Unit – 5: Backtracking

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Unit wise course: What are you learning

UNIT-V: Backtracking: CO3

- *Basic Traversal and Search Techniques, breadth first search and depth first search, connected components.*
- *Backtracking basic strategy, graph coloring, Hamiltonian cycles, 8-Queen's problem, sum of subset problem, Introduction to Approximation algorithm.*
- **Design Dynamic programming and Backtracking Paradigms to solve the real life problems.**

Basic of Backtracking

- Applications: Game Theory
- For every given problem: **Input** is represented in the form of Tuple [x1..xn].
- The solution is generated by selecting proper input from the tuple [x1..xn]. Each selected input must satisfy the **criterion function**.
- Since Backtracking is **"Selection" based solution,** at each stage the selection may be optimized by testing **validity** of criterion function [constraints]

Basics

- **Basic Principle:** Solution is constructed component-wise and at each point the component is tested against the criterion function. The construction of solution continues, if criterion function is satisfied.
- For example, if there are “n” elements then first component can be (x1..xi) is checked against (p1..pi) and if partial solution and partial criterion function are not matching then remaining part of solution is simply ignored.
- *(x1..xi) is partial solution and (p1..pi) is partial criterion function.*

Basics

- Problem solving using backtracking requires that all the solutions must satisfy the complex set of constraints. These constraints can be classified into two classes:
- ***Explicit Constraints***
- ***Implicit Constraints***
- **Explicit Constraints: [Direct]**
- They are set of rules which allows "xi" to take value only from the given set.
- For example: xi >= 0 or xi = 0 or 1

