Laboratory 2

Numerical Analysis

Teacher: Henry Arguello Fuentes
System Engineering

Presented By:
Mendoza Casas Oscar Mauricio 2150087

1 First problem:

1. Modify Programs 1 and 2 to display to display an appropriate error message when (i) division by zero occurs in (1) or (2), respectively, or (ii) the maximum number of iterations, max1 is exceeded.

$$p_k = g(p_{k-1}) = p_{k-1} - \frac{f_{pk-1}}{f'_{pk-1}}$$

$$p_{k+1} = g(p_k, p_{k-1}) = p_k - \frac{(f_{pk-1})(p_k - p_{k-1})}{(f_{pk}) - f_{pk-1}}$$

Solution:
Newton method

```
     NaN       NaN       NaN       NaN
     NaN       NaN       NaN       NaN
     NaN       NaN       NaN       NaN
     NaN       NaN       NaN       NaN
     NaN       NaN       NaN       NaN
Exceeded number of iterations

p0 =

   NaN
```

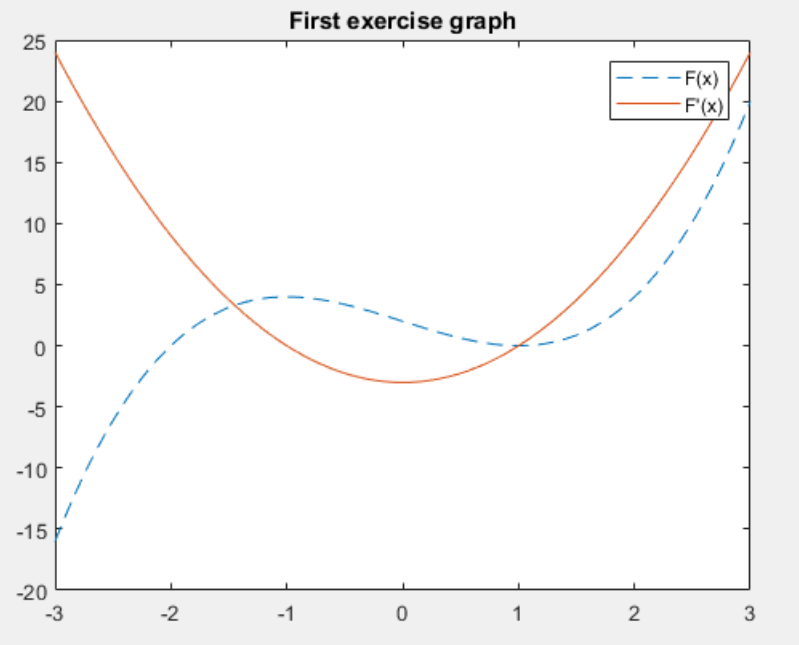
Secant method

```
     NaN       NaN       NaN       NaN
     NaN       NaN       NaN       NaN
Exceeded number of iterations

p0 =

   NaN
```

Using the lemma of division by zero, we have: "Assume that $f(x)$ and its derivates $f'(x), \ldots, f^{(M)}(x)$ are defined and continuous on an interval about $x = p$. We say that $f(x) = 0$ has a root of order M at $x = p$ if and only if :"

$$f(p) = 0, f'(p) = 0, \ldots, f^{(M-1)}(p) \wedge f^{(M)}(p) \neq 0$$

First Graph



The lemma showed before, says the orders of the all roots of f(x), then, in the script "Punto1.m" shows the graph of the function $f(x)$ and print the message when a division by zero occurs in the Newton Method, this happens when the initial point p_0 is assigned in the exactly root with degree major than 1.

The script that should be used for this exercise is First.m

2 Second problem:

2. Modify program 1 to use Newton's square-root algorithm to approximate each of the following square

   roots to 10 decimal places.

Solution:

2.1 Start with p0 = 3 and approximate $\sqrt{8}$

```
3.000000000  0.166666667 -5.000000000 0.193333333
2.833333333  0.004901961 -4.833333333 0.206686717
2.828431373  0.000004248 -4.828431373 0.207106417
Los valores son    2.828427124749380

Con sqrt
ans =

   2.828427124746190
```

