Density Management Diagram for Northeastern Red Spruce and Balsam Fir Forests

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ABSTRACT: *A stand-density management diagram is presented for use in northeastern red spruce and balsam fir forests. The diagram was derived from an extensive archived data set collected during the 1970s from fully stocked stands throughout northern Maine and a more recent study of precommercially thinned stands. The negative exponential relationship between mean stemwood volume per tree and stand density, commonly known as the "self-thinning rule," was formulated to define a biological maximum stand density. The maximum size–density equation can be used to calculate the relative density of any stand and is accurate for thinned and unthinned natural stands as well as plantations. Equations for estimating quadratic mean diameter and stand top height are also derived for unthinned natural stands only. Data used to fit the self-thinning line are substantially above the A-lines on the familiar northeastern stocking guides, suggesting that these guides underestimate maximum density and thus overpredict self-thinning. Examples illustrate how to use the diagram to predict stand development under commercial and precommercial thinning scenarios, as well as natural stand development without thinning. Relevant site index and volume equations are included in an appendix. North. J. Appl. For. 16(1):48–56.*

Even-aged red spruce (*Picea rubens*)–balsam fir (*Abies balsamea*) stands in the northeastern United States typically develop from prolific advance regeneration following overstory removal (Seymour 1992). Subsequent herbicide treatment controls hardwood and herbaceous competition (Newton et al. 1992), but often results in extremely dense stands that exhibit slow diameter growth and heavy competition-induced mortality. Initial attempts at commercial thinning in such dense stands, as the crop trees first reach merchantable size (about 5 in. dbh at ages 30–40), have been uneconomical due to the sheer number of unmerchantable stems. To shorten rotations and the time to merchantability, costly precommercial thinning (PCT) has become common (Seymour 1995). However, few guidelines are available to foresters in the Northeast to develop rigorous stand-density management prescriptions for planting, early spacing, or commercial thinning.

The density management diagram (DMD) has become an increasingly important field guide in many regions of the United States (Drew and Flewelling 1979, Smith 1989, Dean and Jokela 1992). These diagrams are partly based on the self-thinning rule, which describes the consistent relationship between average tree size and density in stands experiencing competition-induced mortality (Westoby 1984, Jack and Long 1996). Because DMDs estimate the *maximum* size–density relationship, they allow foresters to determine *relative density* (RD) for any stand. RD is defined as the ratio of the actual density to the maximum density for stands with the same mean tree size (Curtis 1970). RD has the advantage of being independent of site quality (measured by site index) and stand age over the wide range of densities that follows a similar self-thinning trajectory. This generality is what gives this approach such wide utility. RD has been widely used in growth and yield modeling (Curtis et al. 1981), as a covariate for predicting light extinction coefficients in a stand (Smith 1991), and in developing wildlife habitat prescriptions (Smith and Long 1987). Here, we develop and illustrate common applications of a new DMD and RD measure for even-aged spruce-fir stands in the Northeast.

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Table 1. Species composition expressed as a percentage of the plot basal area in a species.

Data set	Balsam fir	Red spruce	Black spruce	White spruce	Other[a]
Griffin	39.1[b]	51.5	5.9	3.4	0
	(0–100)	(0–100)	(0–93.9)	(0–55.5)	(0)
PCT	84.6	11.3	1.0	2.4	0.6
	(15.0–100)	(0–74.9)	(0–85.0)	(0–43.3)	(0–22.5)

[a] Includes other conifer and hardwood species.
[b] Mean percentage (range).

Methods

Database

A data set was compiled from several sources that spanned the range of ages, soil types, and stand structures typical of even-aged spruce (*Picea* spp.) and balsam fir stands in Maine prior to the 1970s spruce budworm (*Choristoneura fumiferana*) outbreak. The first data set (the Griffin data set) came from several studies used to develop variable density yield tables and site index curves for even-aged spruce-fir stands in western (Brewitt 1971), northern (Johnson 1976), and eastern (Schiltz 1978) Maine. Plot establishment, stand criteria, and sampling methods were held constant across all studies. Single plots supporting only spruce and/or balsam fir were located in stands that met four criteria: (1) they developed without cultural treatment since their origin; (2) they developed free from competing hardwood species; (3) they were free from apparent insect or disease damage; and (4) the codominant and dominant trees differed less than 15 yr in breast height age. Circular plots varied in size to include ten upper crown class trees (dominant or codominant). Lower crown classes (intermediate and overtopped trees) within the plot, having a dbh of at least 0.5 in., were also tallied. A point sample was taken with a basal area factor 10 (BAF) prism, and all count trees were tallied by species, dbh, and crown position. All standing and fallen mortality at least 0.5 in. dbh that originated within the plot was tallied by species and dbh.

All living trees in the plot were felled, limbed, and cut into 4.0 ft sections beginning with a stump height of 0.5 ft. Measurements for each live tree included: (1) species, crown class, total height, and dbh; (2) diameter inside bark at the stump and top of each 4.0 ft section; and (3) ages at the root collar and at breast height. Total bole volume, inside bark, was computed for each tree following the methods in Husch et al. (1972). The stump was treated as a cylinder, and each 4.0 ft section as a frustrum of a paraboloid except the top section, which was treated as a cone.

