Bellman Ford Algorithm

M.B.Chandak

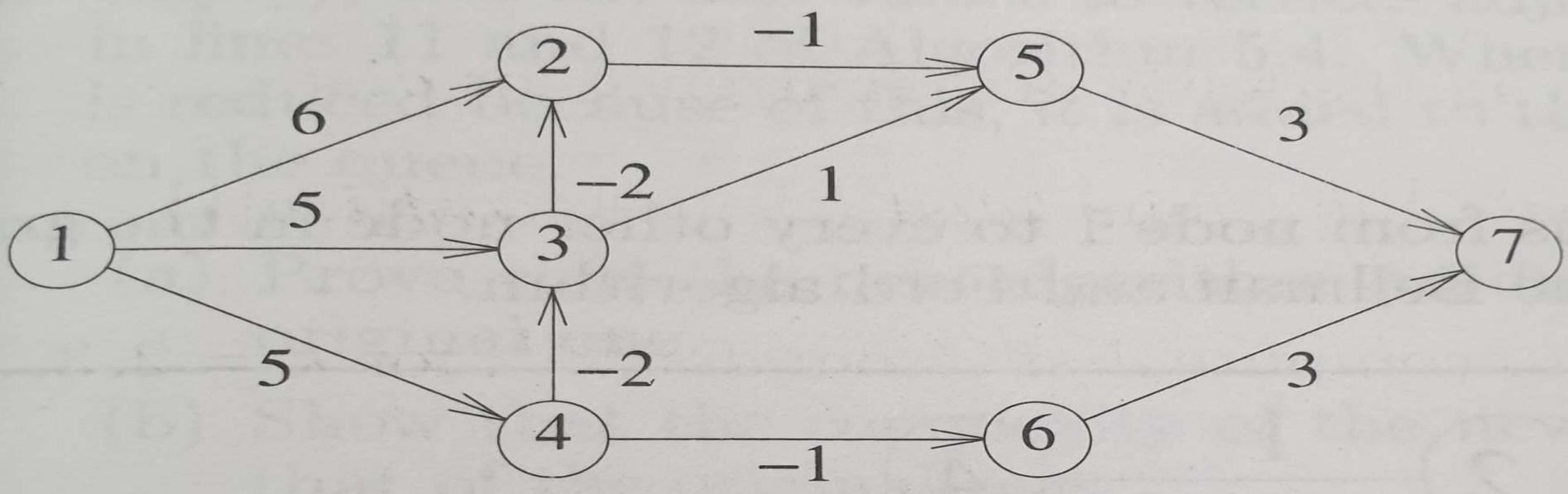
hodcs@rknec.edu

www.mbchandak.com

Bellman Ford Algorithm: Graph with Negative Edges

- Single Source Shortest Path Algorithm
- Permits Graph with Negative Edges
- Significance of Negative Edge
- Negative edges signifies facts like: discount, free offer, schemes etc.
- There is possibility of cycle in a graph. [Problem?]
- Generally with negative edges present in graph, the cost of traversal can be zero or some times negative.
- Negative cost of traversal means, the customer is paid for travelling instead of paying for services.
- Example: Jio plans, Airline traffic etc

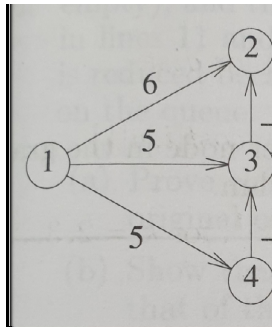
Bellman Ford Algorithm: Graph with Negative Edges