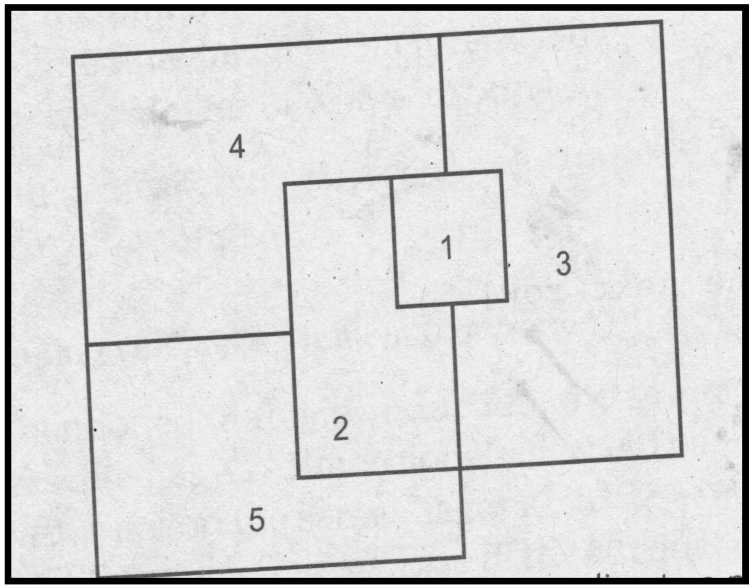
Basics

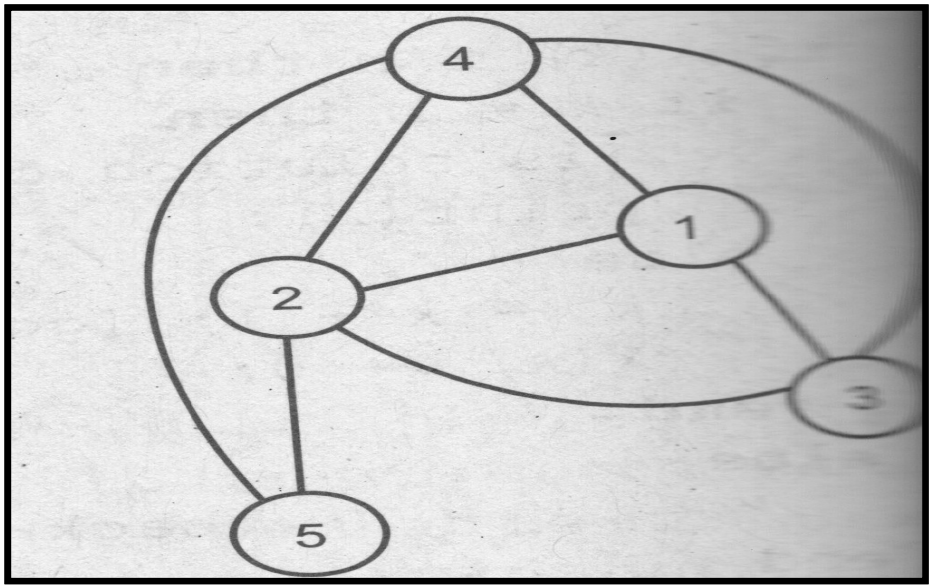
- **Implicit Constraints: [Indirect]**
- These constraints are the rules that decides which of the tuple in the solution space of "i" satisfies criterion function
- Example: Sum of subset problem: a=[12,5,22,9,2]
- Sum=24
- There are various combinations: 9,5,12 or 12,5,9
- Implicit constraints will allow combination 12,5,9 or 22,2 and will not allow 2,22 or 9,5,12 etc..
- For above problem Implicit constraint can be stated as: Use values in array only once and in the order of occurrence in given array.

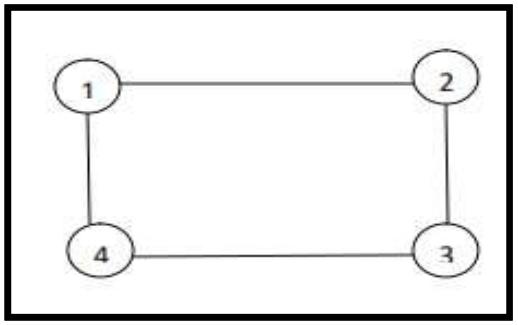
Graph Coloring: Algorithm

- Algorithm solution for problem solved using **BACKTRACKING are RECURSIVE**
- The input to algorithm is vertex number present in the graph
- The algorithm generates the color number assigned to vertex and stores it an array.
- If the constraint are not matched at any point, then remaining part of algorithm is not executed and new cycle is initiated.
- The algorithm terminates with a solution satisfying the constraints.
- ***Application: Area demarcation/Map Coloring/Remote Sensing Applications***

