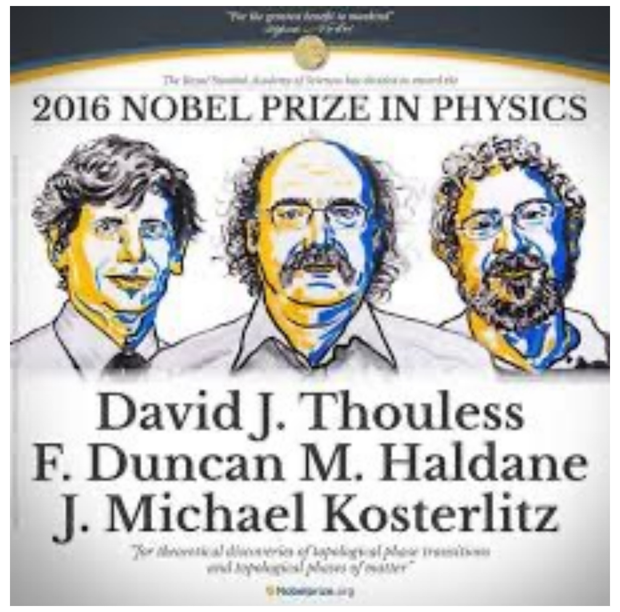
Politecnico di Torino

1859

QTCM computational lab on quantum simulation

Topological phases of matter



possible appearance of “new” particles with **fractional charge** and **statistic**: **the anyons**

beyond the Landau’s classification of phases of matter: the **high** (sometimes long-range) **entanglement** implies descriptions in terms of **non-local order parameters**

edge physics $\neq$ bulk physics: **current** in the system **edges** (2D) or just gapless **edge state** (1D) associated to gapped **insulators** or **superconductors** in the system **bulk**

very **stable** with respect to external **perturbation**: **useful** for topological **quantum computation**

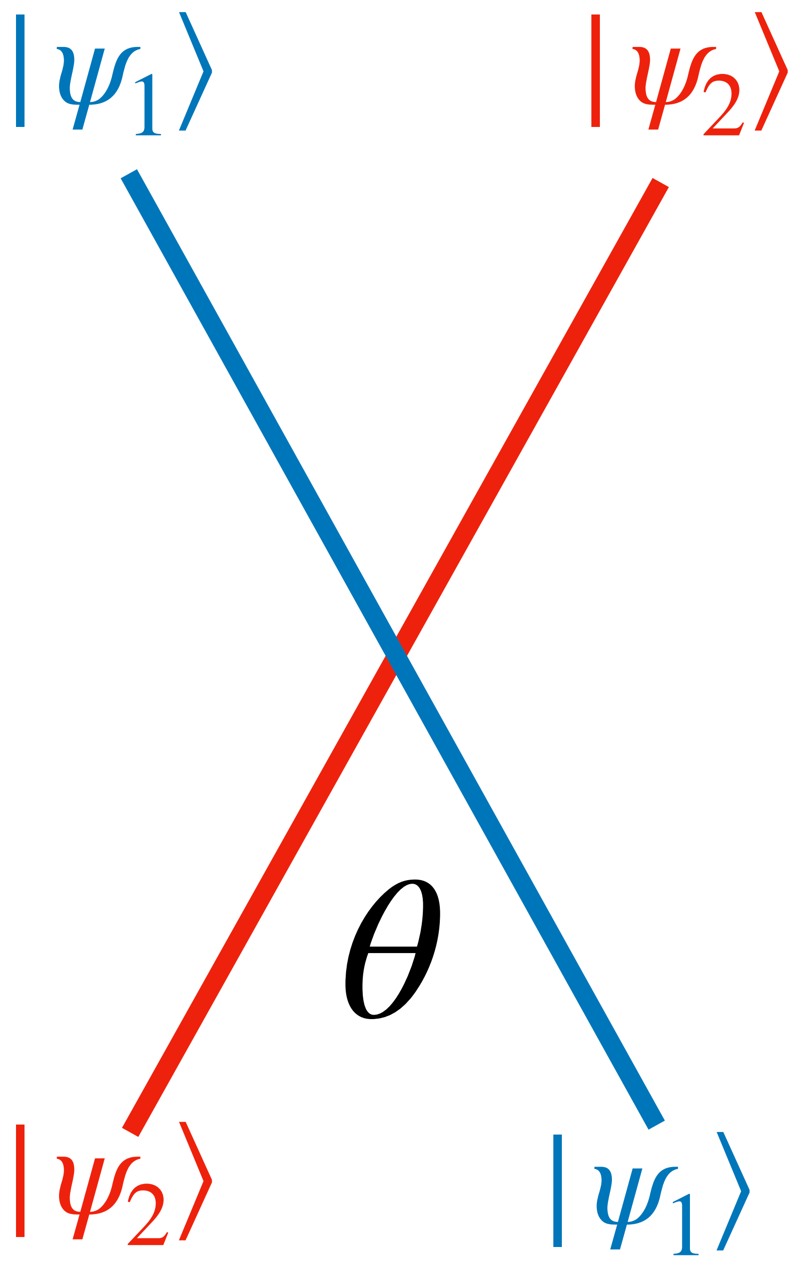
Particle statistics $|\psi_1\psi_2\rangle = e^{\imath\theta}|\psi_2\psi_1\rangle$

$\theta = 0 -> |\psi_1\psi_2\rangle = |\psi_2\psi_1\rangle$

BOSONS $[b_i, b_j^\dagger] = \delta_{i,j}$

$$H_{BH} = -J\sum_i b_i^\dagger b_{i+1} + h.c. + \frac{U}{2}\sum_i n_i(n_i - 1)$$

At density=1 MI-SF phase transition as a function of U/J



$\theta = \pi - > |\psi_1\psi_2\rangle = -|\psi_2\psi_1\rangle$

FERMIONS $\{c_{i,\sigma}, c_{i,\sigma'}^\dagger\} = \delta_{i,j}\delta_{\sigma,\sigma'}$

$$H_{BH} = -J\sum_{i,\sigma} c_{i,\sigma}^\dagger c_{i+1,\sigma} + h.c. + U\sum_i n_{i,\uparrow}n_{i,\downarrow}$$

At density=1 MI for U/J>0 and LE for U/J<0

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Particle statistics $a_j a_k^\dagger - e^{\imath\theta sgn(j-k)} a_k^\dagger a_j = \delta_{jk} \quad a_j a_k = e^{\imath\theta sgn(j-k)} a_k a_j$