(a) A directed graph

Bellman Ford Algorithm: Graph with Negative Edges

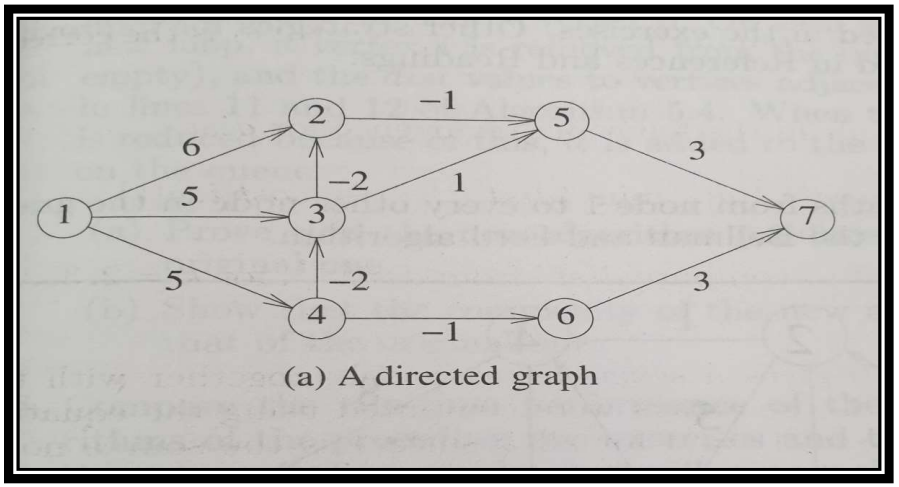
- Stage wise solution is designed
- Stage 1: Select Source Vertex : Given vertex: Vertex 1
- Stage 2: The path of length = 1 will be allowed.
- It means: computations for all the vertices directly reachable from source vertex can be selected as reached vertex [cost will be as per the graph]



K	1	2	3	4	5	6	7
1	0	6	5	5	∞	∞	∞

Bellman Ford Algorithm: Graph with Negative Edges

- Stage 3: Path of length = 2
- All the vertices reachable in path of length=2, are selected and distances are computed as per the graph.



(a) A directed graph

K	1	2	3	4	5	6	7
1	0	6	5	5	∞	∞	∞
2	0	3	3	5	5	4	∞

- All vertices except vertex-7 are reachable.

Bellman Ford Algorithm: Formulation

Formula used for computing Shortest Path:

$dist^k[u] = min \{ dist^{k-1}[u] , min \{ dist^{k-1}[i] + cost[i, u]\}\}$

where "i" represents all the vertices except vertex "u" present in the graph. The vertex "i" should have connection to vertex "u" in the graph.

Stage 4: Path of length = 3

(Sample Calculations)

$dist^3[u] = min \{ dist^2[u] , min \{ dist^2[i] + cost[i, u]\}\}$

Consider Vertex: 2 as destination

1-4-3-2 = [5-2-2=1]

How to avoid complexity of calculation

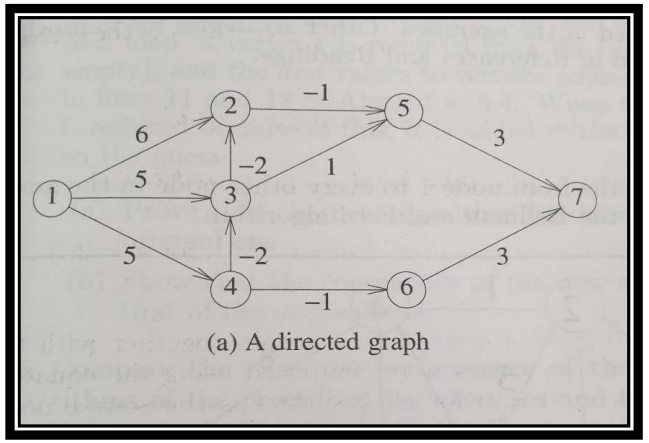
Refer the previous calculation

What is "i" [Either 1 or 3]

If i=1 then distance is 6

If i=3 then distance will be dist[3]+cost[3,2] ➔ previous computations

$dist^3[2] = min \{ dist^2[2] , min \{ dist^2[3] + cost[3,2]\}\} = 1$



(a) A directed graph

Bellman Ford Algorithm: Graph with Negative Edges

- Final matrix for the given graph

K	1	2	3	4	5	6	7
1	0	6	5	5	∞	∞	∞
2	0	3	3	5	5	4	∞
3	0	1	3	5	2	4	7
4	0	1	3	5	0	4	5
5	0	1	3	5	0	4	3
6	0	1	3	5	0	4	3

Bellman Ford Algorithm: Home Work