**Example** $T(n) = 3T\left(\frac{n}{4}\right) + n^2$

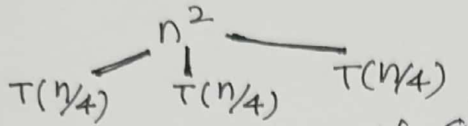
Rem: 4
$4 \times 4$
$4 \times 4 \times 4 \ldots$
$4 \times 4 \times 4 \times 4 \ldots$

Rem: $T(K) = \frac{n}{4^K}$

Rem: At height "K" subproblem size will be $T(1)$
No further subdivision is possible.

---

Step: 1 Function can be expressed as:




$\Rightarrow 3T(n/4) + n^2$

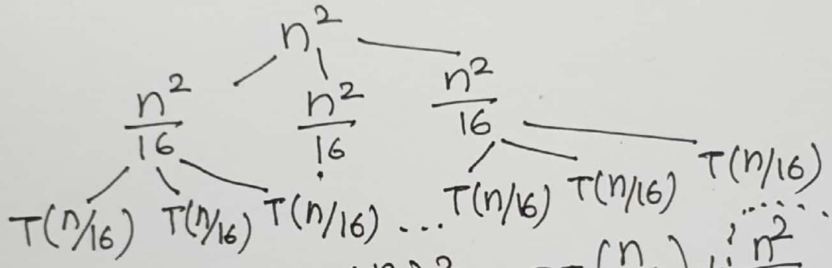
There are three subproblems of size 'n/4' and to merge solution of subproblems $n^2$ time is required.

---

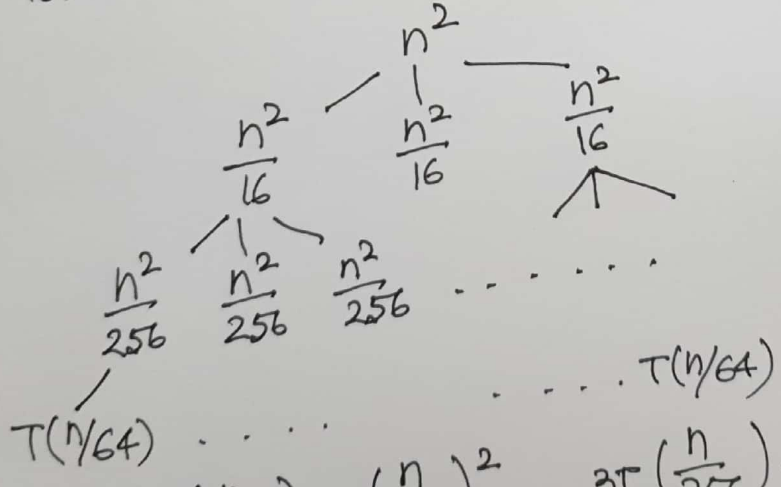
Step 2: Apply further subdivision:

$4^1$ $T\left(\frac{n}{4}\right) = 3T\left(\frac{n}{16}\right) + \left(\frac{n}{4}\right)^2 = 3T\left(\frac{n}{16}\right) + \frac{n^2}{16}$

Becomes root for Merg




$4^2$ $T\left(\frac{n}{16}\right) = 3T\left(\frac{n}{64}\right) + \left(\frac{n}{16}\right)^2 = 3T\left(\frac{n}{64}\right) + \frac{n^2}{256}.$

