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Assignment III 杨舒云 22344020

Problem 7.6

Solution: According to the definition of optical path difference, we can directly calculate by the following formula:

$$OPD = n_{water}l + 2n_{galss}d - l - 2d = 0.0382m$$

while $l = 10cm, d = 0.5cm$.

As for the phase difference at the finishing line, we can find that:

$$\Delta\phi = \frac{2\pi}{\lambda_0}OPD = 1.25 \times 10^5\pi$$

Problem 7.12

Solution: Using phasors, we can get the result shown in the Figure 1:

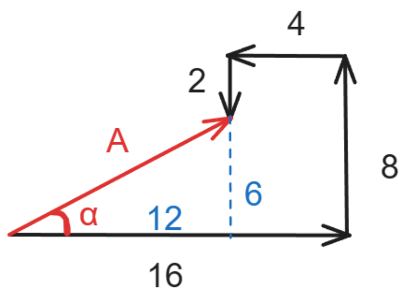

Figure 1: Problem 7.12

Then we find that:

$$A = \sqrt{12^2 + 6^2} = 13.4$$

$$\alpha = \arctan\frac{6}{12} = 0.464rad$$

Problem 7.21

Solution: Using product-to-sum formula, we can find that:

$$\begin{aligned} E &= E_0(1 + a\cos\omega_m t)\cos\omega_c t \\ &= E_0\cos\omega_c t + E_0 a\cos\omega_m t\cos\omega_c t \\ &= E_0\cos\omega_c t + \frac{E_0 a}{2}\cos(\omega_m - \omega_c)t + \frac{E_0 a}{2}\cos(\omega_m + \omega_c)t \end{aligned}$$

thus we get that E is equivalent to the superposition of three waves of frequencies ω_c, $\omega_m + \omega_c$, and $\omega_m - \omega_c$.

According to the relevant references, we can determine that $f_{max} = 2 \times 10^4 Hz$, therefore the $\Delta f = 2f_{max} = 4 \times 10^4 Hz$ is just what we want.

Problem 7.23

Proof: From the problem,

$$v_g = \frac{\mathrm{d}\omega}{\mathrm{d}k} = \frac{\mathrm{d}2\pi\nu}{\mathrm{d}\frac{2\pi}{\lambda}} = -\lambda^2\frac{\mathrm{d}\nu}{\mathrm{d}\lambda}$$

Q.E.D.

Problem 7.39

Solution: By the dispersion equation $\omega^2 = \omega_p^2 + c^2k^2$, we can find that:

$$v_p = \frac{\bar{\omega}}{\bar{k}} = \frac{c}{\sqrt{1-(\frac{\omega_p}{\bar{\omega}})^2}}$$

$$v_g = \frac{\mathrm{d}\omega}{\mathrm{d}k}|_{\omega=\bar{\omega}} = c\sqrt{1-(\frac{\omega_p}{\bar{\omega}})^2}$$

In fact, it can be obtained by differentiating the dispersion equation directly that:

$$2\omega\mathrm{d}\omega = 2c^2k\mathrm{d}k \Rightarrow \frac{\mathrm{d}\omega}{\mathrm{d}k} = c^2\frac{k}{\omega}$$

thus we find that when we take $\omega = \bar{\omega}$ and $k = \bar{k}$ from both sides, $v_pv_g = c^2$, Q.E.D.

Problem 7.56

Solution: When $\bar{\lambda_0} = 446nm$ and $\Delta\lambda_0 = 21nm$, we can find that $\bar{f} = \frac{c}{\bar{\lambda_0}} = 6.73\times10^{12}Hz$; and because $\frac{\Delta\lambda}{\bar{\lambda_0}} = \frac{\Delta f}{\bar{f}}$, so there is $\Delta f = 3.17\times10^{13}Hz$.