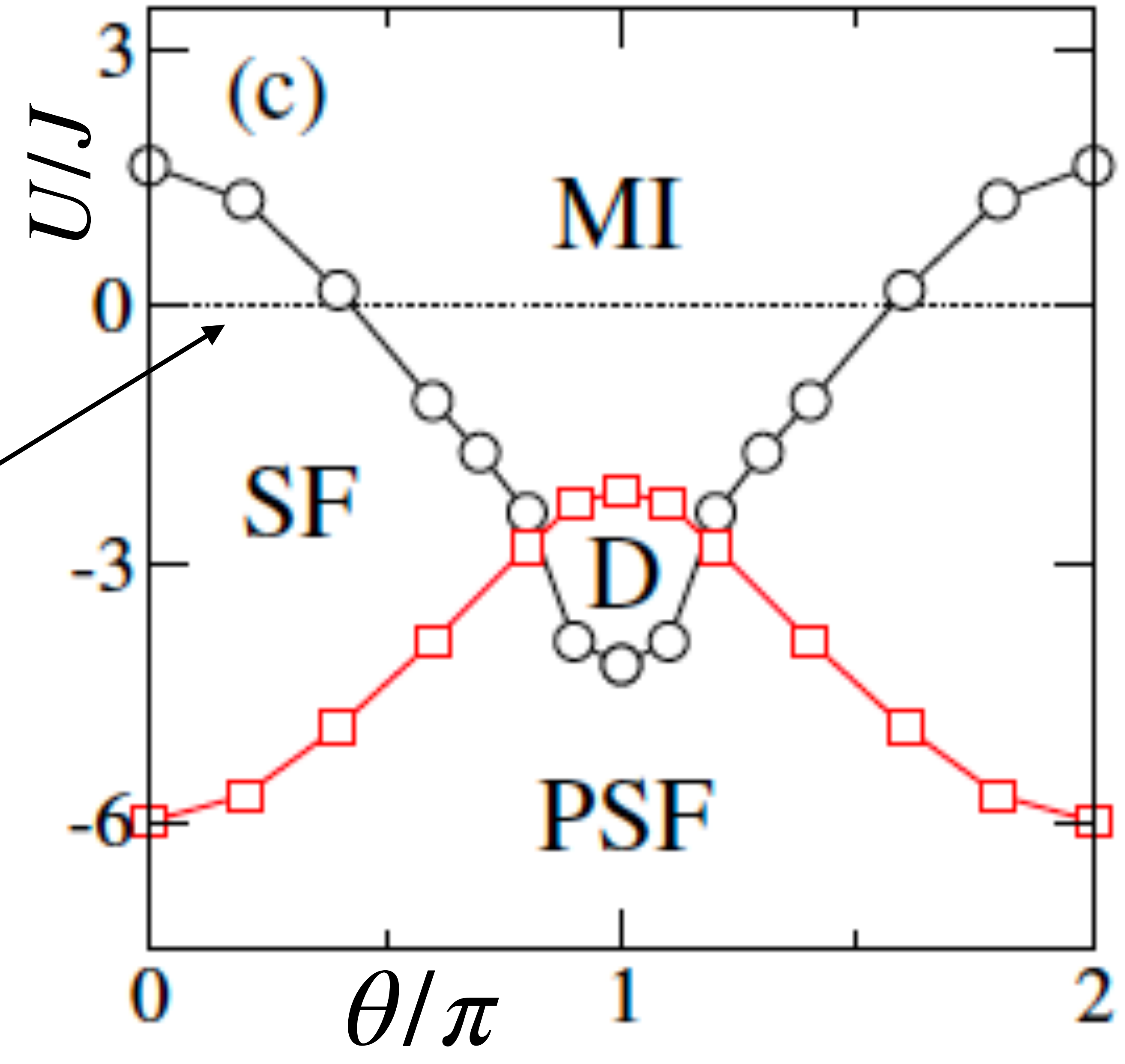
ANYONS

onsite ($j = k$) **bosons**

offsite ($j \neq k$) and $\theta = \pi$ **fermions**

$$H_{AH} = -J \sum_j (b_j b_{j+1}^\dagger e^{\imath\theta n_j} + h.c.) + \frac{U}{2} \sum_j n_j (n_j - 1)$$

At density=1 MI-SF transition even for vanishing interaction!!!



check arXiv:2306.01737 for a recent exp realization

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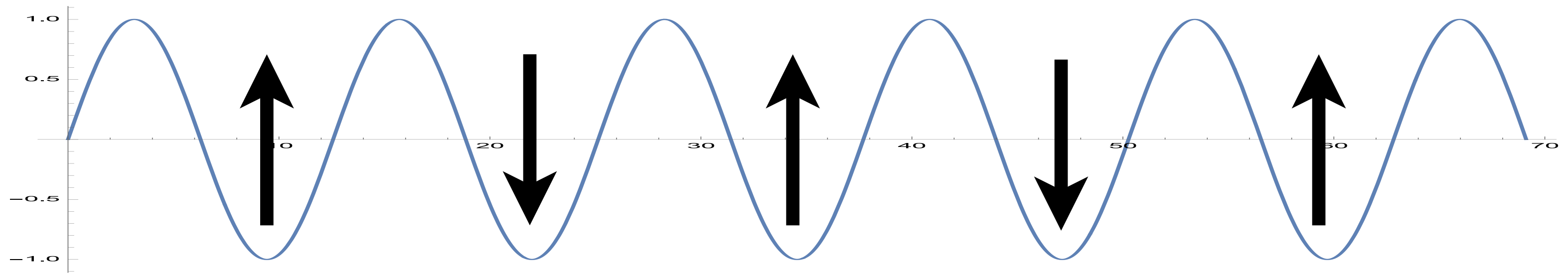
"Normal" phases of matter can be usually described by a **local order parameter** (LOP), reflecting the breaking of some symmetry, e.g. :

$$H = \sum_{j=0}^{L-2} [\frac{J_{xy}}{2}(S_j^+ S_{j+i}^- + S_j^- S_{j+i}^+) + J_{zz} S_j^z S_{j+i}^z]$$

$$J_{zz} \gg J_{xy}$$

antiferromagnetism with broken spin rotational symmetry SU(2)

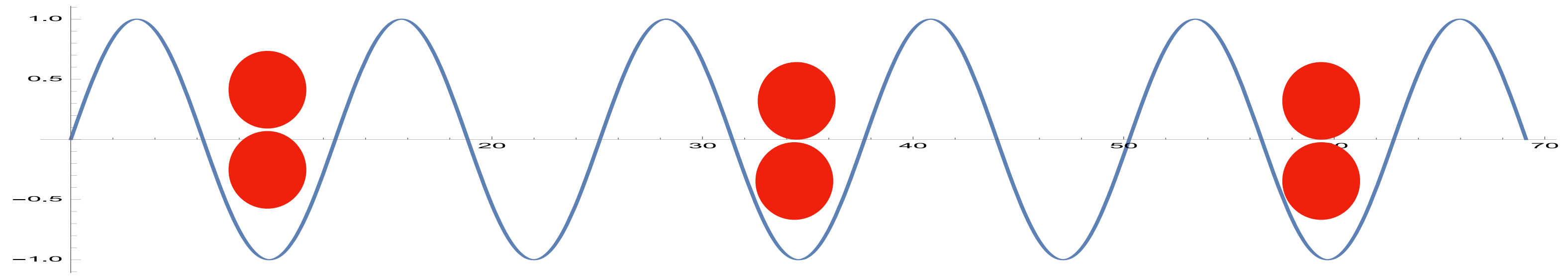


$$\text{LOP } \tilde{S}^z = \frac{1}{L} \sum_i (-1)^i S_i^z$$

$$H_{BH} = -J \sum_i (b_i^\dagger b_{i+1} + b_i b_{i+1}^\dagger) + \frac{U}{2} \sum_i n_i(n_i - 1) + V \sum_i n_i n_{i+1}$$

