Solow-Swan Growth Model

ECON8069 - Lecture Nine

Australian National University

Solow-Swan Growth Model

- Modelling Economic Growth
- Solow-Swan Growth Model
- Policies for Economic Growth

Textbook: Chapter 21, Appendix to Chapter 21, Chapter 22

Modelling Economic Growth

- Recall the Aggregate Production Function from last week:

$$Y = A \times F(K, H)$$

- Y - Income, or GDP
- A - Technology
- K - Physical Capital
- H - Effective Labour (combining labour hours L and average human capital h)

Modelling Economic Growth

- For economic growth, we care about *rates* of growth; i.e. percentage changes

$$\text{Growth}_t = \frac{x_t - x_{t-1}}{x_{t-1}}$$

- Where x is the 'variable of interest'. This will usually be Income Y, or Income per capita y.

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- Where x is the 'variable of interest'. This will usually be Income Y, or Income per capita y.
- Note on notation: *Usually* we use upper case letters (Y, K, etc.) for nominal amounts, and lower case letters (y, k, h, etc.) for per capita amounts.

Solow-Swan Model

The Solow-Swan model is a dynamic macroeconomic model utilising three key components

1. Aggregate Production - $Y_t = A_t K_t^{1/3} H_t^{2/3}$
2. A closed economy with no government, so $Y_t = C_t + I_t$
3. Capital Accumulates from Investment, but also depreciates, so $K_t = K_{t-1} - \delta K_{t-1} + I_t$

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Let's look at these in more detail.

Extra simplifying assumption

- We will also here assume that H does not change between periods
- The model can be extended to allow for this, but we won't do so here
- Also, we really want to work with *per capita* values

Solow-Swan Production Function

- The Solow model uses the Cobb-Douglass production function

$$Y_t = A_t K_t^{1/3} H^{2/3}$$

- The t subscripts are because we have a dynamic model, so Y_t is income (or GDP) at time t.
- This production function satisfies the desired properties from last lecture.

Solow-Swan Production per capita

- As noted, we want everything to be *per capita*. So divide by H.

$$Y_t/H = \frac{A_t K_t^{1/3} H^{2/3}}{H}$$

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- Now we have output per capita (or per effective capita) in terms of technology, and capital per capita.
- This has a nice graph

Closed Economy with no Government

- The usual GDP is $Y_t = C_t + I_t + G_t + NX_t$
- To keep things simple, we ignore Government, and other countries to give

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- We will fix the savings rate s over time

Closed Economy with no Government per capita

- Using $Y_t = C_t + I_t$ and divide by H to give

$$y_t = c_t + I_t / H$$

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- Combining these gives

$$y_t = c_t + sy_t, \text{ or } c_t = (1 - s)y_t$$

Capital Deprecation

- Not all capital lasts forever; each period some proportion of existing capital will depreciate
- Let δ be the proportion of capital that depreciates each period

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- Not all capital lasts forever; each period some proportion of existing capital will depreciate
- Let δ be the proportion of capital that depreciates each period
- How much capital we have today depends on what we had yesterday, how much depreciated, and how much investment there was

$$K_t = K_{t-1} - \delta K_{t-1} + I_t$$

Capital Depreciation per capita

- We want everything to be *per capita*. So divide by H.

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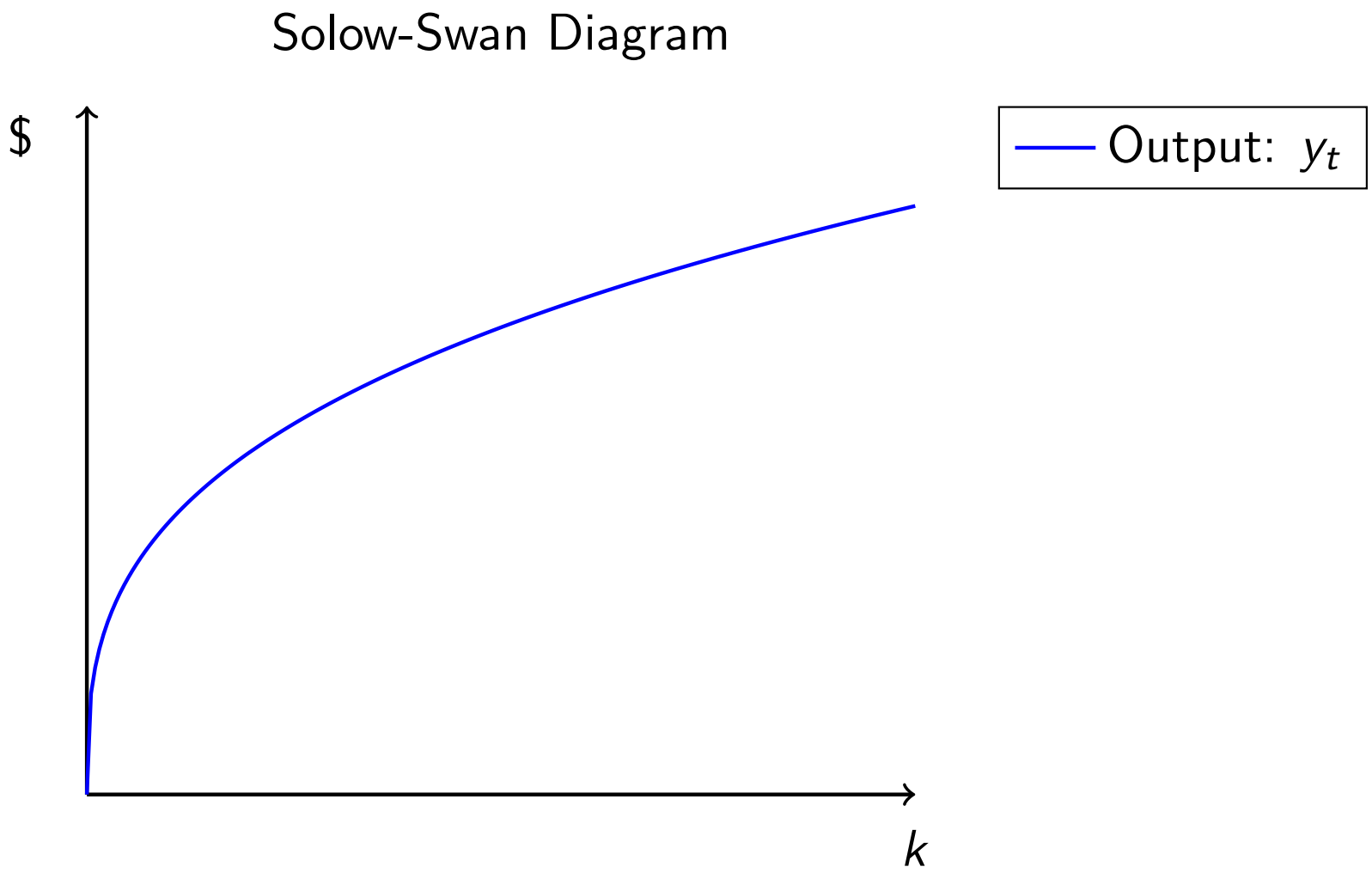
\end{aligned}
$$

- Now we have capital per capita in terms of capital per capita last period, and savings per capita (sy_t) this period

Solow-Swan Model per capita

1. Aggregate Production - $y_t = A_t k_t^{1/3}$
2. A closed economy with no government - $y_t = c_t + sy_t$
3. Capital Accumulation - $k_t = (1 - \delta)k_{t-1} + sy_t$

Solow-Swan Diagram



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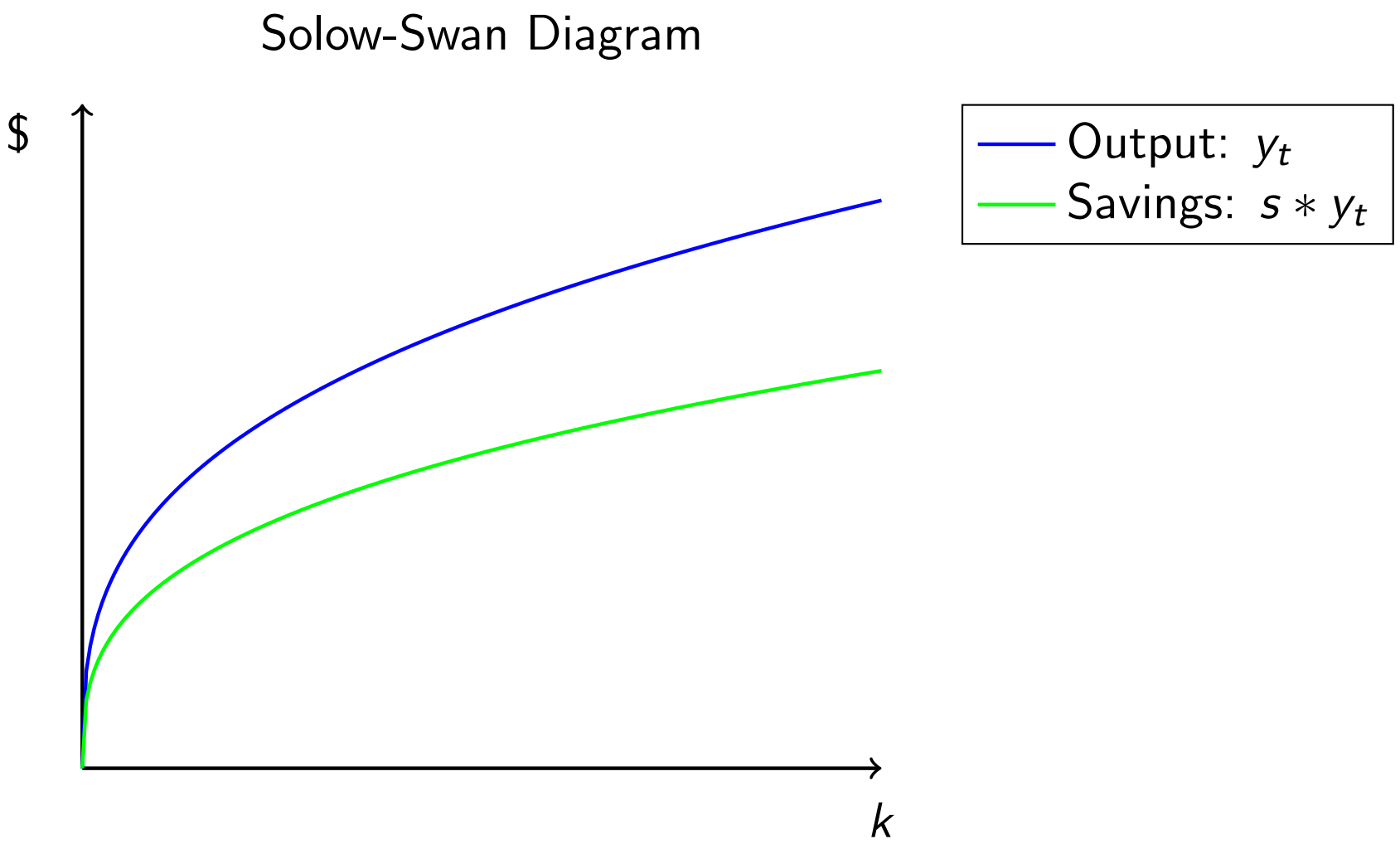

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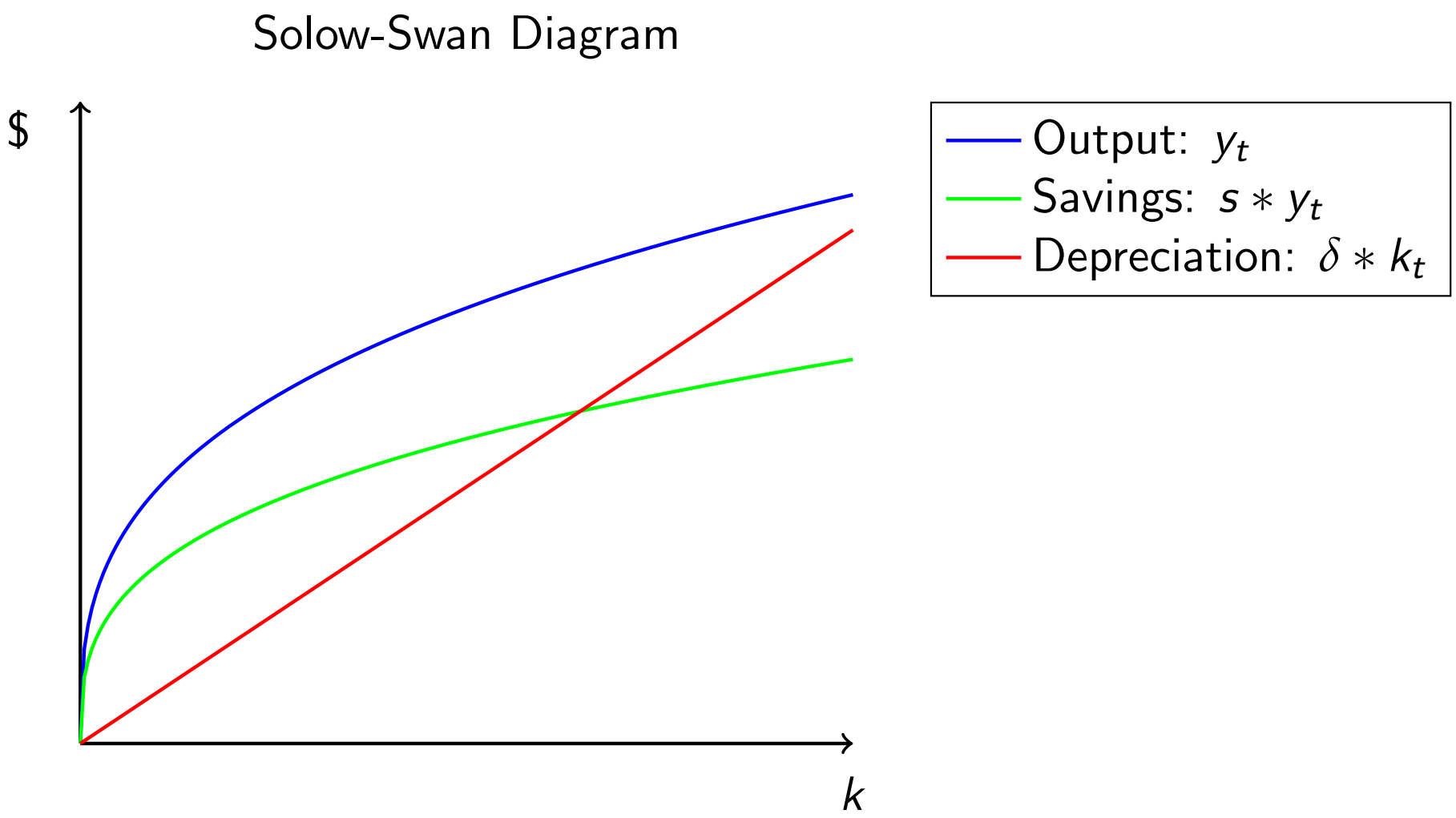

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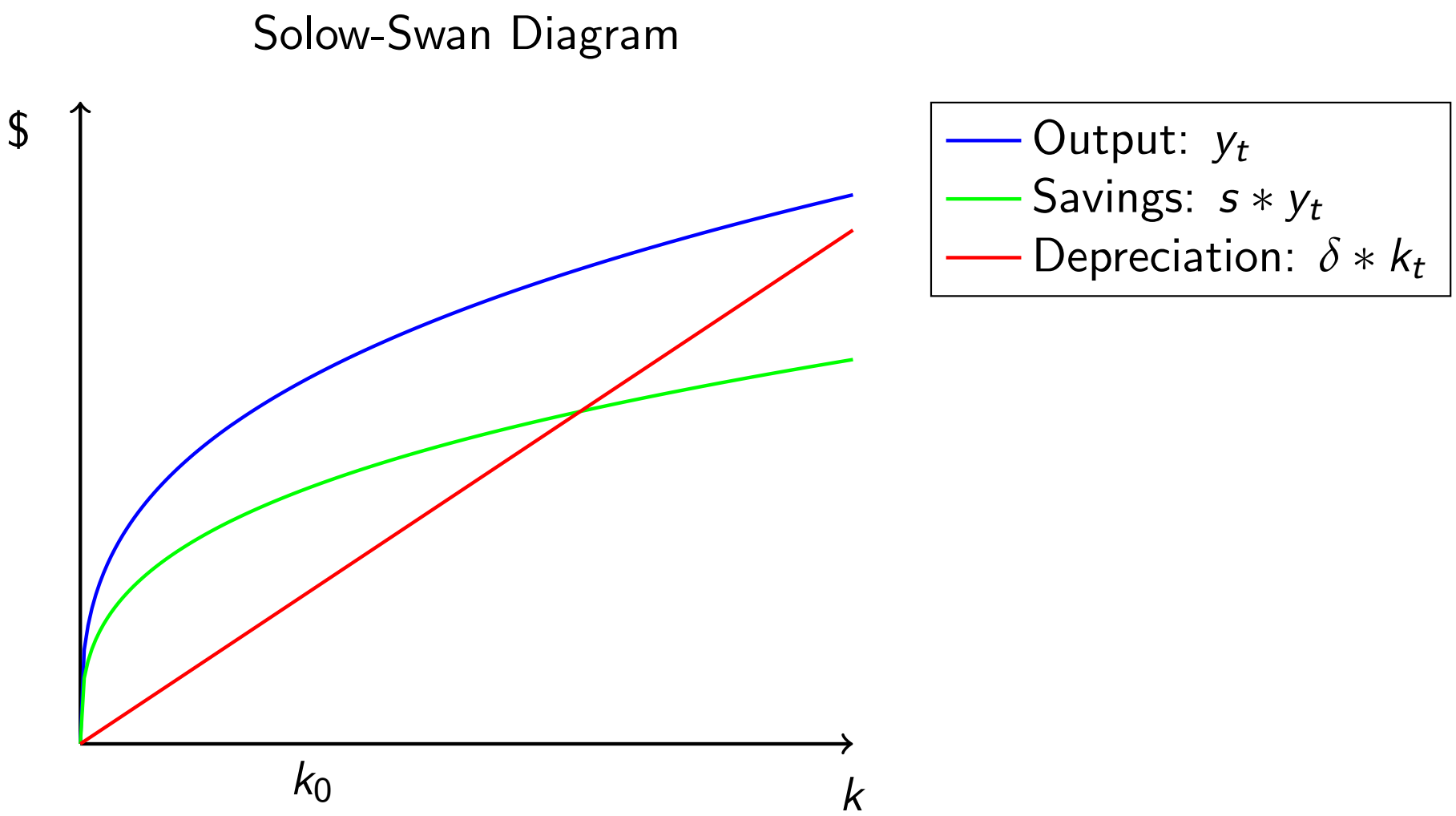

