Measure the business cycle
Breaks down a times series into a **trend** component and a **cyclical** component.

$$x_t = \tau_t^x + v_t^x \tag{1}$$

Where:

- x_t = gdp at time t.
- τ = trend component.
- v = ciclycal component.

This means that the business cycle component (v) of GDD can be captured as the deviation of the GDP serires from a smooth trend fit to the data ($v = x - \tau$).

HP filter
The HP filter constructs the trend that minimizes the loss function:

$$min \sum_{t=1}^{T} (x_t - \tau_t^x)^2 + \lambda \sum_{t=2}^{T-1} [(\tau_{t+1}^x - \tau_t^x) - (\tau_t^x - \tau_{t-1}^x)]^2 \tag{2}$$

The first term puts a penalty on the distance of τ (the trend) from x, while the 2nd term puts a penalty on the changes in the growth rates of τ (we want the trend in subsequent periods to be as small as possible).
The higher lambda is the more the trend will take the shape of a straight line.

Why logs?
Recall the approximation: $log(1+y) \sim y$. When x (which is log of the series) is close to its trend τ we have:

$$v_t^x = x_t - t_t^x = log(X_t) - log(T_t^x) = log(\frac{X_t}{T_t^x}) = log(1 + \frac{X_t - T_t^x}{T_t^x}) \sim \frac{X_t - T_t^x}{T_t^x} \tag{3}$$

where capital X and capital T are equal to exp() of small x and small t. Given that small x and small t are measured in logs, capital X and T represents

the real value of the series.
In this way the ciclycal component v^x represents the **percentage deviation of the series from its trends**, meaning that it can be compared across different series.