A total of 205 plots was established in these studies, with 3,152 stem-analyzed trees. Species composition was primarily red spruce and balsam fir; black spruce (*P. mariana*) and white spruce (*P. glauca*) were present on a few plots (Table 1). Within each region, plots were distributed across a wide range of stand ages, heights, and densities (Table 2). Virtually all soil types in the spruce-fir region of Maine were sampled, except organic soils.

Data from sixty-two 0.05 ac plots in young, even-aged spruce-fir stands in Maine that were precommercially thinned between 1977 and 1987 were included to provide a broader range of densities for young stands (Table 2). Measurements were taken in either 1989 or 1990, when all crop trees were tallied for species, dbh, and total height. Total plot volumes were estimated from volume equations that were constructed from 483 stem analyzed trees (Lemin and Briggs 1993). Further description of this data set is found in Briggs and Lemin (1994). An additional twenty-two 0.05 ac permanent plots from western Maine were included that had been precommercially thinned in 1983 and remeasured in 1994 (Table 2). Plot establishment and measurements were similar to the Briggs and Lemin (1994) study. Stands from both studies were precommercially thinned at a relatively young age; when measured, no plots showed evidence of competition-induced mortality in the crop trees.

Data Screening

Average plot size in the Griffin data set was very small (ten upper crown class trees), and certain plots may have been located in well-stocked patches that were not continuous beyond the plot boundary. The Griffin data set was screened for such a bias by comparing the two measures of basal area (BA) available for each plot: (1) the basal area calculated

Table 2. Mensurational characteristics of the plot data. (See also Figures 1 and 3.)

Source	n[a]	Age at breast ht[b] (yr)	Site index[c] (ft)	Density (trees ac^{-1})	Basal area (ft^2 ac^{-1})	DQ[d] (in.)	Top height[e] (ft)	Stand volume[f] (ft^3 ac^{-1})
Griffin	169	45.7[g]	49.0	2,409	202.2	4.6	44.4	4,271
		(6.2–86.0)	(23–77)	(469–19,784)	(39.4–326.4)	(1.1–9.2)	(12–69)	(350–8,920)
PCT	84	Unavail.	Unavail.	811	56.1	3.6	28.3	657
				(380–1,660)	(20.5–130.1)	(1.6–6.5)	(18–44)	(196–1,903)

[a] Number of plots.
[b] Average of the dominant and codominant trees.
[c] Base age 50 yr at breast height.
[d] Quadratic mean diameter.
[e] Average height of the tallest 40 trees/ac.
[f] Inside bark, including top and stump.
[g] Means with ranges in parenthesis.

directly from the dbh of trees on a plot, expanded to a per acre basis, and (2) the basal area determined from the 10-BAF variable-radius prism count. The small BAF (10) allowed this basal area estimate to be used as a surrogate for the stand conditions encompassing a wider area beyond the fixed area plot.

This variability was compared against a similar data set where both the plot BA and a BAF (10) prism count were estimated for 105, larger fixed area plots (0.03 ac) (Schiltz and Grisi 1980). This analysis was used to measure the variation that could be expected between these two alternate methods of estimating local BA. The Schiltz and Grisi data showed that a range of $\pm$ 75 ft^2 ac^{-1} from the 1:1 relation approximately bounded the variation (Wilson 1996), and plots outside that range were viewed as potentially having an edge bias. This criterion was then applied to the entire data set, resulting in a total of 36 plots being dropped from further analysis.

Constructing the Diagram

Estimating a maximum size–density relationship can be problematic when permanent plot data are not available to verify that stands are self-thinning and when stands are sampled regionally. Results from permanent plot studies have shown that stands sampled from a region typically follow parallel self-thinning trajectories (slopes), but may differ in the intercepts (Drew and Flewelling 1979, Flewelling et al. 1980, Westoby 1984). With permanent plot data, the average trajectory through this self-thinning "zone" (i.e., a range of RDs within which self-thinning occurs) is used to define the limiting density (Curtis et al. 1981, Hester et al. 1989). Where permanent plot data are not available, as in this study, it is thus critical to estimate the self-thinning slope accurately by carefully screening the data to include only stands at maximum stocking. The resulting self-thinning line is thought to be an unbiased estimate of that obtained from remeasurement data.

The maximum size-density relationship was modeled using an equation form:

$$\log_{10} \overline{vol} = b_0 - b_1 \cdot \log_{10} TPA \qquad (1)$$

where $\overline{vol}$ is mean stem volume (total stemwood in ft^3, inside bark) for all trees on the plot, and TPA is the number of stems per acre over 0.5 in. dbh. Twenty-one stands were chosen that appeared to represent a maximum mean size–density relationship for spruce-fir (Figure 1). The parameters in Equation (1) were estimated by fitting a straight line with reduced major axis (RMA) regression (Rayner 1985). Eight plots with no evidence of mortality and 19 plots with evidence of density-independent mortality were excluded as potential stands used to fit the self-thinning relationship [Equation (1)]. Plots were considered to show density-independent mortality if there was at least one dead tree in a diameter class (1 in. classes) equal to or greater than the smallest diameter class of a dominant tree.