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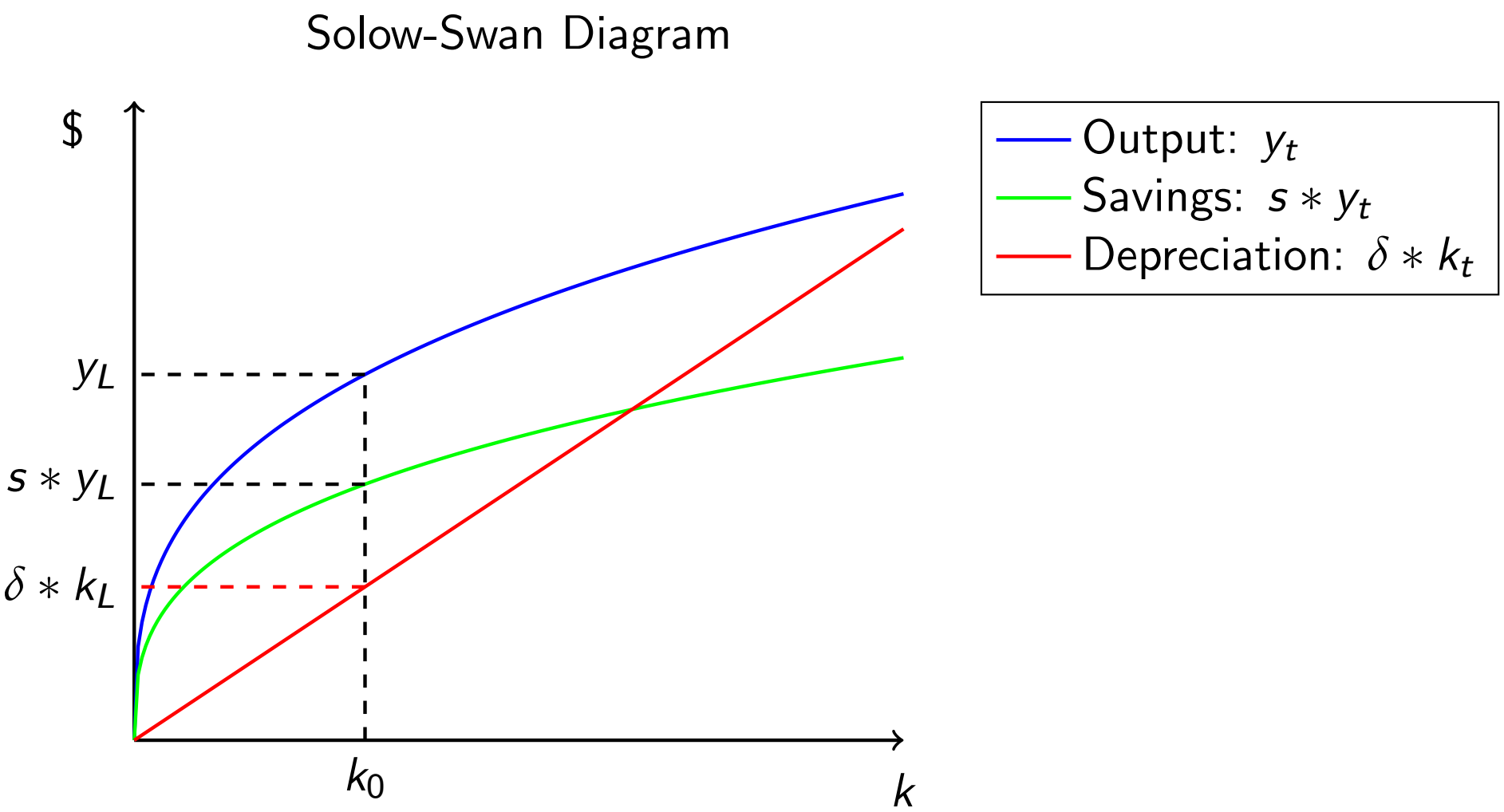

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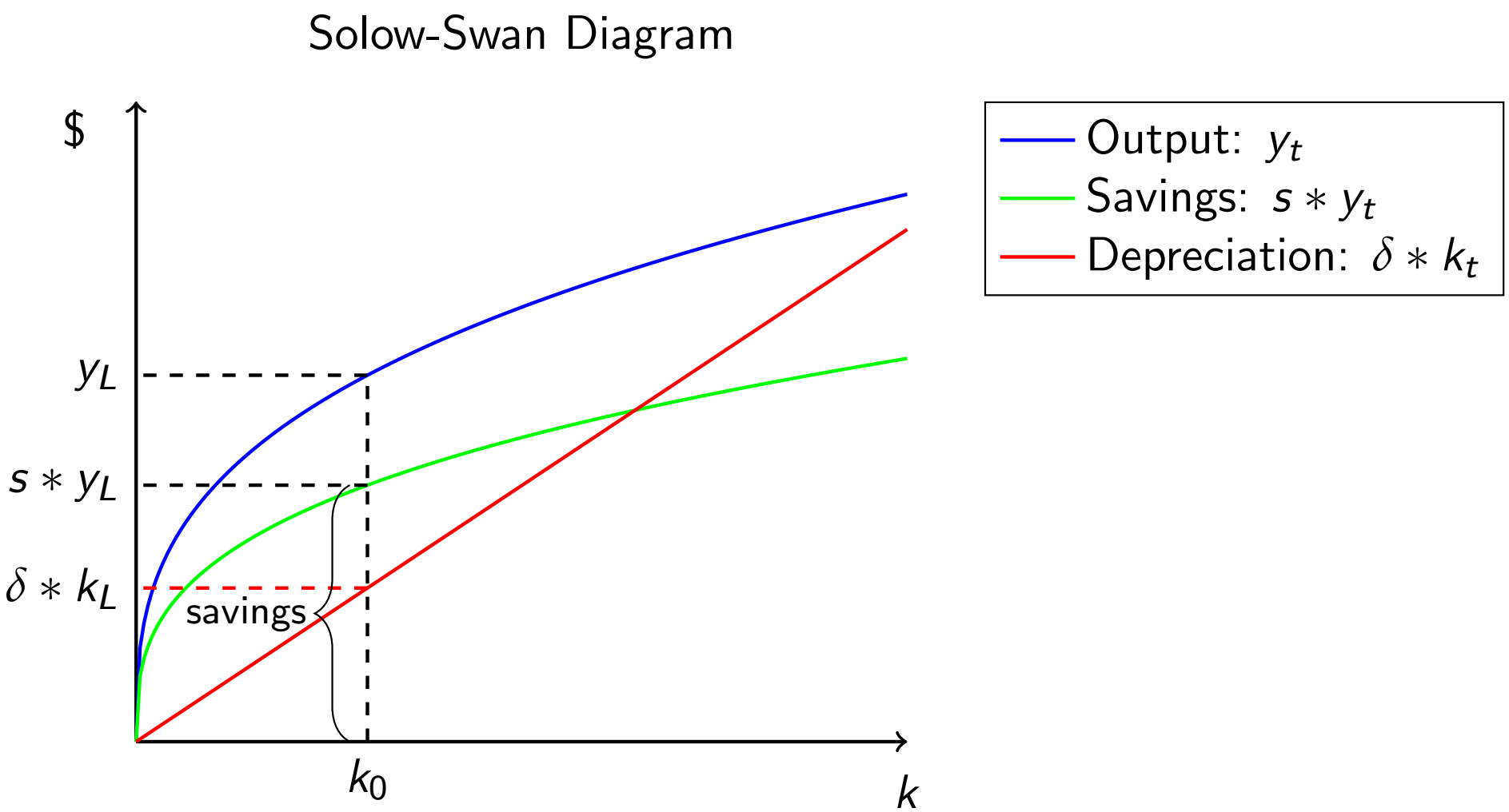

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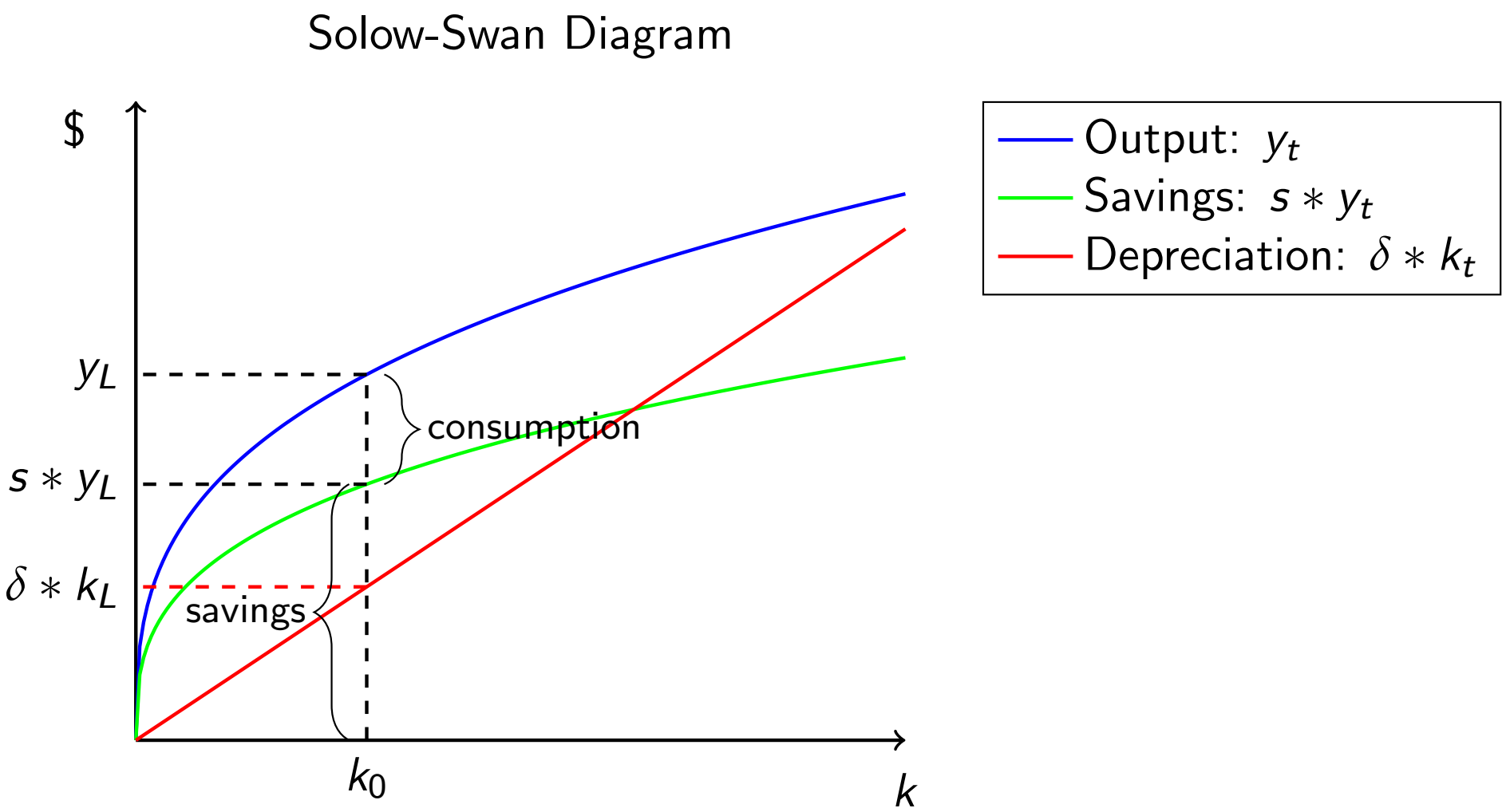

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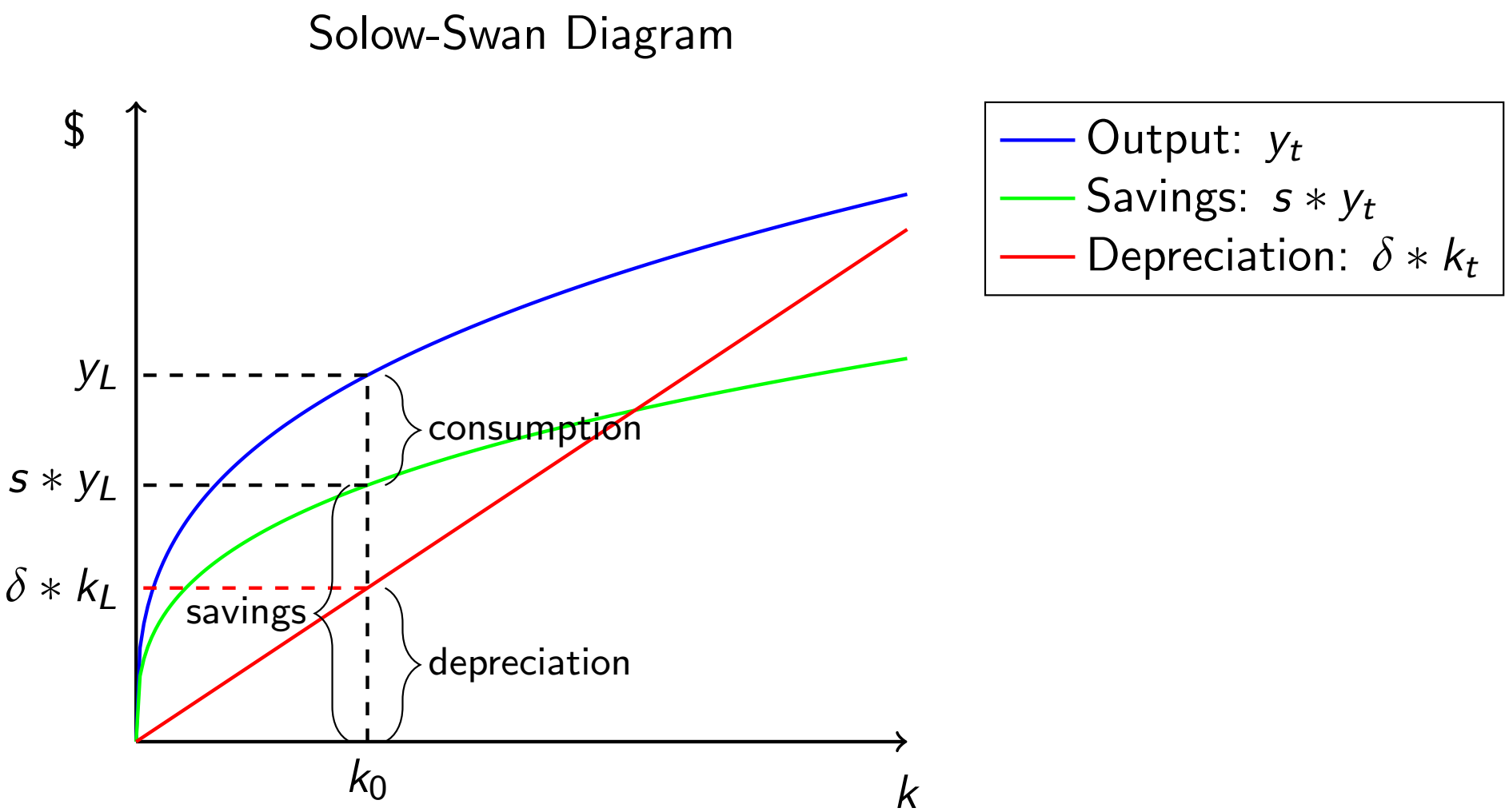

Solow-Swan Model Dynamics

- The Solow-Swan model is in a steady state when capital does not change over time
- That is, when $k_t = k_{t-1}$, or Depreciation = Savings

$$\delta k_{t-1} = s y_t$$

- This is shown on the Solow-Swan diagram
- (Outside this course): Some algebra gives $k^* = (sA/\delta)^{3/2}$

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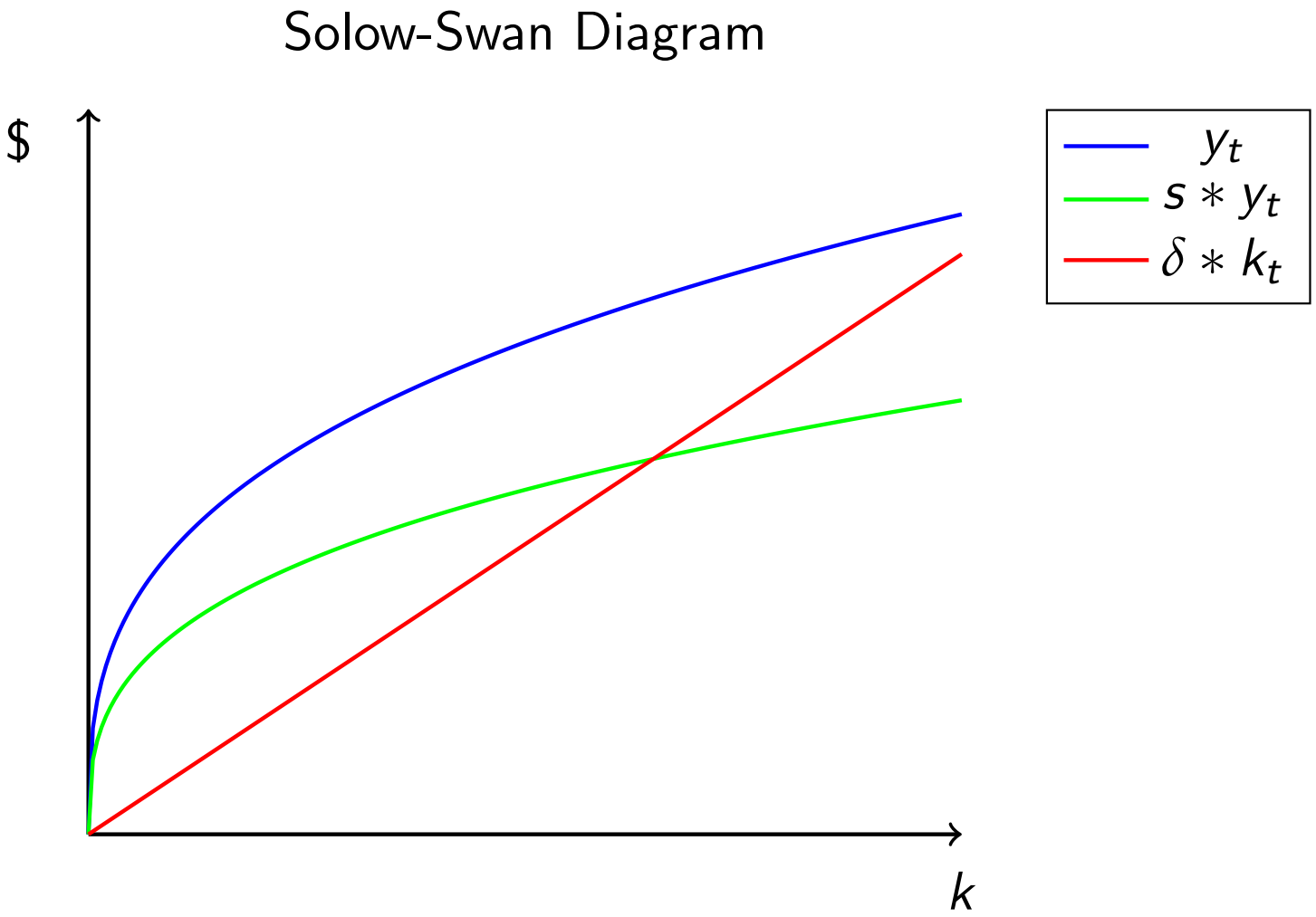
- (Outside this course): Some algebra gives $k^* = (sA/\delta)^{3/2}$
- If Depreciation $<$ Savings, capital will grow over time
- If Depreciation $>$ Savings, capital will shrink over time

Low Capital Stock and Capital Accumulation



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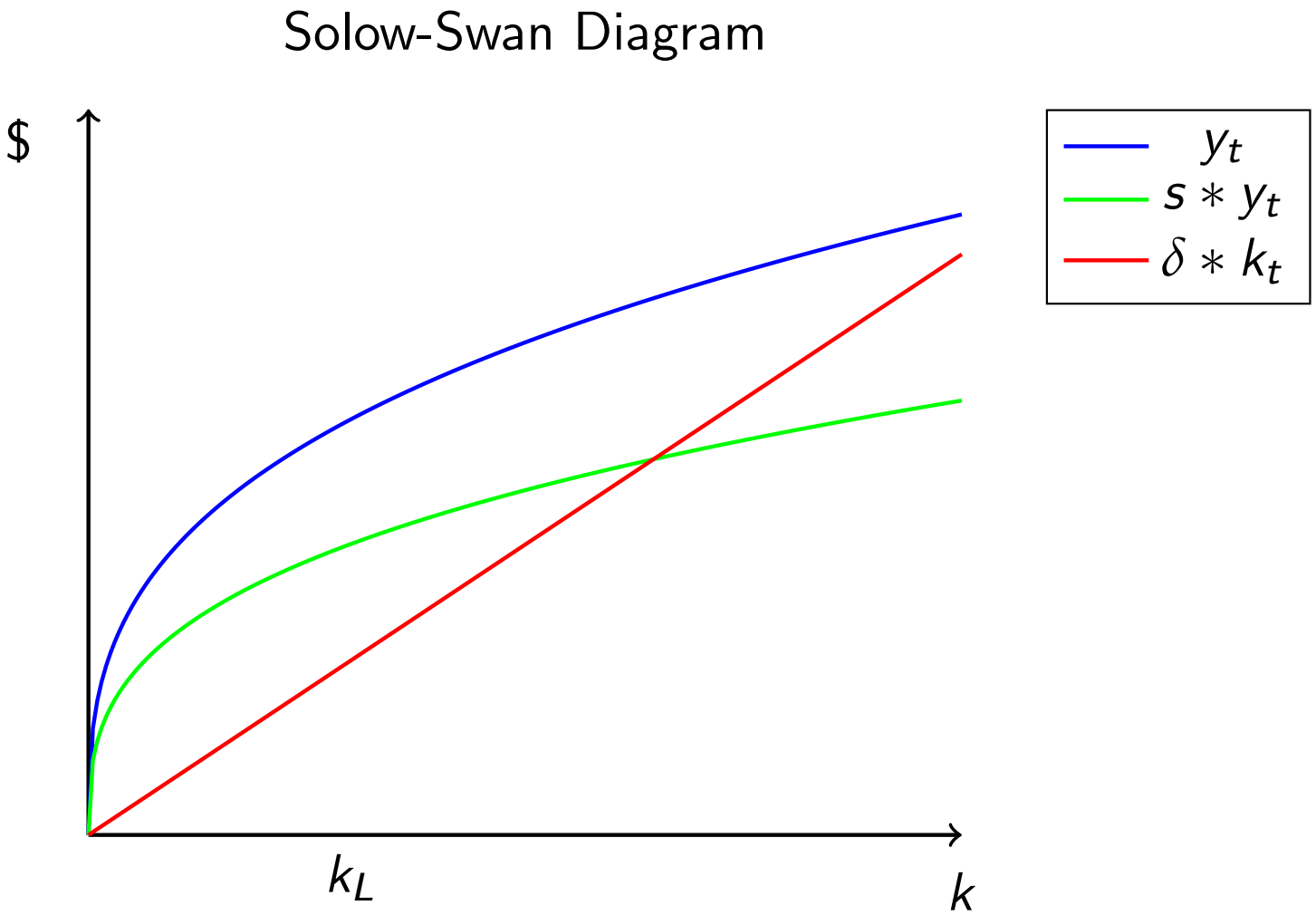

