Regional Stem Taper Equations for Eleven Conifer Species in the Acadian Region of North America: Development and Assessment

Rongxia Li, Aaron Weiskittel, Adam R. Dick, John A. Kershaw Jr., and Robert S. Seymour

ABSTRACT

Taper models to predict upper stem diameters, as well as total tree volume, are presented for 11 major conifer species in the Acadian Forest Region of North America. The Kozak (2004. My last words on taper equations. *For. Chron.* 80:507–514) Model 02 taper equation was used as the base model form. A nonlinear mixed-effects modeling approach was used to account for autocorrelation present among multiple stem analysis observations collected from the same tree. Results show that fitted taper equations can accurately predict both stem form and volume across a range of conditions. The taper models generally had slightly lower bias and root mean square error than the commonly used regional Honer refitted volume equations (1965. A new total cubic foot volume function. *For. Chron.* 41:476–493). The mean absolute bias was reduced up to 28% for certain species using the fitted taper equations compared with the refitted Honer (1965) equations, although the refitted Honer's models are also quite accurate where total stemwood volumes are needed. Independent validation data sets were used to further confirm reliability and accuracy of fitted taper models in predicting tree volume. These data sets indicated that the equations performed well, in general, but were slightly biased in certain thinned stands and in some New Brunswick ecoregions. Additional data are needed to confirm this and potentially improve model behavior. Overall, the models will be useful for predicting both stem form and merchantable and total volume.

Keywords: nonlinear mixed-effects, stem profile, total volume, Maine, New Brunswick, Nova Scotia

Stem taper equations are widely used to estimate diameter inside (diameter inside bark) and outside bark (diameter outside bark) at any given tree height, or conversely to estimate a corresponding height at any given diameter. In addition, total tree volume or merchantable volume to any specified upper stem diameter can be obtained by integrating taper equations along the bole.

Numerous taper equations are proposed in the forestry literature. Conventionally, taper equations are divided into two major categories: segmented taper equations, represented by Max and Burkhart (1976) and Clark et al. (1991), and continuous variable exponent or form taper equations, which are exemplified by Kozak (1988, 2004), Zakrzewski (1999), and Bi (2000). Among them, a model form (Model 02) from Kozak (2004) is found to be one of the most reliable taper equations for various species in terms of predicting both upper stem diameters and tree volume in several comprehensive studies (Rojo et al. 2005, Yang et al. 2009b, Li and Weiskittel 2010).

Taper equations are generally species specific, which means that model accuracy in estimating diameters depends on tree species (Sharma and Zhang 2004). Therefore, for each species a separate set of parameters for a fixed taper equation that identifies the unique bole shape is needed. Stand characteristics, such as stem density, regeneration method, soil type, and geoclimatic attributes also may have a significant impact on tree growth and stem form. However, tree size attributes are generally the most effective variables for predicting stem form (e.g., Muhairwe et al. 1994).

The Acadian Forest Region, which includes Maine, the Maritime Provinces of Canada, and parts of eastern Quebec, is covered with a mixed species forest that marks a transition zone from broadleaf to boreal forest (Braun 1950). Conifer species generally dominate this region, including balsam fir (*Abies balsamea* [L.] Mill.), spruce (red, white, black) (*Picea* spp. A. Dietr.), and a number of pines (*Pinus* spp. L.). A systematic study of the behavior of taper profile models for majority of conifer species in this specific region is lacking. For example, Solomon et al. (1989) only presented taper models for balsam fir and red spruce in Maine, whereas a few studies of stem form analysis on balsam fir, jack pine, white spruce, and black spruce in other regions of Canada (Sharma and Zhang 2004, Lejeune et al. 2009, Yang et al. 2009a) are available. In addition, Westfall and Scott (2010) recently presented taper equations for the primary commercial species in the northeastern United States based on data collected from standing trees.

The Honer (1965, 1967) volume equations have been used over the decades to calculate stand and individual tree volumes for a majority of species present in the Acadian Forest Region. However, previous work has shown that Honer (1965) equations can be biased

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Rongxia Li (li.rongxia@gmail.com), School of Forest Resources, University of Maine, Orono, ME 04496-5755. Aaron Weiskittel and Robert S. Seymour, School of Forest Resources, University of Maine, Orono, ME 04496-5755; Adam Dick, New Brunswick Department of Natural Resources, New Brunswick Growth and Yield Unit, Fredericton, N.B., Canada; John A. Kershaw Jr., University of New Brunswick, Fredericton, N.B., Canada. *Funding for this study was provided by the University of Maine Cooperative Forestry Research Unit and Forest Bioproducts Research Institute. Our thanks also go to the Ontario Ministry of Natural Resources, Quebec Ministry of Natural Resources, Nova Scotia Department of Natural Resources, New Brunswick Growth and Yield Unit, New Brunswick Department of Natural Resources, Acadia Research Forest (Canadian Forest Service), Laura Kenefic, Leah Phillips, Dan Gilmore, Doug Maguire, Micah Pace, and Spencer Meyer for providing access to the data used in this analysis.*

This article uses metric units; the applicable conversion factors are: centimeters (cm): 1 cm = 0.39 in.; meters (m): 1 m = 3.3 ft; cubic meters (m^3): 1 m^3 = 35.3 ft^3.

Table 1a. Data summary statistics of 11 conifer species for diameter outside bark taper models. The data from the fitting data set are presented.

	Natural					Planted				
		DBH (cm)		H (m)			DBH (cm)		H (m)	
Species[a]	No. of trees	Mean	Range	Mean	Range	No. of trees	Mean	Range	Mean	Range
		. . . . . .(cm)	. . . . . .	. . . .(m)	. . . .		. . . . . .(cm)	. . . . . .	. . . .(m)	. . . .
BF	4113	16.3	(3.6, 48.0)	12.6	(3.5, 29.7)					
BS	2289	18.4	(5.1, 53.0)	14.4	(4.1, 28.8)	378	13.3	(6.7, 24.1)	9.1	(5.0, 13.3)
WS	1926	23.8	(4.0, 66.0)	15.8	(4.6, 35.8)	177	13.6	(7.4, 22.6)	9.9	(4.9, 18.9)
RS	2231	14.9	(3.0, 51.6)	12.3	(4.1, 27.3)	11	12.7	(9.1, 15.3)	6.7	(4.9, 10.0)
JP	2808	17.1	(3.7, 42.7)	14.4	(5.3, 27.0)	209	15.8	(7.2, 23.2)	12.0	(8.0, 16.4)
RP	1149	30.0	(5.3, 59.7)	19.6	(4.9, 34.5)					
WP	1511	28.6	(3.9, 72.0)	18.7	(5.2, 35.1)					
NS						87	13.4	(6.4, 29.4)	8.7	(4.1, 20.8)
EH	368	30.1	(9.3, 61.0)	15.2	(5.4, 27.5)					
NWC	749	25.1	(9.0, 77.4)	12.2	(4.6, 21.7)					
TL	380	19.2	(9.0, 44.6)	15.3	(4.8, 21.7)					

Table 1b. Data summary statistics of 11 conifer species for diameter outside bark taper models. The data from the validation data set are presented.