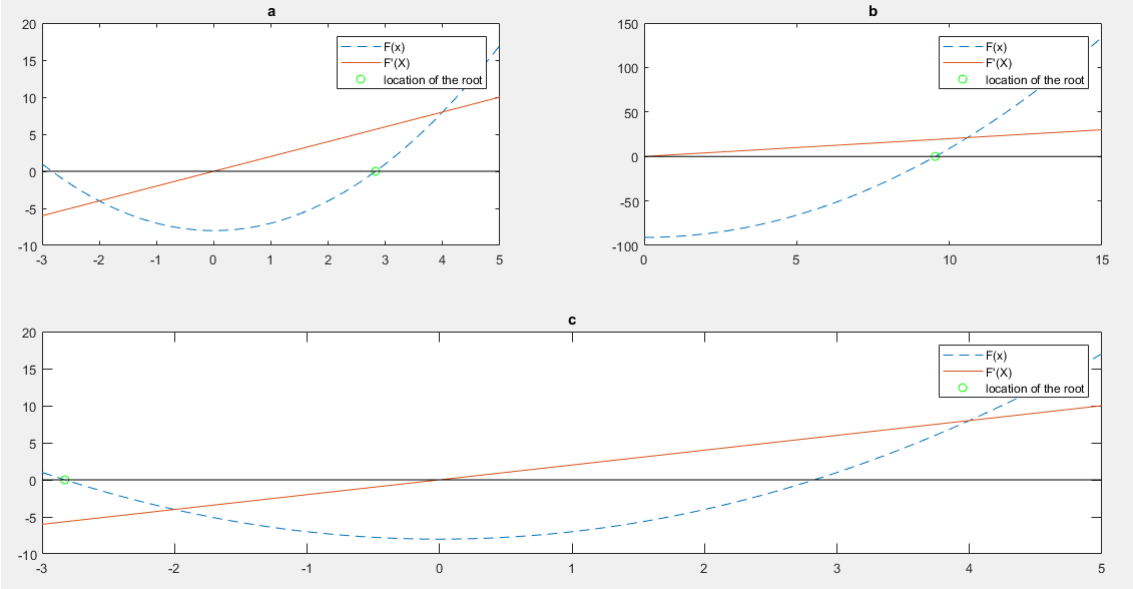
2.2 Start with p0 = 3 and approximate $\sqrt{91}$

```
10.000000000  0.450000000 -12.000000000 0.080208333
9.550000000  0.010602094 -11.550000000 0.086500612
9.539397906  0.000005892 -11.539397906 0.086659590
Los valores son    2.828427124749380

Con sqrt
ans =

   9.539392014169456
```

2.3 Start with p0 = 3 and approximate $-\sqrt{8}$

```
-3.000000000  0.166666667 1.000000000 0.833333333
-2.833333333  0.004901961 0.833333333 1.192941176
-2.828431373  0.000004248 0.828431373 1.207094402
Los valores son    2.828427124749380

Con sqrt
ans =

  -2.828427124746190
```

The graphs for the respective functions are: The respective Script used



in this exercise is Second.m

3 Third problem:

3. Suppose that the equation of the motion for a projectile are

$$y = f(t) = 9600(1 - e^{-t/15}) - 48't$$

$$x = r(t) = 2400(1 - e^{-t/15})$$

3.1 Find the lapsed time until impact accurate to 10 decimal places

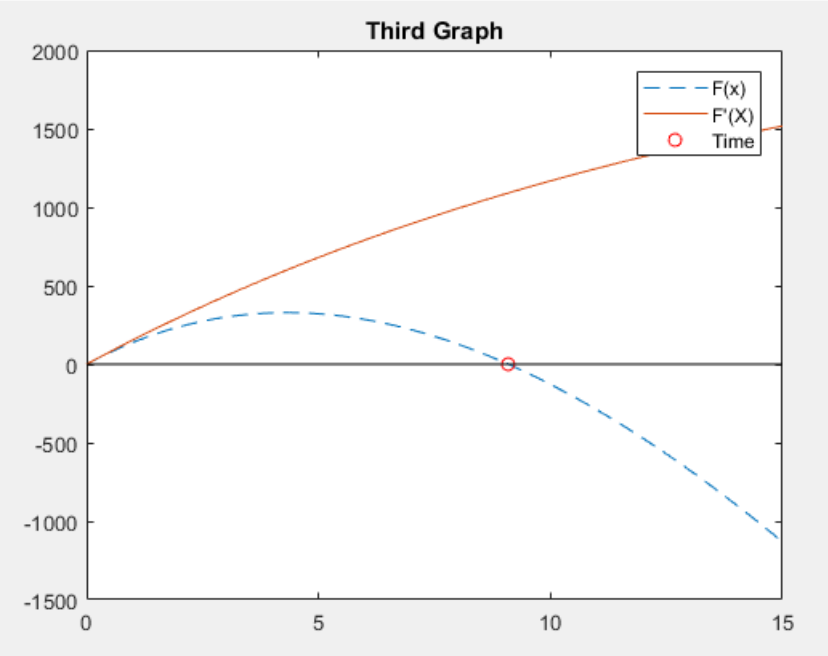
3.2 Find the range accurate to 10 decimal places

In order to find the time lapse before the impact, using the newton method we can find the respective roots of $f(t)$, then we use both equations with the respective solution to find the answers that can be seen in the next figure.

