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Started 13th October 2025

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Chapter 1

Graphs

1.1 Definitions and examples

A *graph* is an ordered pair $(V(G), E(G))$ consisting of a set $V(G)$ of *vertices* and a set $E(G)$, disjoint from $V(G)$, of *edges*, together with an *incidence function* ψ_G that associates with each edge of G an unordered pair of (not necessarily distinct) vertices of G.

If e is an edge and u and v are vertices such that $\psi_G(e) = \{u, v\}$ then e is said to *join* u and v, and the vertices u and v are called the *ends* of e.

We denote the numbers of vertices and edges in G by $v(G)$ and $e(G)$; these two basic parameters are called the *order* and *size* of G respectively.

Example 1
These examples serve to clarify the definition. For notational simplicity, we write uv for the unordered pair $\{u, v\}$.

$$G = (V(G), E(G))$$

where

$$V(G) = \{u, v, w, x, y\}$$
$$E(G) = \{a, b, c, d, e, f, g, h\}$$

and ψ_G is defined by

$$\psi_G(a) = uv \quad \psi_G(b) = uu \quad \psi_G(c) = vw \quad \psi_G(d) = wx$$
$$\psi_G(e) = vx \quad \psi_G(f) = wx \quad \psi_G(g) = ux \quad \psi_G(h) = xy$$

Example 2

$$H = (V(H), E(H))$$

where

$$V(H) = \{v_0, v_1, v_2, v_3, v_4, v_5\}$$
$$E(H) = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}\}$$

and ψ_H is defined by

$$\psi_H(e_1) = v_1 v_2 \quad \psi_H(e_2) = v_2 v_3 \quad \psi_H(e_3) = v_3 v_4 \quad \psi_H(e_4) = v_4 v_5$$
$$\psi_H(e_5) = v_5 v_1 \quad \psi_H(e_6) = v_0 v_1 \quad \psi_H(e_7) = v_0 v_2 \quad \psi_H(e_8) = v_0 v_3$$
$$\psi_H(e_9) = v_0 v_4 \quad \psi_H(e_{10}) = v_0 v_5$$

Drawings

Graphs are so named because