	Pitt and Lanteigne (2008)					Lemin and Briggs (1993)				
		DBH (cm)		H (m)			DBH (cm)		H (m)	
Species	No. of trees	Mean	Range	Mean	Range	No. of trees	Mean	Range	Mean	Range
		. . . . . .(cm)	. . . . . .	. . . .(m)	. . . .		. . . . . .(cm)	. . . . . .	. . . .(m)	. . . .
BF	120	18.6	(9.6, 31.0)	16.9	(10.8, 22.7)	519	9.9	(0.3, 22.1)	6.7	(1.6, 14.3)
RS						68	8.0	(0.8, 16.3)	5.2	(1.6, 8.5)

[a] BF, balsam fir; BS, black spruce; WS, white spruce; RS, red spruce; JP, jack pine; RP, red pine; WP, white pine; NS, Norway spruce; EH, eastern hemlock; NWC, northern white cedar; TL, tamarack/larch.

(Weiskittel et al. 2009, Li and Weiskittel 2010). The development of new taper models to estimate standing tree volume would benefit forest practitioners by improving model accuracy and flexibility in volume predictions while providing a method to estimate stem form.

The purpose of this study was to provide practical guidelines in developing taper profile models for the majority of conifer species in the Acadian Forest Region. The specific objectives were to (1) fit the Honer (1965) volume equation and Kozak (2004) taper equation for 11 conifer species in the Acadian Forest Region of North America using a nonlinear mixed effects modeling approach, (2) assess accuracy and precision of fitted taper models in predicting both diameters and volume outside bark (VOB), (3) evaluate differences in stem taper between naturally regenerated stands and planted stands when possible, (4) assess spatial distribution of bias in the stem taper models among different ecoregions of New Brunswick, and (5) use independent data sets for a subset of species to evaluate taper and volume equation performance across a range of stand conditions.

Methods

Data

The data used in this analysis came from a variety of sources. The majority of fitting data were obtained from three primary data sources: (1) stem analysis data used in the Honer (1965, 1967) study, (2) newly collected taper data from New Brunswick, and (3) stem data collected from Quebec. The study species include balsam fir (*Abies balsamea* [L.] Mill) (BF), black spruce (*Picea mariana* [Mill.] Britton, Sterns & Poggenb.) (BS), white spruce (*Picea glauca* [Moench] Voss) (WS), red spruce (*Picea rubens* Sarg.) (RS), jack pine (*Pinus banksiana* Lamb.) (JP), white pine (*Pinus strobus* L.) (WP), red pine (*Pinus resinosa* Aiton) (RP), Norway spruce (*Picea abies* [L.] Karst.) (NS), eastern hemlock (*Tsuga canadensis* [L.] Carrière) (EH), northern white cedar (*Thuja occidentalis* L.) (NWC), and tamarack/larch (*Larix occidentalis* Nutt.) (TL). Regardless of origin, all data were obtained from the stem analysis of felled trees. The primary data sources are described in brief detail below.

The Honer (1965, 1967) data were originally gathered for construction of regional form-class volume tables for the Ontario Department of Lands and Forests. Sampled plots were located in various sites throughout central and eastern Canada. The forest stands were all naturally regenerated and covered a wide range of stand types and ages. The stem analysis data, including both diameter outside bark and diameter inside bark at stump height (0.3 m), breast height (1.37 m), and each 1/10th section of total height above breast height, were recorded. Total tree height and other tree characteristics were also recorded.

Data from planted stands located in New Brunswick, Canada, were collected during 2009. Sample plots were dispersed among all seven ecoregions of New Brunswick (Zelazny 2007). Total tree height, stump age, dbh, and other relevant tree characteristics were measured and recorded. The stem diameters inside bark were measured at 0.15 (stump height), 0.65, 1.29, and 1.65 m and every 0.5 m up to tree tip. Bark thickness at each section was recorded as well.

The Quebec data were collected by the Ministry of Natural Resources throughout the province. The data consisted of 94,746 observations of diameter outside bark from 4,983 trees. The species sampled included balsam fir (n = 1,949), eastern hemlock (n = 358), jack pine (n = 1207), northern white cedar (n = 670), red pine (n = 211), and white pine (n = 588). Trees were sampled from a range of stand types that varied in age, density, and species composition. On each tree, measurements of diameter outside bark were

Table 2. Estimated parameters and corresponding standard errors of the fitted diameter outside bark taper models for 11 conifer species. The φ and σ are estimates of continuous first-order autoregressive parameter (CAR1) and the variance weighting factor, respectively.

	BF[a]		BS		WS		RS		JP		RP		WP		NS		EH		NWC		TL	
	Estimate	SE	Estimate	SE	Estimate	SE	Estimate	SE	Estimate	SE	Estimate	SE	Estimate	SE	Estimate	SE	Estimate	SE	Estimate	SE	Estimate	SE
α_0	0.791	0.007	0.858	0.009	0.732	0.010	0.876	0.011	1.021	0.009	1.096	0.022	1.020	0.020	1.051	0.044	0.868	0.043	0.902	0.024	0.739	0.029
α_1	0.975	0.004	0.961	0.005	0.958	0.006	0.992	0.004	0.982	0.004	1.006	0.006	0.985	0.007	0.949	0.027	0.916	0.017	0.968	0.010	0.972	0.015
α_2	0.120	0.006	0.105	0.006	0.159	0.008	0.063	0.007	0.015	0.005	−0.0352	0.008	0.015	0.010	0.037	0.022	0.156	0.023	0.085	0.016	0.143	0.020
β_1	0.269	0.004	0.260	0.004	0.264	0.005	0.413	0.004	0.375	0.004	0.500	0.005	0.370	0.005	0.611	0.023	0.407	0.019	0.320	0.011	0.271	0.017
β_2	−0.5513	0.026	−0.3409	0.028	−0.4246	0.032	−0.6877	0.031	−0.7954	0.022	−0.9959	0.033	−0.7512	0.043	−0.3001	0.110	−0.6163	0.151	−0.4336	0.105	−0.4958	0.094
β_3	0.561	0.006	0.480	0.008	0.551	0.008	0.441	0.007	0.499	0.007	0.301	0.007	0.354	0.009	0.373	0.037	0.418	0.026	0.521	0.014	0.651	0.028
β_4	0.901	0.082	0.501	0.114	−0.1269	0.139	1.182	0.090	2.041	0.092	4.636	0.193	3.850	0.214	1.126	0.365	3.626	0.766	0.016	0.429	−0.3887	0.458
β_5	0.126	0.002	0.110	0.002	0.115	0.002	0.113	0.002	0.077	0.001	0.047	0.002	0.107	0.002	0.032	0.007	0.169	0.006	0.137	0.005	0.132	0.005
β_6	−0.6708	0.009	−0.4952	0.009	−0.6249	0.013	−0.4356	0.011	−0.3335	0.008	−0.05	0.013	−0.5131	0.014	−0.0297	0.026	−0.8829	0.038	−0.4585	0.025	−0.7035	0.033
β_7	0.000	0.000	0.097	0.006	0.088	0.009	0.104	0.033	0.041	0.005	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
φ	0.622		0.619		0.713		0.526		0.586		0.765		0.701		0.486		0.669		0.492		0.613	
δ	−0.3913		−0.3436		−0.4531		−0.3346		−0.2114		−0.1986		−0.3293		−0.2323		−0.4178		−0.4367		−0.374	

