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Chapter 1

Graphs

1.1 Definitions and examples

A *graph* is an ordered pair $(V(G), E(G))$ consisting of a set $V(G)$ of *vertices* and a set $E(G)$, disjoint from $V(G)$, of *edges*, together with an *incidence function* ψ_G that associates with each edge of G an unordered pair of (not necessarily distinct) vertices of G.

If e is an edge and u and v are vertices such that $\psi_G(e) = \{u, v\}$ then e is said to *join* u and v, and the vertices u and v are called the *ends* of e.

We denote the numbers of vertices and edges in G by $v(G)$ and $e(G)$; these two basic parameters are called the *order* and *size* of G respectively.

Example 1
These examples serve to clarify the definition. For notational simplicity, we write uv for the unordered pair $\{u, v\}$.

$$G = (V(G), E(G))$$

where

$$\begin{aligned} V(G) &= \{u, v, w, x, y\} \\ E(G) &= \{a, b, c, d, e, f, g, h\} \end{aligned}$$

and ψ_G is defined by

$$\begin{array}{llll} \psi_G(a) = uv & \psi_G(b) = uu & \psi_G(c) = vw & \psi_G(d) = wx \\ \psi_G(e) = vx & \psi_G(f) = wx & \psi_G(g) = ux & \psi_G(h) = xy \end{array}$$

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Example 2

$$H = (V(H), E(H))$$

where

$$V(H) = \{v_0, v_1, v_2, v_3, v_4, v_5\}$$
$$E(H) = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}\}$$

and ψ_H is defined by

$$\begin{array}{llll} \psi_H(e_1) = v_1v_2 & \psi_H(e_2) = v_2v_3 & \psi_H(e_3) = v_3v_4 & \psi_H(e_4) = v_4v_5 \\ \psi_H(e_5) = v_5v_1 & \psi_H(e_6) = v_0v_1 & \psi_H(e_7) = v_0v_2 & \psi_H(e_8) = v_0v_3 \\ & \psi_H(e_9) = v_0v_4 & \psi_H(e_{10}) = v_0v_5 & \end{array}$$

Drawings and more definitions

Graphs are so named because they can be represented graphically. Each vertex is indicated by a point and each edge by a line joining the points representing its ends. Here are the graphs of the two given examples:

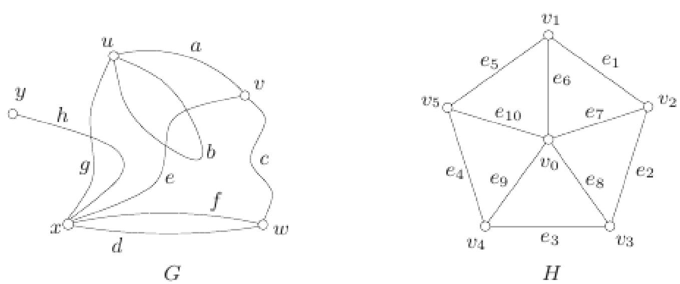

Fig. 1.1. Diagrams of the graphs G and H

There is no single correct way to draw a graph; the relative positions of the points representing vertices and the shapes of lines representing edges usually have no significance. A diagram of a graph merely depicts the incidence relation between its vertices and edges.

The ends of an edge are said to be *incident* with the edge and vice versa. Two vertices incident with a common edge are *adjacent*, as are two edges which are incident with a common vertex, and two distinct adjacent vertices are *neighbours*. The set of neighbours of a vertex v in a graph G is denoted by $N_G(v)$.

An edge with identical ends is called a *loop*, and an edge with distinct ends a *link*. Two or more links with the same pair of ends are said to be *parallel edges*. For instance in the graph G of the first example, the edge b is a loop, and all other edges are links; the edges d and f are parallel edges.
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Cont.

We use G to denote a graph. But when there is no scope for ambiguity, we omit the letter G from graph-theoretic symbols and write, for instance, V and E for $V(G)$ and $E(G)$. We also denote the numbers of vertices and edges of G by n and m respectively.

A graph is $f$inite