

# Assignment 7

## Quantum Ising Model

December 3rd 2024

## QUANTUM ISING MODEL

Consider  $N$  spin-1/2 particles on a one-dimensional lattice, described by the Hamiltonian:

$$\hat{H} = \lambda \sum_i^N \sigma_i^z + \sum_i^{N-1} \sigma_i^x \sigma_{i+1}^x$$

where  $\sigma_x$  and  $\sigma_z$  are the Pauli matrices and  $\lambda$  is the strength of the external field.

- 1) Write a program to compute the  $N \times N$  matrix representation of the Hamiltonian  $\hat{H}$  for different size  $N$ .
- 2) Diagonalize  $\hat{H}$  for different  $N = 1, \dots, N_{max}$  and  $\lambda \in [0, -3]$ . What is the largest is  $N_{max}$  you can reach
- 3) Plot the first  $k$  levels as a function of  $\lambda$  for different  $N$  and comment on the spectrum.

Quantum Ising model is one of the simplest non trivial many-body quantum systems. Let's consider a linear chain of interacting spin-1/2 in the presence of an external field of intensity  $\lambda$ .

The two pieces of the Hamiltonian do not commute:

$$[\mathbb{I}_1 \otimes \sigma_2^z, \sigma_1^x \otimes \sigma_2^x] \neq 0$$

Remember that:

$$\sigma_i^z = \mathbb{I}_1 \otimes \mathbb{I}_2 \otimes \dots \otimes \sigma_i^z \otimes \mathbb{I}_{i+1} \dots \otimes \mathbb{I}_N$$

$$\sigma_i^x \otimes \sigma_i^x = \mathbb{I}_1 \otimes \mathbb{I}_2 \otimes \dots \otimes \sigma_i^x \otimes \sigma_{i+1}^x \otimes \mathbb{I}_{i+2} \dots \otimes \mathbb{I}_N$$

So the overall size is?

CODE DEVELOPMENT

Matrix representation of the operator  $A \otimes B$  , with  $A = (M_r, M_c)$     $B = (N_r, N_c)$ ,  
labeled as:

$$A \otimes B = (M_r \times N_r, M_c \times N_c)$$

$$((ii - 1)N_r + kk, (jj - 1)N_c + mm) \text{ with}$$

$ii \in [1, M_r]$  $jj \in [1, M_c]$  $kk \in [1, N_r]$  $mm \in [1, N_c]$

$$A \otimes B = \begin{bmatrix} A_{11}B & A_{12}B & \dots & A_{1M_c}B \\ A_{21}B & A_{22}B & \dots & A_{2M_c}B \\ \vdots & \vdots & \dots & \vdots \\ A_{M_r1}B & A_{M_r2}B & \dots & A_{M_rM_c}B \end{bmatrix}$$