[a] BF, balsam fir; BS, black spruce; WS, white spruce; RS, red spruce; JP, jack pine; RP, red pine; WP, white pine; NS, Norway spruce; EH, eastern hemlock; NWC, northern white cedar; TL, tamarack/larch.

taken at stump height, breast height, and approximately every 1 m after breast height.

For balsam fir and red spruce, we also obtained additional stem analysis data from other sources, including Gilmore and Seymour (1996), Maguire et al. (1998), Meyer (2005), and Phillips (2002). Gilmore and Seymour (1996) study sites were located on the University of Maine Dwight B. Demeritt Forest, and Maguire et al. (1998) and Phillips (2002) were on the nearby US Forest Service Penobscot Experimental Forest. The Meyer (2005) data were collected from various locations throughout Maine. Details on specific sampling schemes used in each study can be obtained from Li and Weiskittel (2010). However, sampled trees gathered from these various studies are a very small percentage of the data used for this analysis. For eastern hemlock and northern white cedar, additional data were obtained from Kenefic and Seymour (1999) and Hofmeyer et al. (2009), respectively.

The validation data sets were gathered from two different studies. One data set came from Pitt and Lanteigne (2008), which was originally designed to study the long-term responses of balsam fir to precommercial thinning (PCT). Sample plots were located in the Green River watershed of northwestern New Brunswick. In our study, we used their tree-list data to validate accuracy of outside bark volume predictions. Another data set from Lemin and Briggs (1993) that was made up of young balsam fir and spruce in PCT and no-PCT stands in Maine was also used.

A summary of the data, including number of trees, mean, and range of dbh and total tree height, is presented in Tables 1a and 1b.

Data Analysis

The form of the Kozak (2004) Model 02 taper equation is

$$d = \alpha_0 D^{\alpha_1} H^{\alpha_2} X^{(\beta_1 z^4 + \beta_2 (1/e^{D/H}) + \beta_3 X^{0.1} + \beta_4 (1/D) + \beta_5 H^Q + \beta_6 X)}, \qquad (1)$$

where $X = 1 - z^{1/3}/1 - p^{1/3}$, $Q = 1 - z^{1/3}$, $d =$ diameter inside bark or diameter outside bark (cm), $H =$ total tree height from ground (m), $D =$ dbh (cm), $h =$ section height from ground (m), $p = 1.3/H$ (relative breast height), and $z = h/H$ (relative height from ground). The original equation was used to predict diameter inside bark; however, because diameter outside bark generally has a linear relationship with diameter inside bark, we used this equation to predict diameter outside bark because much of our data only had overbark measurements.

Table 3. Mean absolute bias (MAB) and root mean square error (RMSE) of diameter outside bark for 11 conifer species. M1 and M2 are the taper models fitted in the present analysis without and with an indicator for regeneration method, respectively. Westfall refers to the taper equation provided in Westfall and Scott (2010).

Species[a]	Model	MAB (cm)	RMSE (cm)
BF	M1	0.809	1.266
	Westfall	0.790	1.201
BS	M1	0.836	1.279
	M2	0.817	1.262
	Westfall	0.837	1.253
WS	M1	1.281	2.085
	M2	1.253	2.058
	Westfall	1.098	1.779
RS	M1	0.753	1.211
	M2	0.752	1.210
	Westfall	0.869	1.306
JP	M1	0.733	1.061
	M2	0.728	1.055
	Westfall	0.834	1.204
RP	M1	1.088	1.558
	Westfall	1.727	2.463
WP	M1	1.261	1.965
	Westfall	1.396	2.187
NS	M1	0.624	0.825
	Westfall	0.694	0.928
EH	M1	2.024	3.202
	Westfall	1.948	3.164
NWC	M1	1.399	2.369
	Westfall	1.394	2.380
TL	M1	1.146	1.706
	Westfall	1.180	1.841

[a] BF, balsam fir; BS, black spruce; WS, white spruce; RS, red spruce; JP, jack pine; RP, red pine; WP, white pine; NS, Norway spruce; EH, eastern hemlock; NWC, northern white cedar; TL, tamarack/larch.

The fact that stem data were collected at multiple points on the same individual tree presented a form of autocorrelation among observations from the same tree. Neglecting this autocorrelation may result in incorrect statistical inference and confidence intervals. Therefore, nonlinear mixed effects (nlme) modeling techniques were used to fit the above taper equation to account for autocorrelation by incorporating individual tree random effects into the model. The advantages and details of using nlme modeling techniques for hierarchical or longitudinal data in forestry applications have been fully discussed in several publications (e.g., Gregoire et al. 1995, Hall and Clutter 2004). In this study, parameters α_0 and β_3