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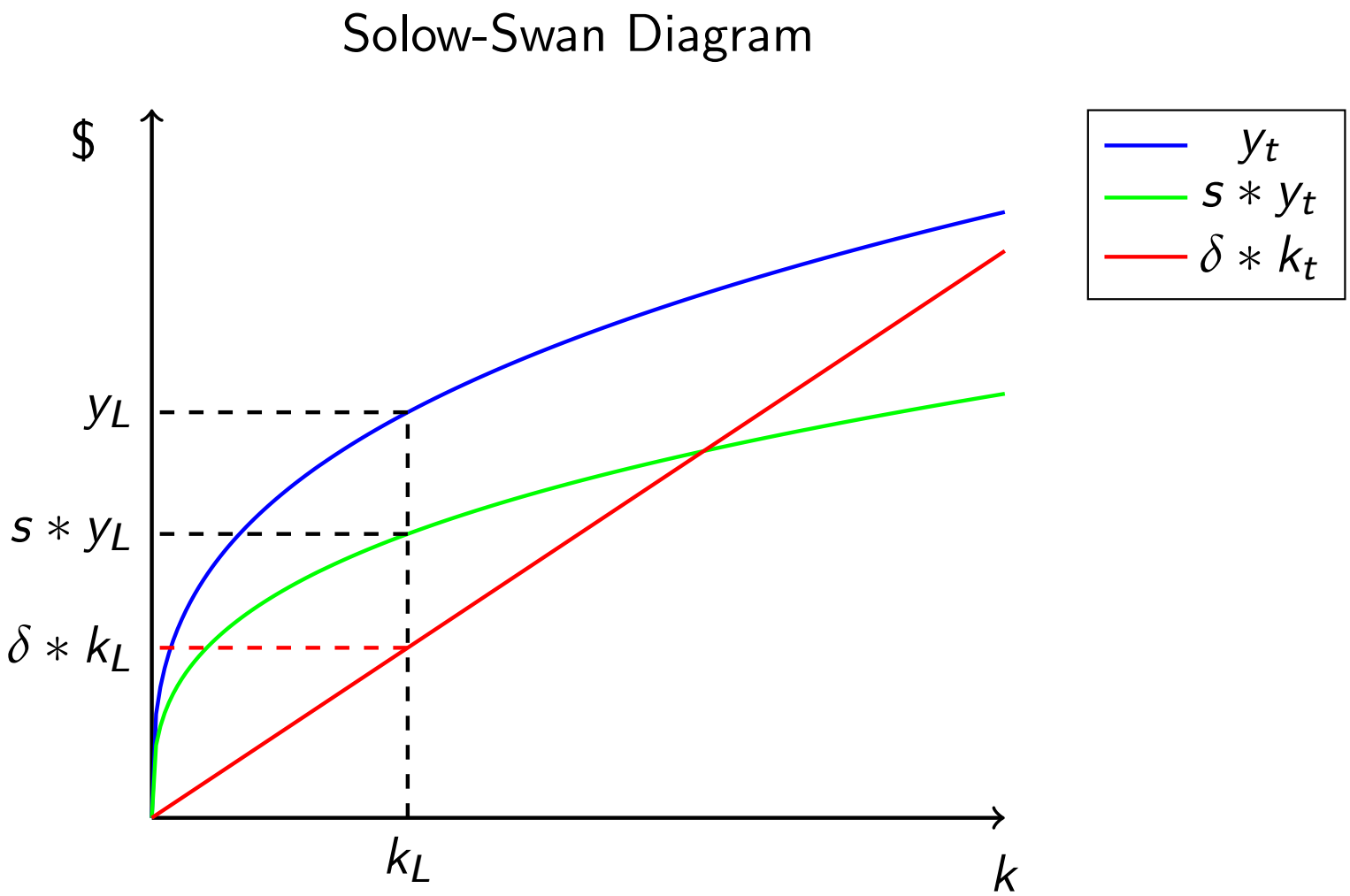

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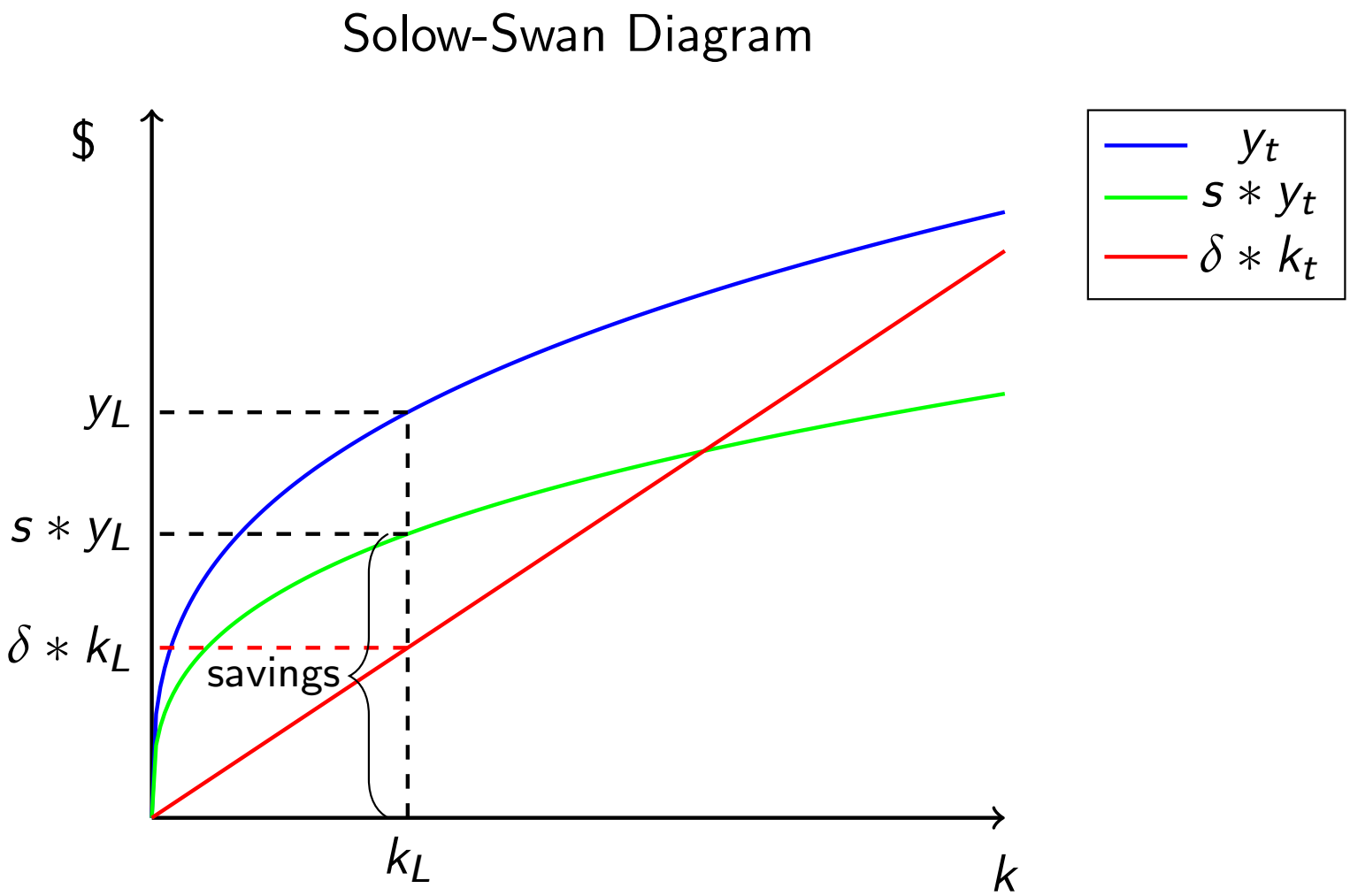

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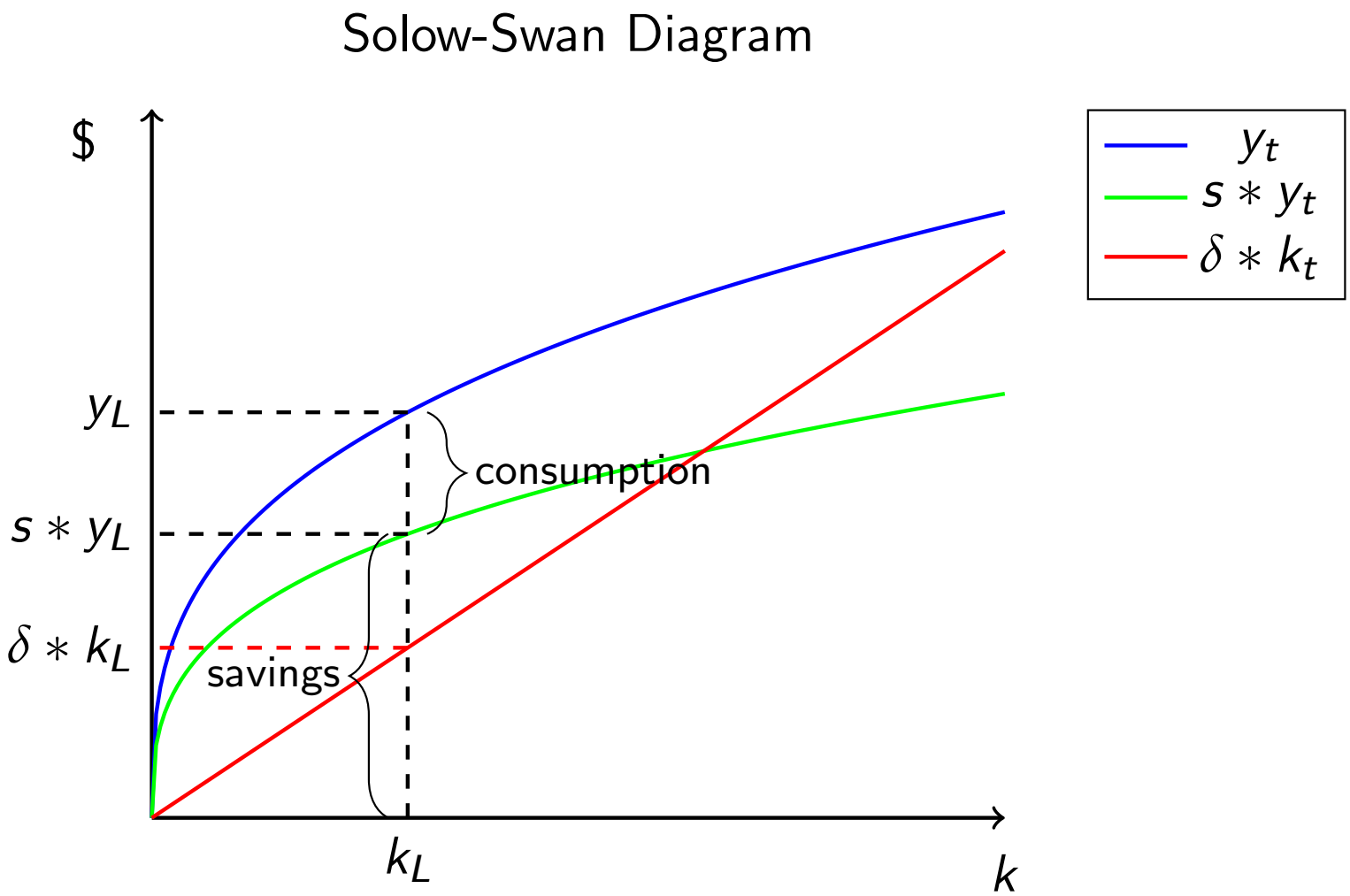

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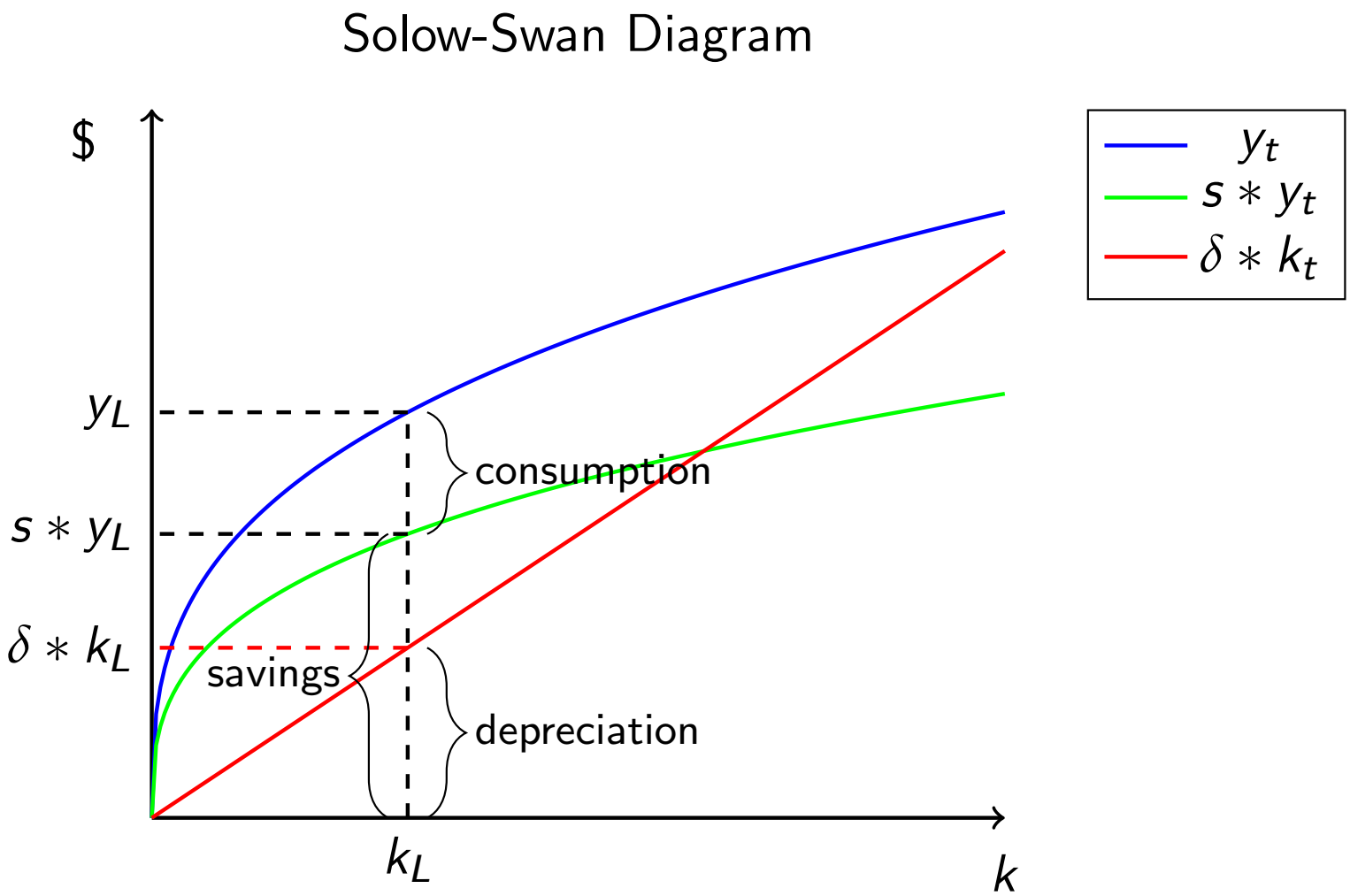

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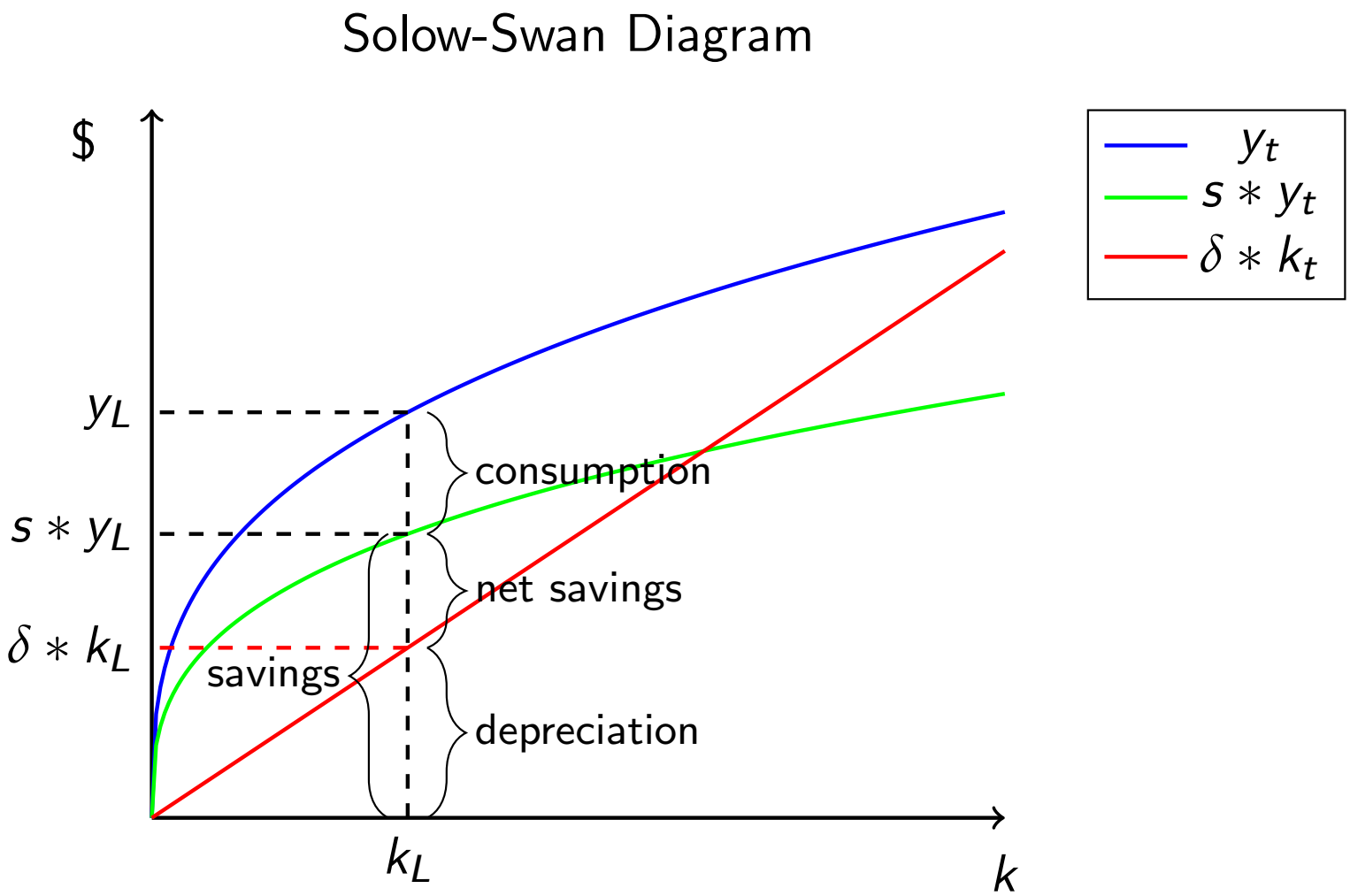