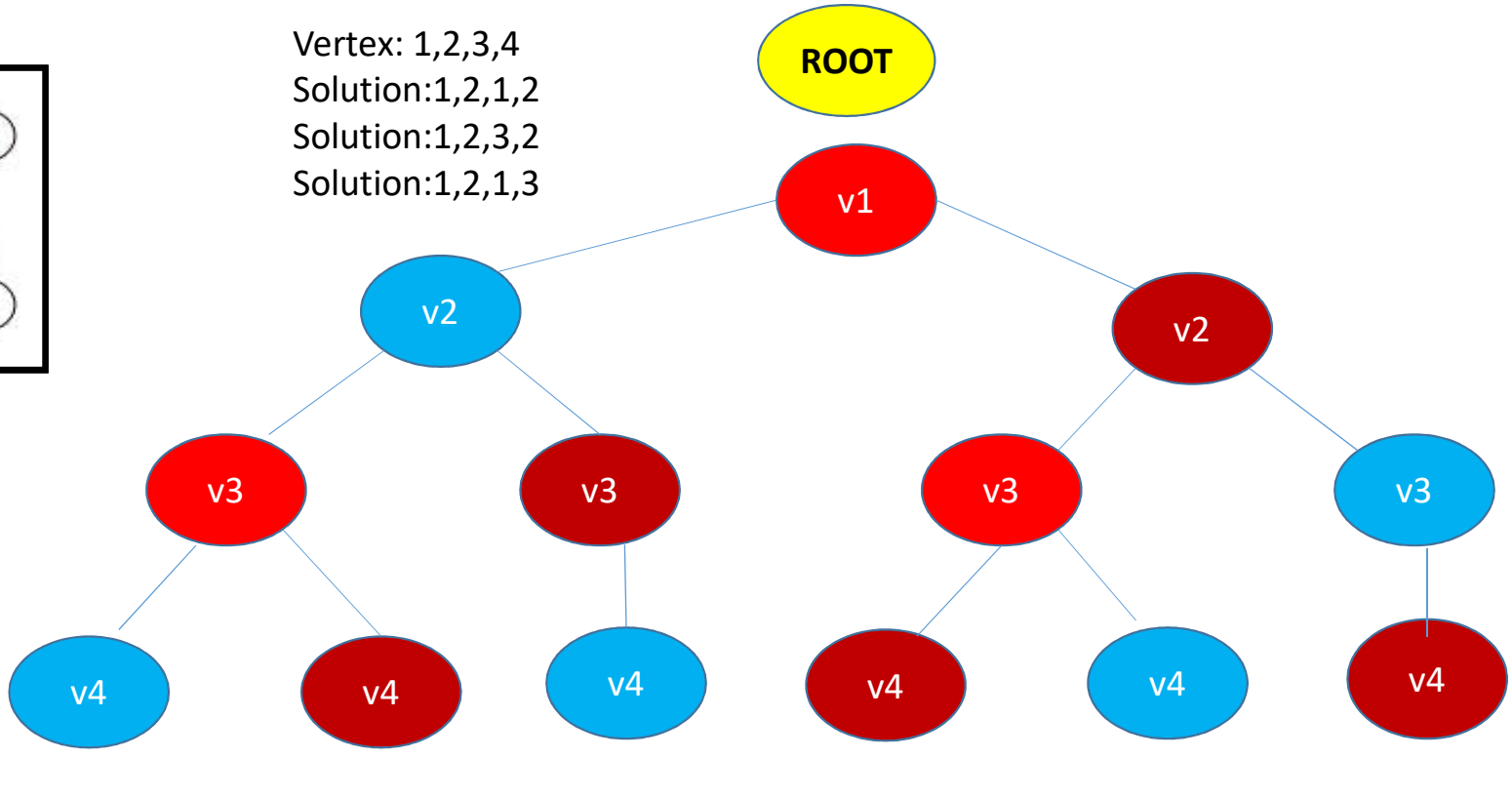
Chromatic Number

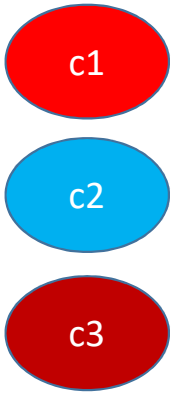






Vertex: 1,2,3,4
Solution:1,2,1,2
Solution:1,2,3,2
Solution:1,2,1,3





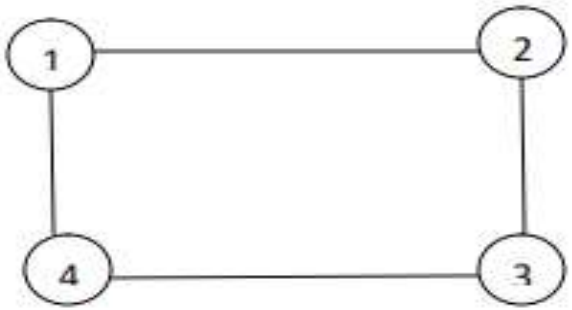
Degree of Vertex=2
Number of colours required will be d+1 = 3 [Generic solutions]. **Optimization is possible in some cases based on density of graph**

Part – I: mcolor

- This component passes the different values of vertex to “nextvalue” algorithm, and accept the value returned by “nextvalue” algorithm.
- The value is checked and if found suitable then stored in output array.
- The output array x[1..n] is solution for the given problem.

ALGORITHM FOR GRAPH COLORING

```
Algorithm mcolor(k)
{
    The graph is represented in the form of matrix nxn
    "k" is an index of vertex to be colored
    {
        repeat
        {
                Nextvalue(k)
                If (x[k] = 0) then return
                If (k=n) then
                        Write(x[1:n])
                Else
                        Mcolor(k+1)
        } until(false)
    }
}
```



Algorithm: nextvalue

- This algorithm will accept the vertex number from “mcolor” and generates color value for the vertex, with constraint satisfaction.
- To check constraints following conditions are tested:
- Let vertex “k” is in process, then all vertices in the graph are tested for adjacency test.
- If the vertices are adjacent, then, should be different colors
- The color information is stored in array “x”
- Hence G[k,j] != 0 and x[k] = x[j] then color is not suitable otherwise suitable.