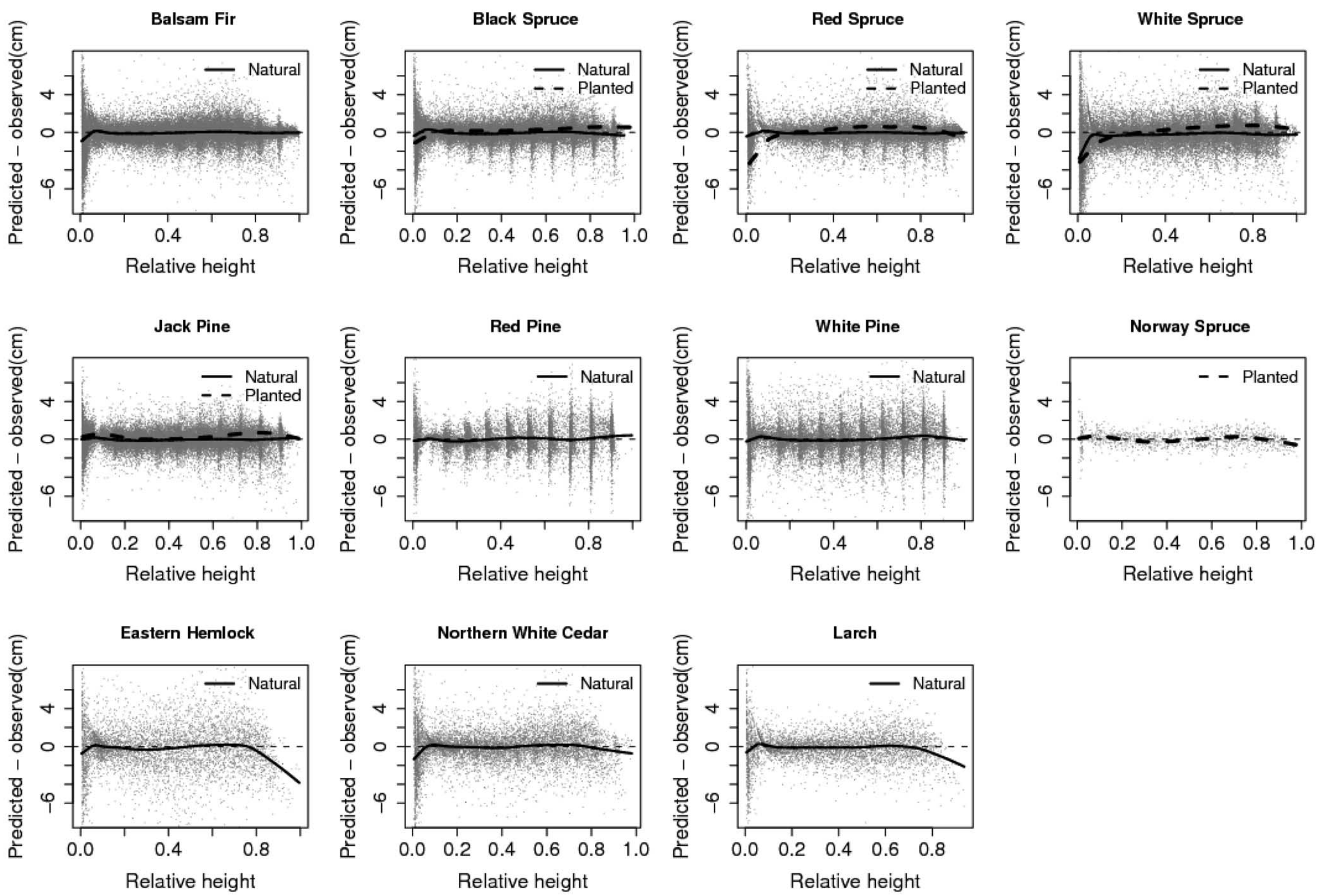

Figure 1. Residual plots (predicted − observed) of fitted models of diameter outside bark for 11 conifer species. Black solid and dashed lines are lowess lines for naturally regenerated and planted stands, respectively.

in Equation 1 were associated with random effects to account for individual tree variation.

The preliminary analysis showed that a first-order continuous autoregressive error structure (CAR1) was needed to further reduce autocorrelation. It is defined as Corr $(\varepsilon_t, \varepsilon_s) = \varphi^{|t-s|}$, where ε_t and ε_s are model residuals of two observations from the same tree, parameter φ represents the correlation between any two observations 1 unit apart, and $|t - s|$ is the height distance between these two observations. A power variance function was specified in the model fitting process to handle heteroscedasticity noted in the residuals during preliminary analysis. This function is defined as $\mathrm{Var}(\varepsilon_i) = \sigma^2|\upsilon_i|^{2\delta}$, where σ^2 is the residual sum of squares, υ_i is the weighting variable (section height in this study), and δ is the variance function coefficient.

All statistical analyses were carried out in R using the nlme library (Pinherio and Bates 2000).

Evaluation Criteria

We chose to use mean absolute bias (MAB) and root mean square error (RMSE) to evaluate model performance for both estimation in the fitting stage and prediction in the validation stage. The formulas of MAB and RMSE are as follows:

$$MAB = \frac{1}{n}\sum_{i=1}^{n} |\hat{Y}_i - Y_i| \qquad (2)$$

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n} (\hat{Y}_i - Y_i)^2}{n}} \qquad (3)$$

where Y_i refers to the observed or measured ith response value (diameter outside bark or VOB), and $\hat{Y}_i$ is the corresponding predicted response value.

Trees sampled from planted stands may present a different stem profile from trees sampled from naturally regenerated stands. To test whether an additional parameter(s) is needed to account for the difference between naturally regenerated and planted stands, a Wald-type test or a likelihood ratio test is generally used. However, because of large-size samples a small difference in model forms could lead to rejection of the null hypothesis. Furthermore, this is an observational study, and there may be other confounding factors, such as different geographic locations, stand ages, and measurement methods. Therefore, a formal significance test is trivial in this situation; however, we still present our model form (Equation 4) here with an additional parameter indicating the stand regeneration method (natural versus planted), and users can choose the most suitable model form based on their own data. We also compared and assessed the improvement of the model performance due to this additional parameter.

$$d = \alpha_0 D^{\alpha_1} H^{a_2} X^{(\beta_1 z^4 + \beta_2(1/e^{D/H}) + \beta_3 X^{0.1} + \beta_4(1/D) + \beta_5 H^Q + \beta_6 X + \beta_7 I)} \qquad (4)$$