$$V \gg U, J$$

density wave with broken Z_2 translational symmetry



$$\text{LOP } \delta N = \frac{1}{L} \sum_i (-1)^i (\bar{n} - n_i)$$

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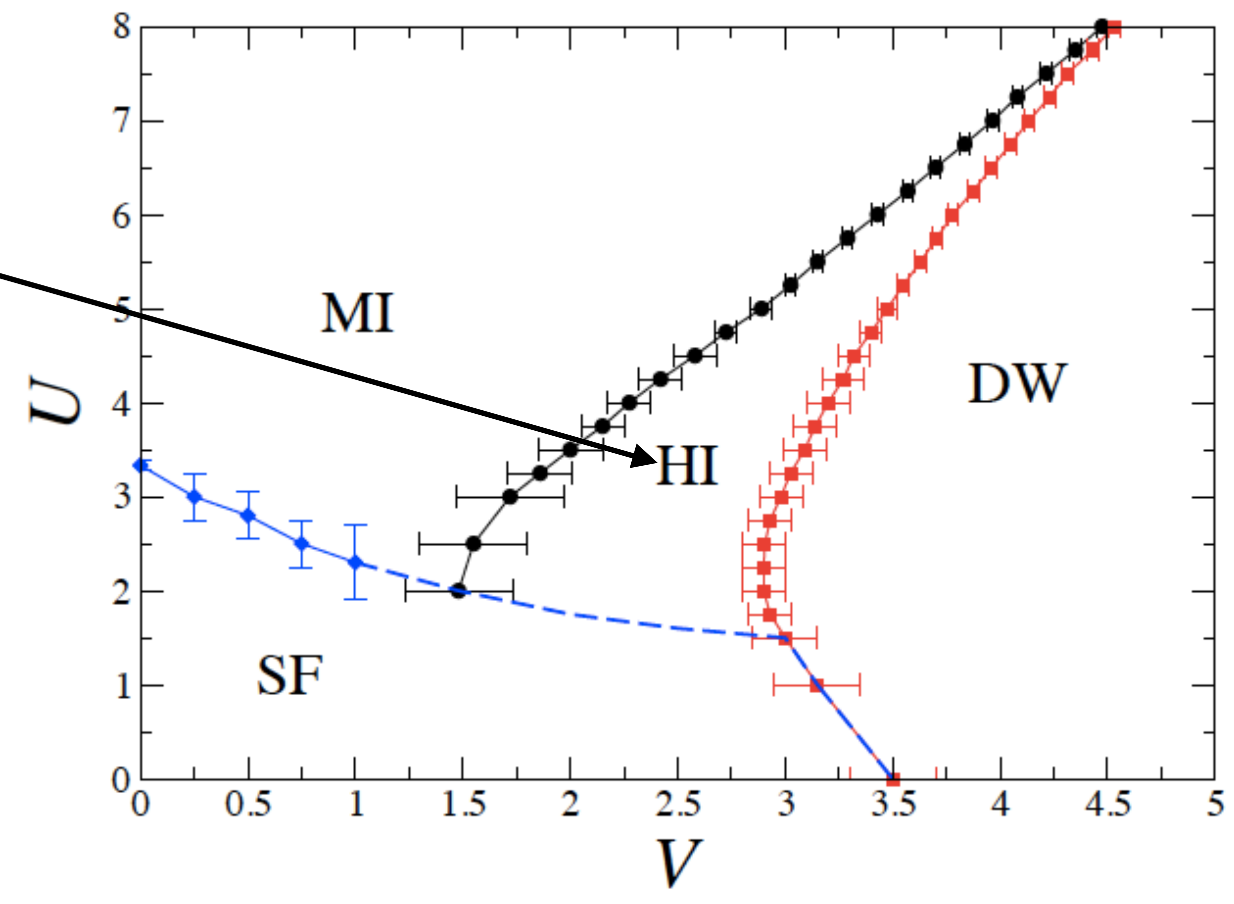
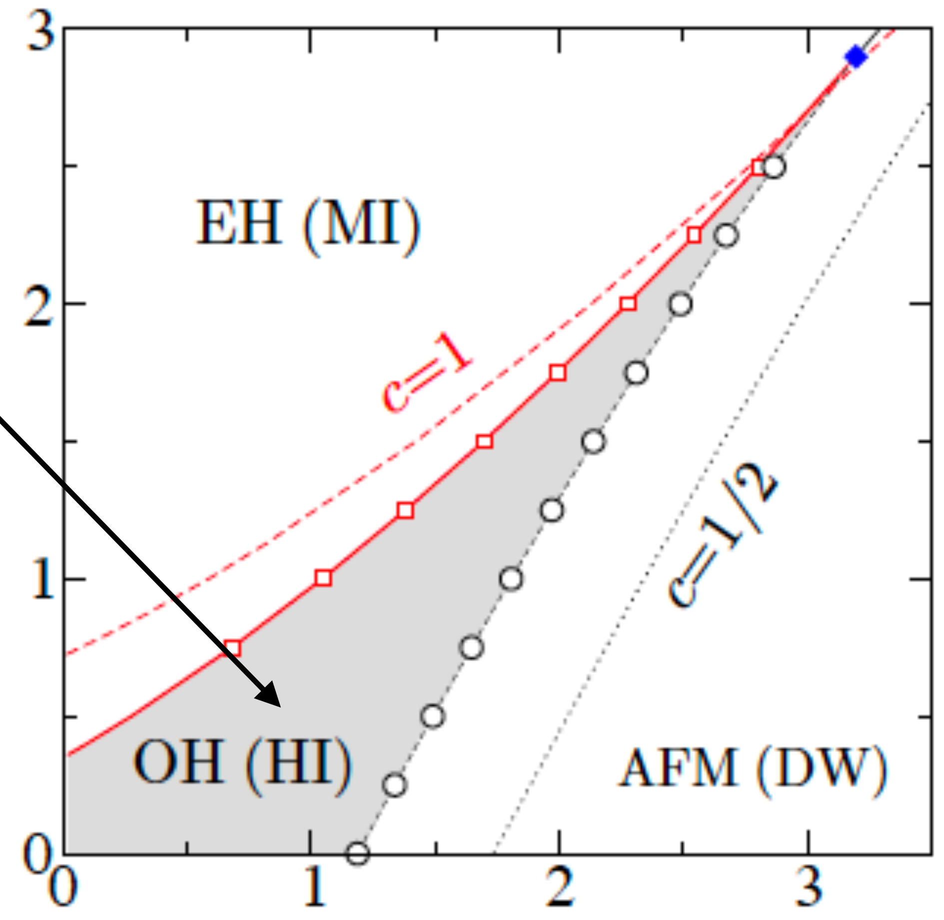
Topological phases of matter can be described uniquely by using **non-local order parameters** (NLOP), usually called strings, reflecting the absence of local order in presence of finite gaps, e.g. **Haldane phase (1D)** or spin liquids and fractional quantum Hall (2D)



$$H_{BH} = -J\sum_i (b_i^\dagger b_{i+1} + b_i b_{i+1}^\dagger) + \frac{U}{2}\sum_i n_i(n_i - 1) + V\sum_i n_i n_{i+1}$$

For U large enough (but not too large), states like $|3\rangle, |4\rangle, \dots |N\rangle$ are irrelevant while the relevant ones are $|0\rangle, |1\rangle, |2\rangle$ —> we have a 3-level system, thus we can map H_{BH} into a spin-1 Hamiltonian



$$S_i^+ = (2\bar{n} - n_i)^{1/2} b_i \quad S_i^- = b_i^\dagger (2\bar{n} - n_i)^{1/2} \quad S_i^z = (\bar{n} - n_i)$$

$$|0\rangle, |1\rangle, |2\rangle \leftrightarrow |1\rangle, |0\rangle, |-1\rangle$$

$$H_{XXZ,S=1} = J\sum_i (S_i^+ S_{i+1}^- + S_i^- S_{i+1}^+) + \frac{U}{2}\sum_i (S_i^z)^2 + V\sum_i S_i^z S_{i+1}^z$$