Logic for m=3 [v1=c1, v2=c2, v3=c1, v4=c2]

Array "X"	**X[1]=V1**	X[2]	X[3]	X[4]
Initial stage	0	0	0	0
Mcolor(1)	Call nextvalue(1)			
	X[1] = 1			
Mcolor(2)		**X[2]=1 [Break]**		
		X[2]=2		
Mcolor(3)			X[3]=1	
Mcolor(4)				X[4]=2

```
Algorithm nextvalue(k)
Assume x[1..k-1] are assigned integer in the range of [1,m] such that no
two adjacent vertices are in the same color
x[k] is assigned the next value such that distinctness is maintained
If no such color exists then x[k]=0
{
    repeat
    {
        x[k] = (x[k] + 1) mod m+1
        if x[k] = 0 then return
        for j = 1 to n do
        {
            If (G[k,j]!=0) and (x[k]=x[j]) then
                Break
        }
        If (j=n+1) then
            Return
    }until (false)
}
```

Hamiltonian Cycle

- It is a cycle generated on graph, with same vertex as source and destination.
- The constraints for generating cycle:
  - Each vertex has to be visited once
  - Any edge can be used only once
  - Some of the edges can be skipped, but vertices cannot be skipped.
  - Applications: Travelling salesman problem
  - One of the approach to solve travelling salesman problem algorithmically

Hamiltonian Cycle

```
Algorithm Hcycle(k)
{
        The graph is represented in the form of matrix nxn
        "k" is an index of vertex to be colored
                repeat
                {
                        Nextvalue(k)
                If (x[k] = 0) then return
                If (k=n) then
                        Write(x[1:n])
                Else
                        Hcycle(k+1)
                } until(false)
        }
}
```

Nextvalue algorithm for Hcycle

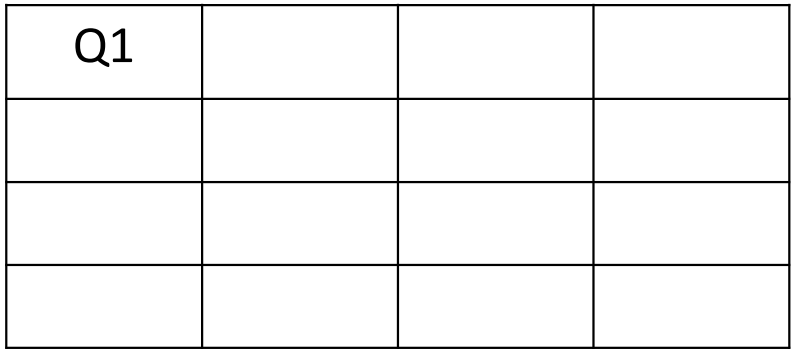
```
Algorithm nextvalue(k)
{
        Repeat
        {
                x[k]=(x[k]+1) mod n+1
                if (x[k]=0) then return
                if(G[x[k-1],x[k]!=0) then
                {
                        For j=1 to k-1 do

                                If(x[j]=x[k] then break
                                If(j=k) then
                                If{(k<n) or [(k=n) and
                                G[x[n],x[1]]!=0)} then
                                        Return
                        }
        } until(false)
}
```

