Homeworks 3 - Random Matrix Theory

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EXERCISE 1

Choose a given type of ecological structure (e.g. mutualistic, or predator-prey, or etc...). Generate random matrices with for completely random and for the chosen ecological structures. You need to create SxS matrices (S is the number of species) with C non-zero entries and 1-C zeros (C is the connectivity between 0 and 1). The non-zero elements are drawn at random from given distributions. Depending on the network structure some symmetries and constraints may hold. Please follow the detailed step-by-step explanation in the "Homework-detail-week3-from-Allesina-Stability-Criteria-2012-Nature.pdf" (Also uploaded in the Google Drive, note folder). Fix C and variance of the interaction and S=100, and calculate the maximum real eigenvalue of the matrix. Do many realizations and calculate each time such maximum real eigenvalue. Do this for different combinations of C and of the variance so to have a good range of values of the control parameter (e.g. for the complete random case is $\sigma * (S * C)^0.5$). Set the self-interaction d=-1

Plot the probability for the maximum real eigenvalue of be smaller than 0, against the corresponding control parameter and see when the transition to instability happens. Compare with analytical prediction if available.

Solution

I've studied the case of full random and predator-prey structures. First, I've analyzed the distribution of the maximum real part of the eigenvalues of the interaction matrices for both cases since this directly portrays the stability of the system (i.e. lower maximum eigenvalue constitutes a lower stability condition). The results of this analysis are shown in Figure 1. The values of the parameters used in this simulation are:

- S = 100
- d = -1
- C = 0.02
- σ = 0.5

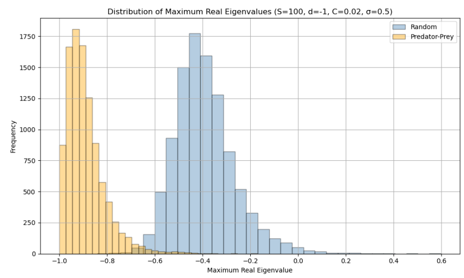

Figure 1: Histograms of the maximum real part of the eigenvalues for the two models (iterations=10000.

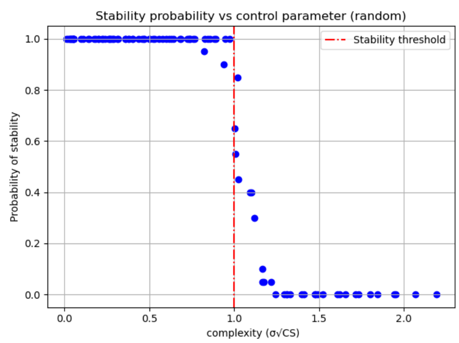

Figure 2: Stability probability as a function of the control parameters C and σ and of the complexity parameter for the full random structure. Stability threshold from literature $t = -d$.

where S is the number of species, d is the self-interaction term, C is the connectance and σ is the square root of the variance of the normal distribution from which the non-diagonal and non-zero interaction coefficients are extracted. As it can be seen in Figure 1, the full random structure is more stable than the predator-prey one, as expected from the theoretical results.

Next, I've repeated the simulations for different values of the control parameters (i.e. C and σ) and calculated for each combination the probability of having the maximum real part of an eigenvalue to be negative. Such probability was trivially defined through a frequentist approach. This probability was then plotted as a function of the complexity parameter, which is defined as equal to $\sigma\sqrt{SC}$. The results are presented in Figures 2, 3, 4 and 5.
As can be seen from the Figures, the results are in agreement with what is expected from the theory.

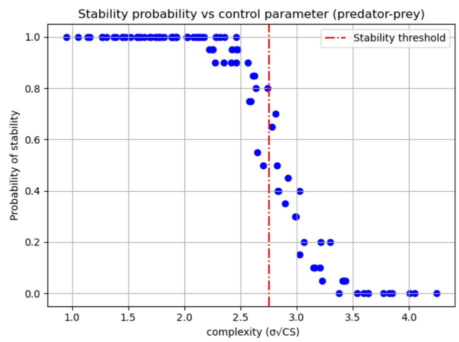

Figure 3: Stability probability as a function of the control parameters C and σ and of the complexity parameter for the predator-prey structure. . Stability threshold from literature $t = -d\pi/(\pi - 2)$.

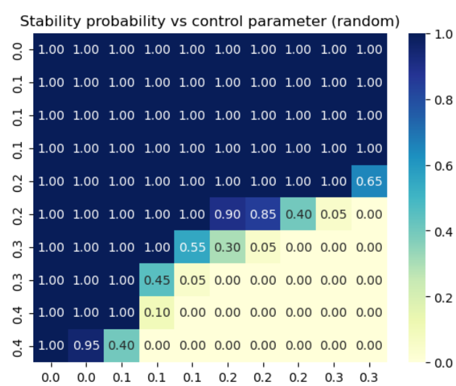

Figure 4: Stability probability as a function of the control parameters C and σ and of the complexity parameter for the random structure.

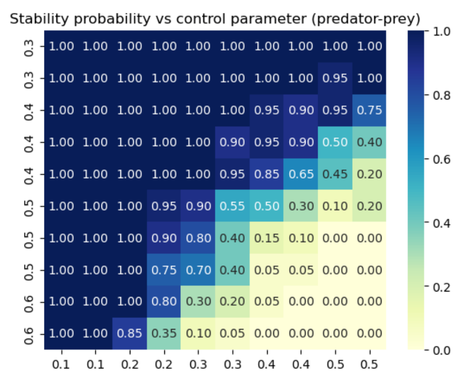

Figure 5: Stability probability as a function of the control parameters C and σ and of the complexity parameter for the predator-prey structure.