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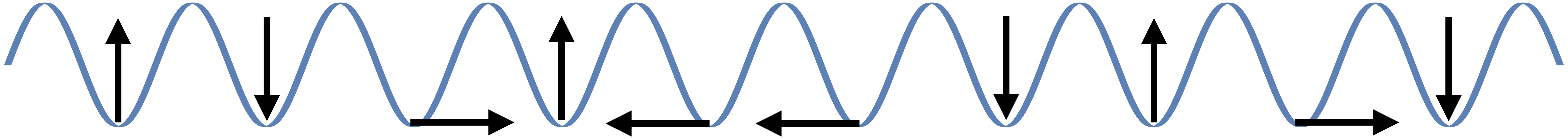
The **Haldane phase** has no local order: 1 single measure of S_i^z will find



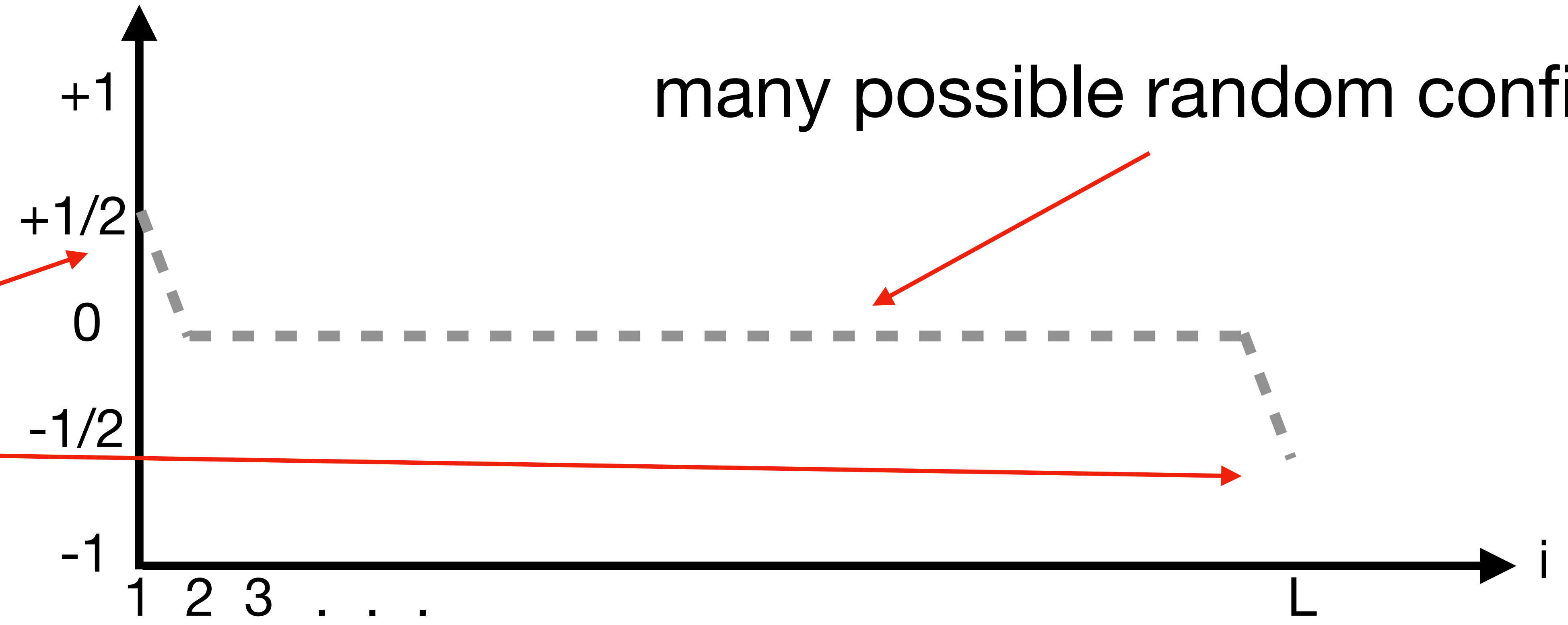
$\langle S_i^z \rangle =$ 1 -1 0 1 0 0 -1 1 0 -1

$\tilde{S}^z = \frac{1}{L}\sum_i (-1)^i S_i^z = 0$ so **no LOP**....but if you plot the **non-local string order parameter**

$O^z(r) = \langle S_i^z e^{\imath\pi \sum_{k=i+1}^{i+r-1} S_k^z} S_{i+r}^z \rangle \neq 0 —>$ HI is characterized **by hidden AF order**, namely after un up spin you always find a down spin with in between a random number of 0 spins

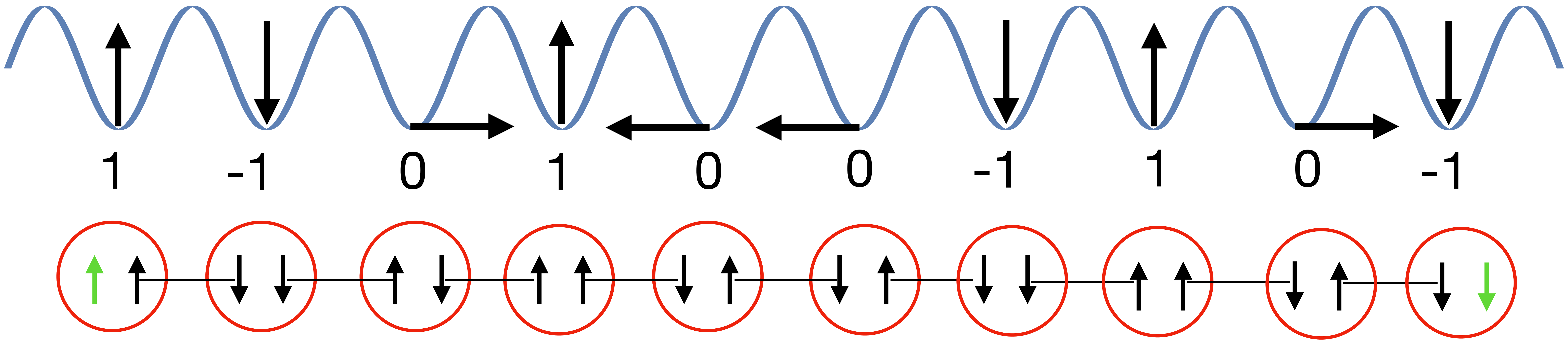
After many measurements S_i^z will look like

many possible random configurations



why a finite edge magnetization???

Affleck-Kennedy-Lieb-Tasaki construction: a spin-1 can be written as the sum/difference of two spin-1/2



Effective lattice dimerization where the even links —— are stronger than the odd links and ↑ ↓ are actually spin-1/2 decoupled from the system bulk—> they are gapless—> they are edge states!!!!

