

# An Introductory Course in Computational Neuroscience—Paul Miller (Notes)

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# Contents

0.1	xLIF . . . . .	2
0.1.1	Modelling the Leaky membrane potential . . . . .	2
0.1.2	Solution for Leaky ODE . . . . .	4
0.1.3	Leaky-Integrate-and-Fire . . . . .	6

## 0.1 xLIF

### 0.1.1 Modelling the Leaky membrane potential

#### Nernst Potential

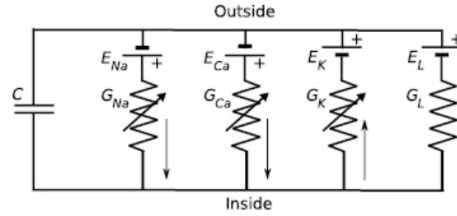
The *Nernst potential*  $E_A$  of an ion  $A$  of charge  $z_A$  with intracellular concentration  $[A_{\text{in}}]$  and extracellular concentration  $[A_{\text{out}}]$  is given by

$$E_A = \frac{k_B T}{z_A q_e} \ln \left( \frac{[A_{\text{out}}]}{[A_{\text{in}}]} \right)$$

where  $T$  is the temperature in Kelvin,  $k_B$  the Boltzmann constant ( $1.39 \times 10^{-23} JK^{-1}$ ) (which converts units of temperature to units of thermal energy).  $q_e$  is the fundamental electronic charge ( $1.60 \times 10^{-19} C$ ).

#### Model

Considering this representation of a neuron's membrane:



If all channels with variable conductance are closed, then the current will only flow through the leak channels (subscript  $L$ ) until the cell membrane is at the leak potential  $E_L$ . The current through a channel is given by

$$I_t = G_t(V_m - E_t)$$

Where  $G_t$  represents conductance and  $E_t$  the nernst potential;  $t$  represents the type of channel.

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### Equilibrium

When the cell is at equilibrium the different currents balance each other out and sum to zero:

$$I_m = \sum_t I_t = \sum_t G_t(V_m - E_t) = 0$$

In the context of this current model this can be rewritten as

$$G_{Na}(V_m - E_{Na}) + G_{Ca}(V_m - E_{Ca}) + G_K(V_m - E_K) + G_L(V_m - E_L)$$

Solving for  $V_m$  we can see that the *resting membrane potential*—where no net current flows, is the weighted average of the individual Nernst potentials:

$$V_m = \frac{G_{Na}E_{Na} + G_{Ca}E_{Ca} + G_K E_K + G_L E_L}{G_{Na} + G_{Ca} + G_K + G_L}$$

The derivation of the resting membrane potential is typically more complicated.

### Leaky membrane potential

Here we consider the passive properties of the cell, where the variable conductance of all channels are fixed. With this we treat the circuit as having a single ‘leak’ conductance and potential.

The membrane potential is generated by the charge stored on the membrane; it depends on both the stored charge and the membrane’s capacitance  $C_m$  via the equation

$$Q = C_m V_m$$

The current is defined as positive when it flows *out* of the cell; with that we have

$$\frac{dQ}{dt} = -I_m = -G_L(V_m - E_L)$$

Fixing the capacitance we obtain the dynamics of the resting membrane potential as

$$C_m \frac{dV_m}{dt} = G_L(E_L - V_m)$$

This is a linear first order ODE.

### 0.1.2 Solution for Leaky ODE

We had the dynamics of the resting membrane potential as

$$C_m \frac{dV_m}{dt} = G_L(E_L - V_m)$$

Expressing in standard form and solving by integrating factor:

$$\begin{aligned} \frac{dV_m}{dt} + \frac{G_L}{C_m} V_m &= \frac{G_L}{C_m} E_L \\ V_m &= \frac{1}{\exp\left(\frac{G_L}{C_m} t\right)} \left( \int \exp\left(\frac{G_L}{C_m} t\right) \cdot \frac{G_L}{C_m} E_L dt + c \right) \end{aligned}$$

To simplify we define the *time constant*  $\tau_m = C_m/G_L$ :

$$\begin{aligned} V_m &= \frac{1}{\exp\left(\frac{t}{\tau_m}\right)} \left( \int \exp\left(\frac{1}{\tau_m} t\right) \cdot \frac{1}{\tau_m} E_L dt + c \right) \\ &= \exp\left(-\frac{t}{\tau_m}\right) \left( \exp\left(\frac{t}{\tau_m}\right) E_L + A \right) \\ &= E_L + \exp\left(-\frac{t}{\tau_m}\right) \cdot A \end{aligned}$$

At initial condition  $V_m(0)$ :

$$V_m(0) = E_L + A \implies A = V_m(0) - E_L$$

With that we have the solution

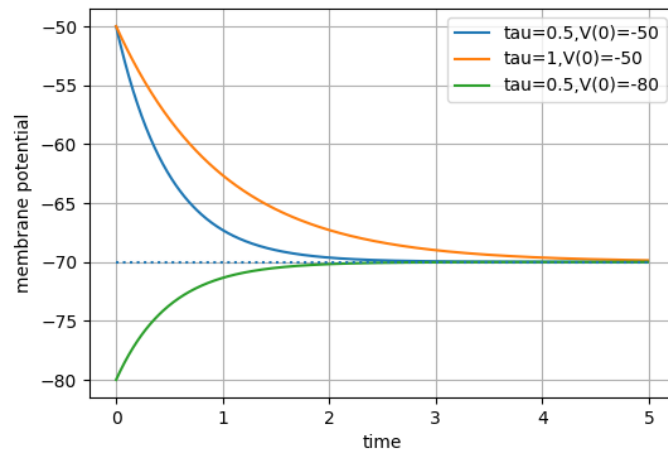
$$V_m = E_L + (V_m(0) - E_L) \exp\left(-\frac{t}{\tau_m}\right)$$

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### Illustrated

See that the equation tends to  $E_L$ , and that  $\tau_m$  dictates how fast this decay occurs (thus the name). Illustrated here using code from `leakymembrane.py`:

$$V_m = E_L + (V_m(0) - E_L) \exp\left(-\frac{t}{\tau_m}\right)$$



### 0.1.3 Leaky-Integrate-and-Fire

The LIF introduces a framework on which we can build more realistic models of neurons. It is essentially the initial model for the leaky membrane with an additional term  $I_{\text{app}}$  modelling an applied current. The spike is modelled by the membrane potential being reset to a low (hyperpolarised) value  $V_{\text{reset}}$  after the potential reaches some threshold  $V_{th}$  (threshold potential):

$$C_m \frac{dV_m}{dt} = G_L(E_L - V_m) + I_{\text{app}}; \text{ if } V_m > V_{th} \text{ then } V_m \mapsto V_{\text{reset}}$$

Notice that an actual ‘spike’ shape hasn’t been modelled, and would have to be put in by hand before  $V_m$  is reset; spike times are recorded at the time when the membrane potential crosses the threshold.