Equations predicting quadratic mean diameter (DQ), top height (HT_{40}), and crown closure were estimated using OLS regression. Top height was defined as the height (in feet) of the tallest 40 trees/ac. Several model forms were evaluated using different combinations of predictor variables, and weighted or unweighted, linear or nonlinear, models. Final equation form was chosen on the basis of residual patterns, residual normality, and an index of fit (Furnival 1961). All nonlinear models performed better than their linear counterparts. Once the maximum size–density line was estimated, RD was calculated for each plot as the ratio of the actual density *(TPA)* to the maximum density defined by the upper self-thinning line, for plots having the same $\overline{vol}$ (Curtis 1970). Relative density is calculated in the original units (not log-transformed).

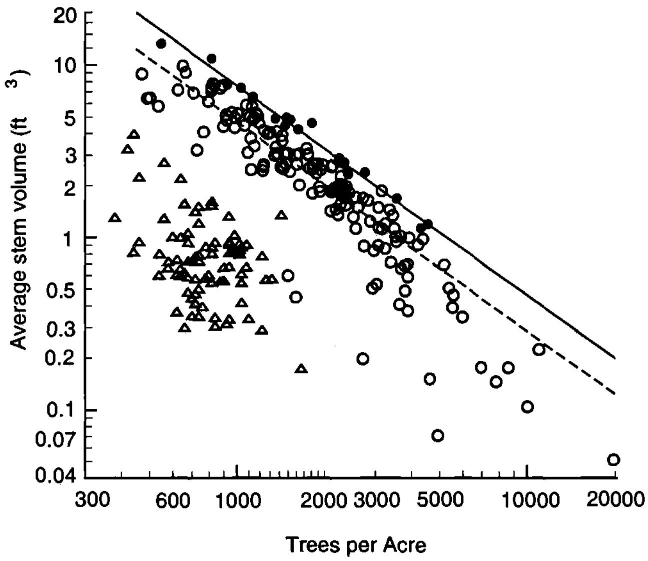

Figure 1. The maximum self-thinning relationship (solid line) was fit using RMA regression to 21 stands (●); the remainder of the Griffin data set is shown as open circles (○). The average-stand trajectory (dashed line) was fit through the geometric mean of the data used to estimate the self-thinning line, retaining the same slope as the self-thinning line. Precommercially thinned plots, shown as triangles (△), were not used in estimating the self-thinning relationship, but were included in fitting the *DQ* and HT_{40} prediction models [Equations (5) and (6)].

The point of complete crown closure was estimated from a crown width–stem volume relationship fit to 168 balsam fir that were precommercially thinned 10 yr previous and were essentially open-grown. These results from individual trees were expanded to a per acre basis to provide a rough projection of when crown closure occurs in young, spaced or natural stands. Average crown width *(CW)* at 100% canopy closure was calculated assuming circular crowns:

$$CW = \left[\frac{(43560)(4)}{\pi \cdot TPA}\right]^{0.5}$$

Results and Discussion

Equations

The self-thinning relationship (Figure 1) was estimated as:

$$\log_{10} \overline{vol} = 4.5222 - 1.2149 \log_{10} TPA; \quad (r^2 = 0.99; n = 21) \qquad (2)$$

Transforming Equation (2) to the original units gives a "reverse-j" nonlinear form:

$$\overline{vol} = 10^{4.5222} TPA^{-1.2149} \qquad (3)$$

The self-thinning, or *maximum* size-density line in Equation (2) represents a regionally estimated limiting relationship for spruce-fir that will, on average, overestimate the potential stocking of an average fully stocked stand. To predict *average* stand development, a second line parallel to Equation (2) was fit through the geometric means of the entire self-thinning data set (TPA = 1813, $\overline{vol}$= 2.26 ft^3). This average-stand trajectory line thus has the same slope as (2) but a lower intercept:

$$\overline{vol} = 10^{4.3138} TPA^{-1.2149} \qquad (4)$$

Equation (4), which has a RD of 0.67, should be used to make stand projections.

Equations for DQ (in.) and HT_{40} (ft) are:

$$DQ = b_1 \overline{vol}^{b2} TPA^{b3} \qquad (5)$$

$$HT_{40} = b_1 \overline{vol}^{b2} TPA^{b3} \qquad (6)$$

Parameter estimates and standard errors for Equations (5) and (6) are given in Table 3. Residuals were not biased by species composition.

Complete crown closure was determined using the relationship:

$$CW = 8.0263 \cdot V^{0.264}$$

where CW is crown width (ft) and V is the estimated stem volume for individual balsam fir (Lemin and Briggs 1993). This equation was fit with weighted nonlinear regression with variance proportional to $V^{0.5}$; $r^2 = 0.762$; MSE = 1.035; and n = 168. The complete crown closure line was then determined algebraically as:

$$\log_{10} \overline{vol} = 5.5688 - 1.897 \log_{10} TPA \qquad (7)$$

This line corresponds to a RD of 0.11 at a $\overline{vol}$ = 0.3 ft^3 (approximately 2 in. dbh), and rises to a RD of 0.32 at a $\overline{vol}$ = 10 ft^3 (8 in. dbh). Because this line is based on a limited sample, we offer it only as an approximation.