The Haldane phase has a NLOP + edge states—> is a topological phase

Haldane phase realized in an atomic quantum simulator

Article

Realizing the symmetry-protected Haldane phase in Fermi–Hubbard ladders

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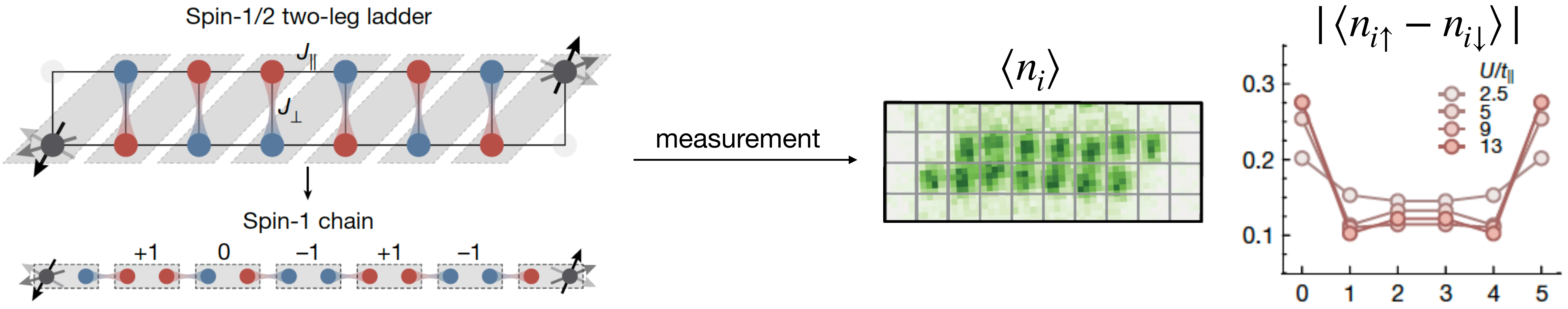
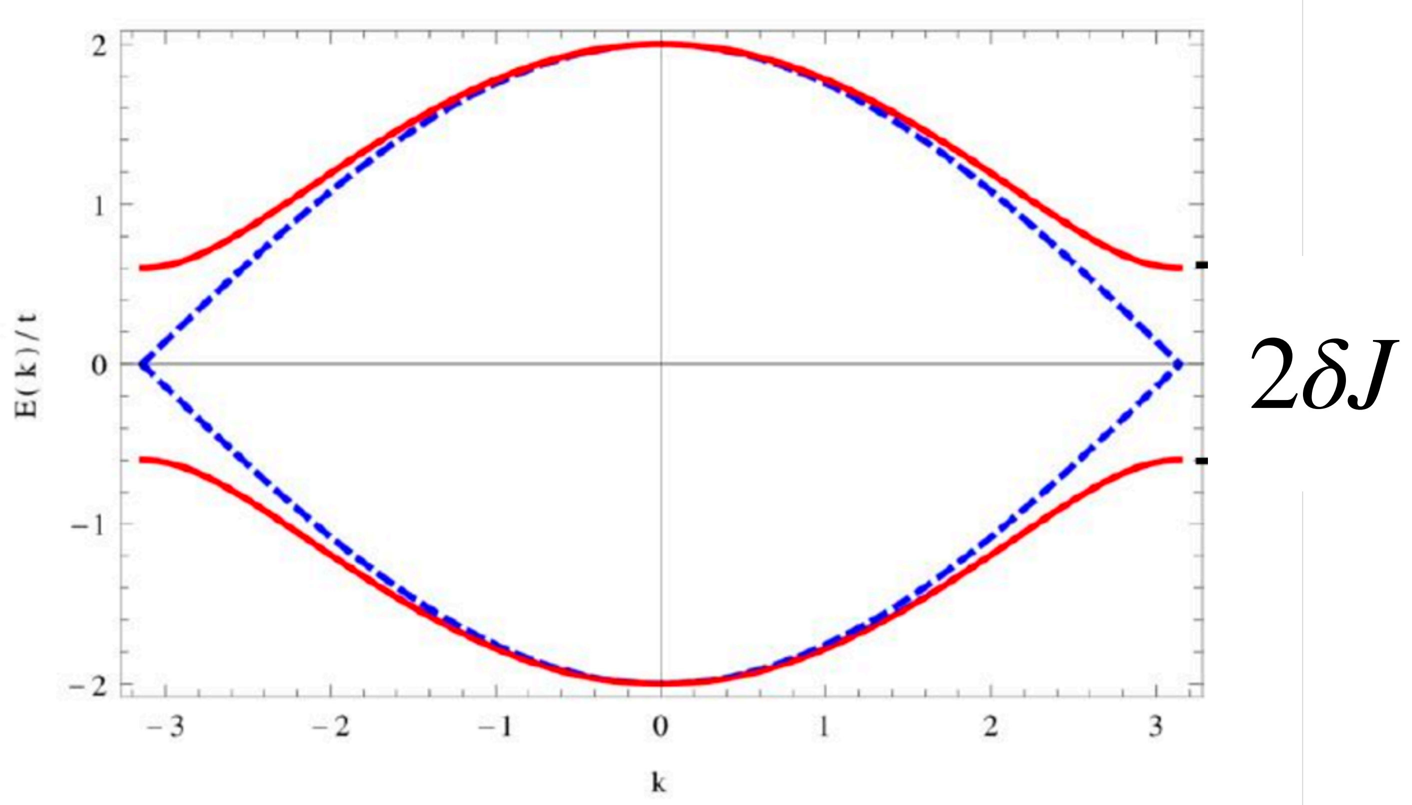
2 coupled 1D lattices (a ladder) filled with **two different fermionic species** (mimicking spin-1/2) at **unit density** and **strong U**



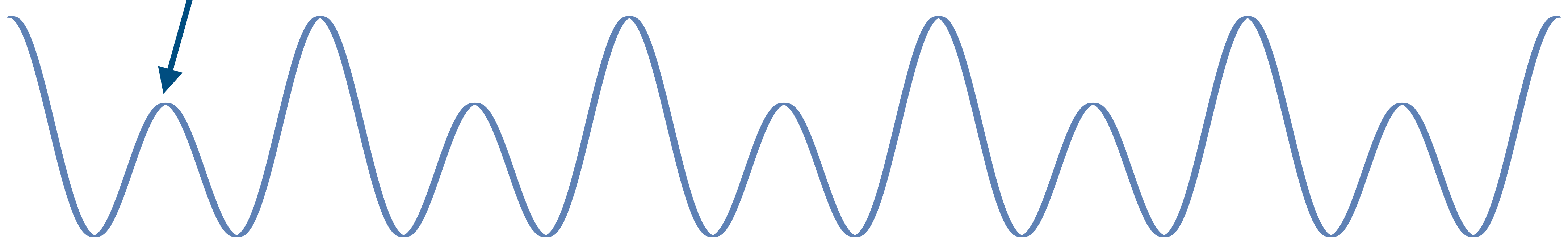
Dimerized lattice: **the Su Schrieffer Heeger (SSH) model** for spineless fermions

$$H = -\sum_{i=1} (J + \delta J(-1)^i)(c_i^\dagger c_{i+1} + c_{i+1}^\dagger c_i)$$