Solution with Third.m:

```
9.087895612  0.000004121 -11.087895612 0.090188473
9.087899733  0.000000066 -11.087899733 0.090188405
9.087899668  0.000000001 -11.087899668 0.090188406
El tiempo transcurrido(raíz) hasta que impacte es:
 9.0878996688
la trayectoria que recorrio en el tiempo anterior es:
 1090.5479602556
```

Third exercise Graph:



4 Fourth problem:

4. Modify Programs 1 and 2 to output a matrix analogous to Table 2.5 and 2.6.
   with the function:

$$A = x^3 - 3 * x + 2$$

with $p_0 = -2.4$ the initial point to $p_0 = -2.6$, and $p_1 = -2.4$ the root is $p = -2$

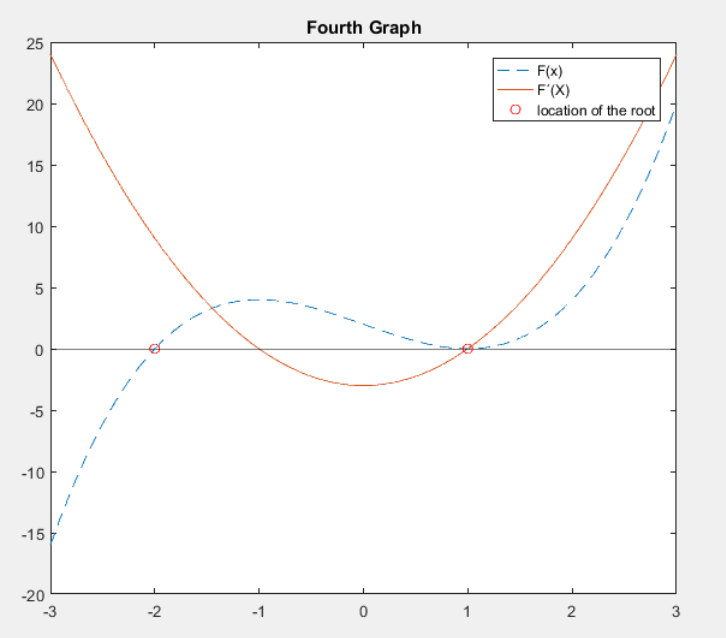
First Matrix:

```
-2.400000000  0.323809524 0.400000000 0.476190476
-2.076190476  0.072594466 0.076190476 0.619469027
-2.003596011  0.003587421 0.003596011 0.664277916
-2.000008590  0.000008590 0.000008590 0.666660828
-2.000000000  0.000000000 0.000000000 0.000000000
```

Second Matrix:

```
1.200000000  0.096969697 -3.200000000 0.303030303
1.103030303  0.050673886 -3.103030303 0.317002882
1.052356417  0.025955603 -3.052356417 0.324829869
1.026400814  0.013143080 -3.026400814 0.328990525
1.013257734  0.006614316 -3.013257734 0.331138262
1.006643418  0.003318043 -3.006643418 0.332229763
1.003325375  0.001661767 -3.003325375 0.332780024
1.001663607  0.000831573 -3.001663607 0.333056296
1.000832034  0.000415959 -3.000832034 0.333194719
1.000416075  0.000208023 -3.000416075 0.333264002
1.000208052  0.000104022 -3.000208052 0.333298662
1.000104029  0.000052014 -3.000104029 0.333315996
1.000052016  0.000026008 -3.000052016 0.333324664
1.000026008  0.000013004 -3.000026008 0.333328999
```

The next figure is the graph for the respective point:



In both cases I used newton method, in the graph the blue line is $F(x)$, and the red one is $F'(x)$ and the red circles are the location of the roots, the respective script is "Fourth.m"

5 Fifth problem:

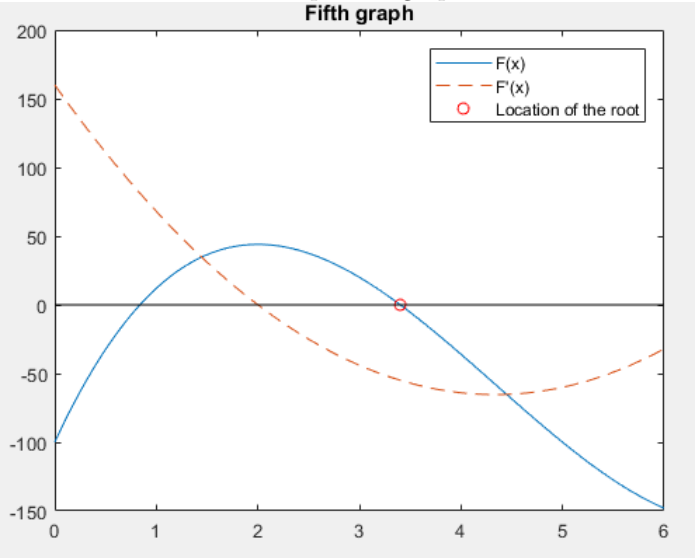
5. An open-top box is constructed from a rectangular piece of sheet metal measuring 10 by 16 inches. Squares of what size (accurate to 0.000000001 inch) should be cut from the corners if the volume of the box is to be 100 cubic inches?.