The complete density management diagram constructed from Equations (2), (4), (5), (6) and (7) is shown in Figure 2.

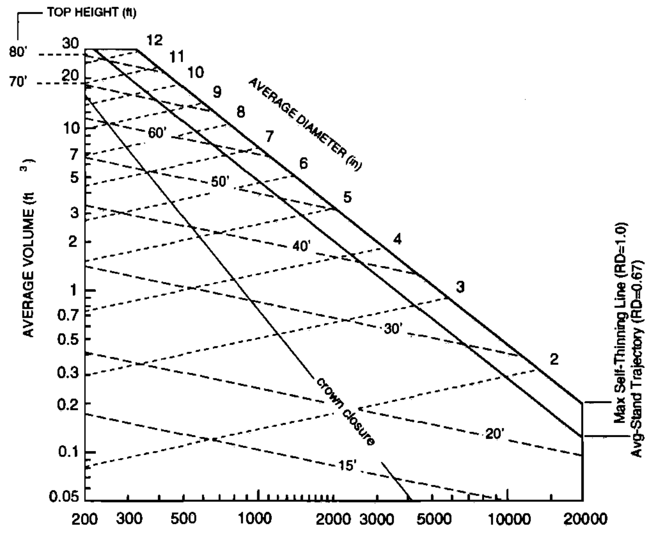

Figure 2. Density management diagram for northeastern red spruce and balsam fir forests. *RD* = relative density.

Any spruce-fir stand can be located on the diagram by the combination of $\overline{vol}$ and TPA. It is important that both $\overline{vol}$ and TPA be based on the same sample of all trees with a dbh greater than 0.5 in. Calculating $\overline{vol}$ normally requires a tally of dbh's and heights and an appropriate total stemwood volume equation. For unthinned, even-aged spruce-fir or young precommercially thinned stands, direct calculation of $\overline{vol}$ is unnecessary, however, in order to use the diagram. If DQ is known, either solve Equation (5) for $\overline{vol}$, or plot the intersection of DQ and TPA directly on the diagram. Quadratic mean diameter (in.) is determined from stand basal area (ft^2 ac^{-1}) and TPA, using the standard formula:

$$DQ = 13.54 \sqrt{\frac{BA}{TPA}}$$

In older stands with a history of thinning, Equations (5) and (6) and their isolines (Figure 2) are not accurate. However, the diagram can still be used to compute RD and predict future self-thinning if volume is calculated directly from stand data. Three volume equations useful for this purpose are compiled in Appendix A.

In stands with RDs below the average self-thinning line, $\overline{vol}$ is expected to increase without significant competition-induced mortality (represented by an upward vertical trajectory in Figure 2). Once a stand reaches the average stand trajectory line (RD = 0.67), $\overline{vol}$ will continue to increase, but

Table 3. Coefficient estimates and fit statistics for Equations (5) and (6).

Equation[a]	n[b]	Coefficient estimates b_1	b_2	b_3	MSE[c]	R^2	FI[d]
(5) – DQ	253	7.5490[e]	0.3137	–0.1024	0.0194	0.985	0.1525
		(0.2071)	(0.0025)	(0.0038)			
(6) – HT_{40}	253	15.3306	0.3306	0.1054	5.1329	0.966	2.2656
		(0.7570)	(0.0044)	(0.0066)			

[a] Equation number and dependent variable.
[b] Number of observations.
[c] Mean squared error. Equation (5) was fit with variance proportional to $\overline{vol}^{0.5}$.
[d] Furnival's (1961) index of fit.
[e] Coefficient estimate with standard error in parenthesis.

this increase is now accompanied by mortality; stand development will proceed diagonally along the line (i.e., upwards and to the left). Examples in the Applications section illustrate its use for developing thinning schedules.

Effect of Species Composition

The potentially confounding effects on estimating the self-thinning relationship from mixed stands of balsam fir and spruce (primarily red spruce) were examined in several ways. The correlation between relative density and species composition (arcsine transformed) was not significant for either the points used to fit the self-thinning line ($P = 0.23$) or the full data set ($P = 0.14$). Furthermore, for plots on the self-thinning line that were greater than 90% spruce or 90% fir by volume (n = 10 and n = 4 respectively), the range of stand densities overlap widely. No pattern is evident that suggests the two species might have separate self-thinning relationships. Both results suggest that balsam fir and spruce have indistinguishable self-thinning relationships and can thus be treated similarly in density management.

Comparison to Stocking Guides

Density management in the Northeast has traditionally used stocking guides following the model developed by Gingrich (1967) for upland oaks. Stocking guides depict stand density in terms of a nomogram that relates *BA* and *TPA* for a given *DQ*. The "A-Line," considered to be the upper limit of density, is often derived by fitting a tree-area ratio equation to data from normal yield tables. The "B-Line," which is intended to define the lower limit of full crown closure, is determined either from crown data of open-grown trees (as in this study) or by simply taking a percentage (about 60%) of the A-Line. It is not known whether Frank and Bjorkbom (1973) or Solomon et al. (1987) followed these procedures to develop their spruce-fir stocking guides; these publications do not describe the methods used to establish A or B lines.