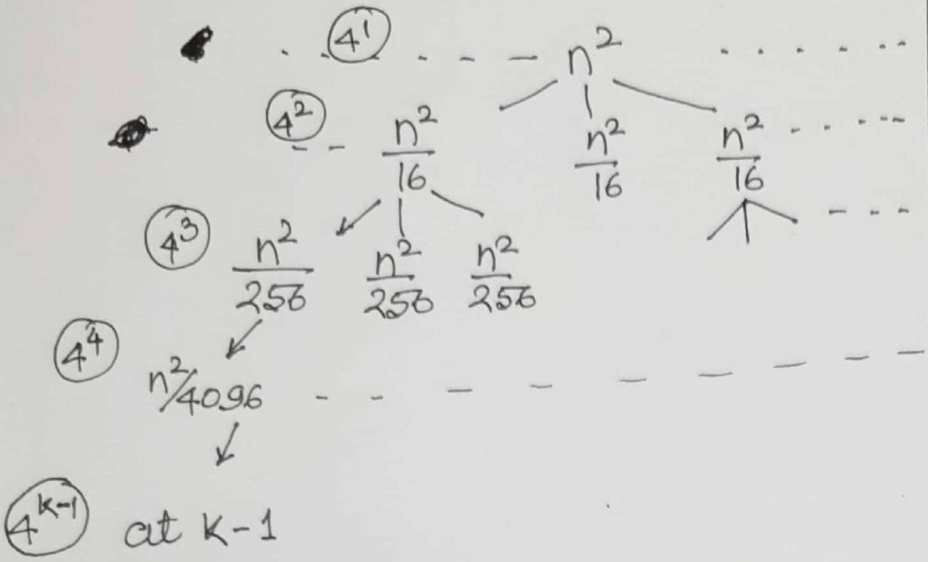



$4^3$ $T\left(\frac{n}{64}\right) = 3T\left(\frac{n}{256}\right) + \left(\frac{n}{64}\right)^2 = 3T\left(\frac{n}{256}\right) + \frac{n^2}{4096}, \rightarrow (A)$

$4^4$ $T\left(\frac{n}{256}\right) = 3T\left(\frac{n}{4096}\right) + \left(\frac{n}{256}\right)^2 = 3T\left(\frac{n}{4096}\right) + \frac{n^2}{65{,}536} \rightarrow (B)$

$4^{K-1}$ $T\left(\frac{n}{4^{K-1}}\right)$ & Finally $4^K \Rightarrow \boxed{T\left(\frac{n}{4^K}\right) = 1}$

## Step-3 : Recurrence Tree




| | |
|---|---|
| $1\ (3^0)$ | $1 \times n^2$ |
| $3\ (3^1)$ | $3 \times n^2/16\ (16^1)$ |
| $9\ (3^2)$ | $9 \times \frac{n^2}{256}\ (16^2)$ |
| $27\ (3^3)$ | $27 \times \frac{n^2}{4096}\ (16^3)$ |
| $3^{k-1}$ | $3^{k-1} * \frac{n^2}{16^{k-1}}\ (16^{k-1})$ |

AT $K^{th}$ Level all subproblem of Size $T(1)$ : $4^K$. $\frac{n}{4^k} = 1 \quad \therefore \quad n = 4^K \quad k = \log_4 n$

$$\frac{3^K}{16^K} * n^2 \Rightarrow \frac{3^K}{16^K} * (4^k)^2 \Rightarrow 3^K \rightarrow \text{At } T(1)$$

$$\text{At } T(1) : 3^K = 3^{\log_4 n} \Rightarrow n^{\log_4 3}$$

At 1 to $k-1$ – Use '$\Sigma$'

$$\sum_{i=0}^{k-1} \left(\frac{3}{16}\right)^i * n^2 \Rightarrow n^2 \sum_{i=0}^{k-1} \left[\frac{3}{16}\right] = n^2 \cdot \left[\left(\frac{3}{16}\right)^0 + \left(\frac{3}{16}\right)^1 + \left(\frac{3}{16}\right)^2 + \dots \left(\frac{3}{16}\right)^{k-1}\right]$$

$$= n^2 \left[\frac{1}{1 - \frac{3}{16}}\right] = \left[\frac{16}{13}\right] n^2$$

