on!

→ 3.training\_testing.ipynb  
Exercise 3.1