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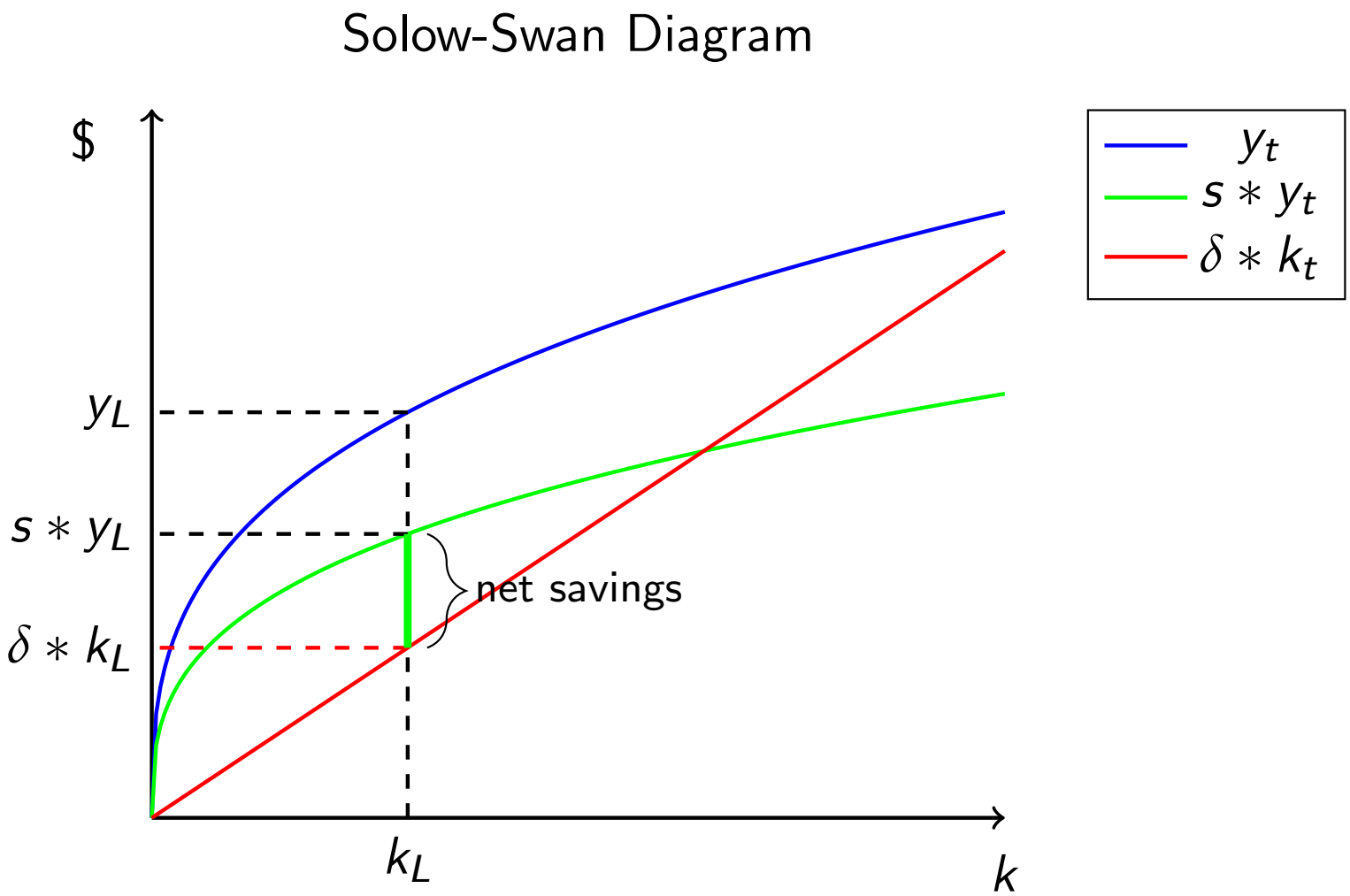

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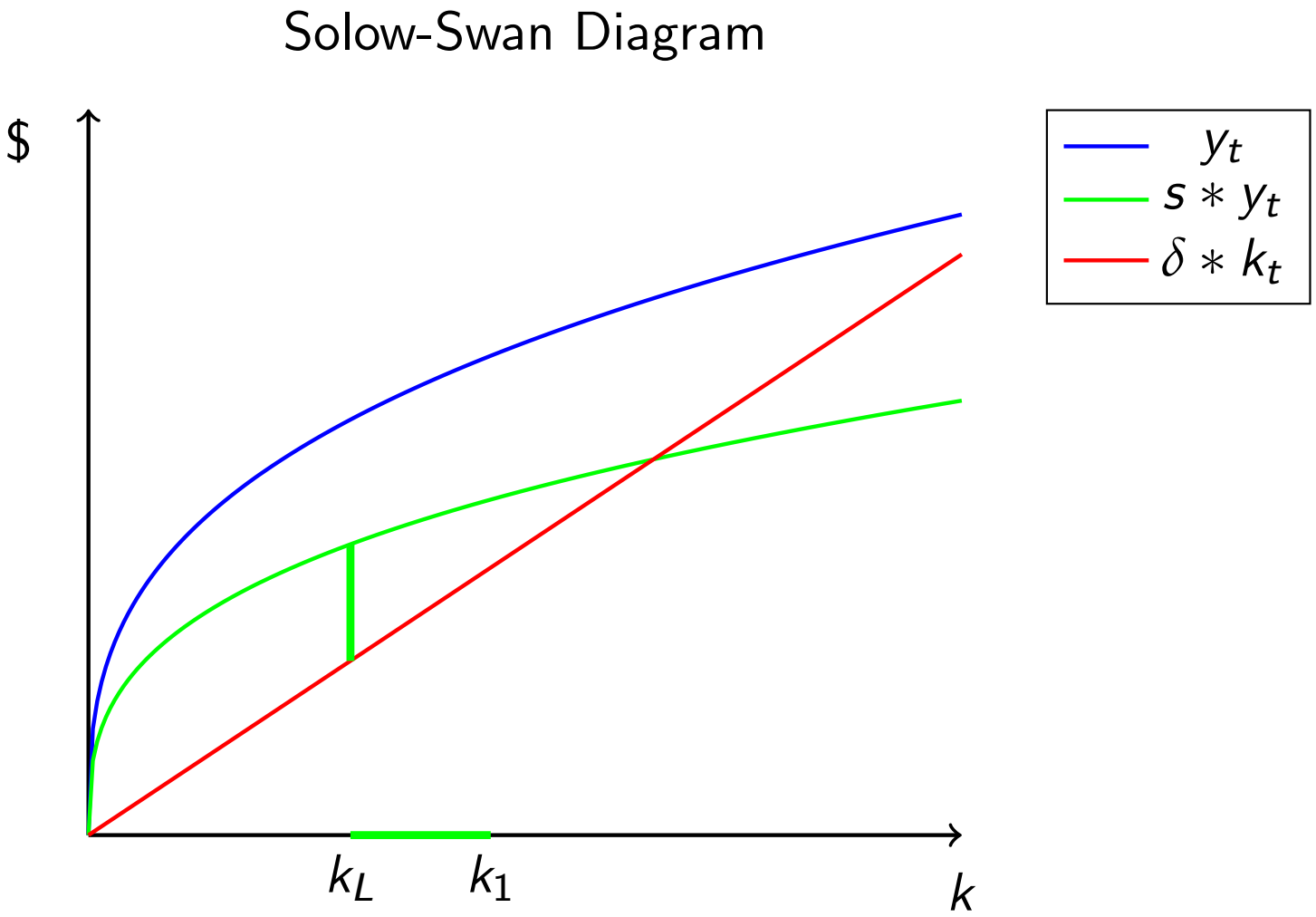

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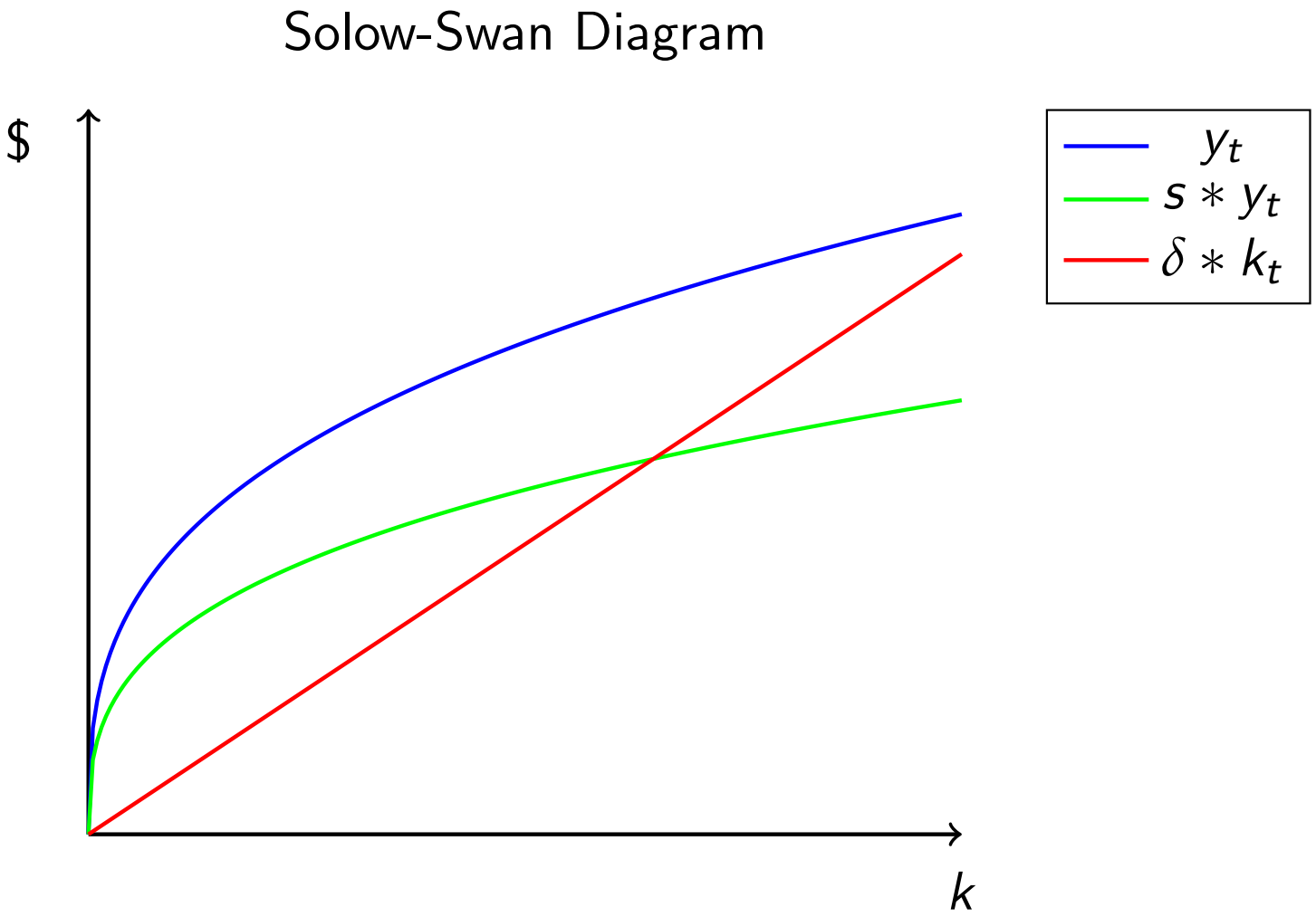

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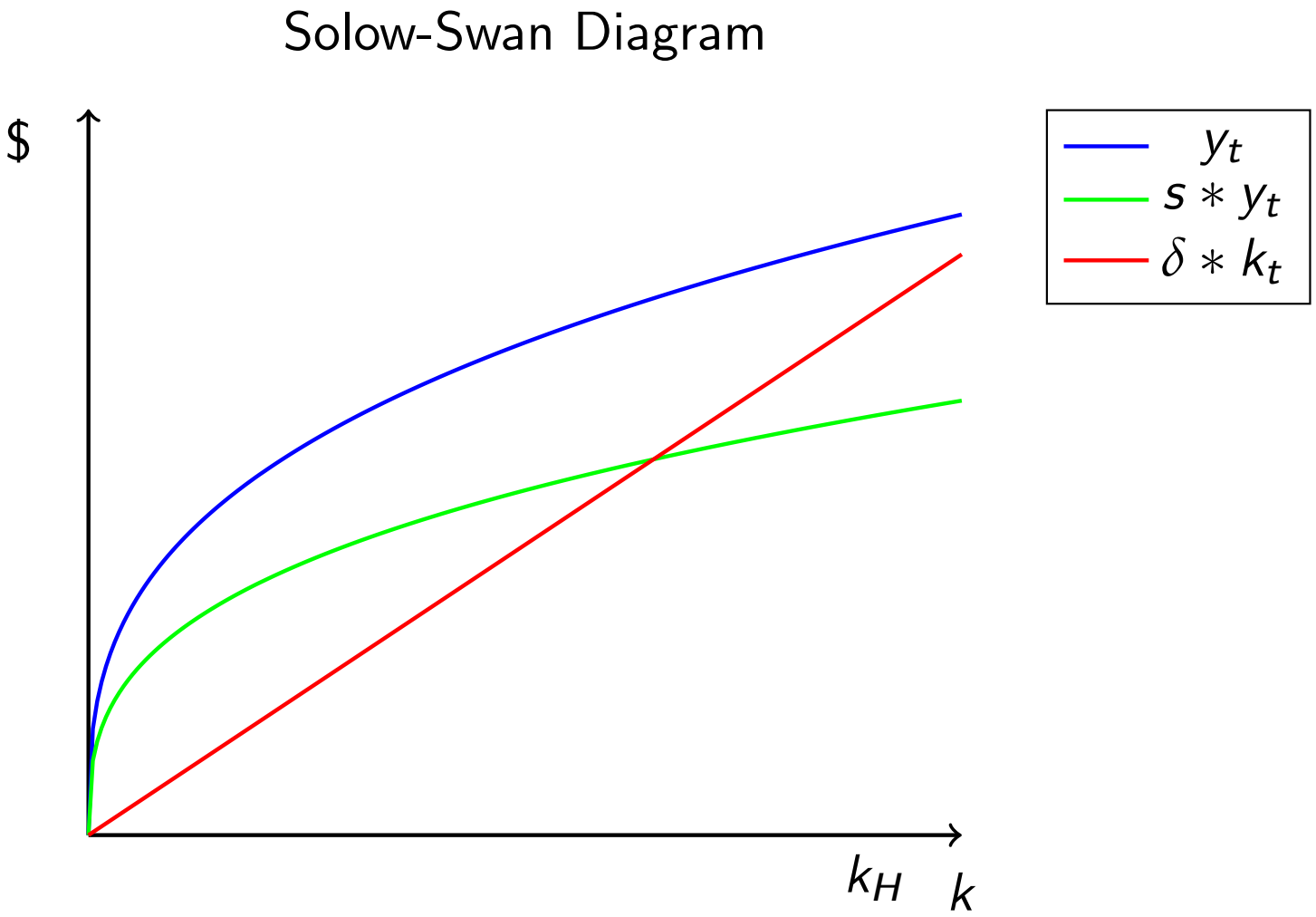

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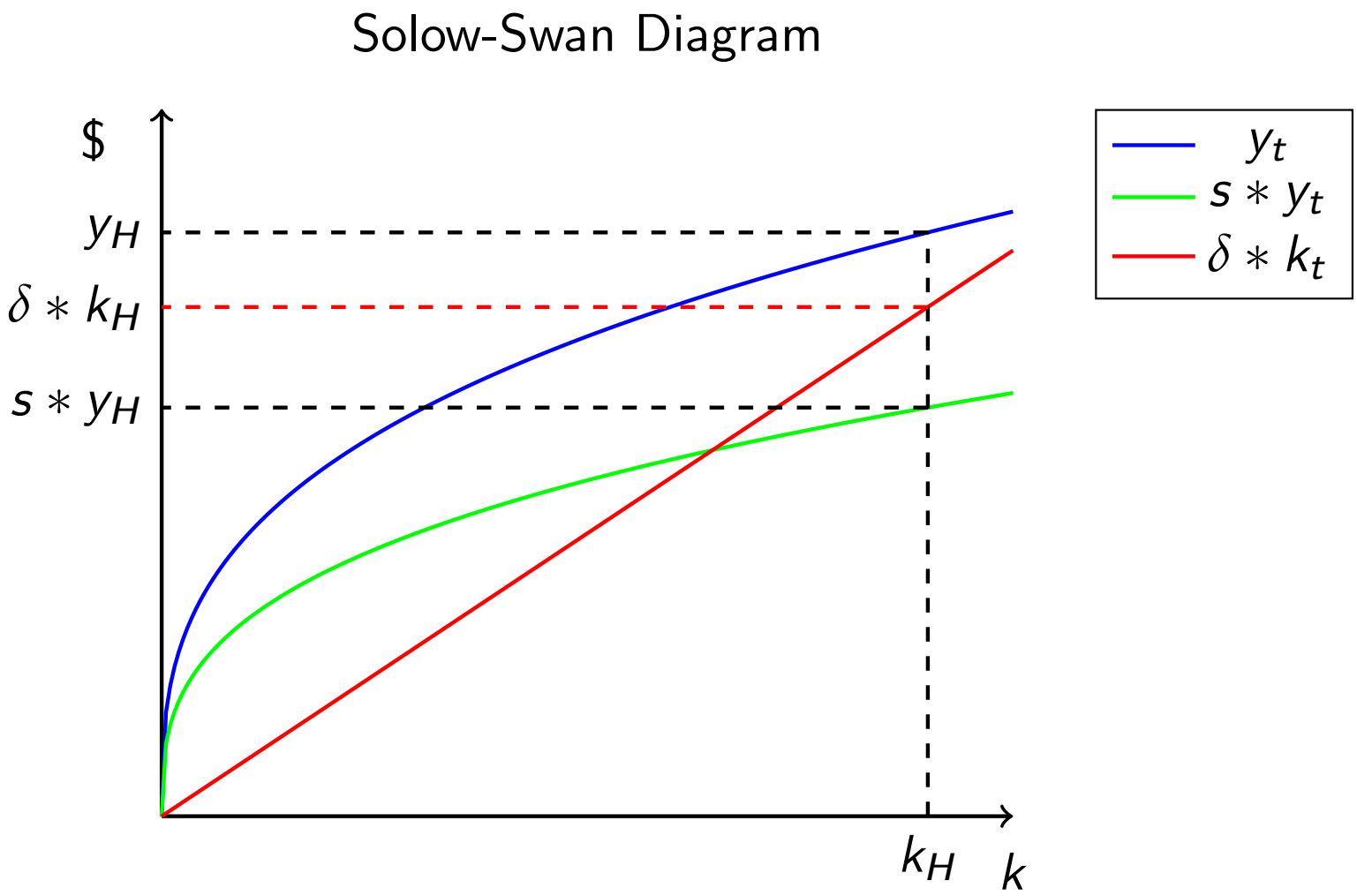

