# Calculus 4-80

Avi Mehra

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## 1

**Problem 1.1.**

$$\int_1^8 2x^{-3}dx$$

We can use the inverse power rule to get an antiderivative:

$$\frac{d}{dx}\left(-x^{-2}\right) = 2x^{-3}.$$

So by the Fundamental Theorem of Calculus,

$$\int_1^8 2x^{-3}dx = \left[-x^{-2}\right]_1^8.$$

Now we plug in our bounds:

$$\begin{aligned}\left[-x^{-2}\right]_1^8 &= -8^{-2} - (-1^{-2}) \\ &= \frac{-1}{64} + 1 \\ &= \boxed{\frac{63}{64}}.\end{aligned}$$

## 2

**Problem 2.1.**

$$\int_{\pi/4}^{\pi/2} \sin x dx$$

Recall that

$$\frac{d}{dx}\left(-\cos x\right) = \sin x.$$

So by the Fundamental Theorem of Calculus,

$$\int_{\pi/4}^{\pi/2} \sin x dx = \left[-\cos x\right]_{\pi/4}^{\pi/2}.$$

Now we plug in our bounds:

$$\begin{aligned} \left[-\cos x\right]_{\pi/4}^{\pi/2} &= -\cos\frac{\pi}{2} - \left(-\cos\frac{\pi}{4}\right) \\ &= 0 - \left(-\frac{\sqrt{2}}{2}\right) \\ &= \boxed{\frac{\sqrt{2}}{2}}. \end{aligned}$$

# 3

**Problem 3.1.**

$$\int_4^9 3\sqrt{x}dx$$

We can use the inverse power rule to get an antiderivative:

$$\frac{d}{dx}\left(2x^{3/2}\right) = 3\sqrt{x}.$$

So by the Fundamental Theorem of Calculus,

$$\int_4^9 3\sqrt{x}dx = \left[2x^{3/2}\right]_4^9.$$

Now we plug in our bounds:

$$\begin{aligned}\left[2x^{3/2}\right]_4^9 &= 2(9)^{3/2} - 2(4)^{3/2}\\ &= 54 - 16\\ &= \boxed{38}.\end{aligned}$$

# 4

**Problem 4.1.**

$$\int_0^2 \left(3x^2 - 6x + 2\right)\,dx$$

We can use the inverse power rule to get an antiderivative:

$$\frac{d}{dx}\left(x^3 - 3x^2 + 2x\right) = \left(3x^2 - 6x + 2\right).$$

So by the Fundamental Theorem of Calculus,

$$\int_0^2 \left(3x^2 - 6x + 2\right)\,dx = \left[x^3 - 3x^2 + 2x\right]_0^2.$$

Now we plug in our bounds:

$$\begin{aligned}\left[x^3 - 3x^2 + 2x\right]_0^2 &= \left(2^3 - 3(2)^2 + 2(2)\right) - \left(0^3 - 3(0)^2 + 2(0)\right)\\ &= (8 - 12 + 4) - 0\\ &= \boxed{0}.\end{aligned}$$

# 5

**Problem 5.1.**

$$2\int_1^3 \left(\frac{3}{x^2}\right)\,dx$$

We can use the inverse power rule to get an antiderivative:

$$\frac{d}{dx}\left(\frac{-3}{x}\right) = \left(\frac{3}{x^2}\right).$$

So by the Fundamental Theorem of Calculus,

$$2\int_1^3 \left(\frac{3}{x^2}\right) dx = 2\left[\frac{-3}{x}\right]_1^3.$$

Now we plug in our bounds:

$$\begin{aligned} 2\left[\frac{-3}{x}\right]_1^3 &= 2\left(\frac{-3}{3} - \frac{-3}{1}\right) \\ &= 2(-1-(-3)) \\ &= 2(2) \\ &= \boxed{4}. \end{aligned}$$

# 6

**Problem 6.1.**

$$\int_{-1}^{8} x^{1/3} dx$$

We can use the inverse power rule to get an antiderivative:

$$\frac{d}{dx}\left(\frac{3}{4}x^{4/3}\right) = x^{1/3}.$$

So by the Fundamental Theorem of Calculus,

$$\int_{-1}^{8} x^{1/3} dx = \left[\frac{3}{4}x^{4/3}\right]_{-1}^{8}.$$

Now we plug in our bounds:

$$\begin{aligned} \left[\frac{3}{4}x^{4/3}\right]_{-1}^{8} &= \frac{3}{4}(8)^{4/3} - \frac{3}{4}(-1)^{4/3} \\ &= \frac{3}{4}(16-1) \\ &= \frac{3}{4}(15) \\ &= \boxed{\frac{45}{4}}. \end{aligned}$$

# 7

We will split the sum into two parts. Then we'll add the answers at the end.

**Note:** there are other tricks that you can use to simplify the problem. Try to find one.

**Problem 7.1.**

$$\int_3^9 (2x - 20)\, dx$$

We can use the inverse power rule to get an antiderivative:

$$\frac{d}{dx}\left(x^2 - 20x\right) = (2x - 20)\,.$$

So by the Fundamental Theorem of Calculus,

$$\int_3^9 (2x - 20)\, dx = \left[x^2 - 20x\right]_3^9.$$

Now we plug in our bounds:

$$\begin{aligned}
\left[x^2 - 20x\right]_3^9 &= \left((9)^2 - 20(9)\right) - \left((3)^2 - 20(3)\right) \\
&= (81 - 180) - (9 - 60) \\
&= (-99) - (-51) \\
&= \boxed{-48}.
\end{aligned}$$

**Problem 7.2.**

$$2\int_{-2}^3 (x - 10)\, dx$$

We can use the inverse power rule to get an antiderivative:

$$\frac{d}{dx}\left(\frac{1}{2}x^2 - 10x\right) = (x - 10)\,.$$

So by the Fundamental Theorem of Calculus,

$$2\int_{-2}^3 (x - 10)\, dx = 2\left[\frac{1}{2}x^2 - 10x\right]_{-2}^3.$$

Now we plug in our bounds:

$$\begin{aligned} 2\left[\frac{1}{2}x^2 - 10x\right]_{-2}^{3} &= 2\left(\left(\frac{1}{2}(3)^2 - 10(3)\right) - \left(\frac{1}{2}(-2)^2 - 10(-2)\right)\right) \\ &= \left(\left((3)^2 - 20(3)\right) - \left((-2)^2 - 20(-2)\right)\right) \\ &= (9-60) - (4-(-40)) \\ &= -51 - 44 \\ &= \boxed{-95}. \end{aligned}$$

So our final answer is

$$-48 + (-95) = \boxed{-143}.$$

# 8

The letter doesn't matter so I'm using $x$ instead of $m$.

**Problem 8.1.**

$$\int_1^{27} \left(9x^{-2/3} - 2x^{-3}\right) dx$$

We can use the inverse power rule to get an antiderivative:

$$\frac{d}{dx}\left(27x^{1/3} + x^{-2}\right) = \left(9x^{-2/3} - 2x^{-3}\right).$$

So by the Fundamental Theorem of Calculus,

$$\int_1^{27} \left(9x^{-2/3} - 2x^{-3}\right) dx = \left[27x^{1/3} + x^{-2}\right]_1^{27}.$$

Now we plug in our bounds:

$$\begin{aligned} \left[27x^{1/3} + x^{-2}\right]_1^{27} &= \left(27(27)^{1/3} + (27)^-2\right) - \left(27(1)^{1/3} + (1)^-2\right) \\ &= \left(81 + \frac{1}{729}\right) - (27+1) \\ &= \boxed{53 + \frac{1}{729}} \end{aligned}$$