Homeworks 2 - SAR and EAR

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24 October 2024

EXERCISE 1

Plot both SAR and EAR as a function of the relative area, using the Log Series as the SAD distribution $P(n) = c(1-m)^n/n$ (where c is the normalization constant), and compare it with a Power Law SAR of exponent $z \in (0.2, 0.3)$ ($S(a) = kA^z$) and tune z and k to be as close as possible to the solution for the random placement case.

Solution

We define the Species-Area Relationship (SAR) as the number of species occupying a territory versus the considered territory's area. If we consider the so-called "null" model for the distribution of the species on the territory (i.e. the species are well mixed), them we can define the SAR as follows:

$$SAR(\alpha) = S\left[\alpha - \sum_{n=1}^{\infty}(1-\alpha)^n P(n)\right]$$

where α is the considered section of the total territory and n the abundance. Similar to the SAR is the Endemic Area Relationship (EAR), which describes the number of species completely contained by the area α and is defined as follows:

$$EAR(\alpha) = S\sum_{n=1}^{\infty}\alpha^n P(n).$$

If we use the Log Series P(n):

$$P(n) = \frac{(1-m)^n}{n|\ln(m)|}$$

where m is the migration rate, as the Species Abundance Distribution (SAD), the SAR becomes:

$$SAR(\alpha) = S \cdot \left(1 - \frac{\log\left(m(1-\alpha)+\alpha\right)}{\log(m)}\right)$$

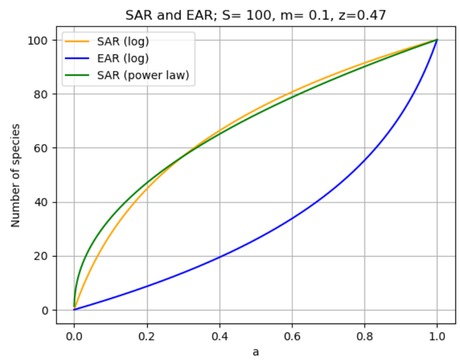

Figure 1: Results using: $k = S$, $m = 0.1$

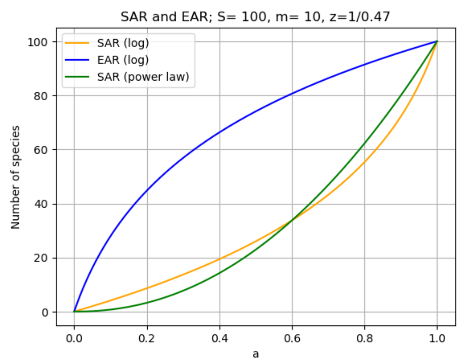

Figure 2: Results using: $k = S$, $m = 10$

and the EAR:

$$EAR(\alpha) = S \cdot \left(\frac{\log\left(1 - \alpha(1-m)\right)}{\log(m)} \right).$$

The SAR is usually approximated with a power law in the form:

$$S(\alpha) = k\alpha^z.$$

The results show that two regimes are distinguishable, based on the value of m ($m > / < 1$), as can be seen in Figures 1 and 2. It can also be noted for $m' = 1/m$ the EAR and SAR curves are essentially swapped.
Moreover, an analysis was conducted for finding the best value for z and that resulted in $z_{best} = 0.47$ and $z_{best} = 1/0.47$ respectively for $m = 0.1$ and $m = 10$. This is also graphed in Figures 3 and 4. It is clear from these results that the best value for z falls outside the expected interval of $(0.2, 0.3)$.

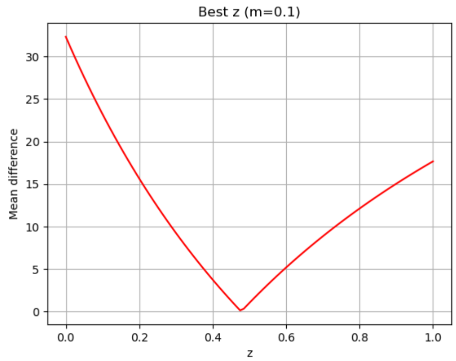

Figure 3: Mean difference between the SAR and the power law, using: $k = S$, $m = 0.1$

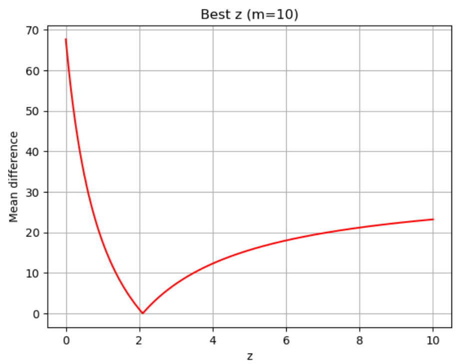

Figure 4: Mean difference between the SAR and the power law, using: $k = S$, $m = 10$

EXERCISE 2

If you remove 1/3 of the area, how many species go extinct? Do the estimation both using the SAR and the EAR.

Solution

The two relationships ($[SAR(\alpha = 1) - SAR(\alpha = 2/3)]$, $[EAR(\alpha = 1/3)]$)give the same expected number of extinct species as a result of the removal of 1/3 of the area, amounting to the 15% of the total number of species.