So that:

$$\Delta l_c = c\Delta t_c = \frac{c}{\Delta f} = 9.47\times10^{-6}m$$

Problem 7.59

Solution: From the Problem, using that $\frac{\Delta f}{\bar{f}} = 2\times10^{-10}$ and $\bar{f} = \frac{c}{\bar{\lambda_0}}$, we can find that $\Delta f = 9.48\times10^4Hz$, then:

$$\Delta l_c = c\Delta t_c = \frac{c}{\Delta f} = 3.16\times10^3m$$

Problem 7.61

Solution: When $\bar{\lambda_0} = 600nm$ and $\Delta\lambda_0 = 10^{-10}m$, we can find that:

$$\Delta l_c = c\Delta t_c = \frac{c}{\Delta f} = \frac{\bar{\lambda_0}^2}{\Delta\lambda_0} = 3.6\times10^{-3}m$$

Assignment IV 杨舒云 22344020

Problem 8.13

Proof: It is noted that formula 8.25 is $T_l = T_0 \cos^2\theta + T_{90}\sin^2\theta$.

It is also known from the problem that $T_l = (T_0 - T_{90})\cos^2\theta + T_{90} = T_0\cos^2\theta + T_{90}(1-\cos^2\theta) = T_0\cos^2\theta + T_{90}\sin^2\theta$.

Q.E.D.

Problem 8.39

Solution: Note that unlike calcite, quartz has less $n_o = 1.5443$ than $n_e = 1.5534$, which means that the optical path diagram will look like the one shown above(Figure 2).

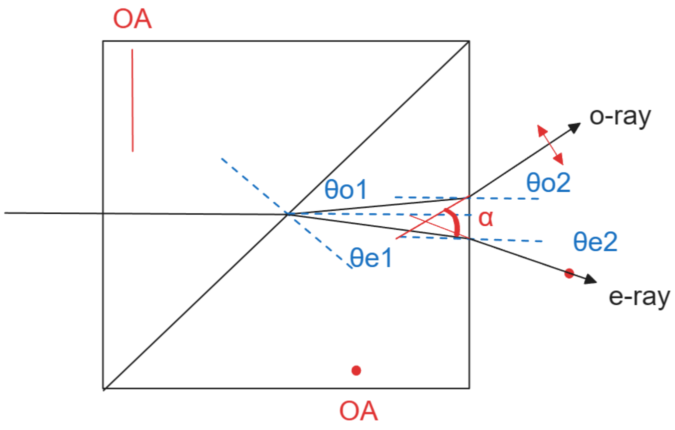

Figure 2: Problem 8.39

Refraction of o-ray across interface between two quartz prisms:

$$n_{o1}\sin\theta_{i1} = n_{o2}\sin\theta_{o1}$$

$$1.5534\sin 45^\circ = 1.5443\sin\theta_{o1}$$

$$\Rightarrow \theta_{o1} = 45.34^\circ$$

Refraction of e-ray across interface between two quartz prisms:

$$n_{e1}\sin\theta_{i1} = n_{e2}\sin\theta_{e1}$$

$$1.5443\sin 45^\circ = 1.5534\sin\theta_{e1}$$

$$\Rightarrow \theta_{e1} = 44.66^\circ$$

We can see from the Figure 2, $\theta_{i2} = \theta_{o1} - 45^\circ = 45^\circ - \theta_{e1} = 0.34^\circ$. Therefore, we can find that:

The second refraction of o-ray:

$$n_{o2}\sin\theta_{i2} = n_{air}\sin\theta_{o2}$$

$$1.5443\sin 0.34^\circ = (1)\sin\theta_{o2}$$

$$\Rightarrow \theta_{o2} = 0.525^\circ$$

The second refraction of e-ray:

$$n_{e2}\sin\theta_{i2} = n_{air}\sin\theta_{e2}$$

$$1.5534\sin 0.34^\circ = (1)\sin\theta_{e2}$$

$$\Rightarrow \theta_{e2} = 0.528^\circ$$

The angle separating the two is: $\alpha = \theta_{o2} + \theta_{e2} = 1.053^\circ$.

Problem 8.53

Solution: From $\lambda_n = \lambda_0/n$, we can find that:

E-ray:

$$\lambda_n^{(e)} = \frac{\lambda_0}{n^{(e)}} = \frac{589.3nm}{1.5534} = 379.4nm$$

O-ray:

$$\lambda_n^{(o)} = \frac{\lambda_0}{n^{(o)}} = \frac{589.3nm}{1.5443} = 381.6nm$$

The two kinds of light have the same frequency: $f = \dfrac{c}{\lambda_0} = 5.091\times 10^{14}Hz$.