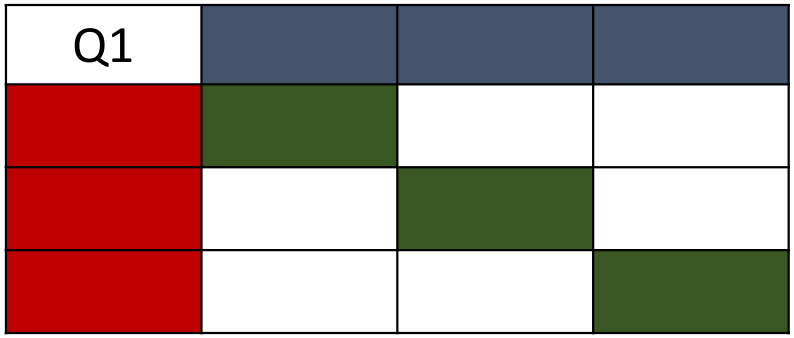
N-Queen Problem

- Basic: Given a chess board of size nxn, it is required to place “n” queen on the chess board with following constraints:
- 1. No two queens should be in same row
- 2. No two queens should be in same column
- 3. No two queens should be diagonally opposite

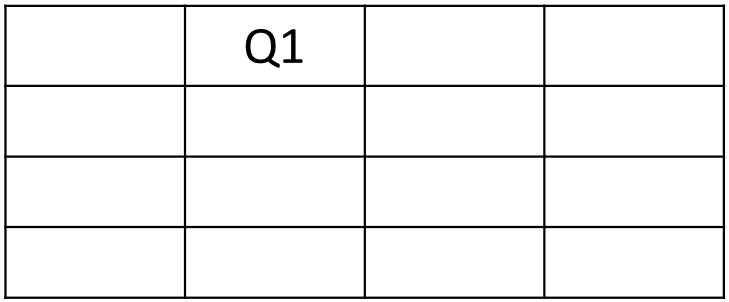
Why backtracking: if any of above constraint does not satisfies at “k” queen, then it is required to adjust 1..k-1 queens.