When the data from this study are displayed on these spruce-fir stocking guides, the comparison is striking (Figure 3). The A-line in both stocking guides falls considerably below most of the plot values used in estimating the self-thinning line. Maximum density lines for two recently published density diagrams for boreal black spruce (Newton and Weetman 1994, Sturtevant et al. 1998) are also significantly lower than that estimated in this study. Using either approach (spruce-fir stocking guides or the boreal forest diagrams) would appear to underestimate potential stocking and yields of spruce-fir stands in Maine. For example, the precommercially thinned plots are expected to increase in basal area well beyond the limit imposed by either of the A-lines before competition-induced mortality begins (Figure 3). This comparison highlights the importance of using density management diagrams developed for the specific region and species of interest.

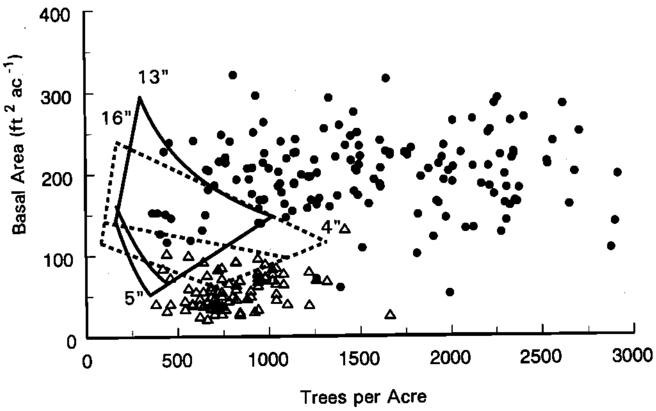

Figure 3. Comparison of the DMD data to spruce-fir stocking guides in the Gingrich (1967) format. Plots from the Griffin and precommercially thinned data sets are shown as circles (●), and triangles (△), respectively. Only dominant, codominant, and intermediate crown class trees were used in calculating density and plot basal area. Plots with densities over 3000 trees/ac were omitted for clarity. The stocking guide developed by Frank and Bjorkbom (1973) has solid, curvilinear lines. The stocking guide developed by Solomon et al. (1987) has dashed, straight lines. Quadratic mean diameter isolines radiate from the origin and are only shown for the boundaries of the stocking guides.

Applications

Sample Stand Description

Permanent plot data from a young, even-aged spruce-fir stand on the Penobscot Experiment Forest, near Bangor, ME, illustrates a typical application. The stand was regenerated by a shelterwood establishment cut in 1957; the overwood was partially removed in 1967 and completely removed in 1972 This resulted in a very dense, even-aged spruce-fir stand with a component of white pine (*Pinus strobus*), eastern hemlock (*Tsuga canadensis*), and hardwoods. Average stand development is shown on the diagram with points indicated by the inventory dates (Figure 4). Density increased markedly between 1980 and 1990 due to ingrowth of established seedlings into the 1 in. dbh class. The 1990 stand was located on the diagram by its DQ = 1.84 in., and average density of 8,023

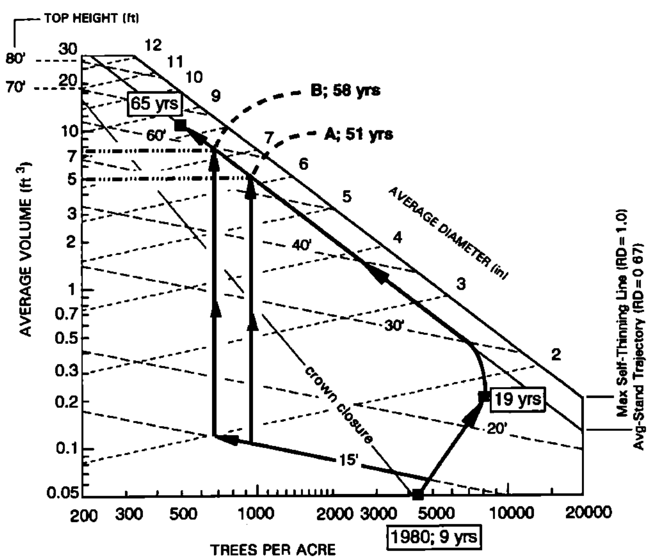

Figure 4. Natural stand development projected using the diagram for a young, shelterwood regenerated spruce-fir stand, site index 53. Inventory dates are shown on the diagram, and future stand development projected to a top height of 65 ft. Arrows show the direction of change through time. Also shown are residual densities for precommercially thinned stands, 945 and 677 trees/ac, which are expected to reach final, average stem volumes of 5.0 and 7.5 ft³, respectively (vertical solid lines). Points A and B are referenced in the text.

TPA. Site index was estimated as 53 ft (base age 50 yr at bh) using an unpublished site index equation fit to the Griffin data set (Appendix A).