Problem 8.56

Solution: Briefly, for natural light, the wave plate does not affect its light intensity (irradiance), whereas the polarizer affects its light intensity.Therefore, it can be determined by observing the change of light intensity separately.

Problem 8.57

Solution: For a 135-degree linearly polarized light, we have $\mathcal{P} = E_0\hat{i}\cos\theta - E_0\hat{j}\cos\theta$.When it passes through a $\dfrac{\pi}{2}$-phase retarder with a fast axis in the vertical direction, it means that the phase of the y-direction polarization will lag behind $\dfrac{\pi}{2}$.

$$E_0\hat{i}\cos\theta - E_0\hat{j}\cos\left(\theta - \frac{\pi}{2}\right) = E_0\hat{i}\cos\theta - E_0\hat{j}\sin\theta = \mathcal{L}$$

This means that it becomes left-handed circular polarization.

If the incoming ray is polarized parallel to the slow axis, it will not rotate.

Problem 8.64

Solution: Considering the Jones vector and the matrix,

Incident light:

$$\mathscr{P}_i = (1, 0)$$

Wave plate (fast axis with x clip $\frac{\pi}{8}$):

$$\mathbf{J} = \begin{bmatrix} \cos^2\left(\frac{\pi}{8}\right) + i\sin^2\left(\frac{\pi}{8}\right) & (1-i)\sin\left(\frac{\pi}{8}\right)\cos\left(\frac{\pi}{8}\right) \\ (1-i)\sin\left(\frac{\pi}{8}\right)\cos\left(\frac{\pi}{8}\right) & \sin^2\left(\frac{\pi}{8}\right) + i\cos^2\left(\frac{\pi}{8}\right) \end{bmatrix}$$

Then we can get emergent light:

$$\mathscr{P}_e = \mathbf{J}\mathscr{P}_i = (\cos^2\left(\frac{\pi}{8}\right) + i\sin^2\left(\frac{\pi}{8}\right), (1-i)\sin\left(\frac{\pi}{8}\right)\cos\left(\frac{\pi}{8}\right))$$

From this we can obtain phasors in both polarization directions very easily (in polar coordinates):

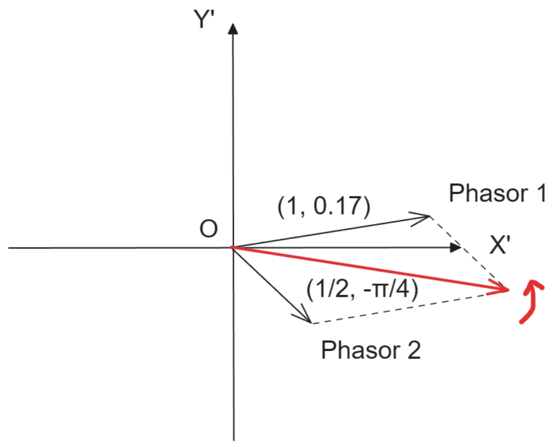

Figure 3: Problem 8.64

Problem 8.92

Solution:

$$\mathbf{J} = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

1. According to the problem,

$$\mathscr{P}_{1i} = (\cos\theta, \sin\theta)$$

The emergent light:

$$\mathscr{P}_{1e} = \mathbf{J}\mathscr{P}_{1i} = (\cos(\theta - \alpha), \sin(\theta - \alpha))$$

It means the change from one linear polarization to another.

2. Similarly, it can be obtained that:
$$\mathscr{P}_{2i} = \mathscr{P}_{\mathcal{L}} = \frac{1}{\sqrt{2}}(1, i)$$
$$\mathscr{P}_{2e} = \mathbf{J}\mathscr{P}_{2i} = \frac{e^{i\alpha}}{\sqrt{2}}(1, i)$$
The emergent light is a left-handed circular polarization with a change in phase.