   With $V = 100 = x * y * z$
   $y = 16 - 2x$
   $z = 10 - 2x$

   Then the final equation is $0 = x(16 - 2x)(10 - 2x)$ this one is the respective function used in the script "Fifth.m"

```
p0 =

   3.401748647492449


err =

     1.271347516151877e-08


k =

     5


y =

     1.421085471520200e-14
```

And the respective graphic is:



6 Sixth problem:

6. A catenary is a curve formed by hanging cable. Assume that the lowest point is (0, 0); Then the formula for the catenary is $y = Ccosh(x/C) - C$. To determine the catenary that goes through (-+a,b) we must solve the ecuation $b = Ccosh(a/C) - C$ for C.

   Show that the catenary Through (+-10,6) is $y = 9.1889cosh(x/9.1889) - 9.1889$.

   Find the catenary that passes through (+-12,5).

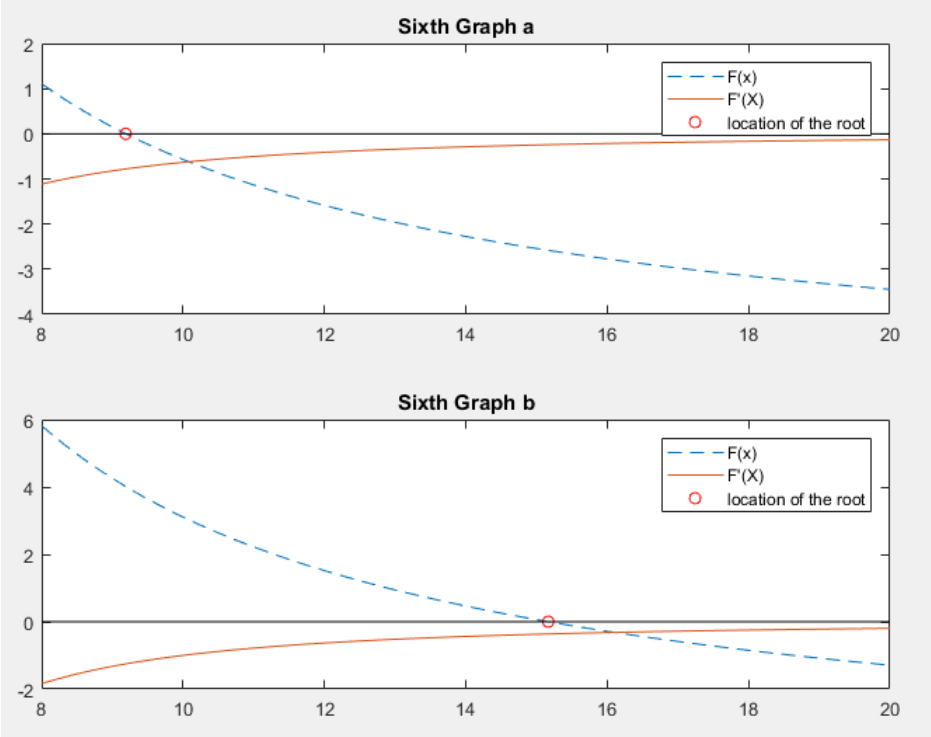
Solution:
With (+-10,6)

```
9.000000000  0.184055606 -11.000000000 0.092430212
9.184055606  0.004882533 -11.184055606 0.089452038
9.188938140  0.000003263 -11.188938140 0.089374012
El valor de c para (10,6) es:
 9.1889414022
```

With (+-12,5)

```
9.188941402  3.205785576 -11.188941402 0.114980828
12.394726978  2.185509781 -14.394726978 0.080017295
14.580236759  0.560753535 -16.580236759 0.062352584
15.140990294  0.025986893 -17.140990294 0.058428133
15.166977187  0.000051155 -17.166977187 0.058251547
El valor de c para (12,5) es:
 15.1670283415
```

The graph where we can see the result is the next one:



The script used in this point is "Sixth.m"