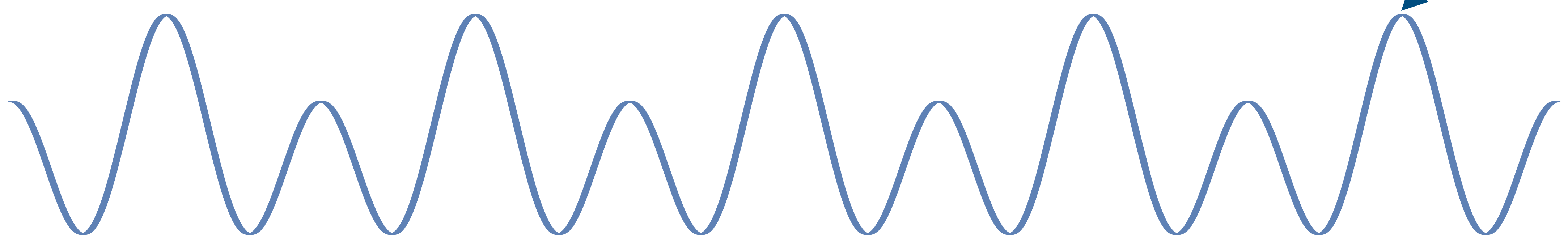
$-(J+\delta J)$

$\delta J < 0$ ***trivial***

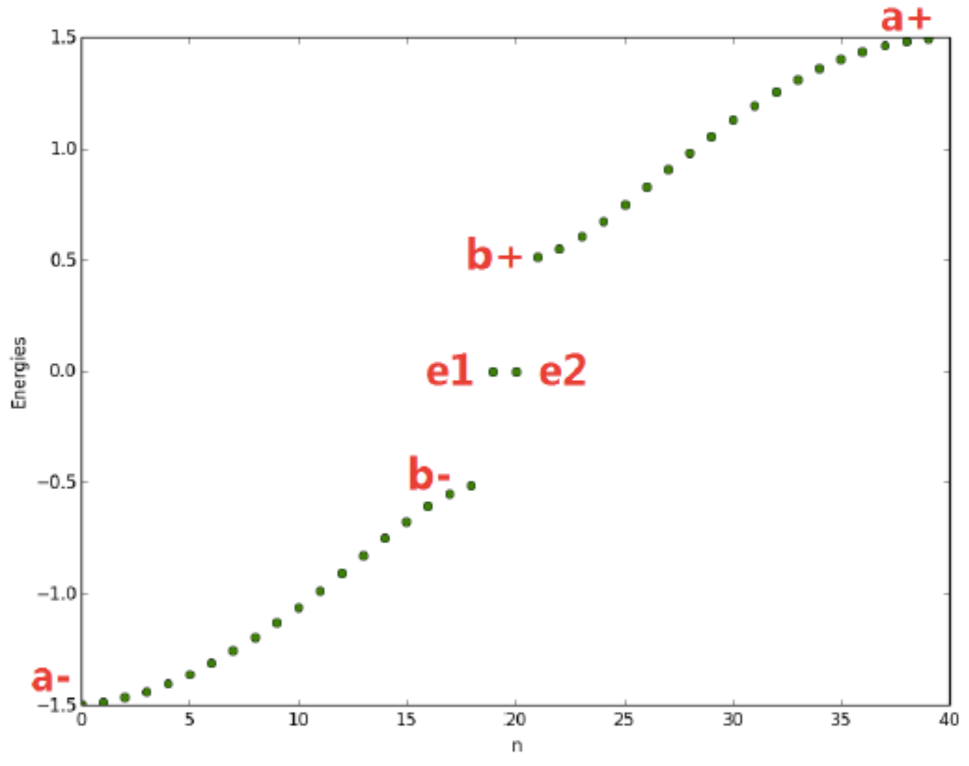


$\delta J > 0$ ***topological***

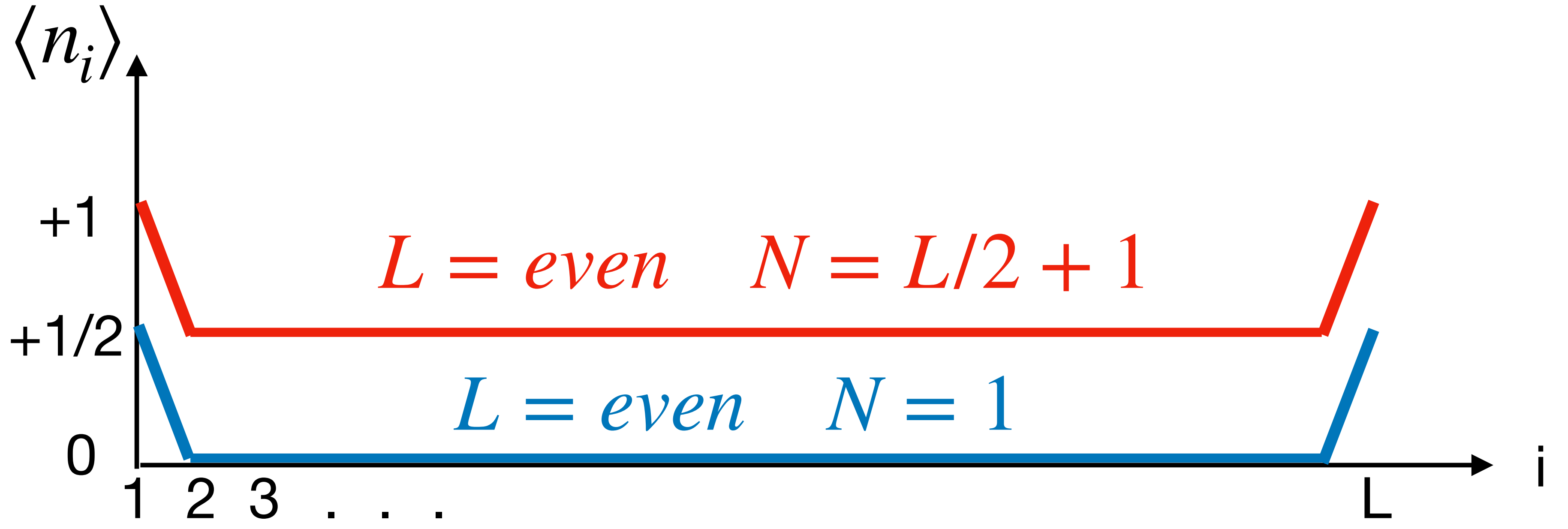
$-(J-\delta J)$



For 1 particle topology should manifest in the presence of 0 energy edge states in the energy spectra and in the particle density operator $\langle n_i \rangle$…so check it with Quspin