3. Tautologically, we can find that:
$$\mathscr{P}_{3i} = \mathscr{P}_{\mathcal{R}} = \frac{1}{\sqrt{2}}(1, -i)$$
$$\mathscr{P}_{3e} = \mathbf{J}\mathscr{P}_{3i} = \frac{e^{-i\alpha}}{\sqrt{2}}(1, -i)$$
The emergent light is a right-handed circular polarization with a change in phase.

4. In summary, this optical element changes linear polarization into linear polarization with a change in direction, left-handed circular polarization into left-handed circular polarization with a change in phase, and right-handed circular polarization into right-handed circular polarization with a change in phase, and left-handed circular polarization and right-handed circular polarization change in phase. Therefore, it is a polarimetric element, and it can be produced by using some optically-active substances or Faraday effects, such as the Faraday polarimetric device that we are more familiar with.

Problem 8.94

Solution: We consider a wave plate that delays the phase δ and whose fast axis is at an Angle θ from the X-axis. Its Jones Matrix will be:
$$\mathbf{J}^{(wp)}_{(\delta,\theta)} = \begin{bmatrix} \cos^2\theta + e^{i\delta}\sin^2\theta & \sin\theta\cos\theta(1 - e^{i\delta}) \\ \sin\theta\cos\theta(1 - e^{i\delta}) & \sin^2\theta + e^{i\delta}\cos^2\theta \end{bmatrix}$$

So, for a quarter wave plate, $\delta = \dfrac{\pi}{2}$.

When $\theta = \dfrac{\pi}{4}$,
$$\mathbf{J}^{(wp)}_{(\frac{\pi}{2},\frac{\pi}{4})} = \frac{1}{\sqrt{2}}e^{i\frac{\pi}{4}} \begin{bmatrix} 1 & -i \\ -i & 1 \end{bmatrix} = \mathscr{A}_2$$

When $\theta = -\dfrac{\pi}{4}$,
$$\mathbf{J}^{(wp)}_{(\frac{\pi}{2},-\frac{\pi}{4})} = \frac{1}{\sqrt{2}}e^{i\frac{\pi}{4}} \begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix} = \mathscr{A}_1$$

Therefore, the two filters are half wave plates with an Angle between the fast axis and the X-axis of negative forty-five degrees and positive forty-five degrees, respectively.

Assignment V 杨舒云 22344020

Problem 9.5

Solution: Using the two-slit interference model, the two peaks shown in the figure are selected to estimate the fringe spacing $\Delta x = 41.5cm$. From the given conditions, we can get the equivalent slit width is $d = 15cm$ and the equivalent gap between the slit and the screen is $a = 1.5m$. Then we can find that:

$$\Delta x = 3\frac{a\lambda}{d} \Rightarrow \lambda = 0.0138m$$

$$\Rightarrow f = \frac{v}{\lambda} = 2.4795 \times 10^4 Hz$$

This is the approximate driving frequency of the speaker.

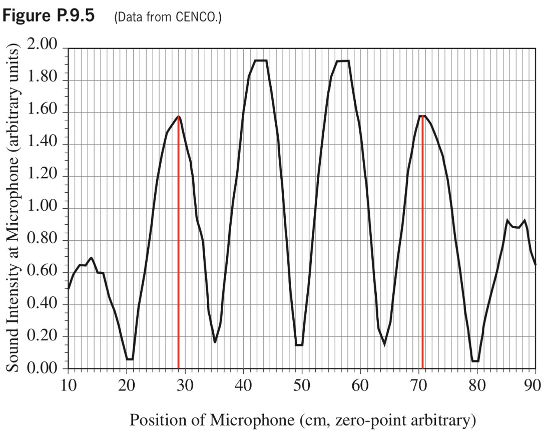

Figure 4: Problem 9.5

Explain why the center is minimal: (according to the two-slit interference model described earlier) This is apparently because the phase difference between the two wave sources is just π (**that is, they are out of phase**).