To compute RD for this stand in 1990, first substitute *DQ* and *TPA* into Equation (5) and solve for $\overline{vol} = 0.21$ ft^3. Next, substitute this same $\overline{vol}$ into Equation (3) to determine the *TPA* of a comparable stand at maximum density (i.e., at RD = 1.0). The result (19,055 *TPA*), divided by the actual density (8,023), gives a RD of 0.42. Note that one cannot visually estimate RD (as the horizontal proportion between the Y-axis and self-thinning line) because *TPA* axis is logarithmic.

Scenario 1: No Thinning

Because this stand is well below the average-stand trajectory line, stand growth is expected to continue without significant mortality (i.e., vertically), assuming ingrowth has ceased. If the stand were significantly clumped, some mortality would occur below the average-stand trajectory line until all patches within the stand were self-thinning. The stand will reach the average-stand trajectory line at a top height of 29 ft [interpolated from the diagram, or calculated from Equation (6)].

Future stand development is projected using a site index equation [Equation (8), Appendix A]. As the stand develops along the average-stand trajectory line, we can expect a HT_{40} of 65 ft when the stand reaches a breast height age of 65. Corresponding values for $\overline{vol}$, *TPA*, and *DQ* are either read off the diagram, or calculated using the appropriate equations. In this example, the density when the stand reaches an HT_{40} of 65 ft is calculated by substituting the right hand portion of Equation (4) for $\overline{vol}$ into Equation (6). Then setting $HT_{40} = 65$ ft and solving, gives the projected density of 497 *TPA* (Table 4). This projected density is inserted into Equation (4) and solved for $\overline{vol} = 10.91$ ft^3; these values are then used to calculate *DQ* from Equation (5) (Table 4). Total stand volume is simply the $\overline{vol}$ multiplied by the density.

These growth projections were compared to a widely used stand growth model (GNY) developed for fully stocked conifer stands in Nova Scotia (NSDNR 1993; Table 4). Comparisons were made for stands with the same HT_{40}, because GNY uses a different height development curve. Average stem volumes were nearly identical, with the diagram and the GNY model calculating 10.91 ft^3 and 10.95 ft^3 respectively (Table 4). Stand yields were higher with GNY owing to the higher *TPA* and thus, higher projected RD (0.85) in the GNY model vs. the diagram (0.67).

Scenario 2: Precommercial Thinning

A major use of the diagram is developing spacing and residual density guidelines for precommercial thinning (PCT) operations designed to shorten the time to merchantability of dense naturally regenerated stands. Here, the target residual density is the maximum number of trees expected to reach a minimum merchantable size. Assume, for example, that harvesting costs require a minimum piece size of 5.0 ft^3 (approximately 17 stems per cord). First, locate this $\overline{vol}$ on the Y-axis and move horizontally to the right until intersecting the average-stand trajectory line (Figure 4, point A); then drop vertically to the Y-axis to locate the corresponding density (approximately 900 *TPA*). Alternately, inserting 5.0 ft^3 into Equation (4) and solving gives a more accurate value of 945 *TPA*. If this PCT were done at a height of 15 ft when

Table 4. Predicted stand conditions for Scenarios 1–3 (described in text) based on the Density Management Diagram (DMD), compared to the GNY model for fully stocked conifer stands in Nova Scotia.

Projection method	Stand ht[a] (ft)	Effective age at bh[b] (yr)	Density (trees ac^{-1})	DQ[c] (in.)	Relative density	Basal area (ft^2 ac^{-1})	Ave tree vol (ft^3)	Total stand vol (ft^3 ac^{-1})	MAI[d] (ft^3 ac^{-1} yr^{-1})
Initial conditions (1990)									
Both	23.6	19	8,023	1.8	0.42	147	0.21	1,685	70.2
Scenario 1: No Thinning (conditions during self-thinning)									
DMD	54	51	945	6.2	0.67	198	5.0	4,725	84.4
DMD	59	58	677	7.0	0.67	181	7.5	5,078	80.6
DMD	65	65	497	8.5	0.67	194	10.91	5,424	77.5
GNY	62[e]	75	624	8.3	0.85	236	10.95	6,834	85.4
Scenario 2: Precommercial thinning									
DMD	NA	NA	945	NA	0.67	NA	5.0	4,725	NA
GNY	42	40	945	6.9	0.66	229	4.9	4,592	114.8
DMD	NA	NA	677	NA	0.67	NA	7.5	5,078	NA
GNY	45	45	677	8.2	0.68	235	7.6	5,114	113.6
Scenario 3: Commercial thinning									
Original	45	41	1,672	4.7	0.67	202	2.5	4,180	90.9
Residual	Same	Same	460	6.1	0.26	93	3.7	1,702	NA
Final (DMD)	NA	NA	460	NA	0.67	NA	12.0	5,520	NA
Final (GNY)	59	60	460	9.3	0.72	241	12.9	5,943	125.8[f]

NOTE: NA = cannot be calculated; equations not accurate after thinning.

[a] HT_{40} for DMD data; average of all trees over 9.5 cm dbh for GNY runs.

[b] Iterative solution of Equation (8) for bh age in DMD examples; stump age—5 in GNY simulations.

[c] Quadratic mean diameter.

[d] Mean annual increment (volume divided by total age, assuming 5 yr to breast height).