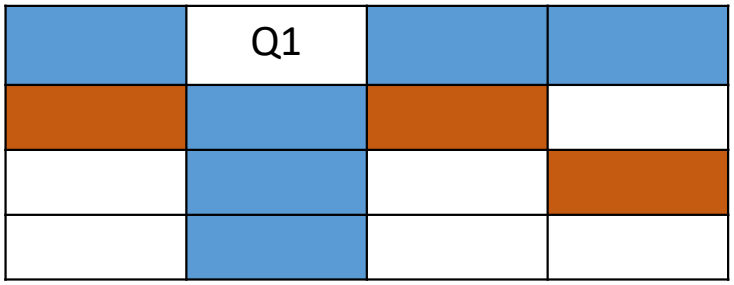
Sub-Problem: 4 Queen problem

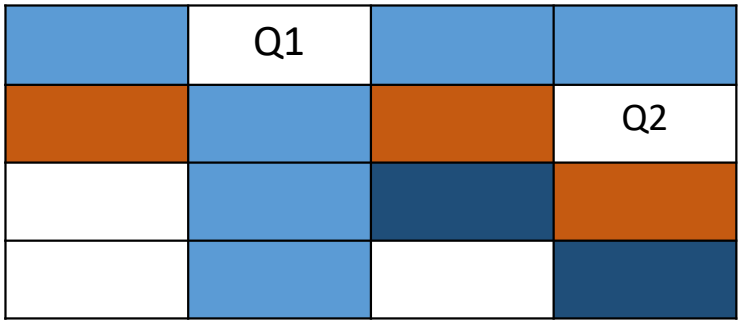


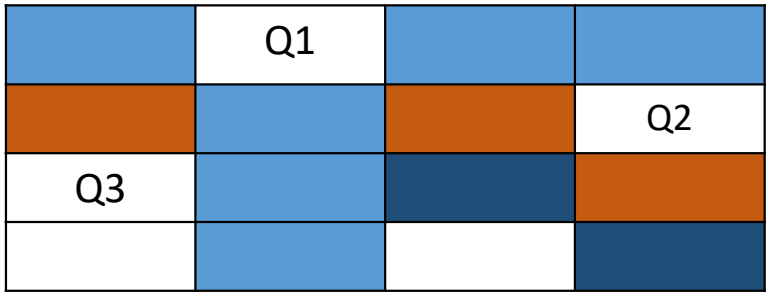


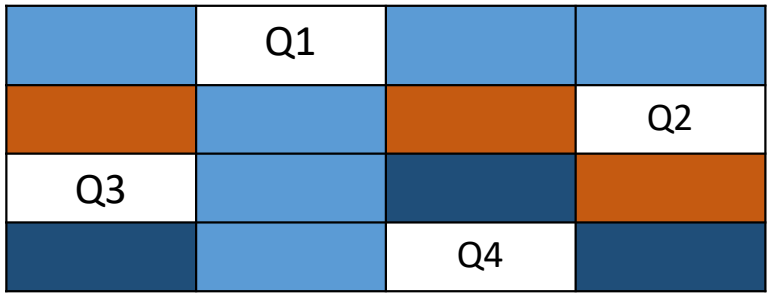
Sub-Problem: 4 Queen problem



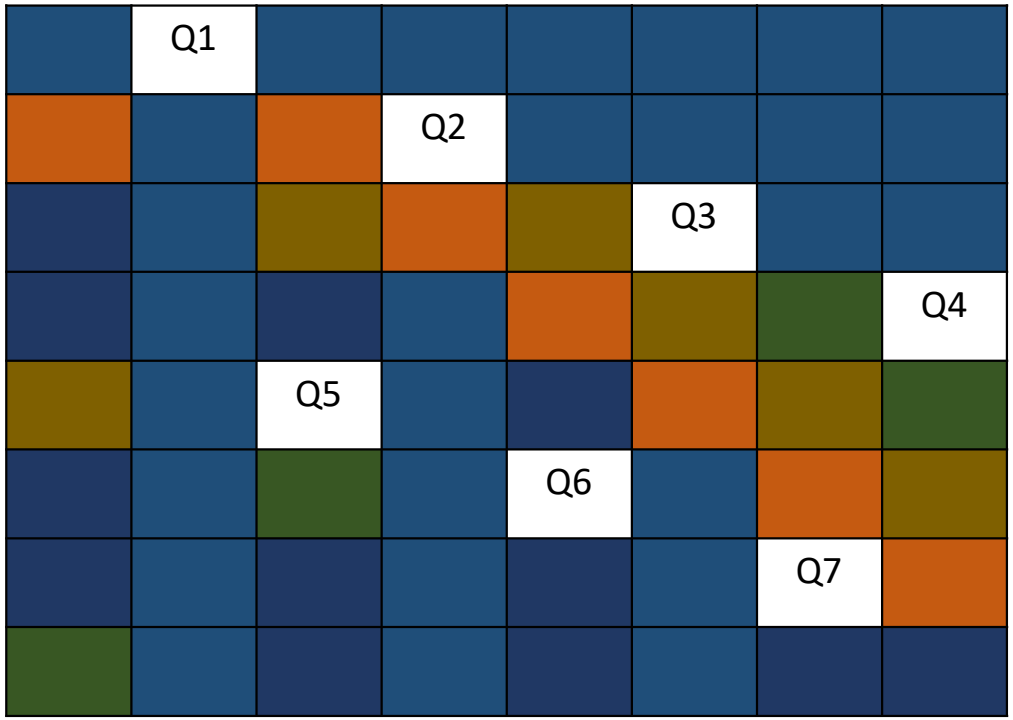




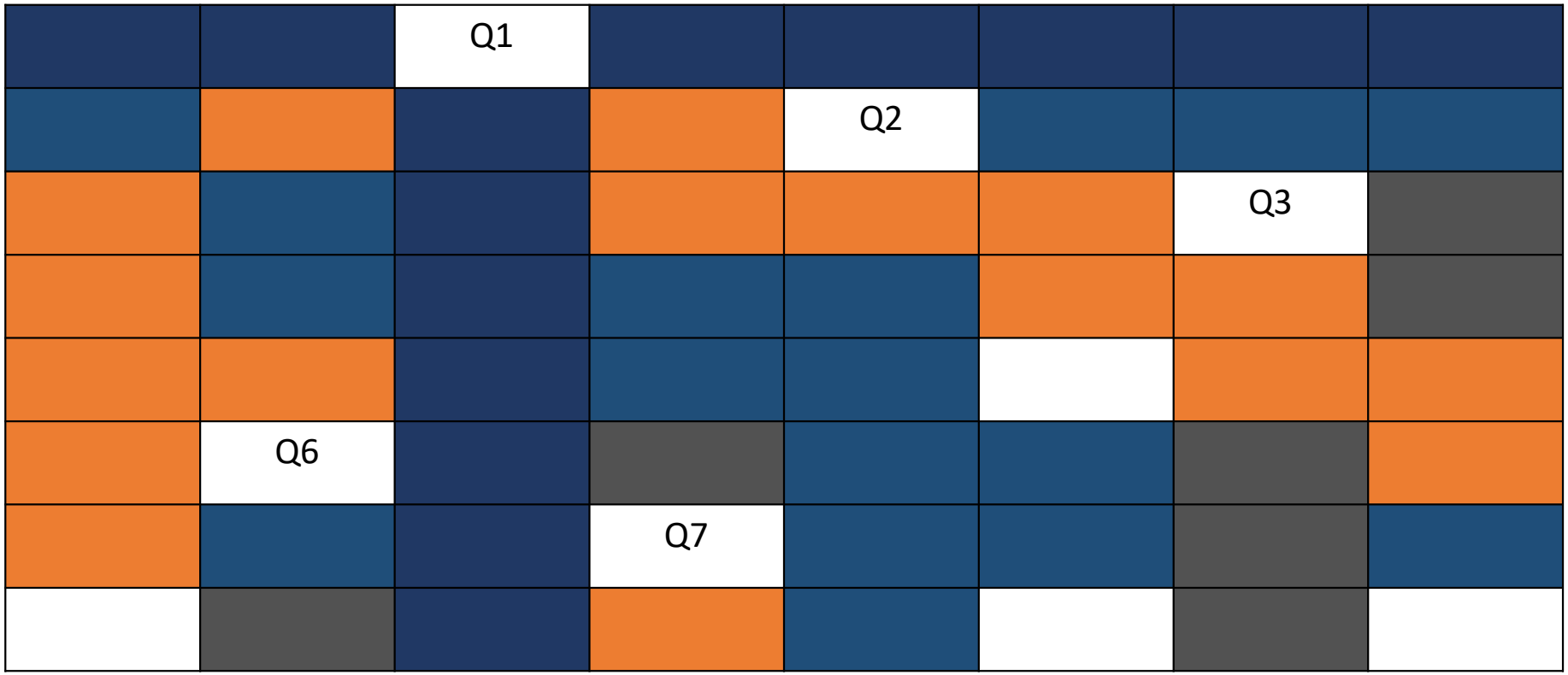




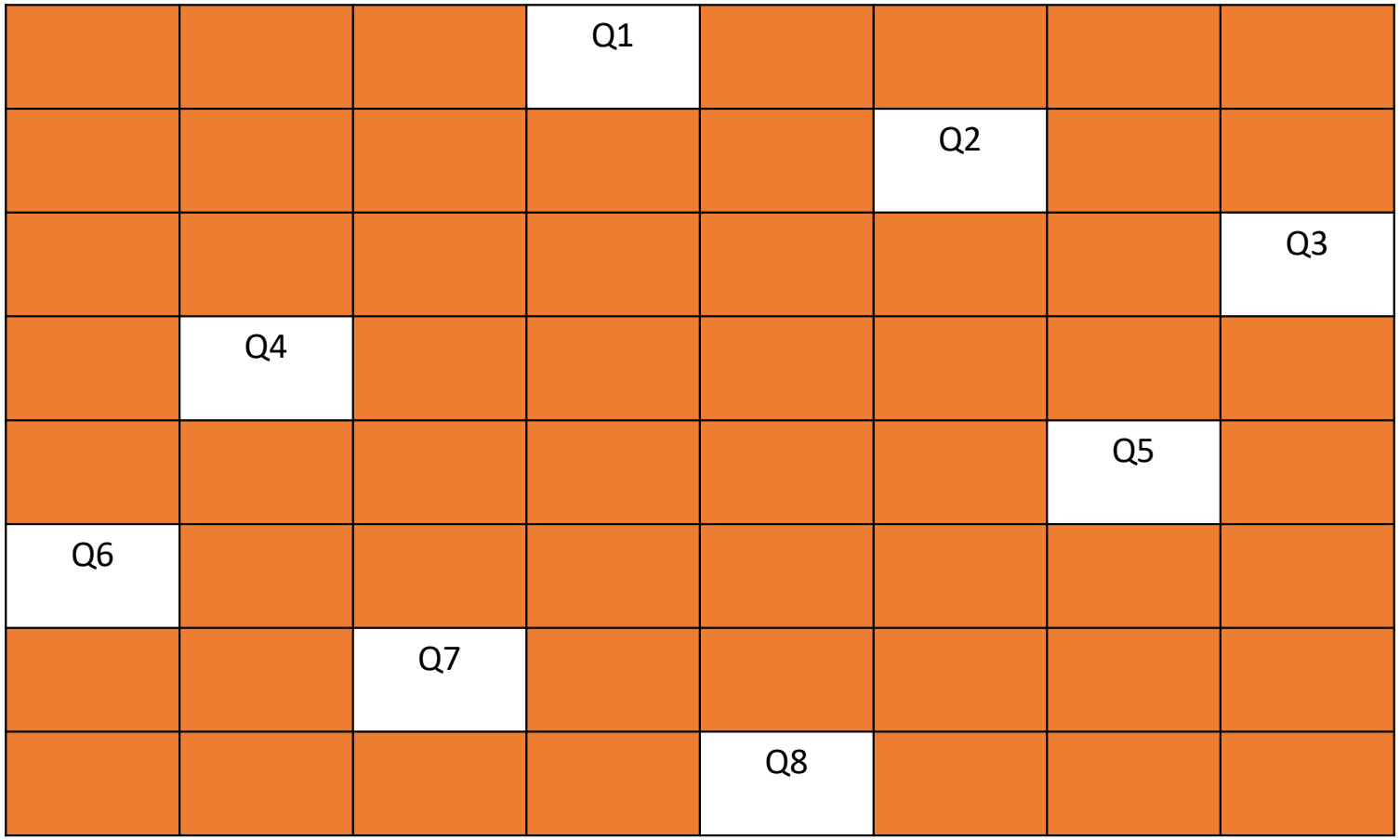
Solution to 8 Queen Problem



Second Attempt



Solution to 8 Queen Problem



Solution to 8 Queen Problem: Constraint logic

	1	2	3	4	5	6	7	8
1								
2								
3								
4					*			
5								
6								
7								
8								

Algorithm: Part - I

- The algorithm will generate array x[1..n] to store the column address of the queens and index of array “x” represents row address.