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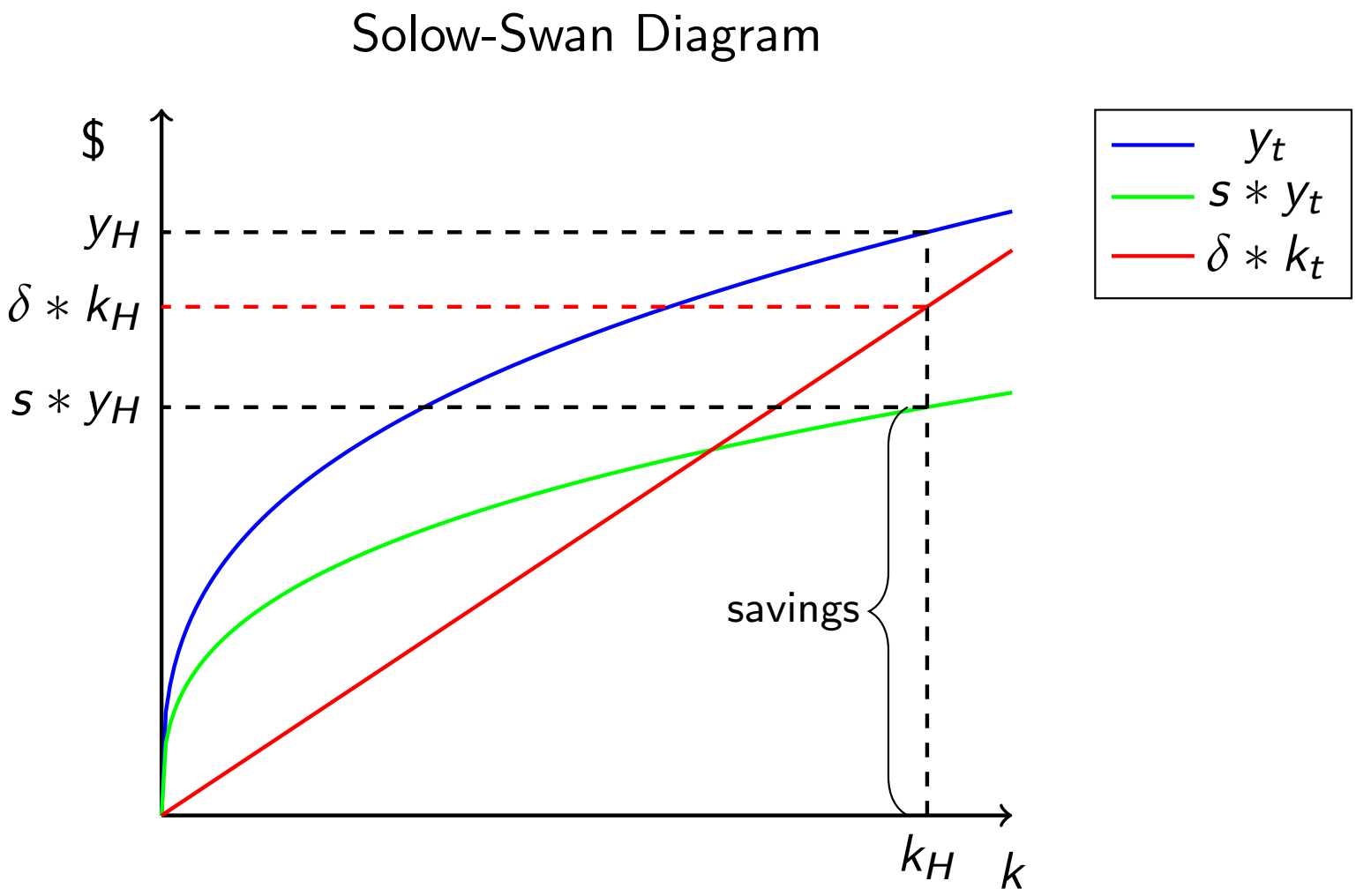

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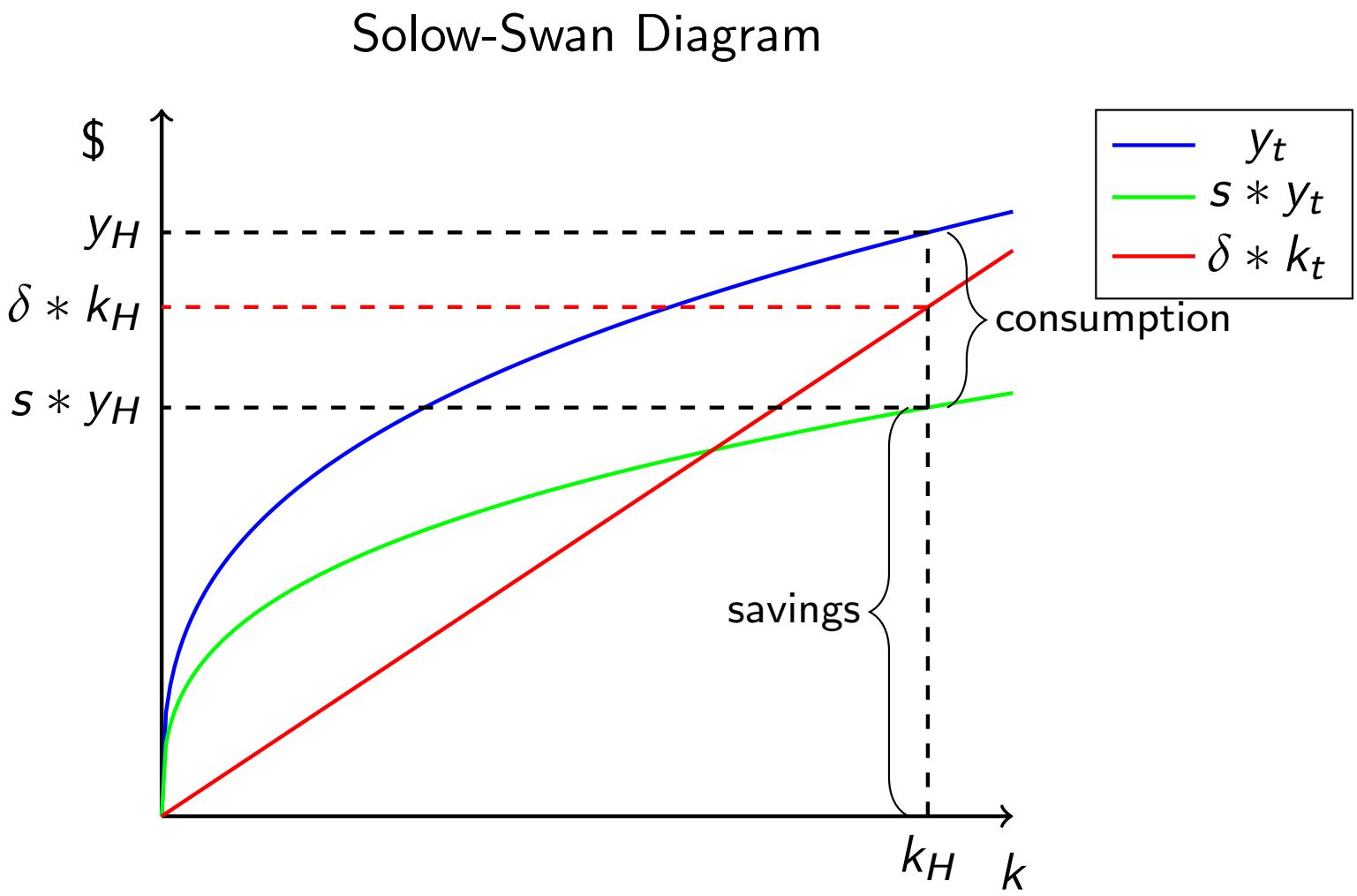

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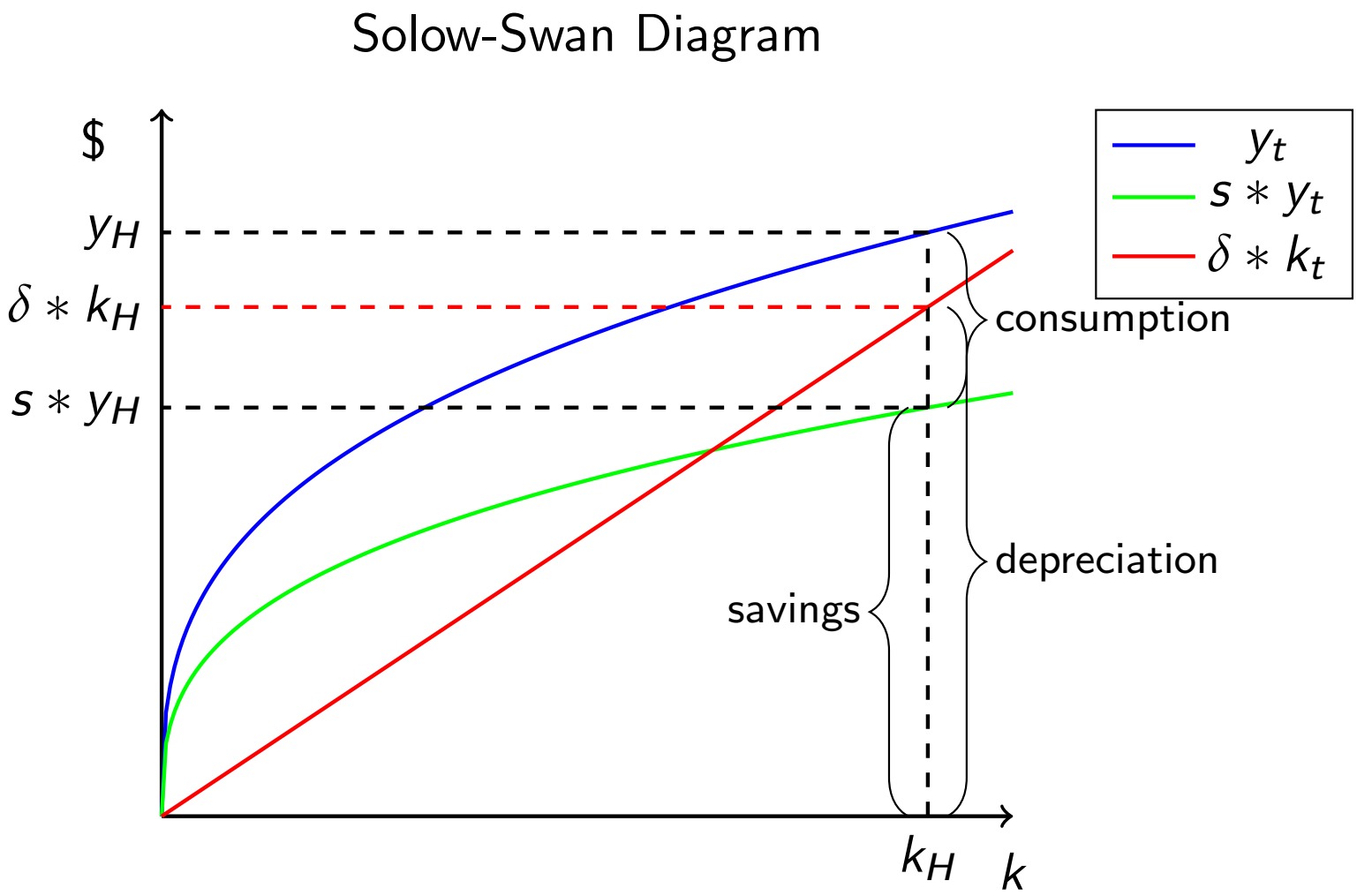

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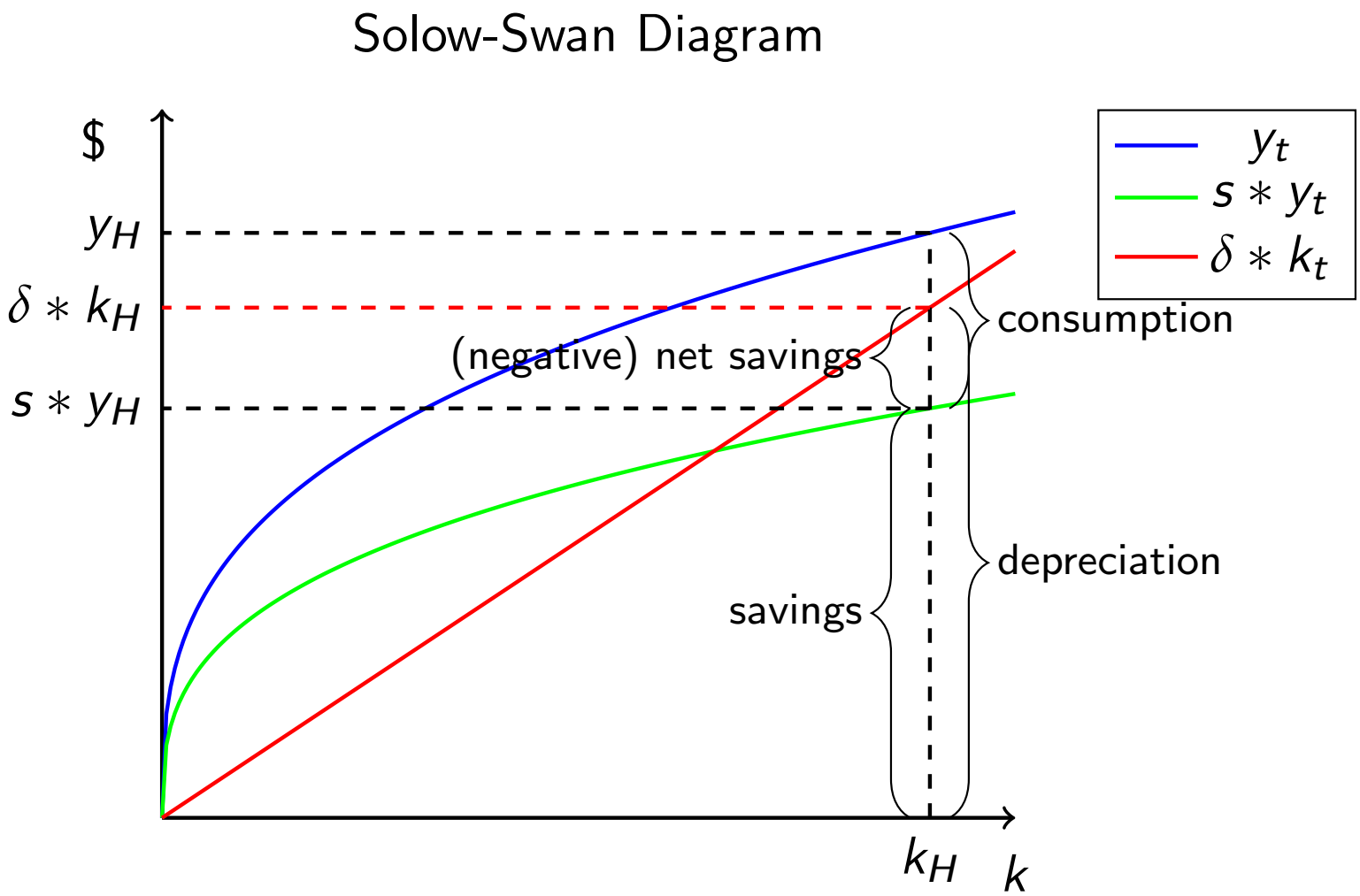

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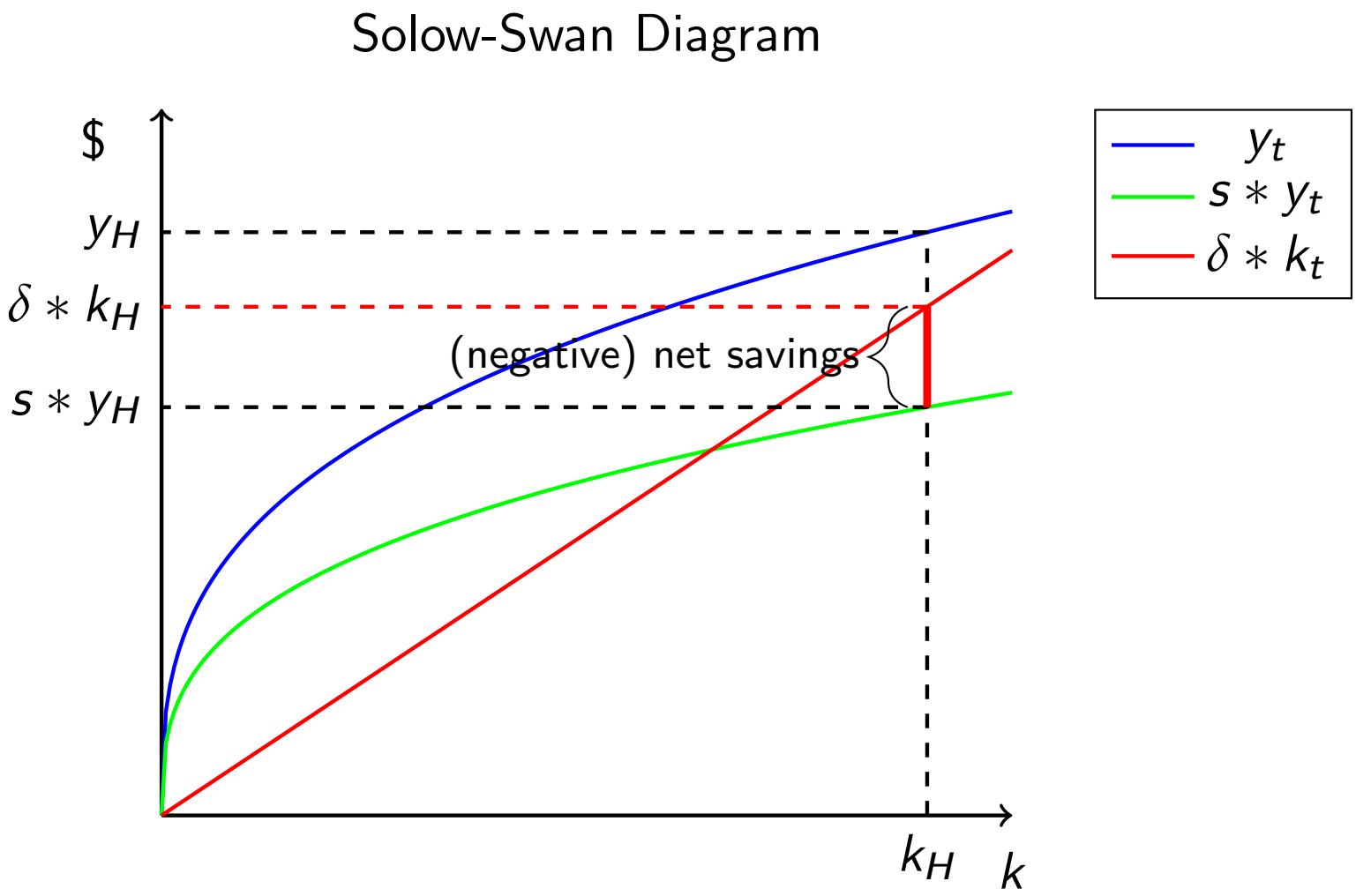

