

Velleman

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Chapter 1

Logic

1.0.1 Logic Factsheet

De Morgan's laws

$\neg(P \wedge Q)$ is equivalent to $\neg P \vee \neg Q$

$\neg(P \vee Q)$ is equivalent to $\neg P \wedge \neg Q$

Commutative laws

$P \wedge Q$ is equivalent to $Q \wedge P$

$P \vee Q$ is equivalent to $Q \vee P$

Associative laws

$P \wedge (Q \wedge R)$ is equivalent to $(P \wedge Q) \wedge R$

$P \vee (Q \vee R)$ is equivalent to $(P \vee Q) \vee R$

Idempotent laws

$P \wedge P$ is equivalent to P

$P \vee P$ is equivalent to P

Distributive laws

$P \wedge (Q \vee R)$ is equivalent to $(P \wedge Q) \vee (P \wedge R)$

$P \vee (Q \wedge R)$ is equivalent to $(P \vee Q) \wedge (P \vee R)$

Absorption laws

$P \vee (P \wedge Q)$ is equivalent to P

$P \wedge (P \vee Q)$ is equivalent to P

Double Negation law

$\neg\neg P$ is equivalent to P

1.0.2 Set operation definitions

The *intersection* of two sets A and B is the set $A \cap B$ defined as follows:

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$

The *union* of A and B is the set $A \cup B$ defined as follows:

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$

The *difference* of A and B is the set $A \setminus B$ defined as follows:

$$A \setminus B = \{x \mid x \in A \text{ and } x \notin B\}$$

See that

$$x \in A \cap B = x \in \{y \mid y \in A \text{ and } y \in B\}$$

where y is a dummy variable. So we can also write that

$$x \in A \cap B = x \in A \wedge x \in B$$

The same can be shown for the union and difference.

1.0.3 Distributivity of set operations

We show

$$x \in A \cap (B \cup C) \text{ is equivalent to } x \in (A \cap B) \cup (A \cap C)$$

By analysing their logical forms:

$$\begin{aligned} x \in A \cap (B \cup C) \\ &= x \in A \wedge x \in (B \cup C) \\ &= x \in A \wedge (x \in B \vee x \in C) \end{aligned}$$

and

$$\begin{aligned} x \in (A \cap B) \cup (A \cap C) \\ &= x \in (A \cap B) \vee x \in (A \cap C) \\ &= (x \in A \wedge x \in B) \vee (x \in A \wedge x \in C) \\ &= [(x \in A \wedge x \in B) \vee x \in A] \wedge [(x \in A \wedge x \in B) \vee x \in C] \\ &= x \in A \wedge [(x \in A \vee x \in C) \wedge (x \in B \vee x \in C)] \\ &= [x \in A \wedge (x \in A \vee x \in C)] \wedge (x \in B \vee x \in C) \\ &= x \in A \wedge (x \in B \vee x \in C) \end{aligned}$$

We can also show, in a similar manner, that

$$x \in A \cup (B \cap C) \text{ is equivalent to } x \in (A \cup B) \cap (A \cup C)$$

1.0.4 Distributivity of difference operation