Complete Solution : $n^{\log_4 3} \cdot T(1) + \left[\frac{16}{13}\right] n^2$

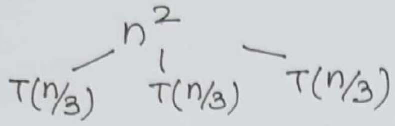
: $O(n^2)$

Ex:2 $T(n) = 3T\left(\frac{n}{3}\right) + n^2$

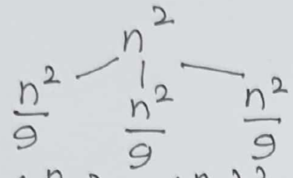
Rem: 3
3×3
3×3×3 ..
3×3×3×3

$\frac{n}{3^k} = 1$ , $k = \log_3 n$

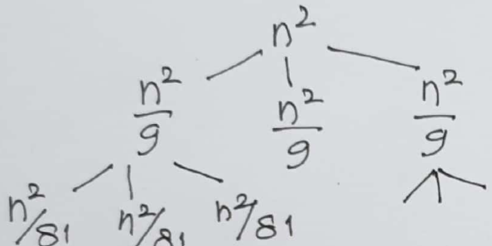
Step.1



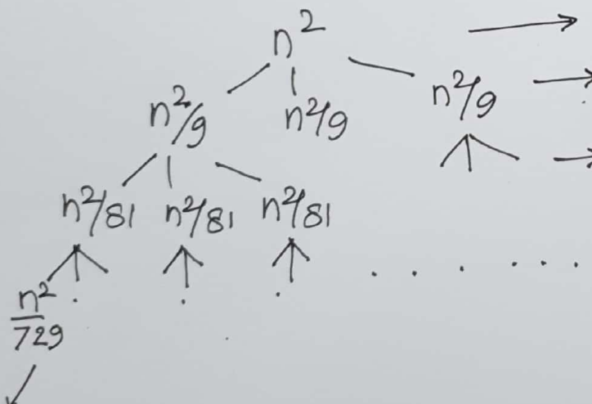
$T(n/3) = 3T\left(\frac{n}{9}\right) + \left(\frac{n}{3}\right)^2 = 3T\left(\frac{n}{9}\right) + \frac{n^2}{9}$



$T\left(\frac{n}{9}\right) = 3T\left(\frac{n}{27}\right) + \left(\frac{n}{9}\right)^2 = 3T\left(\frac{n}{27}\right) + \frac{n}{81}$



$T\left(\frac{n}{27}\right) = 3T\left(\frac{n}{81}\right) + \left(\frac{n}{27}\right)^2 = 3T\left(\frac{n}{81}\right) + \frac{n}{729}$



$\rightarrow 1\ (3^0)$ $n^2$

$\rightarrow 3\ (3^1)$ $n^2/9 * 3 = n^2/3$

$\rightarrow 9\ (3^2)$ $n^2/81 * 9 = n^2/9$

$\rightarrow 27\ (3^3)$ $n^2/729 * 27 = n^2/27$

* $i^{th}$ level $\Rightarrow n^2/3^i$

at k-1 $\frac{n^2}{9^{k-1}}$ . . . . $\rightarrow 3^{k-1}$ $\frac{n^2}{9^{k-1}} * 3^{k-1} = \frac{n^2}{3^{k-1}}$

At Level "K" : $\boxed{\frac{n}{3^k}} = 1$ $\therefore \boxed{k = \log_3 n}$ $\boxed{n^2 = (3^k)^2}$

$\frac{n^2}{9^k} * 3^k = \frac{n^2}{3^k} = \frac{(3^k)^2}{3^k} = \boxed{3^k} = 3^{\log_3 n} = n$

For level 1 to k-1

$\sum_{i=0}^{k-1} n^2 \times \left(\frac{1}{3^i}\right) = \boxed{n^2 \sum_{i=0}^{k-1} \frac{1}{3^i}}$

G. Progression Series :

$$n^2 \sum_{i=0}^{k-1} \left(\frac{1}{3^i}\right) =$$

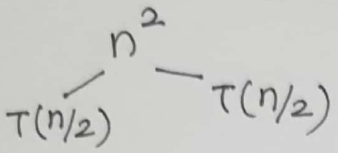
$T(n) = 2T\left(\frac{n}{2}\right) + n^2$

Rem:

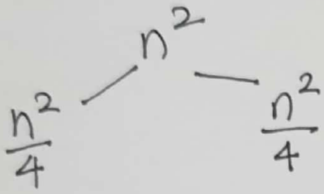
2
2x2
2x2x2 ..
2x2x2x2 ..