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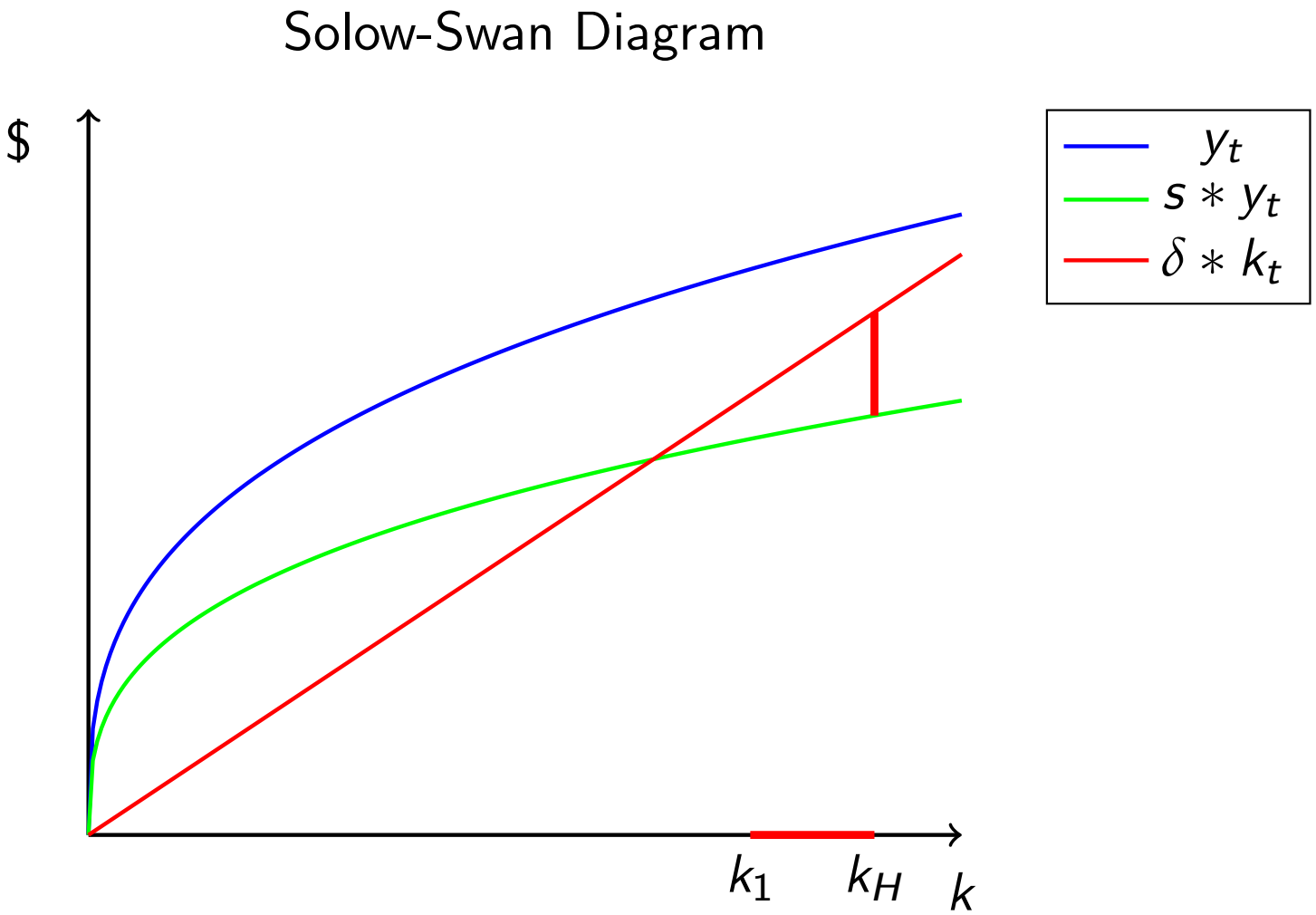
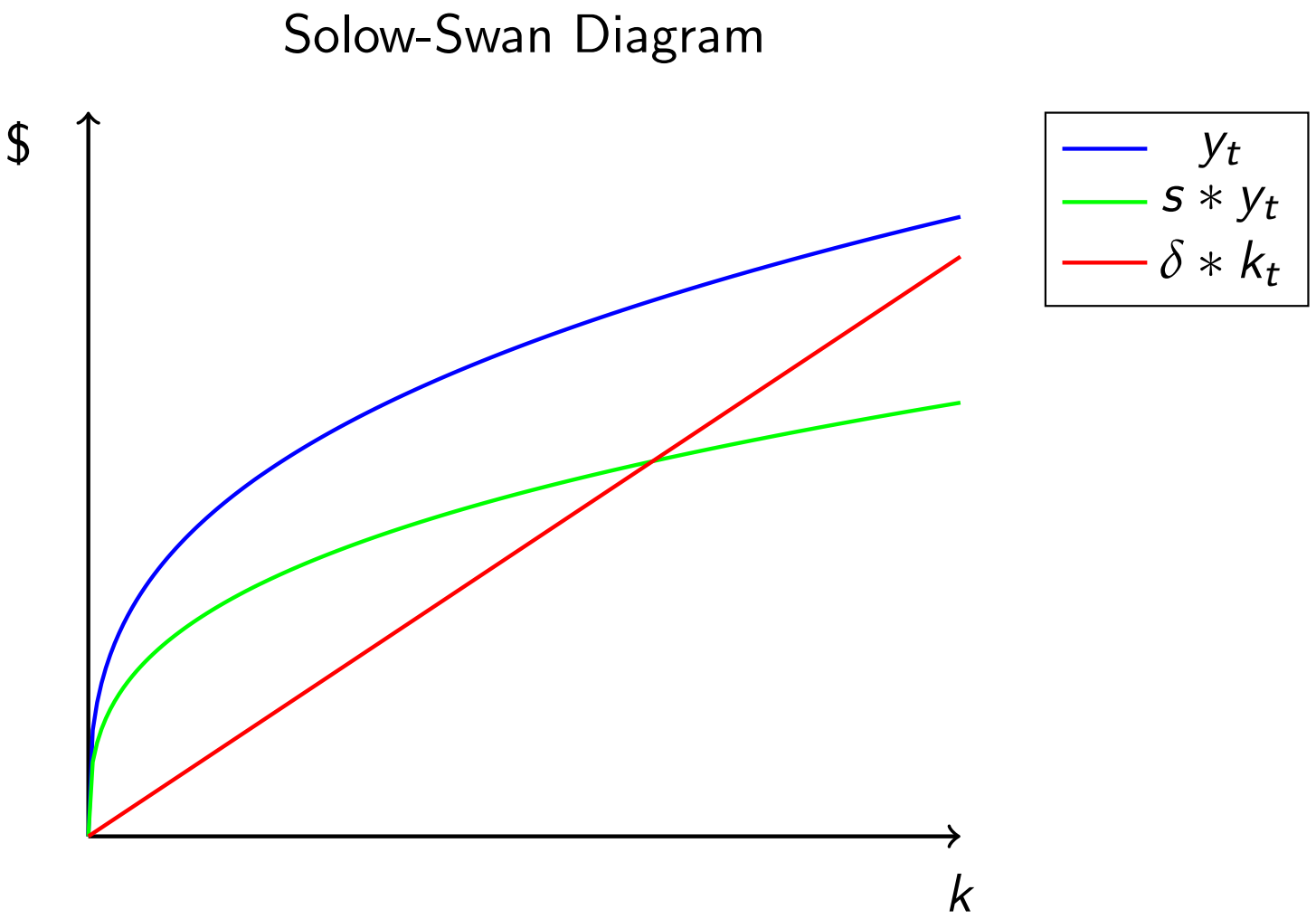


Equilibrium Output in Solow-Swan



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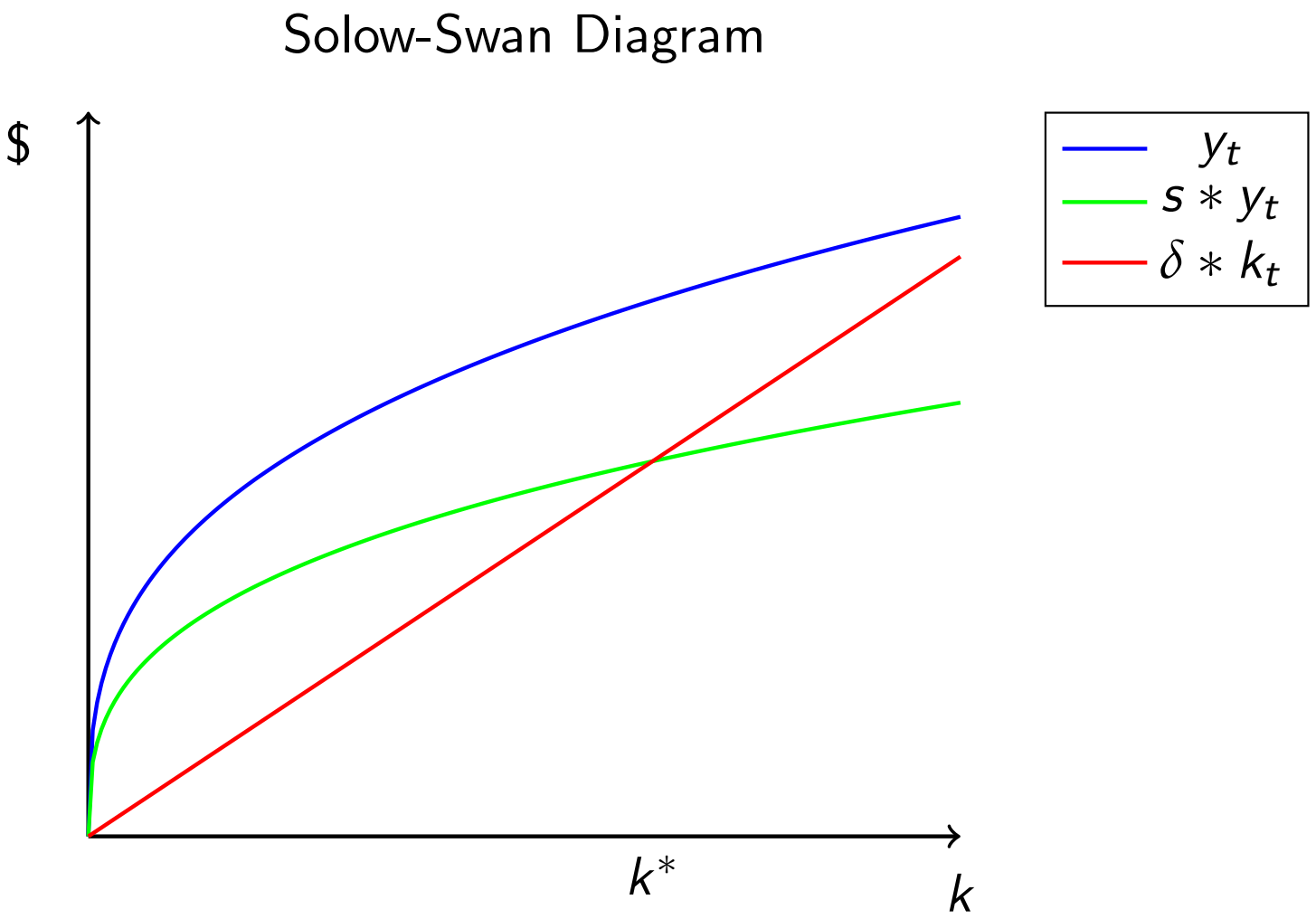

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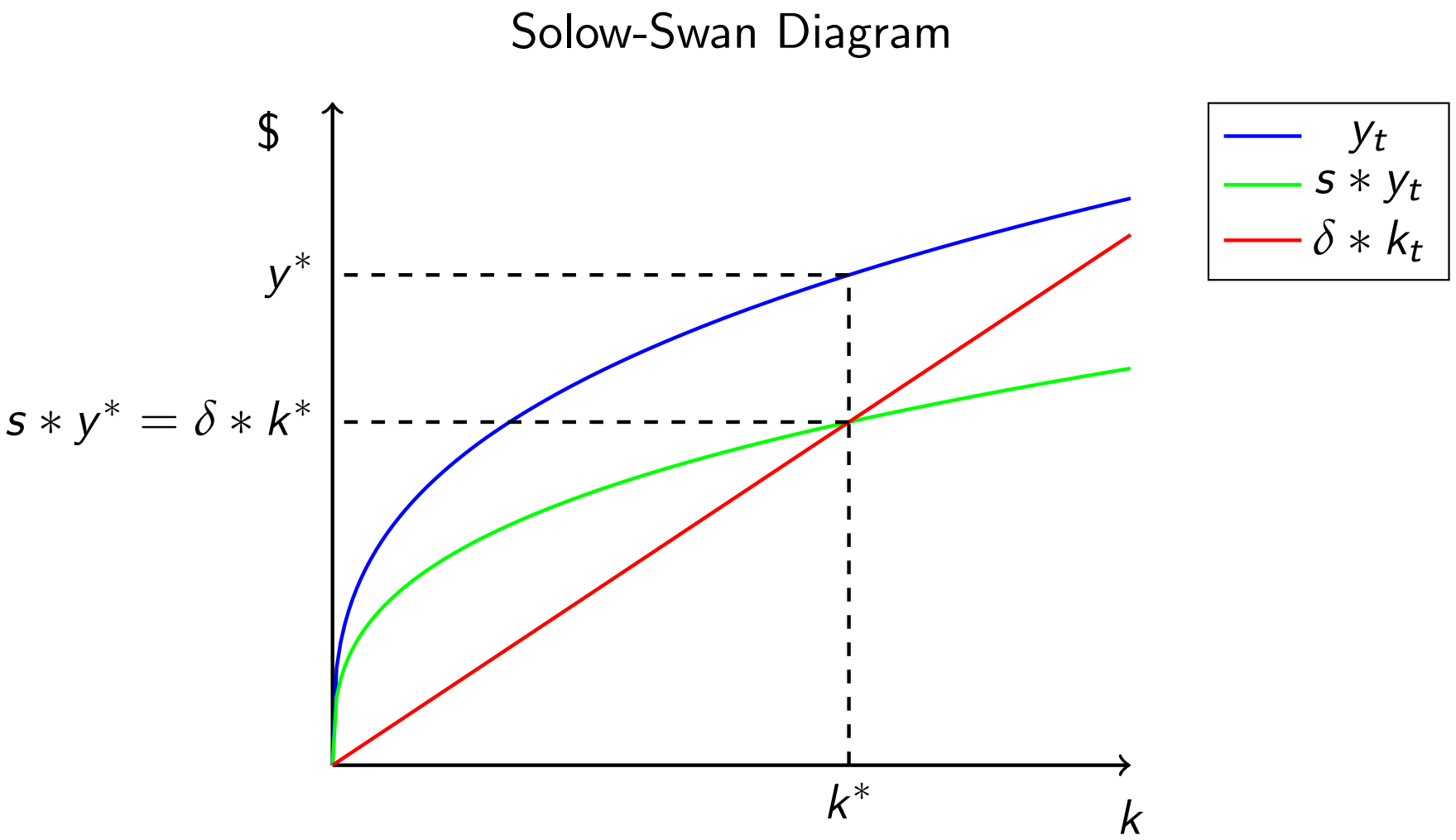

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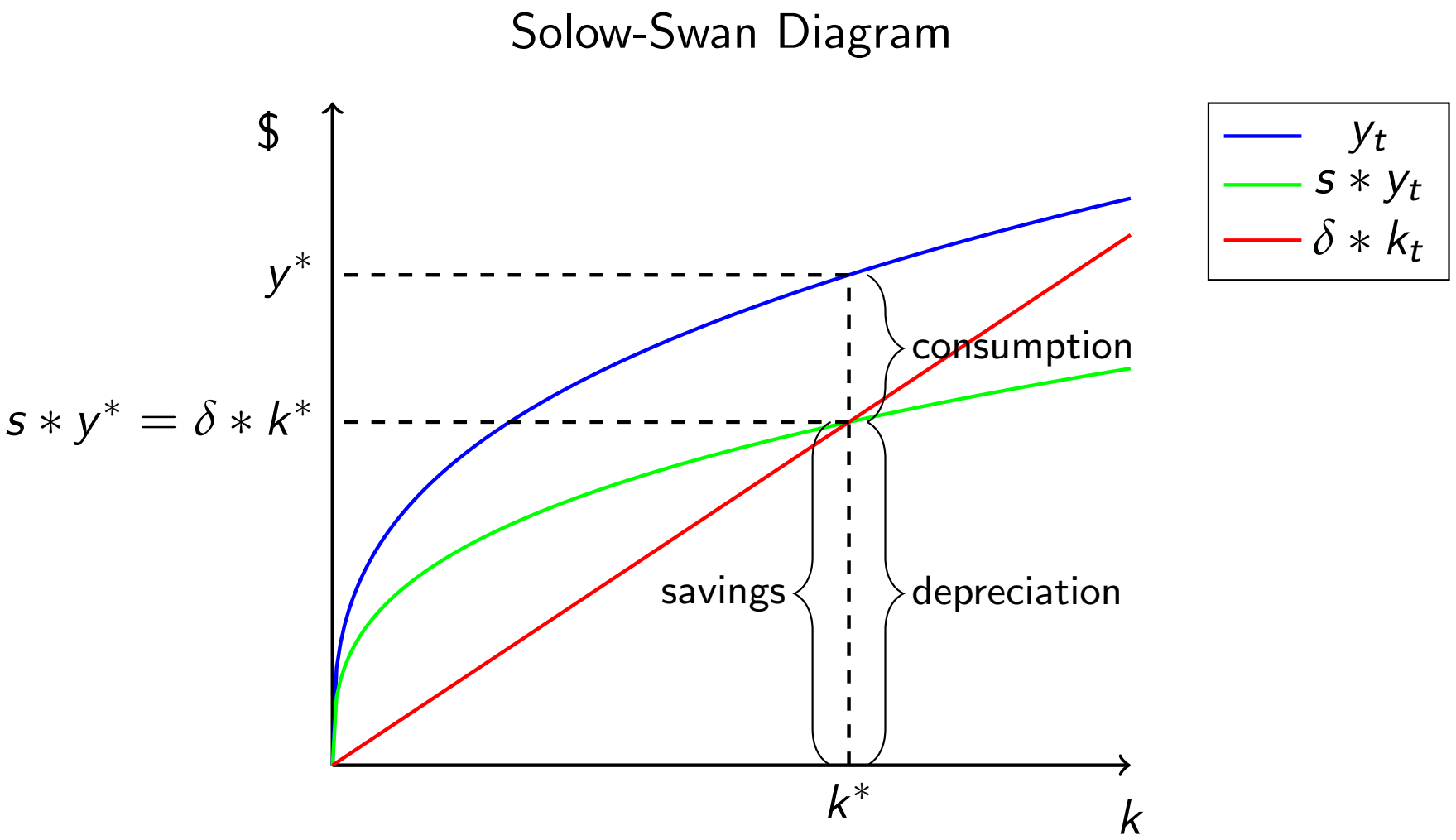

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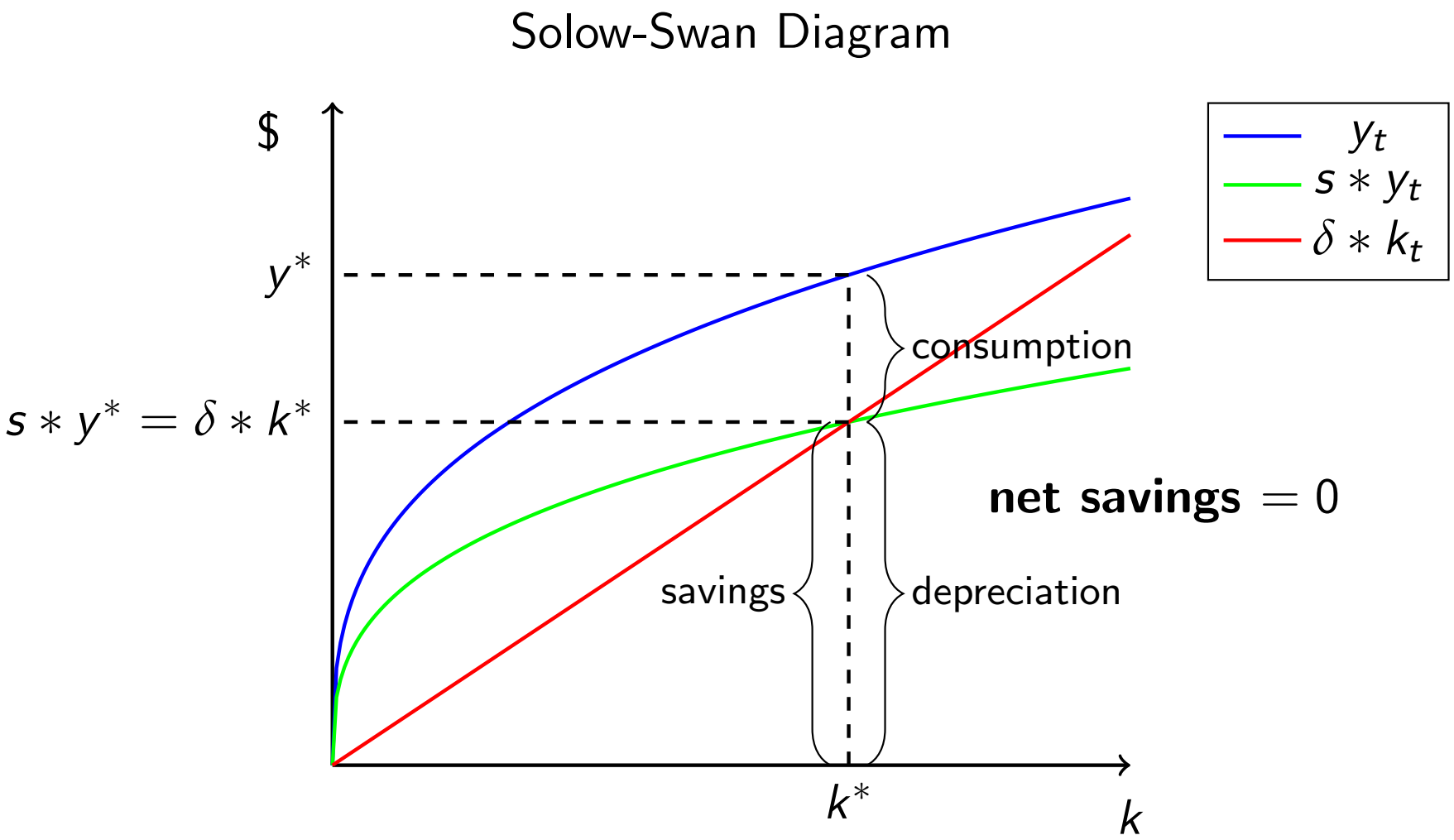