Problem 9.11

Solution: This problem also uses the two-slit interference model, and the fourth-order bright fringe satisfies $x = 4\frac{a\lambda_0}{d}$. And since they gave us a very small angle θ=1 degree we can roughly say that $\theta \approx \frac{x}{a}$. As shown in the figure, we can find that:

$$\theta \approx \frac{x}{a} = 4\frac{\lambda_0}{d} \Rightarrow d = 4\frac{\lambda_0}{\theta} = 1.45 \times 10^{-4}m$$

This is the gap width they want us to find.

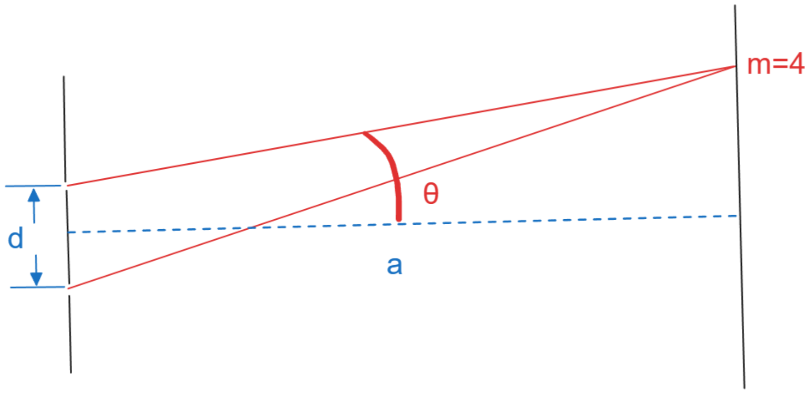

Figure 5: Problem 9.11

Problem 9.17

Solution: Borrowing the diagram from problem 9.11, we can think of $d = 2.70mm$, $a = 4.60m$, and $x = 5.00mm$ (for $m = 5$). Even though it's a dark stripe. Therefore, we can find that:

$$x = 5\frac{a\lambda}{d} \Rightarrow \lambda = \frac{xd}{5a} = \frac{(5 \times 10^{-3}m)(2.7 \times 10^{-4}m)}{5(4.60m)} = 587nm$$

This is the wavelength of light.

Problem 9.19

Solution: As shown in the figure, notice that the light emitted by the two slits at this time has a phase difference, so we can get:

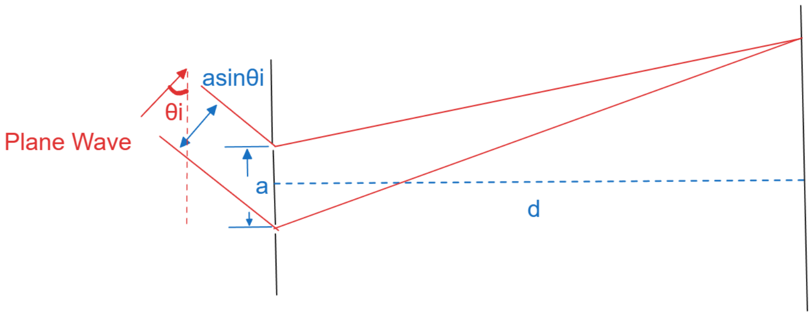

Figure 6: Problem 9.19

$$OPD = a\sin\theta_i + \frac{xa}{d}$$

$$\frac{2\pi}{\lambda}OPD = m(2\pi)$$

$$\Rightarrow \theta \approx \frac{x}{d} = \frac{m\lambda}{a} - \sin\theta_i$$

Problem 9.51

Solution:

$$f = \frac{c}{\lambda}$$

$$\Delta\lambda_0 = \frac{c}{f^2}\Delta f = \frac{\lambda_0^2}{c}\Delta f$$

$$\Delta f = \frac{1}{\Delta t_c}$$

$$\Delta l_c = c\Delta t_c$$

$$\Delta\lambda_0 = \frac{\lambda_0^2}{\Delta l_c}$$

Next, we can further calculate D:

$$\Delta l_c = 2D$$

$$D = \frac{\lambda_0^2}{2\Delta\lambda_0} = 0.1594m$$

Problem 9.52

Solution:

The Jamin interferometer is an optical instrument designed for studying the interference patterns of light waves to measure wavelength, the uniformity of materials, and physical conditions such as temperature and pressure variations in a medium. The device operates on the principle of interference, which occurs when two waves superimpose to form a resultant wave of greater, lower, or the same amplitude.