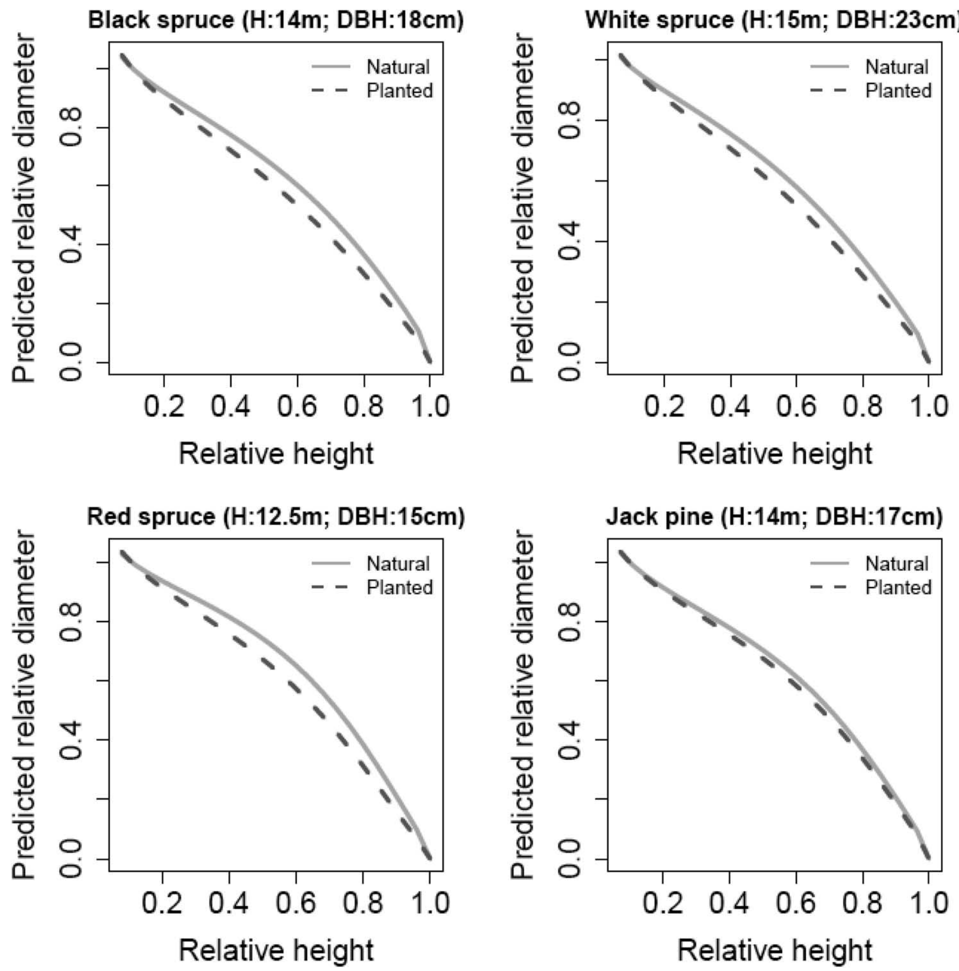

Figure 2. Predicted relative outside diameter over relative height for typical trees with average total tree height and average dbh in each of four species.

where $I = 0$ if the stands are naturally regenerated and $I = 1$ if the stands are planted.

Both "observed" and predicted tree volumes were derived using the Smalian formula. For observed volume, we used measured diameters at each section to obtain section volume and then summed to acquire total tree volume. For predicted tree volume, we divided each tree into 100 sections, predicted diameters at each section height, and then obtained section volume using the Smalian formula, which were then summed. As a comparison, the Honer (1965, 1967) regional equations were refitted to obtain individual tree volume for all the species. The Honer (1965, 1967) volume equation was given as

$$V_e = \frac{D_e^2}{\alpha_1 + \alpha_2/H_e} \tag{5}$$

where D_e represents dbh outside bark in inches, H_e is total tree height in feet, and V_e refers to individual tree volume in cubic feet. English units were used for this portion of the analysis to maintain consistency with the original Honer (1965, 1967) parameters.

For further evaluation and comparison, we also calculated diameter outside bark and VOB predictions for each of study species using a recently published taper equation (Westfall and Scott 2010).

Validation

Although there are no set of specific standards or tests that can be easily applied to determine the "appropriateness" of a model, we need to establish a minimum validation procedure to ensure reliability and reasonable performance of a new model (Huang et al. 2003). One approach to validate a statistical fitted model is to apply the fitted results to an independent data set. Because of the difficulty in collecting new stem analysis data, in this study we validated the fitted models based on volume predictions. The data we used for validation consisted of dbh, total height, and volume calculated from stem analysis (the original stem disc data were unavailable). Because volume predictions were obtained through the integration of diameter predictions based on the fitted diameter models, we believe this validation process is reasonable and can provide insight on how well and reliable the fitted taper models are. All validation statistics were estimated using only the fixed effects of the fitted nlme equation.

Results

Model Development

The estimated parameter values and their corresponding standard errors of the fitted taper equations for diameter outside bark models are listed in Table 2 (see Appendix A for estimates for the diameter inside bark model). All parameters are significant at the

Table 4. Mean absolute bias (MAB) and root mean square error (RMSE) of stem volume outside (VOB) bark for 11 conifer species. M1 and M2 are stem taper models fitted in the present analysis without and with an indicator for regeneration method, respectively. Honer represents the refitted regional Honer (1965, 1967) volume equations. Westfall refers to the taper equation provided in Westfall and Scott (2010).

Species[a]	Model	MAB	RMSE
		. . . .(m^3)	. . . .
BF	M1	0.0138	0.0268
	Honer	0.0156	0.0282
	Westfall	0.0153	0.0282
BS	M1	0.0193	0.0404
	M2	0.0191	0.0395
	Honer	0.0182	0.0364
	Westfall	0.0197	0.0389
WS	M1	0.0302	0.0548
	M2	0.0302	0.0547
	Honer	0.0328	0.0614
	Westfall	0.0330	0.0632
RS	M1	0.0137	0.0315
	M2	0.0137	0.0315
	Honer	0.0190	0.0370
	Westfall	0.0167	0.0353
JP	M1	0.0147	0.0254
	M2	0.0148	0.0256
	Honer	0.0159	0.0269
	Westfall	0.0174	0.0294
RP	M1	0.0529	0.0912
	Honer	0.0571	0.0926
	Westfall	0.1175	0.1864
WP	M1	0.0862	0.2120
	Honer	0.1013	0.1975
	Westfall	0.0955	0.2116
NS	M1	0.0049	0.0090
	Westfall	0.0060	0.0125
	Honer	0.0051	0.0100
EH	M1	0.0880	0.1892
	Honer	0.0922	0.1981
	Westfall	0.0909	0.1890
NWC	M1	0.0382	0.1022
	Honer	0.0415	0.0944
	Westfall	0.0381	0.1003
TL	M1	0.0252	0.0447
	Honer	0.0272	0.0473
	Westfall	0.0318	0.0584

[a] BF, balsam fir; BS, black spruce; WS, white spruce; RS, red spruce; JP, jack pine; RP, red pine; WP, white pine; NS, Norway spruce; EH, eastern hemlock; NWC, northern white cedar; TL, tamarack/larch.

level of 0.05, except α_2 for WP ($P = 0.14$) and NS ($P = 0.09$); β_4 for WS ($P = 0.36$), NWC ($P = 0.97$), and TL ($P = 0.40$); and β_6 for NS ($P = 0.26$). The corresponding fit statistics, MAB and RMSE, are summarized in Table 3.

Both the bias statistic (MAB) and RMSE indicate that diameter outside bark taper models using the Kozak (2004) equation performed well for all 11 conifer species (Table 3). The MAB ranged from 0.6 to 2.1 cm, and the RMSE ranged from 0.8 to 3.3 cm, depending on species. All species showed relatively small bias, except EH. The residual plots also gave further evidence that all fitted models behaved well (Figure 1). Comparing the Kozak (2004) taper equation with the Westfall and Scott (2010) taper equation, we found there was relatively small differences in diameter outside bark predictions. The Kozak (2004) equation predicted diameter outside bark slightly better for species of BS, RS, JP, RP, WP, and NS, whereas the Westfall and Scott (2010) equation provided slightly better predictions for BF, WS, EH, NWC, and TL.