Long-run Economic Growth in Solow Model

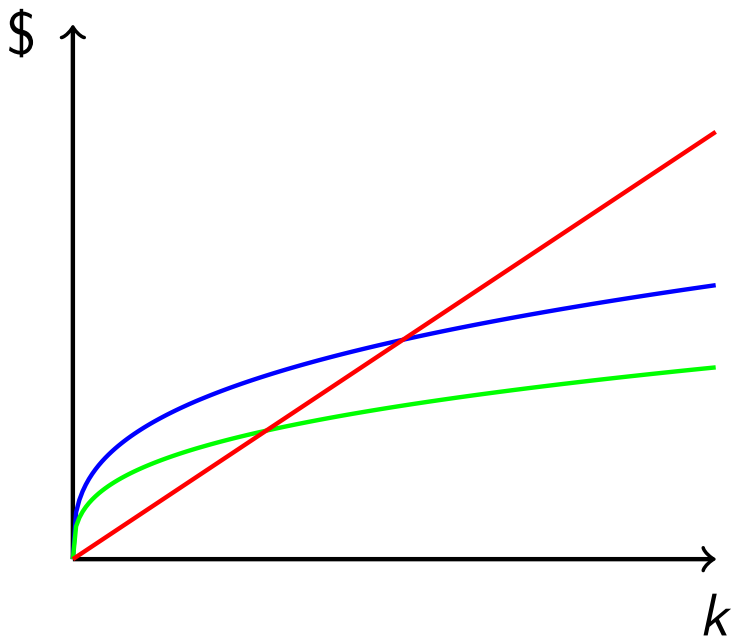
- We want to increase the *steady-state* output level
- MAIN MESSAGE: Capital Accumulation will not work

Long-run Economic Growth in Solow Model

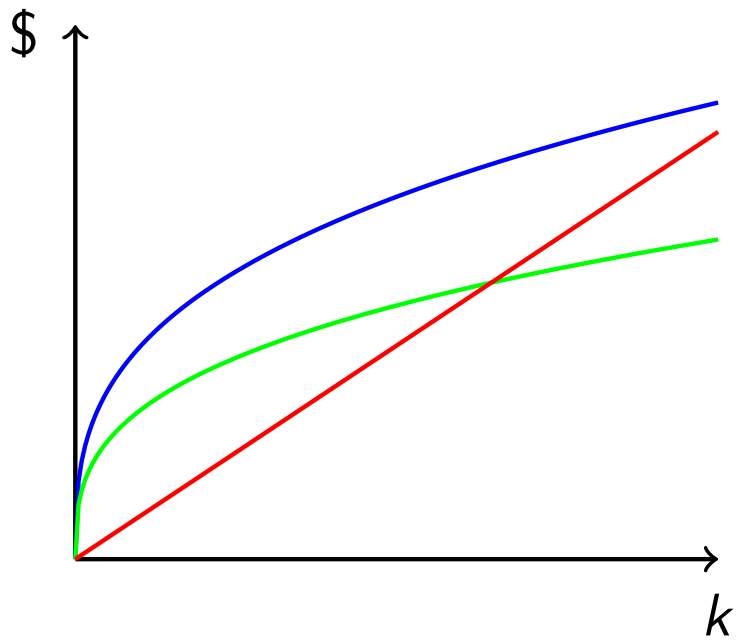
- We want to increase the *steady-state* output level
- MAIN MESSAGE: Capital Accumulation will not work
- Other options:
  - Technological Progress - Success
  - Decrease Depreciation - Works if it could be implemented
  - Increasing the Savings Rate - Partial Success, interesting

Technology Growth and the Steady-State

Low Technology



High Technology



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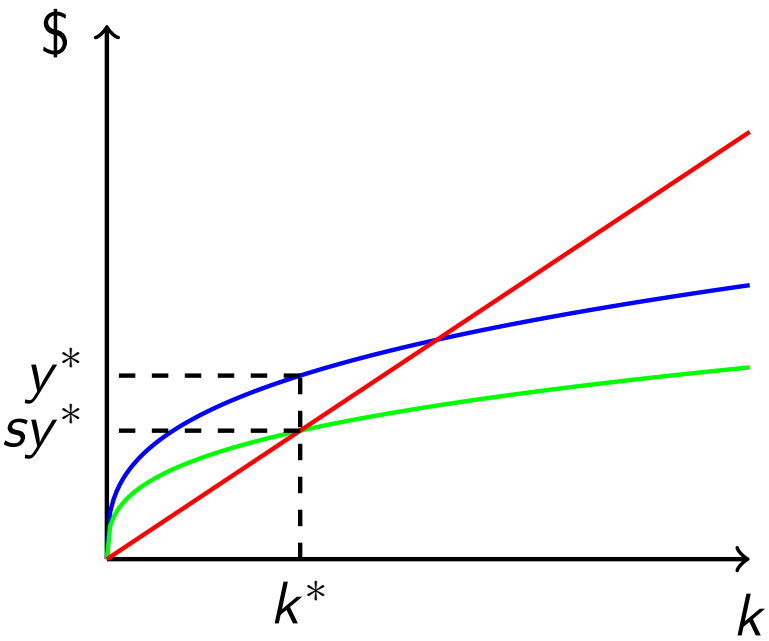

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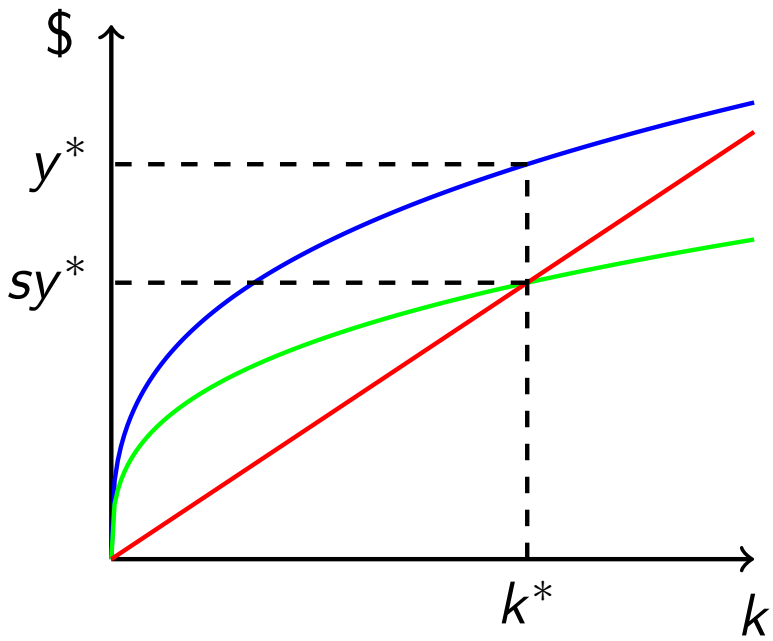
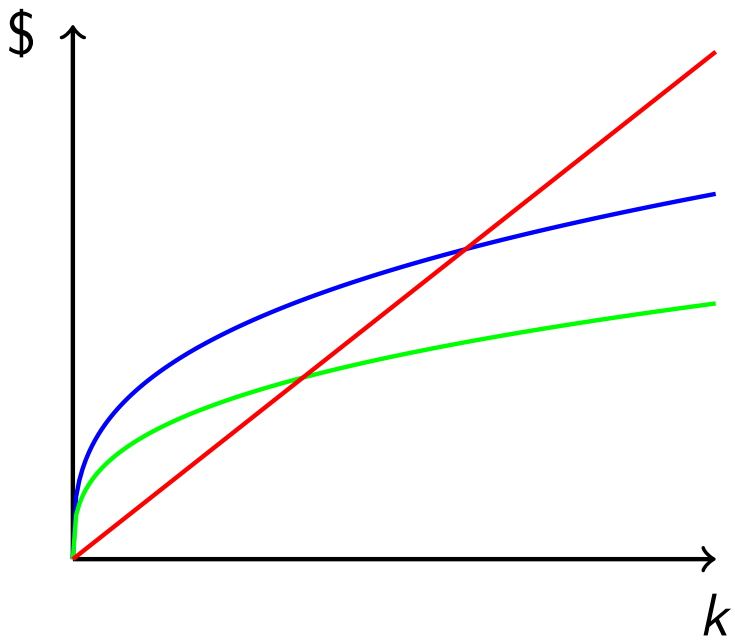
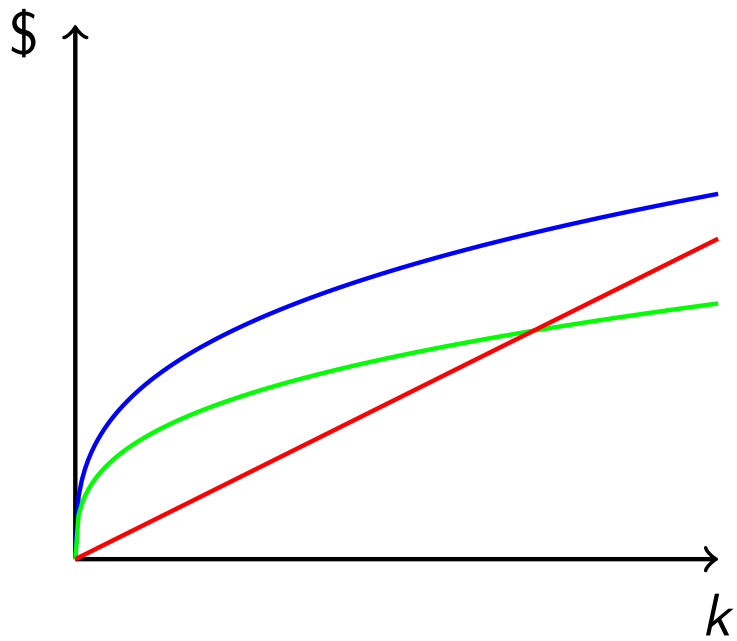


Depreciation Rate and the Steady-State

High Depreciation

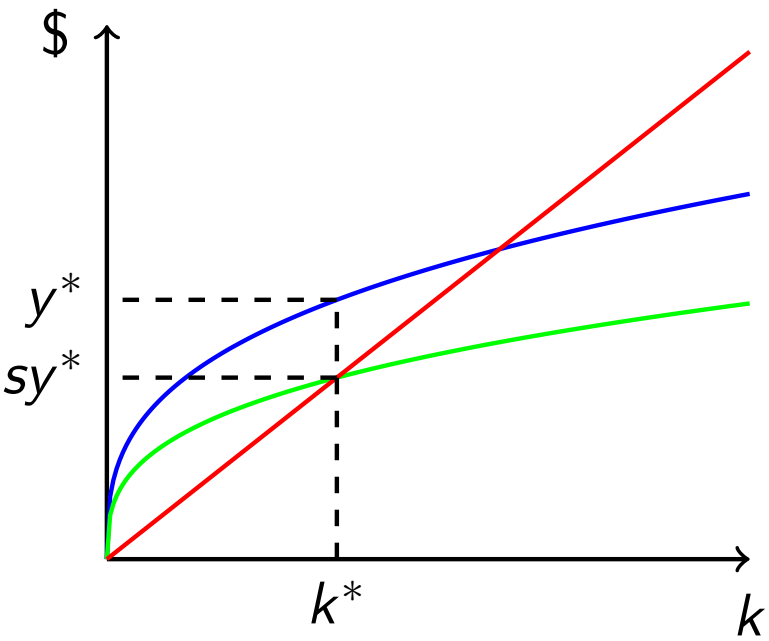


Low Depreciation



Depreciation Rate and the Steady-State

High Depreciation



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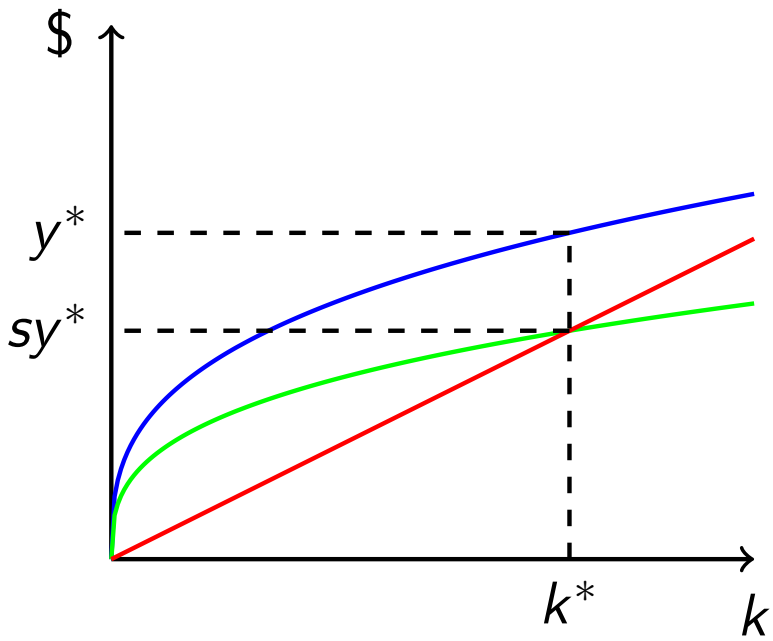
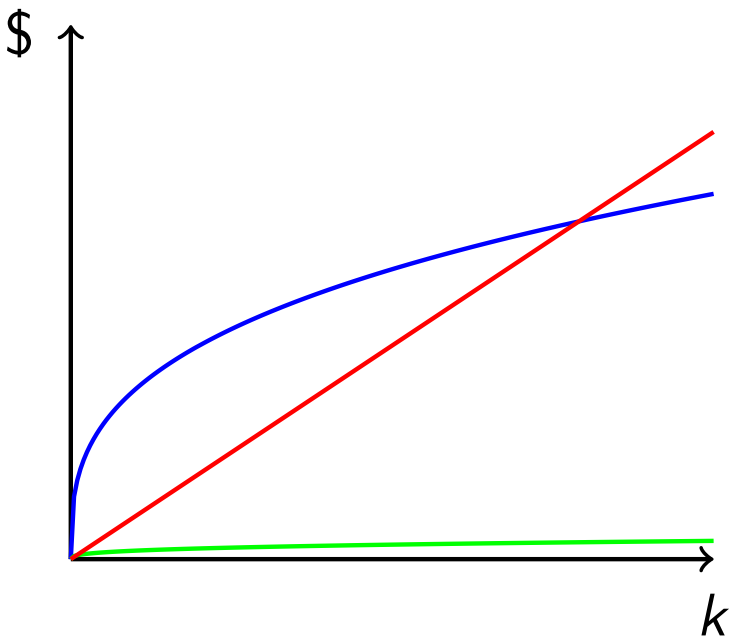


Savings Rate and the Steady-State

Low Savings Rate $s = 0.05$



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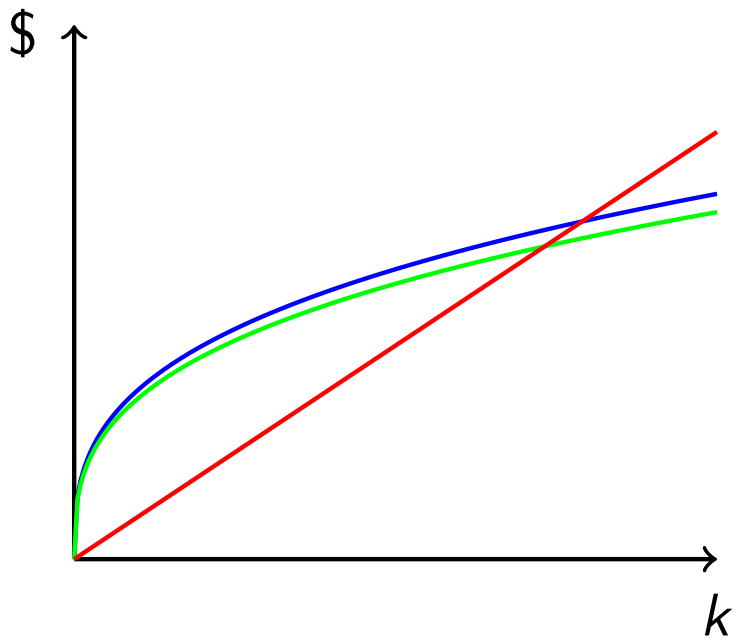

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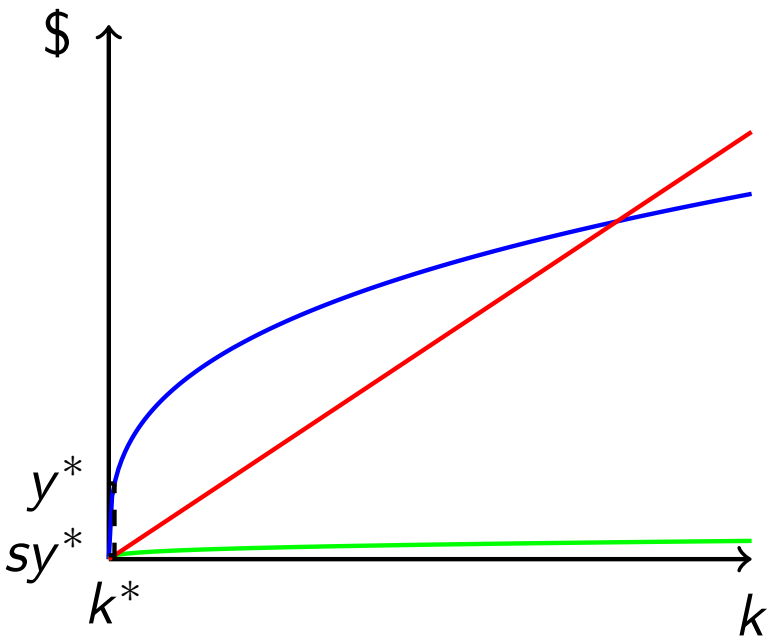