- For example if at location x[3]=2, then one of the location in matrix is [2,3] for storing third queen.
- The algorithm is divided into two parts:
  - Part 1: to pass the values of Q’s
  - Part 2: to generate the locations.

Algorithm: Part - I

```
Algorithm Nqueen(k,n)
{
   //Initial value of k=1 and n=8
   for j = 1 to n do
   {
        if place(k,j) then
            x[k] = j
        if (k=n) then write x[1..n]
        else
            Nqueen(k+1, n)
    }
}
```

Algorithm: Part 2

- The place(k,j) accepts different values of “j“ for “k” queen and check the constraints, if satisfied then returns true value. (“j“ represent column)
- Condition checked:
- For the “k” queen, in “x” array from [1..k-1] the location should not be used.
- Similarly diagonal condition is checked.

Algorithm: Part 2

Algorithm place(k,j) //passing queen number and column number

{ x = {4,6,8, assume j=1 placing Q4 [k=4]}

//Assume that k-1 locations are already computed in array "x". Absolute function is used in the algorithm

for i = 1 to k-1 {

if(x[i]=j) or

(abs(x[i]-j = abs(i-k)) then

return false }

return true;

}

X[i]=4,6,8 and j=1 so x[i]-j=3,5,7

Abs(i-k) = 1-4, 2-4,3-4 ➔ 3,2,1

Condition matched, so j=1 is not valid. {Return False}

Applications:

Game designing

Security

Generating obstacles in space

Construction planning