[e] GNY height (all sawlog trees) assumed to equal DMD height (tallest 40/ac).

[f] Includes 2,234 ft removed by GNY in thinning at bh age 40.

the stand had approximately 5000 *TPA* (interpolated off the diagram), RD would be reduced from 0.09 to 0.03, while increasing *DQ* from 1.3 to 1.8 in. For a desired minimum $\overline{vol}$ of 7.5 ft^3, the target residual density would be 677 *TPA* (Figure 4, point B). Once a stand precommercially thinned to a residual density of 677 *TPA* reaches a $\overline{vol}$ of 7.5 ft^3, significant mortality of merchantable trees will occur, as the stand self-thins along the average-stand trajectory line (Figure 4). Clearly, the stand should be harvested or thinned commercially before this point to avoid losses of merchantable volume.

These estimates from the diagram closely match growth projections from the GNY model, with predicted $\overline{vol}$s of 4.9 and 7.6 ft^3 for the higher and lower residual densities, respectively (Table 4). (Stands were PCT in the GNY model when they were 10 yr old; comparisons were made using values from the GNY model when mortality began.) Because the height isolines are not accurate after PCT, we cannot determine from the diagram or its equations how long it will take the precommercially thinned stands to reach the average-stand trajectory line. GNY predicts that the stands will begin self-thinning again at ages 40 (945 *TPA*) or 45 (677 *TPA*), 11–13 yr sooner than the unthinned stand (Table 4).

The procedures above could also be used to establish a planting density in an artificial regeneration scenario. Neither red spruce nor balsam fir are commonly planted in the Northeast for timber production, however. Most conifer plantations are black or white spruce due to these species' more rapid early growth and resistance to spruce budworm damage. Although this diagram provides a point of departure for spruce plantations, we caution against assuming more precision than can be justified by the natural-stand data.

Scenario 3: Commercial Thinning

Another application of this diagram is to help foresters schedule and prescribe commercial thinning operations. The diagram provides a framework to answer questions such as: "At what age will a commercial entry become possible, given a minimum piece size?" Using the diagram, several alternative ages for commercial thinning can be evaluated against each other, and against the no-management scenario. The target residual density for a thinning can also be evaluated using the projected final $\overline{vol}$ and total volume as criteria.

If, for example, a $\overline{vol}$ of 2.5 ft^3 (about 34 trees/cord) is the earliest point when a stand can be feasibly thinned, then the density, HT_{40}, and age of a stand when it reaches this minimum size, can be readily determined. Using the example above, inserting 2.5 ft^3 into Equation (4) and solving gives a value of 1,672 *TPA*, or 4,180 ft^3 per acre. Substituting into Equations (5) and (6), we obtain a $DQ = 4.7$ in., and a $HT_{40} = 45$ ft, which corresponds to a bh age of 41 yr [Equation (8), Appendix A] and a mean annual increment of 90.9 ft^3 per acre per year (Figure 5; Table 4). These attributes can, of course, be estimated less accurately directly from the diagram and site index curves.

Target residual density for a commercial thinning is determined identically to the PCT example. Assuming a target final $\overline{vol}$ of 12 ft^3 (approximately 7 stems/cord) and a single commercial thinning, the appropriate residual density is 460 *TPA* [calculated from Equation (4)]. If this thinning is made at age 41 when $DQ = 4.7$, we would expect to remove 1672 – 460 = 1,212 stems/ac, or 2,478 ft^3, assuming an increase in average tree size from 2.5 to 3.7 ft^3 per tree (i.e., thinning from below). [These are computed by solving Equation (6) for $\overline{vol}$ given $TPA = 460$ and $HT_{40} = 45$, then substituting into Equation (5) to obtain *DQ*.] The residual stand would have the same HT_{40} and a RD of 0.26, a moderately heavy thinning that could experience losses from windthrow. The final harvest would remove another 5,520 ft^3; the exact rotation age cannot be determined from the diagram because the *DQ* and HT_{40} isolines are not accurate once the stand has been thinned. GNY, using a similar commercial thinning scenario (not shown), predicts that the stand would reach a $\overline{vol}$ of 12.9 ft^3 at a bh age of 60.

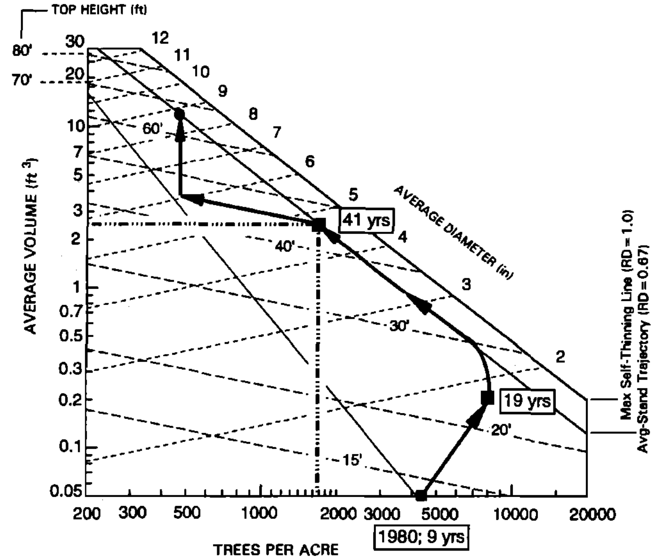

Figure 5. Projected entry timing for commercial thinning, based on a minimum average stem volume of 2.5 ft^3 is a bh age of 41. Arrows show stand development through time. The residual stand is plotted on the diagram using the calculated residual density (460 *TPA*) and the prethinning top height. This stand is expected to increase in mean stem volume without significant mortality until the average-stand trajectory line is reached (12.0 ft^3).