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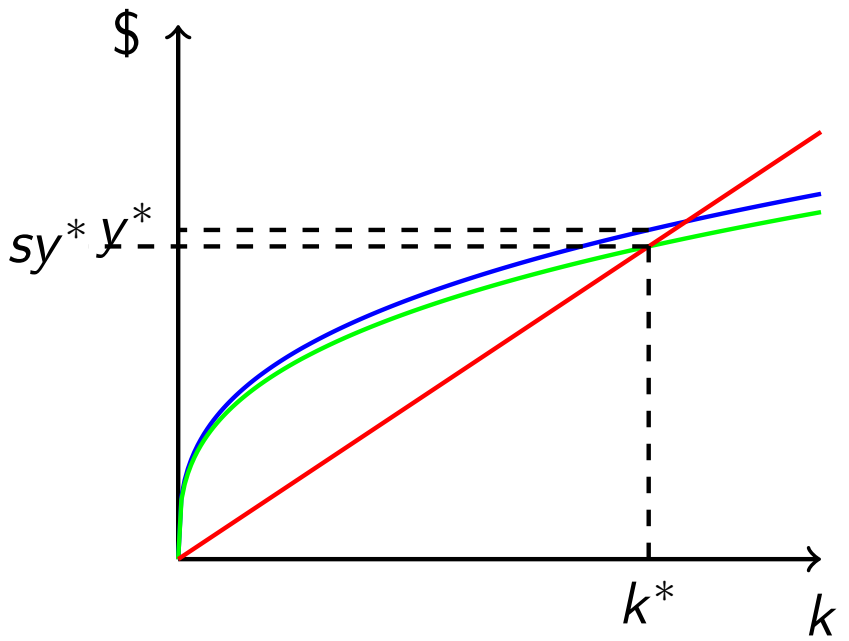
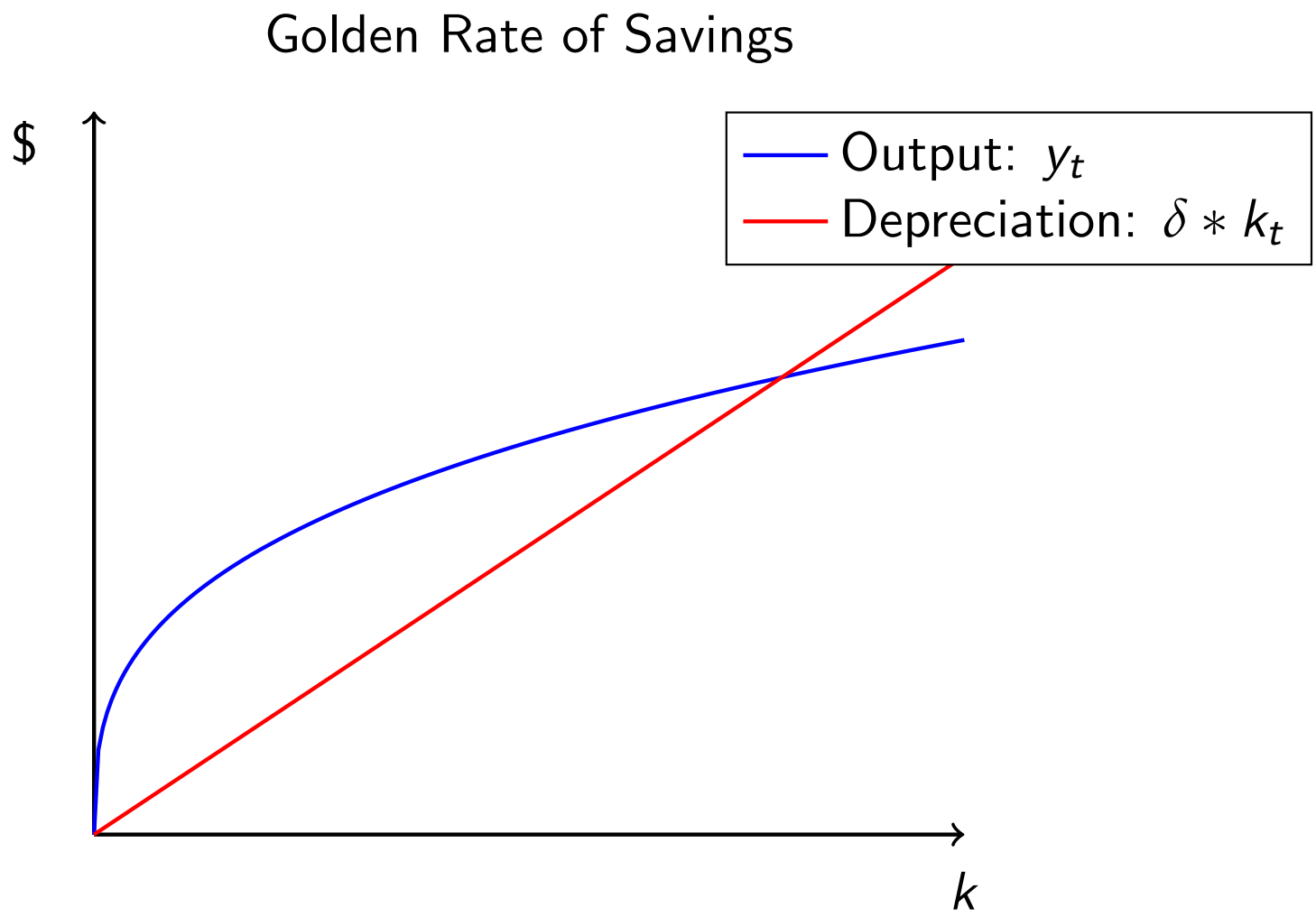


Golden Rate of Savings

- However, we don't even really care (that much) about production
- We care about consumption per person $c_t = C_t / H_t$
- Graphically, c_t is the gap between y_t and sy_t
- There is a level of savings which maximises this gap
- Call this level of savings the Golden Rate of Savings

Choosing Savings to Maximise Consumption



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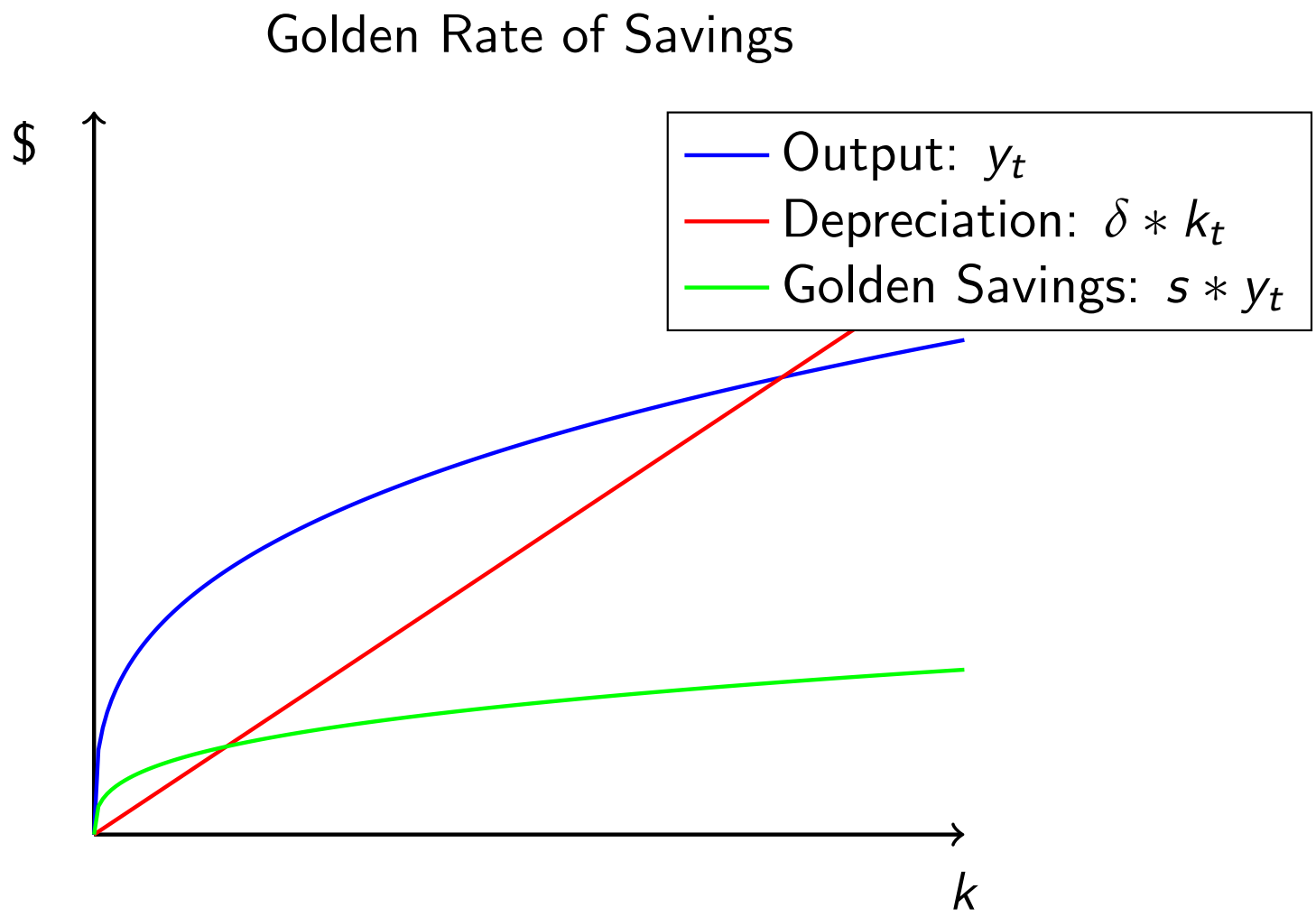

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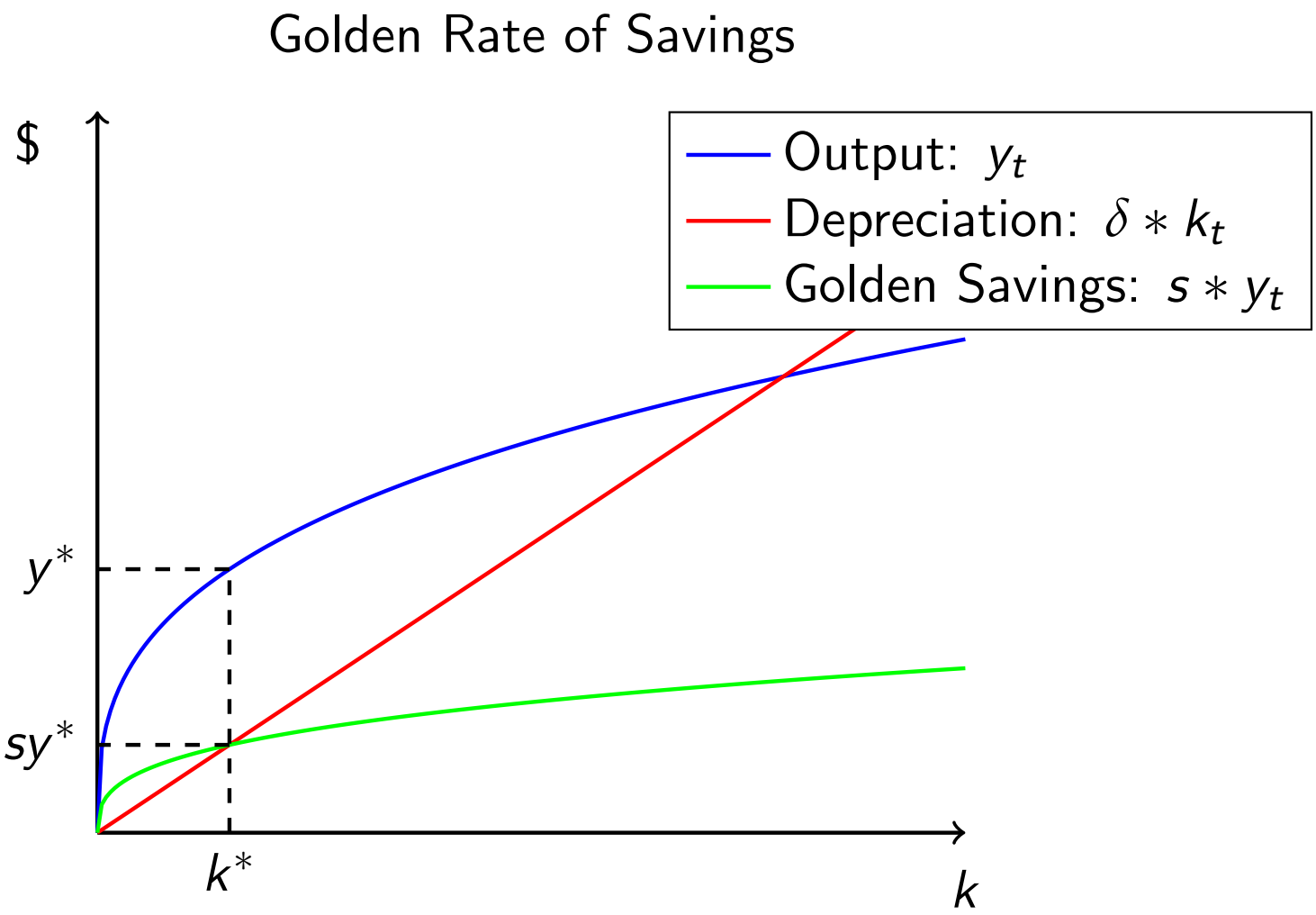

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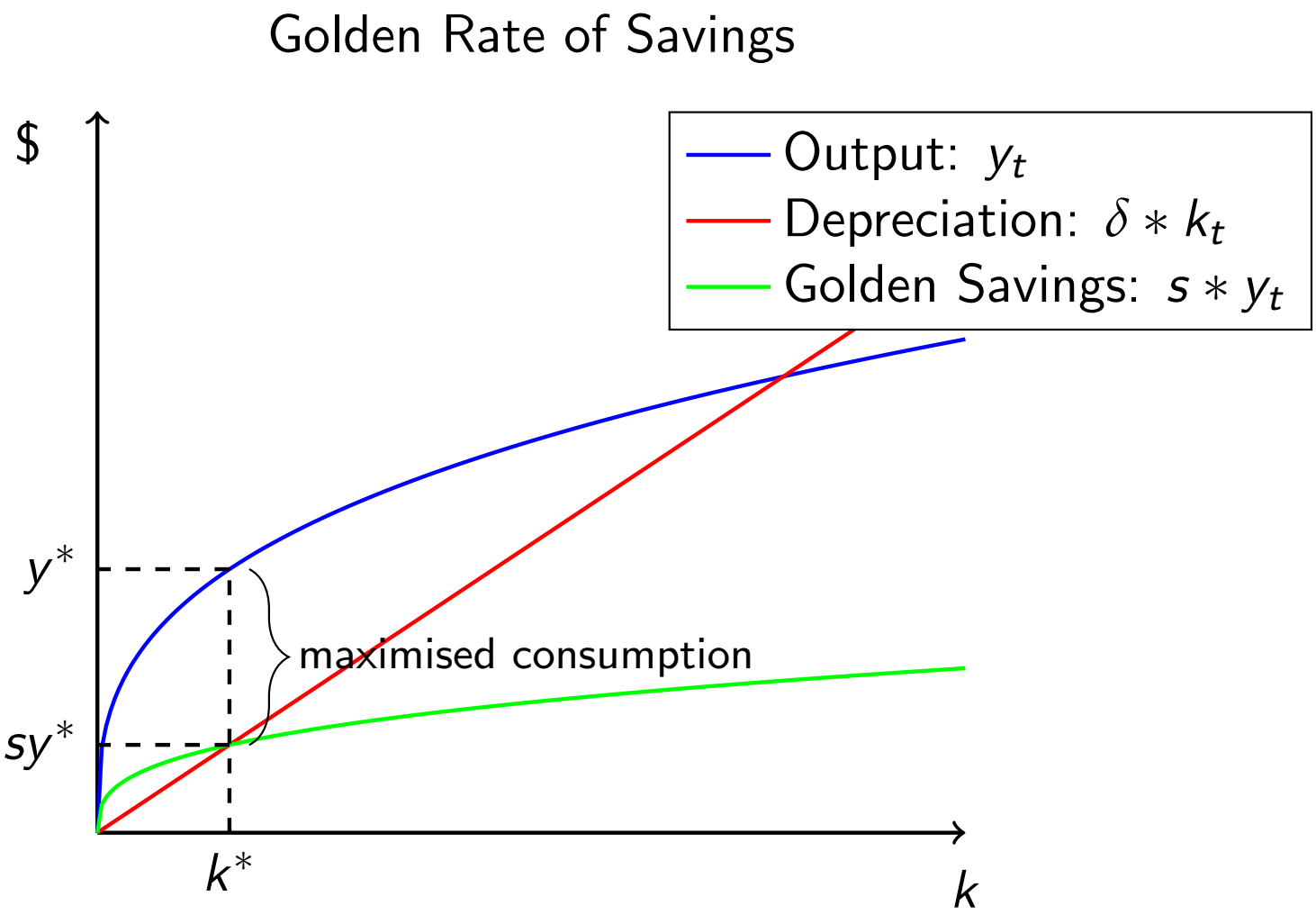

Extensions to this model

There is a huge literature extending this model

- The original model was in continuous time, allowed (exogenous) changes in H_t, and did not have technology A
- The Romer model works with human capital in a more substantial way
- The Ramsey-Cass-Koopmans model endogenises the savings rate, and allows it to change over time
- Many many others use Solow-Swan as the starting point

Economic Growth and Public Policy

Various government policies can encourage economic growth:

- encourage savings (and investment)
- encourage investment from overseas
- encourage education, health, and nutrition
- maintain political stability
- promote research and development

Growth - Encourage Savings

- As we saw earlier, this will increase steady-state production, but may fail to increase well-being
- Also, there is definitely a short-run reduction in well-being
- Savings are encouraged by the government changing the *interest rate*, more on this later

Growth - Overseas Investment

- Investment funds may also come from external investors
- This is outside of our (simplified) Solow model
- Investment might be in physical capital (foreign direct investment), or financial capital (foreign portfolio investment)
- Foreign countries invest because they expect a return (or profit) on their investment

Growth - Education

- In our Solow model, increasing education increases human capital h
- It's a little hard to analyse, but this will increase well-being in the steady-state
- Still has diminishing returns
- But, there may be positive externalities associated with higher education (outside our model)

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- The 'brain drain' effect may be significant, especially for poorer countries

Growth - Health and Nutrition

- All else equal, a healthier workforce is more productive
- This could be modelled either as part of Technology A, or more likely human capital h
- Health is a 'virtuous cycle', healthier workers are more productive, which gives more production, which can be used to further increase health

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- Obesity issues are starting to break this virtuous cycle

Growth - Political Stability

- Rule of Law and Enforceable Contracts

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- Rule of Law allows Market Formation
- These are all **institutions**

Growth - Research and Development

- Technology growth is the main (only?) factor driving long-run growth
- Research and Development is a major source of technology (though strong institutions also play a role
- R & D includes both public and private expenditure
- Given large positive externalities, many economists suggest government should subsidise R & D
- Patent systems to encourage innovation are important, but tricky
  - It has been suggested that current intellectual property settings are so restrictive, that they are worse than nothing at all

Growth - Geography Hypothesis

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- Even so, this hypothesis is not common, and leaves much to be explained, e.g. Singapore

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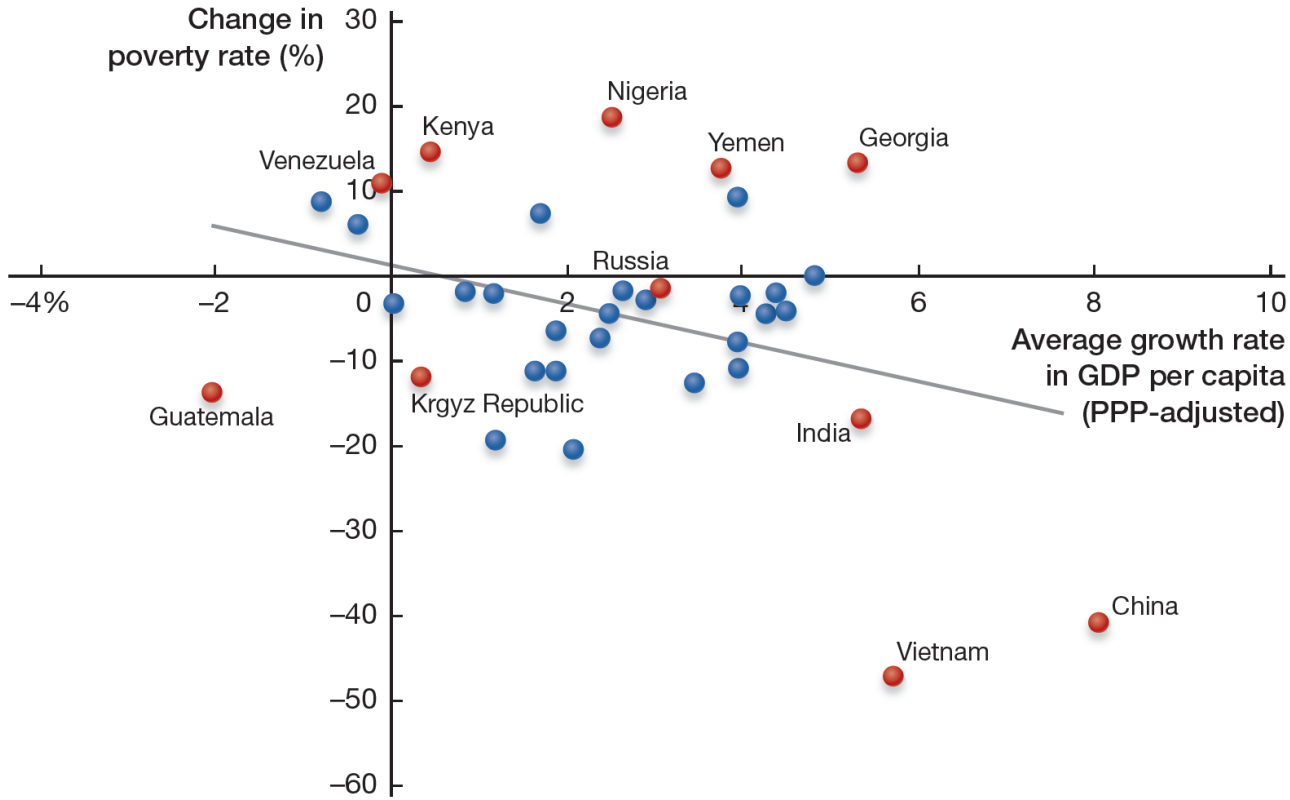
- This claims that different values and cultural beliefs drive differences in prosperity
- Tends to be more overtly bigoted or racist than the geography argument
- Rarely considered a strong argument

Growth - Poverty

- Economic growth does not always mean everyone becomes better off
- Rapid increases in GDP are often (though not always) associated with increases in inequality



Source: Economics by Acemoglu et. al.