Separating planted and natural stands led to a modest gain in model performance. However, this gain was not great as the maximum reduction in the MAB and RMSE was around 0.03 cm for WS. Figure 2 depicts predicted taper profiles using the fitted taper equations on a typical tree with average dbh and HT values for BS, WS, JP, and RS. We found that naturally regenerated stands tended to have greater diameter estimates in the middle section of tree stem compared with planted stands. However, this difference was minimal for JP and modest for BS, WS, and RS. Also, diameters in natural stands may be slightly better predicted than planted stands using Equation 4 (Figure 1).

Table 5. Estimated parameters and corresponding standard errors of the refitted Honer (1965, 1967) volume equations (volume outside bark [VOB] and volume inside bark [VIB]) for 11 conifer species.

	VOB				VIB			
	α_1		α_2		α_1		α_2	
Species[a]	Estimate	SE	Estimate	SE	Estimate	SE	Estimate	SE
BF	0.980	0.336	357.53	2.79	2.158	0.575	365.81	3.27
BS	0.746	0.196	360.13	3.49	1.443	0.510	379.55	6.57
WS	0.533	0.049	375.87	3.12	0.691	0.072	404.62	4.50
RS	0.935	0.080	332.18	3.44	2.278	0.594	341.26	2.67
JP	−0.020	0.063	392.50	3.15	−0.176	0.076	437.28	3.86
RP	−0.075	0.064	391.38	4.46	−0.405	0.074	459.47	5.25
WP	0.971	0.090	346.08	5.22	0.397	0.073	410.15	4.34
EH	1.449	0.284	349.34	13.21	3.137	1.621	378.19	27.08
NS	1.982	0.244	298.27	7.85	1.943	0.263	346.58	8.75
NWC	3.093	0.244	306.51	9.71	3.088	0.552	328.45	24.26
TL	0.621	0.227	392.56	12.66				

[a] BF, balsam fir; BS, black spruce; WS, white spruce; RS, red spruce; JP, jack pine; RP, red pine; WP, white pine; NS, Norway spruce; EH, eastern hemlock; NWC, northern white cedar; TL, tamarack/larch.

Table 4 gives the bias statistics of integrated stem VOB using the fitted taper equations (see Appendix B for an example of how to obtain individual tree volume by integrating fitted taper models). As a comparison, the bias in the tree volume calculated by the refitted regional Honer (1965) volume equations is also presented here. We found the accuracy in predicting VOB was improved for all the species, except BS, by using taper equations over the refitted Honer (1965) volume equations in terms of MAB. The RMSE was also reduced in the stem volume predictions using fitted taper models. However, readers need to be aware that this improvement is at the expense of the complexity of the taper model form. Parameter estimates and corresponding standard errors for refitted Honer (1965) equations are presented in Table 5. Comparing the Kozak (2004) equation with the Westfall and Scott (2010) taper equation, we found that our equation gave relatively less bias than the Westfall and Scott (2010) equation for all the species, except NWC.

Model Assessment

Figure 3 shows the difference of bias percentage in predicting VOB among seven ecoregions in New Brunswick for sampled trees from planted stands. Note that only four species (BS, WS, JP, and NS) were sampled from planted stands and these sampled trees were not evenly distributed in each of the seven ecoregions. For example, NS was only found in ecoregions 2, 3, 4, and 5. We found that for BS, VOB for trees in region 1 and 4 was underestimated and for trees in region 7 was slightly overestimated; for WS, most ecoregions showed signs of underestimated VOB; for JP, VOB was not biased in all sampled regions; and for NS, only ecoregion 2 showed an indication of under-estimation of VOB.

We also assessed the bias of VOB on two independent data sets (Table 6). Because of the lack of data for other species, we only

