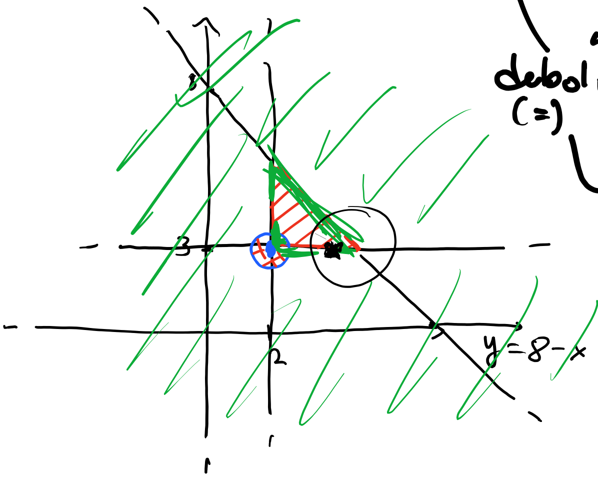
MM

mercoledì 19 novembre 2025 16:10

$$S = \{(x,y) : x \geq 2, y \geq 3, x+y < 8\} \subseteq \mathbb{R}^2$$

deboli (=) stretta

né aperto
né chiuso



• $\forall (x_0,y_0)\in S \quad \exists\, \varepsilon>0$ t.c. $B((x_0,y_0),\varepsilon) \subseteq S$

APERTURA

$$B((x_0,y_0),\varepsilon) = \{(x,y)\in\mathbb{R}^2 : (x-x_0)^2+(y-y_0)^2 < \varepsilon^2\}$$

FALSO : Prendo $\underbrace{(x_0,y_0)=(2,3)}_{\exists}$, $\underbrace{\text{qualsiasi}}_{\forall}$ raggio $\varepsilon>0$

è t.c. $B((2,3),\varepsilon) \not\subseteq S$

Infatti, preso $\varepsilon>0$, il punto $\left(2-\frac{\varepsilon}{2},3\right) \in B((2,3),\varepsilon)$ ma

$\left(2-\frac{\varepsilon}{2},3\right) \notin S$, quindi $B \not\subseteq S$

$2-\frac{\varepsilon}{2} < 2$

CHIUSURA : mostriamo similmente a sopra che $\mathbb{R}^2 - S$
(complementare) NON è aperto

Hint: $(x_0,y_0)=(5,3)$

(1) $\begin{cases} \dot{x} = 3x + 2 \\ \dot{y} = 3x + y \end{cases} \rightsquigarrow \begin{cases} \dot{x} = 0 \\ \dot{y} = 0 \end{cases} \iff \begin{cases} 3x + 2 = 0 \\ 3x + y = 0 \end{cases} \iff \begin{cases} x = -\frac{2}{3} \\ y = -3x = 2 \end{cases}$

$\Rightarrow (x^*, y^*) = \left(-\frac{2}{3}, 2\right)$

$\begin{cases} \dot{x} = f(x,y) = 3x + 2 \\ \dot{y} = g(x,y) = 3x + y \end{cases}$

$$J = \begin{bmatrix} 3 & 0 \\ 3 & 1 \end{bmatrix}$$

Supponiamo

$$J = \begin{bmatrix} 3x & 0 \\ 4 & y/2 \end{bmatrix} \Longrightarrow J(x^*, y^*) = \begin{bmatrix} -2 & 0 \\ 4 & 1 \end{bmatrix}$$

$$\begin{cases} x_{t+1} = 3x_t^2 - y_t \\ y_{t+1} = 4x_t - 2y_t \end{cases} \Rightarrow \begin{cases} x = 3x^2 - y \\ y = 4x - 2y \end{cases}$$

RISOLVI e trovi (x^*, y^*)

$$J = \begin{bmatrix} 6x & -1 \\ 4 & -2 \end{bmatrix}$$

Exercise 3. [5 Points]. Consider the following dynamical system in continuous time:

$$\begin{cases} \dot{x} = 3x(x-4) \\ \dot{y} = y^2 - x^2 - 9 \end{cases}$$

a) Find the stationary solutions.
b) Verify if the stationary solutions are locally asymptotically stable or, if not, define their nature.

$$\begin{cases} 3x(x-4) = 0 \\ y^2 - x^2 - 9 = 0 \end{cases} \Rightarrow \begin{cases} 3x^2 - 12x = 0 \\ y^2 = x^2 + 9 \end{cases} \Rightarrow \begin{cases} \boxed{x = 0} \lor \boxed{x = 4} \\ \boxed{y = \pm 3} \lor \boxed{y = \pm 5} \end{cases}$$

$(0,3)$, $(0,-3)$, $(4,5)$, $(4,-5)$

$$J = \begin{bmatrix} 6x - 12 & 0 \\ -2x & 2y \end{bmatrix}$$

$$J(0,3) = \begin{bmatrix} \cdot & \cdot \\ 0 & 6 \end{bmatrix} \rightsquigarrow -12, 6 \implies (0,3) \text{ sella}$$

$$J(0,-3) = \begin{bmatrix} -12 & 0 \\ 0 & -6 \end{bmatrix} \rightsquigarrow \text{stabile (asintotic.)}$$

$\dot{x} = 3x e^t$ $\quad g(x)=3x, f(t)=e^t$ $\quad \boxed{x(0)=1}$

$\dot{x} = g(x) \cdot f(t)$

$\frac{dx}{dt} = 3x \cdot e^t$

$\frac{1}{3x} dx = e^t dt$

$\int \frac{1}{3x} dx = \int e^t dt$

$\frac{1}{3} \log|x| = e^t + C$

$\frac{1}{3} \log|1| = e^0 + C \Rightarrow C = \frac{1}{3} \cdot 0 - 1 = -1$

$\Rightarrow \frac{1}{3} \log|x| = e^t - 1$

$\log|x| = 3e^t - 3$

$|x| = e^{3e^t - 3}$

$x = \pm e^{3e^t - 3}$

$$\dot{x} + a(t) \cdot x = b(t)$$

$$x(t) = \frac{1}{e^{\int a(t)dt}} \cdot \int e^{\int a(t)dt} \cdot b(t) dt + C$$

$x(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t}$

dimostra che $x(t)$ è la soluzione a

$$\ddot{x} + a\dot{x} + bx = 0$$

con λ_1, λ_2 radici di $\lambda^2 + a\lambda + b = 0$

$$\lambda_{1,2} = \frac{-a \pm \sqrt{a^2 - 4b}}{2}$$