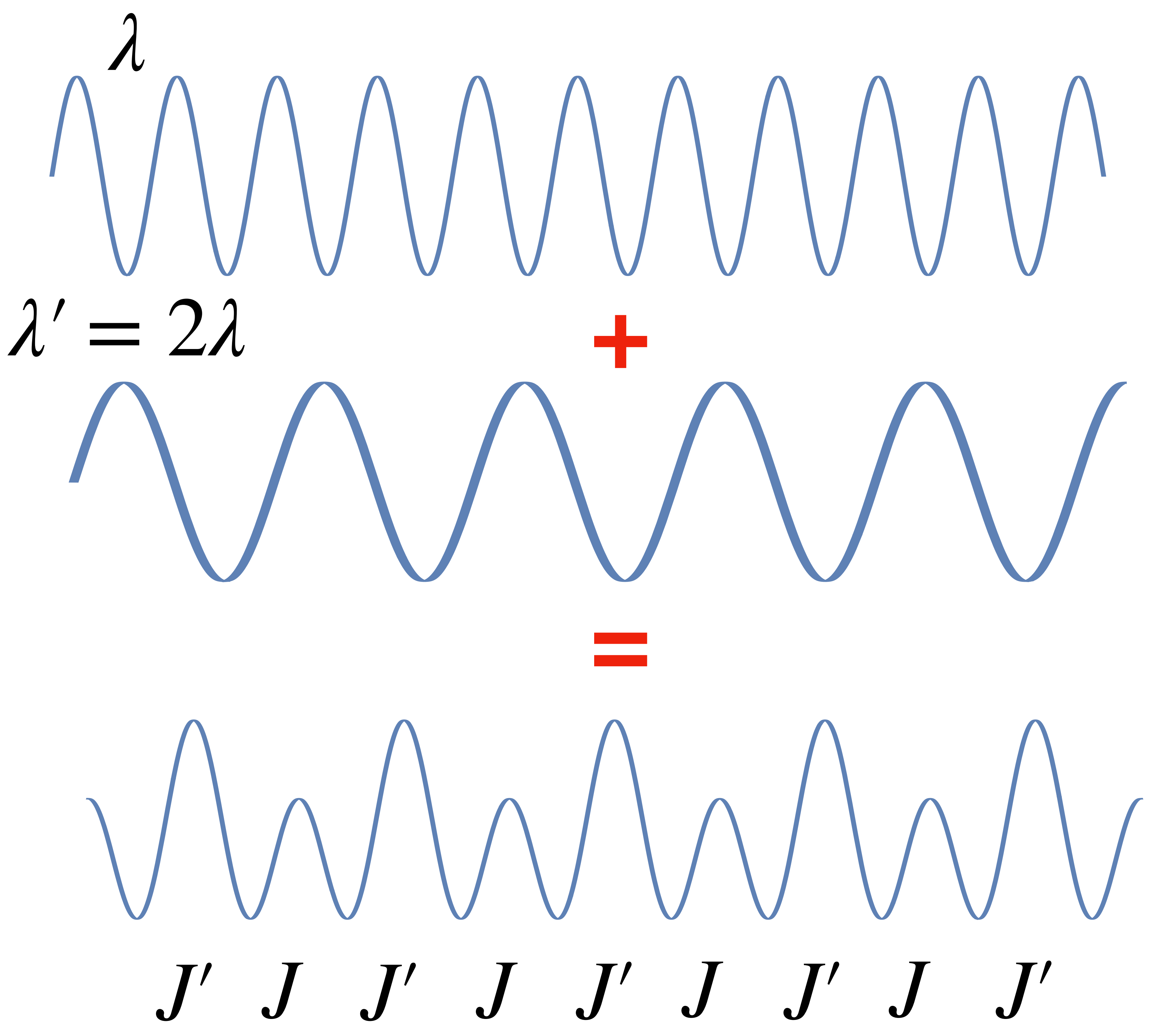


eigen-energy



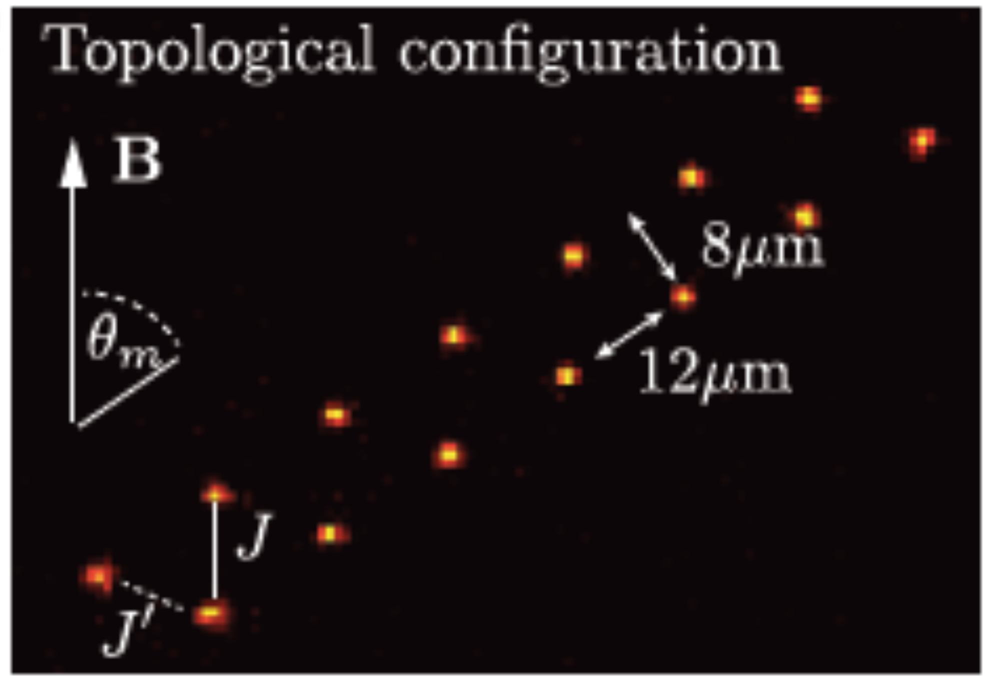
The Su Schrieffer Heeger (SSH) model realized in atomic quantum simulators

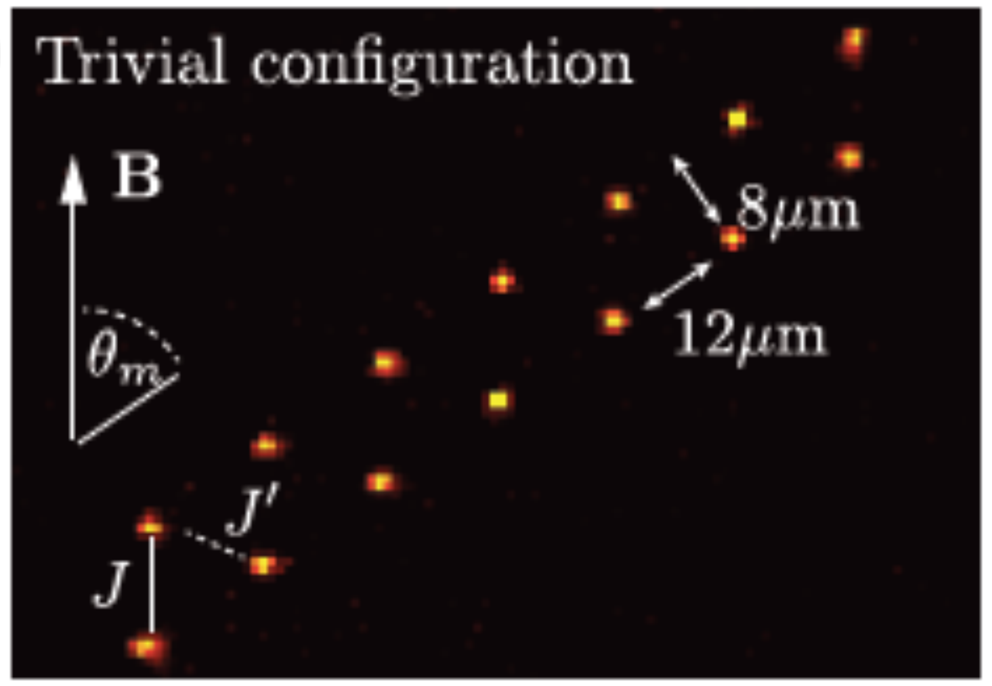
Two optical lattices with one having a wave length twice that of the other