In the Jamin interferometer, a light source emits a beam that is split by a beam splitter into two paths. The beam splitter is typically a partially reflective mirror. One beam reflects off the mirror at an angle, travels a certain path, and reflects back towards a merging mirror. The second beam passes straight through the beam splitter, travels a different path, often through a sample or the medium being studied, reflects off another mirror, and also heads back towards the merging mirror. The two beams are then recombined, and because they have traveled different paths, they acquire a phase difference.

The recombined beams interfere constructively or destructively, creating an interference pattern that can be observed or recorded. By analyzing this pattern, it is possible to determine the characteristics of the medium through which one of the beams has passed. For example, changes in the refractive index due to temperature, pressure, or composition can be measured since they affect the phase difference between the two beams.

Jamin interferometers are used in a variety of scientific and industrial applications. They can be employed in:

1. Refractometry: Measuring the refractive index of gases, liquids, and solids.
2. Metrology: Precise measurements of optical path lengths, which are essential in the calibration of other optical devices.

3. Fluid dynamics: Studying changes in the refractive index due to variations in temperature and pressure, which can be related to fluid flow characteristics.

4. Material science: Investigating the homogeneity and stress distribution within transparent materials.

The accuracy and sensitivity of the Jamin interferometer make it particularly useful in research environments where precise measurements are required.

Problem 9.57

Solution: If not coating, we have:

$$r_0 = \frac{n_s - n_0}{n_s + n_0}$$

After coating, $n_s > n_1 > n_0$, we have:

$$r = \frac{n_0 n_s - n_1^2}{n_0 n_s + n_1^2} < \frac{n_1 n_s - n_1^2}{n_1 n_s + n_1^2} = \frac{(n_s - n_1)^2}{(n_s + n_1)^2} < \frac{n_s - n_0}{n_s + n_0} = r_0$$

That is, the reflection coefficient is reduced.

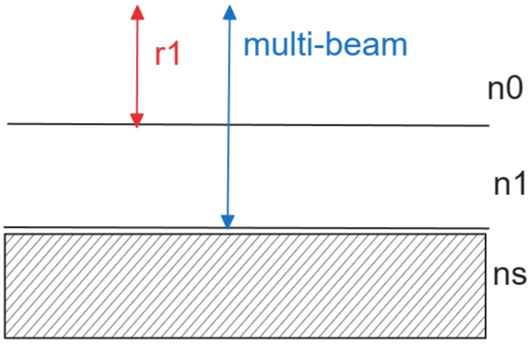

Figure 7: Problem 9.57

The following proves that this is equivalent to the waves reflected back from the two interfaces cancelling each other out.

Interface 1:

$$r_1$$

Interface 2 (Multi-beam):

$$tr_2t' + tr_2r_1'r_2t' + \ldots = tr_2t'(1 + r_2r_1' + \ldots) = \frac{tr_2t'}{1 - r_2r_1'}$$

Cancel out, using that $r_1 = \frac{n_1 - n_0}{n_1 + n_0} = -r_1'$, $t = \frac{2n_0}{n_0 + n_1}$, $t' = \frac{2n_1}{n_0 + n_1}$, $r_2 = \frac{n_s - n_1}{n_s + n_1}$

$$\Rightarrow r_1 - \frac{tr_2t'}{1 - r_2r_1'} = \frac{n_1^2 - n_0n_s}{n_1^2 + n_0n_s}$$

Problem 9.61

Solution: Considering the case of forward incidence and minimum thickness, when $d\cos\theta = (2m+1)\dfrac{\lambda}{4}$, we can find that:

$$d = \frac{\lambda_0}{4n} = \frac{500nm}{4 \times 1.30} = 96nm$$

Assignment VI 杨舒云 22344020

Problem 10.7

Solution: Using the far-field criterion$|r'| >> \frac{(\xi^2+\eta^2)_{max}}{\lambda}$, for single-slit diffraction, the value is taken as $1.5m >> \frac{(0.10mm)^2}{461.9nm} = 0.022m$ (Seam width b=0.10mm). Therefore, the diffraction pattern it produces is a far-field diffraction pattern.

