Math Review

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1 Basic Operations

2 Sets

In Naive Set Theory, a set is just a collection of anything. The set of all integers between 1 and 4, inclusive, can be written as

$$\{1, 2, 3, 4\},$$

and the set of the pizza on the counter can be written as

$$\{\text{the pizza on the counter}\}.$$

Sets don't have any order structure; the first set may very well be written as

$$\{3, 2, 4, 1\}$$

without changing the meaning whatsoever.

2.1 Set terminology

Definition 2.1. Anything inside of a set is called an element of the set or a member of the set. If x is an element of S, we write $x \in S$ to show this relation. This is sometimes written in reverse as $S \ni x$.

Definition 2.2. A set is called finite if it has a natural number of elements.

Definition 2.3. For a finite set S, its cardinality $|S|$ is the number of elements it contains.

Definition 2.4. A set is called countable if you can write down a (possibly infinite) list of its elements without missing any. The natural numbers are countable, because you can list each natural number just by counting up by 1 from 0. All finite sets are countable.

Definition 2.5. A set is countably infinite if it is countable and infinite.

Definition 2.6. We say a set is uncountable if it is not countable. All uncountable sets are infinite.

Definition 2.7. For a finite set F, countably infinite set C, and uncountable set U, their cardinalities are ordered as follows

$$|F| < |C| < |U|.$$

That is, the cardinality of a finite set is always less than the cardinality of a countably infinite set, which is always less than the cardinality of an uncountable set.

2.2 Set relations

Definition 2.8. If every element of S is also an element of T, we write $S \subseteq T$ and say S is a subset of T. If $S \subseteq T$ and $S \neq T$, we write $S \subset T$ and say S is a proper subset of T.

Definition 2.9. The relationship $S \subseteq T$ is the same as $T \supseteq S$, which is read "T is a superset of S." Similarly, $S \subset T$ is the same as $T \supset S$, which is read "T is a proper superset of S."

2.3 Commonly encountered sets

Name	Symbol	Expansion
Integers	$\mathbb{Z}$	$\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$
Natural Numbers	$\mathbb{N}$	$\{0, 1, 2, 3, 4, 5, 6, \ldots\}$
Positive Integers	$\mathbb{Z}^+$	$\{1, 2, 3, 4, 5, 6, 7, \ldots\}$
Real Numbers	$\mathbb{R}$	Any "decimal" number, uncountable

2.4 Set representations

There are, however, other ways of expressing sets. For example, the first set we looked at can be rewritten as

$$\{n \in \mathbb{Z} \mid 1 \leq n \leq 4\}.$$

This notation reads "the set of all integers n where $1 \leq n \leq 4$."

We can use this notation to write the set S of real numbers between 0 (inclusive) and 1 (exclusive):

$$S = \{x \in \mathbb{R} \mid 0 \leq x < 1\}.$$

However, it can also be written as

$$S = [0, 1).$$

In this notation, we use $[a, b)$ to represent the range $\{x \in \mathbb{R} \mid a \leq x < b\}$ between a and b. Square brackets represent an inclusive bound while parenthesis represent an exclusive bound. We can use also use the exclusive bounds $+-\infty$: the set of all real numbers greater than 6 is $(6, \infty)$ and the set of all real numbers less than 2 is $(-\infty, 2)$.

2.5 Set operations

Definition 2.10. Given sets A and B, the union $A \cup B$ is defined as the set of all elements in either set:

$$A \cup B := \{s \mid s \in A \text{ or } s \in B\}.$$

Definition 2.11. Given sets A and B, the intersection $A \cap B$ is defined as the set of all elements common to both sets:

$$A \cap B := \{s \mid s \in A \text{ and } s \in B\}.$$

Definition 2.12. Given sets A and B, the set difference (or asymmetric difference) $A \setminus B$ is defined as the set of all elements in A but not in B:

$$A \setminus B := \{s \mid s \in A \text{ and } s \notin B\}.$$

Definition 2.13. Given sets A and B, the symmetric difference (or disjunctive union) $A \triangle B$ is defined as the set of all elements in either set but not both:

$$A \triangle B := \{s \mid s \in A \text{ xor } s \in B\} = A \cup B \setminus A \cap B.$$

3 Functions

1. Continuity
2. Intermediate value theorem
3. Mean value theorem
4. Invertible
5. method for calculating inverses

4 Equations and Equality

When you have an equation, both the LHS and RHS are equivalent, so any function or operation can be applied to both of them while preserving the equality. This is because you are **applying the function on the *same thing***; of course the results will be the same.

4.1 expansion, dist property, foil, binomial thm

5 Euclidean geometry

1. angles
   - right
   - acute
   - obtuse
2. triangles
   - area
   - perimeter
   - similarity
   - heron's formula
   - centers
     - incenter
     - circumcenter
   - sum of internal angles
     - angle chasing

right, acute, obtuse

pythagorean theorem, pythagorean inequalities

scalene, isosceles, equilateral

isosceles and equilateral area problems

3. circles

   area

   circumference

   area and circumference of sectors

   radius is orthogonal to tangent vector

   angle properties

4. solids

   prism

   volume = bh

   surface area

   cylinders

   pyramids

   volume = bh/3

   surface area

   cone

   cross-sections

5. transversal theorem

6. vertical, complementary, supplementary angles

6 solving linear equations

7 systems of linear equations

1. algebraic solution by substitution

2. algebraic solution by reduction

8 exponentiation (powers)

8.1 raising to the natural power

8.2 raising to the integer power

8.3 raising to the real power

8.4 nth roots

9 solving polynomial equations

9.1 factoring

9.2 completing the square

9.3 quadratic formula

10 parity behavior of polynomial equations

11 exponentiation (exp)

11.1 logarithm

12 systems of equations

1. solutions by graphing
2. analytic (algebraic) solutions

13 inequalities

1. graphing
2. solving

14 systems of linear inequalities

1. graphing
2. solving

15 probability

1. union and intersection formulas
2. independence