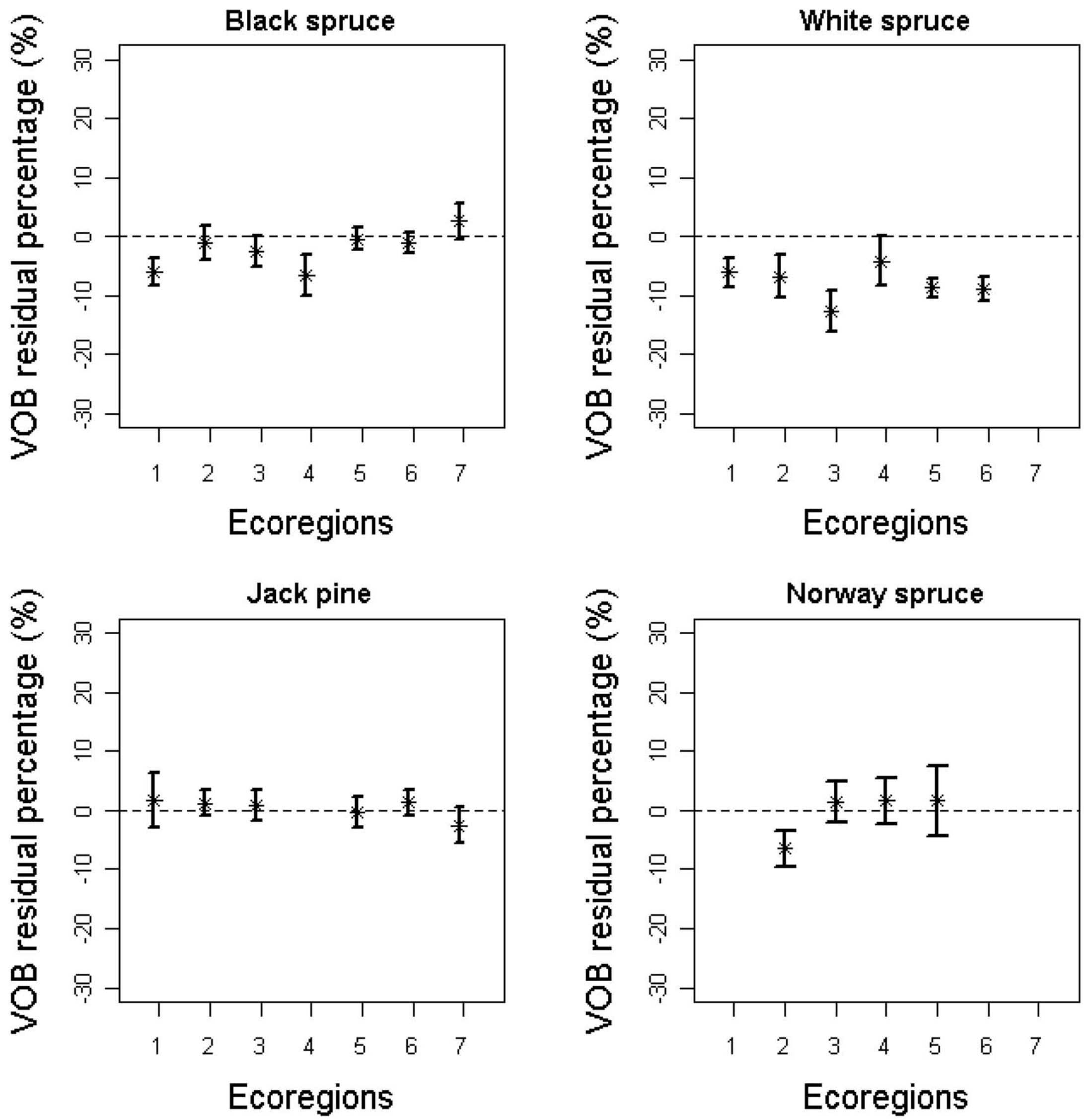

Figure 3. Mean percentage bias for total volume outside bark (VOB) fitted residuals (predicted − observed) and the corresponding 95% confidence intervals for black spruce, white spruce, jack pine, and Norway spruce in different ecoregions of New Brunswick (Zelazny 2007) (1: Highlands; 2: Northern Uplands; 3: Central Uplands; 4: Fundy Coast; 5: Valley Lowlands; 6: Eastern Lowlands; 7: Grand Lake).

Table 6. Bias assessment of volume outside bark predictions on three independent validation data sets for balsam fir and red spruce. M1 and M2 are taper models fitted in the present analysis without and with an indicator for regeneration method, respectively. Westfall refers to the taper equation provided in Westfall and Scott (2010).

Data source	Species[a]	Model	MAB (m^3)	RMSE (m^3)
Pitt and Lanteigne (2008)	BF	M1	0.0240	0.0330
		Honer	0.0344	0.0464
		Westfall	0.0128	0.0188
Lemin and Briggs (1993)	BF	M1	0.0028	0.0036
		Honer	0.0032	0.0040
		Westfall	0.0024	0.0038
	RS	M2	0.0011	0.0015
		Honer	0.0013	0.0016
		Westfall	0.0025	0.0039

[a] MAB, mean absolute bias; BF, balsam fir; RS, red spruce.

evaluated model predictions on BF and RS. Results show that all volume predictions using the Kozak (2004) taper equation were less biased than those using the refitted Honer (1965) volume equations. The MABs were decreased by 30% and 13% for BF on Pitt and Lanteigne (2008) and Lemin and Briggs (1993) data sets, and 15% for RS on Lemin and Briggs (1993) data set, compared with predictions from the Honer (1965) refitted equations. However, it seems that the Westfall and Scott (2010) taper equation performed relatively better than the Kozak (2004) taper equation for BF in the Pitt and Lanteigne (2008) data set.

Discussion

This study represents a systematic regional approach for development of taper profile models for the major conifer species in the Acadian Forest Region of North America. We have demonstrated and presented the analysis of diameter and volume estimation using the widely used Kozak (2004) taper equation. We chose the Kozak (2004) Model 02 taper equation as our model base function because it has been shown by several studies that it can provide reliable and accurate predictions for both diameters and tree volume (merchantable volume and total tree volume) (e.g., Li and Weiskittel 2010). In addition, our preliminary analysis also provided adequate evidence that the Kozak (2004) equation is less biased than other widely used taper equations, such as the segmented equation (Clark et al. 1991), the volume compatible equation (Fang et al. 2000), and the cubic taper equation (Goodwin 2009), as well as several volume equation forms, including Honer (1965, 1967). Solomon et al. (1989) used the Max and Burkhart (1976) equations to predict diameter and volume inside bark for balsam fir and red spruce in this region, but a previous study showed greater bias using the Max and Burkhart (1976) equations on spruce and fir trees (Li and Weiskittel 2010). Moreover, the Kozak (2004) equation has the merits of being flexible and easy to fit, which is important in practical terms. In spite of these advantages, it should be noted that the Kozak (2004) equation can still have a relatively large bias at the bottom and the tip of a tree.

In our analysis, we compared the Kozak (2004) equation with the recently published Westfall and Scott (2010) taper equation and found minimal difference in modeling stem form and a significantly better performance (up to 55% mean bias reduction) using the Kozak (2004) equation in modeling total tree volume. The Westfall and Scott (2010) taper equation is based on a switching model formulated by Valentine and Gregoire (2001), which originally required measurements of height to live crown base as an independent input variable. However, the modified equation in the work of Westfall and Scott (2010) removed this limitation and also relaxed the restriction of fixed joining points. The equally good performance of the Kozak (2004) and Westfall and Scott (2010) equations in predicting diameter outside bark indicates little regional difference in the stem form for the study species.

In recent years, nlme modeling techniques have been applied to many areas in forest growth and yield modeling because of their ability to account for between- and within-subject variations (Gregoire and Schabenberger 1996, Tasissa and Burkhart 1998, Fang and Bailey 2001, Garber and Maguire 2003). As indicated by the name, mixed-effects models include both fixed effects and random effects parameters. In our analysis, the species-level trend was described by fixed parameters, and individual tree variation was taken into account by random effects. It is beyond the scope of this study to provide individual tree calibration for future predictions; however, readers who are interested in tree-level prediction calibration can find more information in other forestry literature (Hall and Bailey 2001, Huang et al. 2009, Yang et al. 2009b).

Overall, the taper equations developed here represent a modest improvement to the Honer (1965) regional volume equations. In addition to providing an ability to estimate total stem volume, the equations can be used to estimate merchantable volume to any desired specification based on minimum top diameter or length. Although some bias in predictions due to geographic region and thinning method existed, the models can be easily locally calibrated with the collection of additional information. Regardless, the models predicted quite well despite the wide range of stand conditions embodied by the available data. However, further testing of these equations is needed. For example, Guiterman et al. (2011) found the developed taper equations from this analysis effective for predicting both eastern white pine stem form and total volume across a range of thinning treatments. Additional data are needed to confirm this result for other species.

Appendix A

Estimated parameters and corresponding standard errors of the fitted diameter inside bark taper models for 10 conifer species. The φ and σ are estimates of continuous first-order autoregressive parameter (CAR1) and the variance weighting factor, respectively.

