General Physics A (II) – Spring Semester 2019

Homework Set 3

Due: 4/10/2019 (Wed.)

Problems 1, 2, & 3 (60 pts)

[Benson] (P=Problem)

Ch 22: P2

Ch 23: P19

Ch 24: P9

Problem 4 (40 pts)

In this exercise, we are going to look at the electric field distribution within a hollow region for given boundary potentials. Please do the following:

1. Key concept: For a point $P = (x, y, z)$ in free space, consider its neighbor points along the x, y, and z axes, respectively: $Q_{1,2} = (x \pm \delta, y, z)$, $Q_{3,4} = (x, y \pm \delta, z)$, and $Q_{5,6} = (x, y, z \pm \delta)$. (a) Treat δ as an infinitesimal distance and expand the potentials $V(Q_i)$ for $i = 1, \cdots, 6$ with respect to P to the second order, δ^2. (b) Show that up to the second order, the potential at P can be written as: $V(P) = \overline{V(Q_i)}$, the average of the potential values of its neighbors.

2. Now as an example, we consider a uniform infinitely long rectangular box (along the z direction), whose cross-section is square with 4 boundary sides of different potentials, V_1, V_2, V_3, and V_4 in clockwise order. Here are the guidelines to numerically calculate the potential and the electric field inside the hollow region for boundary V_i we choose:

   (a) Since the system is uniform along the z direction, the field has no z component. Therefore we only need to consider the x, y components.

   (b) For visualization of the electric field, we may construct a 30×30 two-dimensional square array. Each element is a vpython object `sphere`. These spheres are regularly placed over the cross-section, which you may assume to be a unit square (size= 1). Remember that the radius of a sphere needs to be small enough to avoid overlap with each other. Use the `sphere` property `pos` x and y to store its location, and use `pos` z to record the potential value at this location.

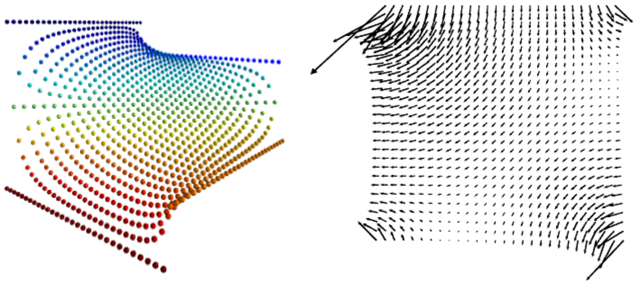

Figure 1: (Left) The potential surface. (Right) The field distribution. Here we choose $V_1 = 1$ V, $V_2 = -2$ V, $V_3 = -3$ V, and $V_4 = 0$ V.

(c) Initialize the potential values (`pos z`) of the boundary spheres according to V_1, V_2, V_3, and V_4 in clockwise order. Choose your own values for $V_{1,\cdots,4}$. Set the potential values of all others to zero.

(d) Keeping the boundary ones unchanged, compute and update the potentials for rest of the array iteratively according to $V(i,j) = \frac{1}{4}[V(i-1,j)+V(i+1,j)+V(i,j-1)+V(i,j+1)]$ until the potential values converge to a constant profile. Here, i and j in $V(i,j)$ indicate the x and y indices of the array. This profile represents the potential distribution within the hollow region.

(e) To compute the electric field, we need to take 2-component derivatives (called gradient) of the potential. The mathematical expression is

$$E_x\hat{x} + E_y\hat{y} = -\frac{\partial V(x,y)}{\partial x}\hat{x} - \frac{\partial V(x,y)}{\partial y}\hat{y}.$$

For numerical analysis, we may use

$$\frac{\partial V}{\partial x}(i,j) \approx \frac{V(i+1,j)-V(i-1,y)}{2\delta}$$
$$\frac{\partial V}{\partial y}(i,j) \approx \frac{V(i,j+1)-V(i,y-1)}{2\delta}.$$

Plot the field strengths and directions on a chosen projected x-y plane. You may use the vpython object `arrow` to indicate the field (strength=arrow length, which can be rescaled) inside the region. Please check online help for its usage.

(f) For this part, you are required to output two plots showing the potential surface and the field diagram as shown in Fig. 1.