Signals And Systems by Alan V. Oppenheim: Notes

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0.1 Introduction

0.1.1 Signal Energy and Power

Motivation and definition
In many but not all, applications, the signals considered directly related to physical quantities capturing power and energy in a physical system. (for instance v^2/R for the power across a resistor)

As such it is a common and worthwhile convention to use similar terminology for power and energy for *any* continuous-time signal, denoted $x(t)$, or any discrete-time signal $x[n]$. In this case, the total energy over the time interval $t_1 \leq t \leq t_2$ in a continuous signal $x(t)$ is defined as

$$\int_{t_1}^{t_2} |x(t)|^2 dt$$

where $|x|$ denotes the magnitude of the (possibly complex) number x; see that the time-averaged signal can be obtained by dividing by $(t_2 - t_1)$. Similarly for a discrete signal $x[n]$ over the interval $n_1 \leq n \leq n_2$ the total energy is

$$\sum_{n=n_1}^{n_2} |x[n]|^2$$

with the average power calculated by dividing by $(n_2 - n_1 + 1)$.

It is important to remember that the terms 'power' and 'energy' are used here *independently* of their relation to physical energy (they clearly don't correlate since their units or scalings would differ). Nevertheless we will find it convenient to use these terms in a general fashion.

Power and energy over infinite intervals
Considering signals over an infinite time interval, meaning for $-\infty < t < +\infty$ or $-\infty < n < +\infty$. Here we define the total energy as the limits of the aforementioned equations increase without bound; in continuous time,

$$E_\infty \triangleq \lim_{T \to \infty} \int_{-T}^{T} |x(t)|^2 dt = \int_{-\infty}^{+\infty} |x(t)|^2 dt$$

and in discrete time,

$$E_\infty \triangleq \lim_{N \to \infty} \sum_{n=-N}^{+N} |x[n]|^2 = \sum_{n=-\infty}^{+\infty} |x[n]|^2$$

Note that these expressions may not converge; for instance say $x(t)$ or $x[n]$ equal some nonzero constant for all time: such signals have infinite energy, while signals with $E_\infty < \infty$ have finite energy.
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Cont.
Analagously, we can define the time-averaged power over an infinite interval as

$$P_\infty \triangleq \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$$

and

$$P_\infty \triangleq \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{+N} |x[n]|^2$$

In continuous and discrete time respectively.

Intuition
See that with these definitions, we can identify three classes of signals: first those with finite total energy, meaning $E_\infty < \infty$. See that such a signal would have zero average power:

$$P_\infty = \lim_{T\to\infty} \frac{E_\infty}{2T} = 0$$

Second would be signals with finite average power P_∞; see from the above expression that for $P_\infty > 0$, this requires that $E_\infty = \infty$.

Last would be signals for which neither P_∞ nor E_∞ are finite. An example of this might be $x(t) = t$.

0.1.2 Even and Odd signals

Definition
A continuous-time signal is *even* if

$$x(-t) = x(t)$$

while a discrete-time signal is *even* if

$$x[-n] = x[n]$$

These signals are referred to as *odd* if

$$x(-t) = -x(t)$$
$$x[-n] = -x[n]$$

Note that an odd signal must be 0 at $t = 0$ or $n = 0$ since the equations require that $x(0) = -x(0)$ and $x[0] = -x[0]$.

Decomposition
An important fact is that any signal can be broken into a sum of two signals, where one is even and the other odd. To see this, consider

$$\mathrm{Ev}\{x(t)\} = \frac{1}{2}[x(t) + x(-t)]$$

which is referred to as the *even part* of $x(t)$. Similarly, the *odd part* of $x(t)$ is given by

$$\mathrm{Od}\{x(t)\} = \frac{1}{2}[x(t) - x(-t)]$$

See that $x(t)$ is the sum of the two. Exactly analagous definitions hold in the discrete time case.

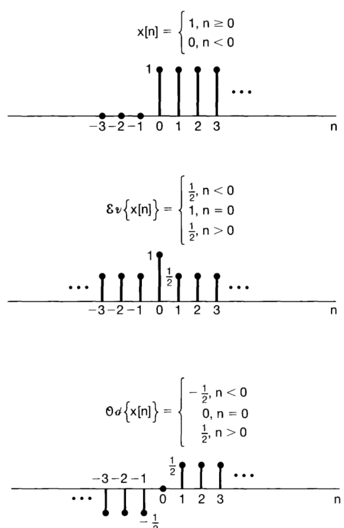

Figure 1.18 Example of the even-odd decomposition of a discrete-time signal.

0.1.3 More on Complex Exponential and Sinusoidal Signals

Periodic Complex Exponential and Sinusoidal Signals