	BF		BS		WS		RS		JP		RP		WP		NS		EH		NWC	
	Estimate	SE	Estimate	SE	Estimate	SE	Estimate	SE	Estimate	SE	Estimate	SE	Estimate	SE	Estimate	SE	Estimate	SE	Estimate	SE
α_0	0.881	0.010	0.805	0.011	0.758	0.014	0.898	0.012	0.932	0.010	0.972	0.022	1.049	0.028	0.931	0.041	0.960	0.294	0.861	0.047
α_1	1.015	0.006	1.008	0.006	0.985	0.009	1.006	0.005	1.008	0.005	1.001	0.008	1.008	0.011	0.974	0.028	1.008	0.165	0.982	0.027
α_2	0.020	0.008	0.056	0.009	0.100	0.012	0.017	0.008	−0.004	0.006	−0.016	0.009	−0.046	0.015	0.035	−0.025	−0.025	0.248	0.057	0.041
β_1	0.420	0.005	0.355	0.005	0.365	0.006	0.495	0.004	0.431	0.005	0.511	0.006	0.381	0.007	0.651	0.025	0.825	0.160	0.407	0.021
β_2	−0.672	0.034	−0.413	0.042	−0.515	0.050	−0.634	0.032	−0.864	0.028	−0.974	0.038	−0.860	0.049	−0.304	0.115	1.963	1.235	−0.055	0.301
β_3	0.543	0.009	0.415	0.010	0.559	0.012	0.384	0.008	0.512	0.009	0.258	0.008	0.344	0.011	0.378	0.039	0.415	0.144	0.478	0.044
β_4	1.482	0.092	1.117	0.171	0.759	0.214	1.414	0.087	2.233	0.105	4.753	0.217	4.608	0.230	1.188	0.383	−5.062	4.172	−1.325	1.339
β_5	0.065	0.002	0.099	0.002	0.070	0.002	0.089	0.002	0.060	0.002	0.059	0.002	0.112	0.002	0.031	0.007	0.010	0.020	0.154	0.011
β_6	−0.347	0.009	−0.410	0.011	−0.449	0.014	−0.298	0.009	−0.332	0.010	−0.124	0.015	−0.552	0.016	−0.032	0.027	−0.096	0.148	−0.537	0.054
β_7	0.000	0.000	0.114	0.007	0.078	0.011	0.152	0.027	0.040	0.005	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
j	0.716		0.736		0.820		0.678		0.657		0.811		0.859		0.515		0.773		0.465	
d	−0.190		−0.224		−0.319		−0.209		−0.131		−0.131		−0.175		−0.242		−0.011		−0.304	

Appendix B: Predicting Volume Outside Bark Using the Fitted Kozak Taper Equation

To predict the volume outside bark (VOB) using the fitted Kozak taper equation, we need two steps: (1) predicting the diameter outside bark (diameter outside bark) for multiple points along the bole by substituting the estimated Kozak taper parameter values into Equation 4; and (2) numerically integrating the above predicted diameters or using Smalian's formula to obtain the VOB for that individual tree. In the following, we use red spruce as an example to illustrate the process.

Assuming we have a red spruce tree from naturally regenerated stand with the dbh $D = 24.1$ cm and the total height $H = 18.6$ m, we first calculate the diameter outside bark for each given height through the fitted Kozak equation (Equation 4). From Table 2, we obtain the estimated parameters of Equation 4 for red spruce: $\alpha_0 = 0.8758$, $\alpha_1 = 0.9920$, $\alpha_2 = 0.0633$, $\beta_1 = 0.4128$, $\beta_2 = -0.6877$, $\beta_3 = 0.4413$, $\beta_4 = 1.1818$, $\beta_5 = 0.1131$, $\beta_6 = -0.4356$, and $\beta_7 = 0.1042$. We then divide the tree bole into 100 equal length sections with the corresponding section heights $h_1, h_2, h_3, \ldots, h_{100.}$ The ground level height is referred to as h_0, which is equal to zero. For h_0

to h_{100}, we calculate $d_0, d_1, \ldots, d_{100}$ as follows:

$$z_0 = h_0/H = 0 \qquad X_0 = \frac{1-(h_0/18.6)^{1/3}}{1-(1.3/18.6)^{1/3}} = 1.7004 \qquad Q_0 = 1 - z^{1/3} = 1 - 0 = 1$$

$$\begin{aligned} d_0 &= \alpha_0 D^{\alpha_1} H^{a_2} X_0^{\beta_1 z_0^4 + \beta_2(1/e^{D/H}) + \beta_3 X_0^{0.1} + \beta_4(1/D) + \beta_5 H^{Q_0} + \beta_6 X_0 + \beta_7 I}, \\ &= 0.876 \times 24.1^{0.992} \times 18.6^{0.063} \times 1.7^{0.413\times 0 - 0.688(1/e^{24.1/18.6}) + 0.441\times 1.7^{0.1} + 1.182\times(1/24.1) + 0.113\times 18.6^{1} - 0.436\times 1.7 + 0.104\times 0} \\ &= 60.54 \end{aligned}$$

Similarly, we obtain $d_1 = 31.64$, $d_2 = 28.88 \ldots$, and$d_{99} = 0.94$. Because h_{100} is equal to the total tree height, we force the diameter at this point being zero, i.e., $d_{100} = 0$. Next, we use the Smalian formula to calculate the volume for each of those 100 sections, v_1, $v_2, \ldots$, and v_{100}.

$$\begin{aligned} v_1 &= (A_0 + A_1)/2 * L_1 \\ &= \frac{\pi}{8}(0.0001 \times (d_0^2 + d_1^2)) \times \frac{1}{100} \\ &= \frac{\pi}{8}(0.0001 \times (60.54^2 + 31.64^2)) \times \frac{1}{100} \times 15 \\ &= 0.0341\ m^3 \end{aligned}$$

Similarly, we can obtain $v_2 = 0.0134$ m^3, $v_3 = 0.0116$ m^3, . . . , and $v_{100} = 6.49 \times 10^{-6}$. By summing up these 100 section volumes, we obtain the total tree volume as 0.464 m^3.

We can use the above steps to calculate the merchantable volume or any section log volume. The only difference is how the section heights $h_0, h_1, h_2, \ldots, h_{100}$ are derived. For example, if the stump height (sH) is 1 m, and the top height (tH) is specified as 3 m, then we have $h_0 = 0 + \text{sH} = 1$ m, $h_1 = 1/100 \times \text{L} + \text{sH}$, and so on, where L is the length of merchantable log, i.e., L = tH − sH. Once section heights are known, we can follow the same procedure listed above to calculate diameters at each section height and then obtain the merchantable volume through Smalian's formula.

Although it is tedious to calculate the volume by hand, it is relatively easy to use a spreadsheet or other computer program to perform the calculation. A Microsoft Excel spreadsheet for calculating individual tree total and merchantable volume inside and outside bark is available on request.

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