Center maximum angle width:

$$\beta_{max} = \pm\pi = \frac{kb\sin\theta}{2} \Rightarrow \theta = 2\arcsin\frac{\lambda}{b} = 0.009238 = 0.529degree$$

θ is the maximum angular width of the center.

Problem 10.34

Solution: Find the radius of the Airy Disk and then the magnitude of the Airy Disk:

$$q_1 = 1.22\frac{\lambda f}{D} = \frac{1.22 \times 540nm \times 140cm}{15cm} = 6.1488 \times 10^{-6}m$$

$$\Rightarrow A = \pi q_1^2 = 1.18777 \times 10^{-10}m^2$$

If $D' = 2D$, it will be $q_1' = 3.0744 \times 10^{-6}m$. That is, it gets smaller.

Problem 10.40

Solution: Similarly, we can calculate:

$$q_1 = 1.22\frac{\lambda R}{D} = \frac{1.22 \times 632.84nm \times 376 \times 10^3 km}{2mm} = 145148m$$

$$\Rightarrow A = \pi q_1^2 = 6.61869 \times 10^{10}m^2$$

Problem 10.46

Solution: Once again, very similarly, we calculated a series of results.

Angular resolution limit:

$$\Theta = 1.22\frac{\lambda}{D} = 1.32087 \times 10^{-7} = 7.55 \times 10^{-6}degree = 2.72 \times 10^{-2}arcsec$$

The Mount Palomar telescope:

$$s = R\Theta = 50.7m$$

The human's eye:

$$2q_1 = 1.22\frac{\lambda R}{D'} \Rightarrow s' = D' = 1.22\frac{\lambda R}{2q_1} = 64483.1m$$

Problem 10.50

Solution: Without any distinction, we calculate that:

$$L = 1.22\frac{\lambda R}{D} = 0.168m$$

Problem 10.58

Solution: Notice that $a = \dfrac{1}{5.9 \times 10^5 groove/m} = 1.695 \times 10^{-6} m/groove$, then we can find that:
When $\lambda_1 = 400nm$ and $m = 1$,

$$\sin\theta_1 = \frac{m\lambda_1}{a} = 13.65degree$$

When $\lambda_2 = 720nm$ and also $m = 1$,

$$\sin\theta_2 = \frac{m\lambda_2}{a} = 25.14degree$$

Therefore, the angular width of the first-order spectrum is:

$$\Delta\theta = \theta_2 - \theta_1 = 11.49degree$$

Problem 10.83

Solution: First we verify that it does not conform to the far-field criterion $|r'| >> \dfrac{(\xi^2 + \eta^2)_{max}}{\lambda}$:

$$d = 250cm < \frac{a^2}{\lambda} = 36m$$

Therefore, we need to consider Fresnel diffraction. We use the following substitution: $u = y\sqrt{\dfrac{2}{\lambda d}} = y\sqrt{\dfrac{2}{694.3nm \times 250cm}}$, $v = z\sqrt{\dfrac{2}{694.3nm \times 250cm}}$. When y and z go from -0.0025m to -0.0025m, u and v go from -2.68356 to 2.68356.

$$I(0) = \frac{I_u}{4}((2C(2.68356))^2 + (2S(2.68356))^2)^2$$

Note that $I_u = 10W/m^2$ and numerically calculate the integral of C and S, we can find:

$$I(0) = \frac{10W/m^2}{4}(0.772621^2 + 0.936271^2) = 5.42835W/m^2$$

Problem 10.91

Solution: For single-slit Fresnel diffraction, we use the following substitution: $u = y\sqrt{\dfrac{2(\rho_0 + r_0)}{\lambda\rho_0 r_0}} = 2538.59y/m$. When y goes from -0.0005m to -0.0005m, u goes from 0.1269 to 0.1269. Then we can find that:

$$\frac{I_0}{I_u} = \frac{1}{2}((2C(0.1269))^2 + (2S(0.1269))^2) = 0.0322054$$