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# Chapter 1

# Permutations and Combinations

## 1.1 Counting principles

**Definition 1.** Let $S$ be a set. A *partition* of $S$ is a collection $S_1, S_2, \ldots, S_m$ of subsets of $S$ such that each element of $S$ is in exactly one of those subsets:

$$S = S_1 \cup S_2 \cup \cdots \cup S_m,$$
$$S_i \cap S_j = \emptyset, \quad (i \neq j).$$

See that the sets $S_1, S_2, \ldots, S_m$ are pairwise disjoint with union $S$. The subsets $S_1, S_2, \ldots, S_m$ are the *parts* of the partition. Note that in this definition a part of a partition may be empty, but there is usually no advantage to this. The *number of objects* in a set $S$ is denoted $|S|$ and is sometimes called the *size* of $S$.

(The following are results of set theory, but are intuitive enough to be given without proofs)

**Theorem 1.** *(Addition principle) Suppose that a set $S$ is partitioned into pairwise disjoint parts $S_1, S_2, \ldots, S_m$. The number of objects in $S$ can be determined by finding the number of objects in each of the parts, and adding the numbers so obtained:*

$$|S| = |S_1| + |S_2| + \ldots + |S_m|$$