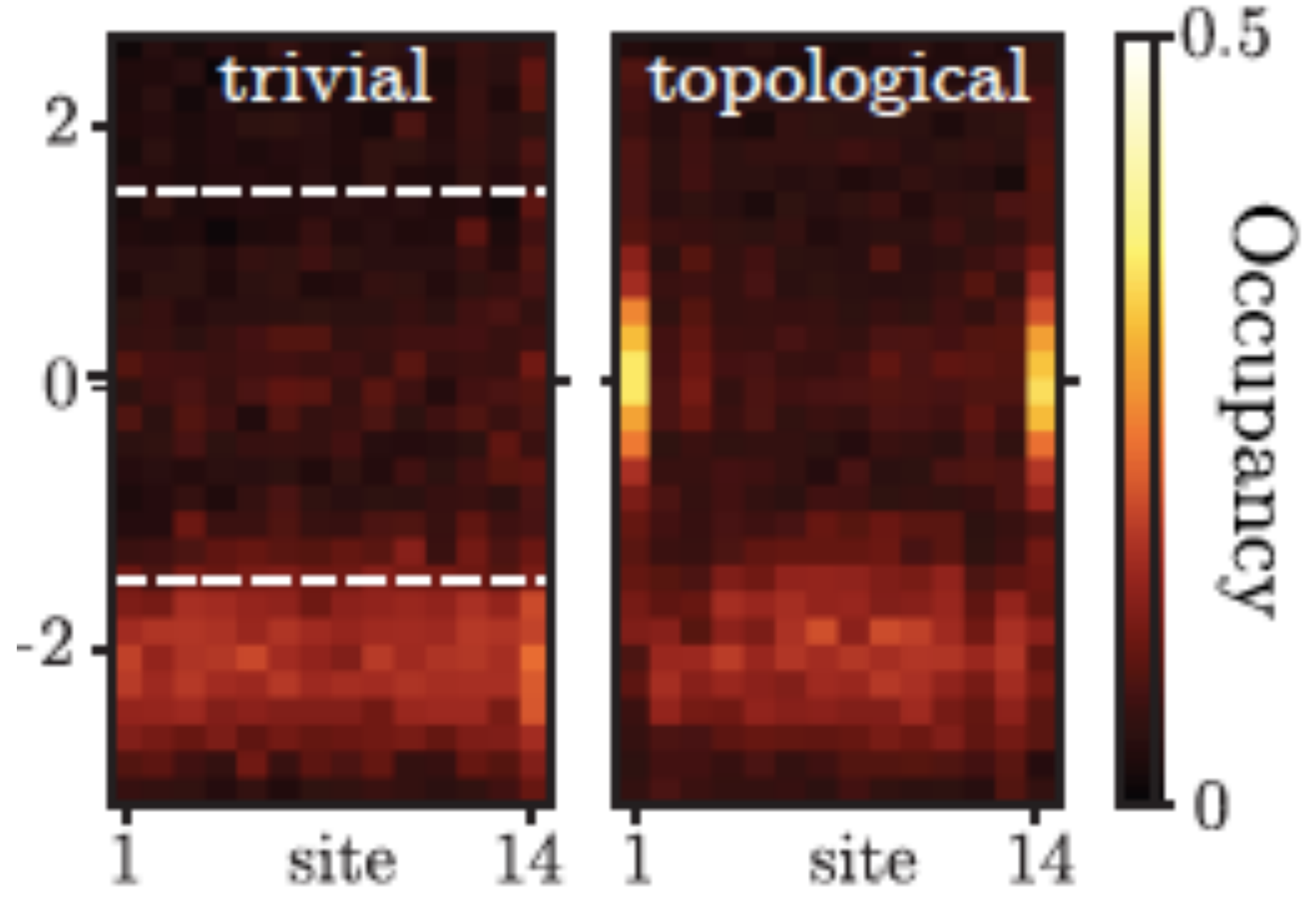


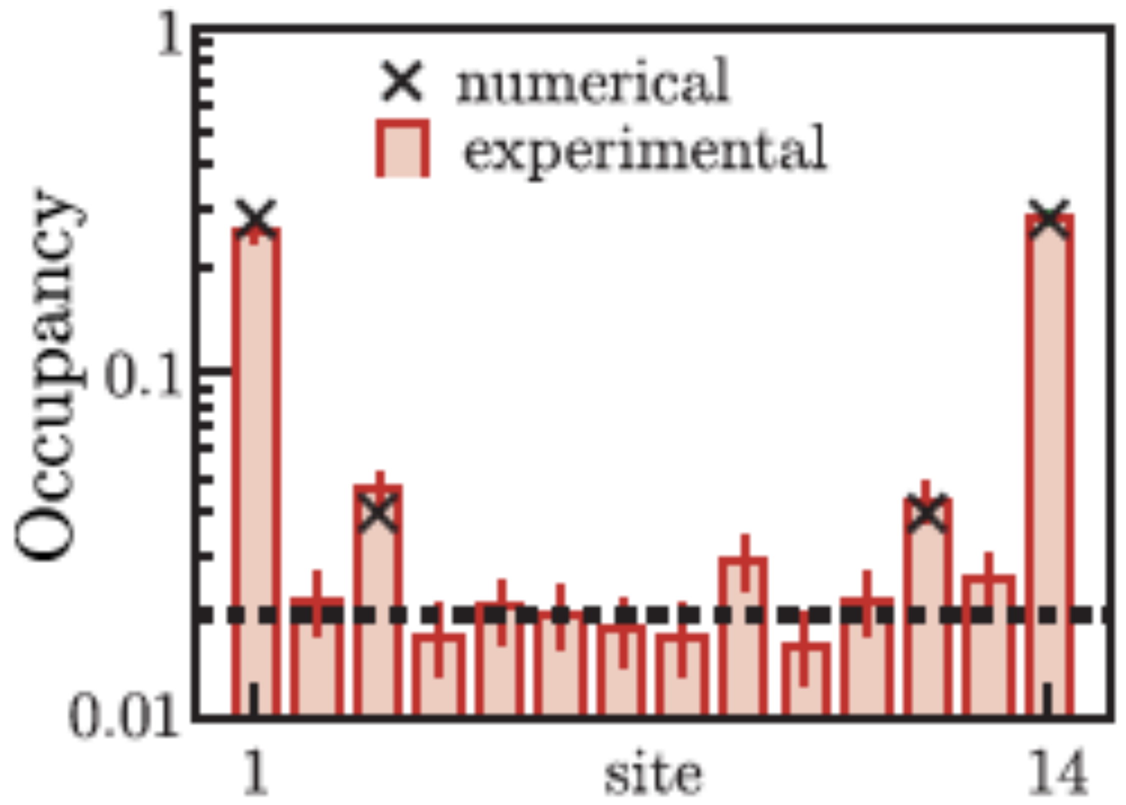
Or

Use a triangular geometry and play with angle in the dipolar interaction so to reach an effective dimerization









What about the **bosonic SSH** model?

$$H = -\sum_{i=1} (J + \delta J(-1)^i)(b_i^\dagger b_{i+1} + b_{i+1}^\dagger b_i) + \frac{U}{2}\sum_i n_i(n_i - 1)$$

*Is the system **always gapped** or there is a **critical U** such that the system develops a gap? once the system is gapped, do we also have **topological properties**? check it with Quspin….*

tips

1. use **PBC** with N=L/2 (L even)
2. **check** that the GS energy does not change under the **transformation** $\delta J \leftrightarrow -\delta J$
3. **calculate** the usual **charge gap** $\Delta_c(\infty, \infty)$ (in the thermodynamic limit!!!)
4. for one value of U such that $\Delta_c(\infty, \infty) \neq 0$, open the chain (**OBC**) and check that the transformation $\delta J \leftrightarrow -\delta J$ does not hold anymore
5. for the same value of U, choose δJ (with the **appropriate sign**!!!!) and fix N=L/2+1
6. plot the **expectation value of** $\langle n_i \rangle$

do you find edge states?!