Additional Considerations

This diagram provides an objective, quantitative framework for density management prescriptions in natural, even-aged red spruce–balsam fir stands. Thinning schedules are but one of many considerations in formulating silvicultural systems. As with any field guide or model, this diagram is not meant to replace professional judgment.

Although the self-thinning lines are independent of initial density and can thus be used to predict mortality and RD of any stand, the *DQ* and HT_{40} isolines are accurate only for natural or young precommercially thinned stands. They have not been tested for plantations. Further, they are not accurate for tracking stand development after thinning, because the relationships between $\overline{vol}$, *DQ*, and HT_{40} (in effect, stem taper) are all strongly affected by initial or residual density. For example, in plantations or older thinned stands, the diagram will tend to overestimate $\overline{vol}$ for stands with a given *DQ* (because the trees are shorter at a given *DQ*), and will underestimate $\overline{vol}$ for stands with a given HT_{40} (because dbh's are larger). Isolines appropriate for plantations, or for projecting long-term development of thinned stands, could be constructed using the procedures outlined above if appropriate data sets were available (Smith 1989).

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APPENDIX A. Equations from Related Research Useful for Making Stand Development Predictions from the Diagram

As part of a Ph.D. dissertation, Steinman (1992) fit a site index equation to the Griffin data for spruce and fir combined. Subsequently, Steinman (unpublished) fit separate equations for spruce and fir, using two different five-parameter Chapman-Richards models.

To predict total height from site index and age:

$$H = 4.5 + (b_0 + b_1 S) \bullet [1 - \exp(b_2 A)]^{(b_3 S^{b_4})} \quad (8)$$

where H = total height of dominants and codominants (feet); S = site index (total height at a breast height age of 50); and A = age at breast height.

To predict site index from height and age:

$$S = [b_0 (H - 4.5)^{b_1}] \bullet [1 - \exp(b_2 A)]^{(b_3 (H-4.5)^{b_4})} \quad (9)$$

where b_i are as in Table A1.

To apply these equations, collect a sample of heights and bh ages from representative dominant trees. Substitute into Equation (9) to estimate S for each tree, and compute the mean for the stand. Then substitute S into Equation (8) along

Table A1.

	Equation (8) (height)		Equation (9) (site index)	
b_i	Spruce	Balsam fir	Spruce	Balsam fir
b_0	54.08	11.18	7.8531	1.19373
b_1	1.6697	1.416	–0.62589	–0.77673
b_2	–0.009722	–0.021459	–0.00010125	–0.00000137
b_3	6.02251	7.75063	–0.22073	–0.29571
b_4	–0.42122	-0.43778	0.33689	0.22433

with a range of bh ages to generate a height development curve. To determine A when H is known, either graph Equation (8) and estimate visually, or input the formula into a spreadsheet and increment A until the corresponding H is obtained. As a check on both equations, the H at $A = 50$ should be very close to the S, although the match is not exact. An Excel worksheet for this purpose is available via e-mail upon request from the corresponding author.

To compute stemwood volumes, several equations are available. The most commonly used model in the region was developed by Honer (1967):

$$VIB = \frac{D^2}{\left(b_0 + b_1 \cdot \frac{1}{H}\right)} \tag{10}$$

where VIB = total stemwood volume inside bark (ft^3); D = dbh (in.); and H = total height (ft). Honer fit separate parameters for each species as in Table A2.

Table A2.

b_i	Balsam fir	Red spruce	White spruce	Black spruce
b_0	2.139	1.226	1.440	1.588
b_1	301.634	315.832	342.175	333.364

For predicting volumes of small diameter trees (1–8 in., spruce and fir combined), Lemin and Briggs' (1993) equation works well:

$$VIB = 0.0826 + 0.00239(D^2H) \tag{11}$$

$$VOB = 0.1041 + 0.00262(D^2H) \tag{12}$$

where VIB = volume inside bark (ft^3) and VOB = volume outside bark (ft^3).

Reams and Brann (1981) fit several different volume equations to the Griffin data set and found the Schumacher model to be best (see Table A3):

$$VOB = b_0 D^{b_1} H^{b_2} \tag{13}$$

Unfortunately, no inside-bark equations were published; one would need to reduce values from Equation (13) by an appropriate bark percentage (about 10%) in order to be comparable with the Y-axis of the diagram.

Table A3.

b_i	Balsam fir	Red spruce	White spruce	Black spruce
b_0	0.0018214	0.0027293	0.0029629	0.0035705
b_1	1.7644	1.7545	1.7562	1.8630
b_2	1.2225	1.1411	1.0124	1.0124