$T(k) = \frac{n}{2^k}$ $n = 2^k$
$k = \log_2 n$

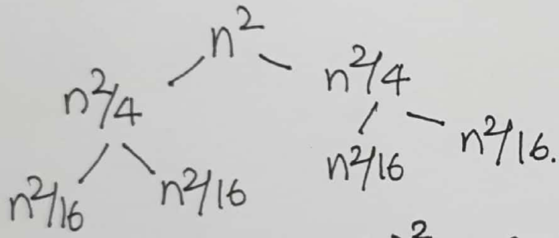
Step-1



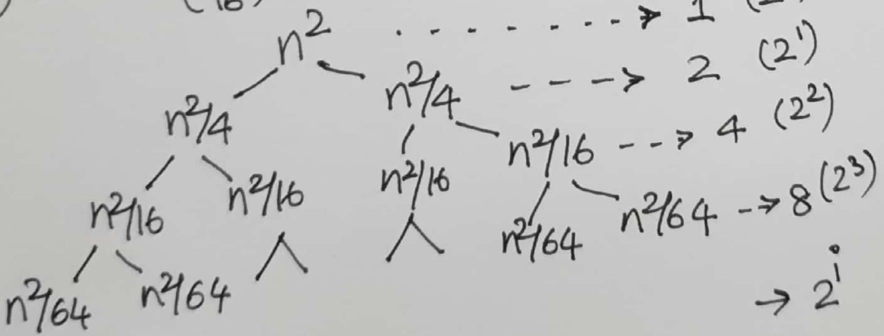

$T(n/2) = 2T\left(\frac{n}{4}\right) + \left(\frac{n}{2}\right)^2 = 2T\left(\frac{n}{4}\right) + \frac{n^2}{4}$




$T(n/4) = 2T\left(\frac{n}{8}\right) + \left(\frac{n}{4}\right)^2 = 2T\left(\frac{n}{8}\right) + \frac{n^2}{16}$




$T(n/8) = 2T\left(\frac{n}{16}\right) + \left(\frac{n}{8}\right)^2 = 2T\left(\frac{n}{16}\right) + \frac{n^2}{64}$




$n^2 \times 1$
$2 \times n^2/4 = n^2/2$
$4 \times n^2/16 = n^2/4$
$8 \times n^2/64 = n^2/8$

→ At ith level → $2^i$ $\quad 2^i \times \frac{n^2}{(2^i)^2}$

At k-1 level → $2^{k-1}$ $\quad \cancel{n^2/2^{k-1}}$ $\quad 2^{k-1} * \left(\frac{n^2}{2^{k-1}}\right)^2$

At kth level $\frac{n}{2^k} = 1$ $\quad n = 2^k$ $\quad k = \log_2 n$ → $2^k * \frac{n^2}{(2^k)^2} = \frac{n^2}{2^k}$

Similarly $\frac{n^2}{2^k} = \frac{(2^k)^2}{2^k} = 2^k$

For $\underline{K^{th} \text{ level}}$

$$2^{K} * T(1) = 2^{\log_2 n} * T(1) = n * T(1)$$

For $\underline{1 \text{ to } K-1}$ level

$$\sum_{i=0}^{K-1} n^2 * \frac{1}{2^i} = \boxed{n^2 \sum_{i=0}^{K-1} \frac{1}{2^i}}.$$