# Some Notes on Gödel, Escher, Bach: an Eternal Golden Braid

Ibraheem Khan

February 2019

# Chapter 1

# Preface

"In a word, *GEB* is a very personal attempt to say how it is that animate beings can come out of inanimate matter." In many ways this book could have been "Gödel's Theorem and the Human Brain" as opposed to *GEB* as it is not concerned with these three men, nor particularly in connections between math, art, and music, but rather in the investigation of "strange loops" or "tangled hierarchies" which arise from meaningful patterns of meaningless symbols in particular systems that ultimately beget inanimate matter and later animate beings. The book is a study of consciousness (although I posit this idea could be extended for existence and the birth of the world in general). Hofstadter has one main point to make here: the connection between Godel's Theorem and Consciousness:

> The Gödelian strange loop that arises in formal systems in mathematics (*i.e.*, collections of rules for churning out an endless series of mathematical truths solely by mechanical symbol-shunting without any regard to meanings or ideas hidden in the shapes being manipulated) is a loop that allows such a system to "perceive itself", to talk about itself, to become "self-aware", and in a sense it would not be going too far to say that by virtue of having such a loop, a formal system *acquires a self.*

Another central thesis of the book is that the aforementioned "meaningless symbols" are not completely meaningless. In his own words, "meaning cannot be kept out of formal systems when sufficiently complex isomorphisms arise." For example, suppose we create some symbol, "a", this symbol may undergo some isomorphism $f$ (which implies $\exists\ f^{-1}$ as $f$ is an isomorphism on a), we could imagine this isomorphism being sufficiently complex in that it "tracks" or "mirrors" some phenomena in the real world, this complexity thus has already applied meaning to it. It is then a question of whether or not all such sufficiently complex isomorphisms on symbols must always model real world phenomena, if the real world is or isn't restricted to isomorphic actions on symbols, or if it is necessary for the functions themselves to be completely isomorphic (are morphisms enough?). In any case, meaning arises from a sufficiently complicated description- the less complicated description, the less meaningful. It is another question if description is enough to ascribe meaning.

Hofstadter continues to elaborate that consciousness is not unique to humans nor does it require some particular and undiscovered biological prerequisite- instead consciousness arises from certain patterns, the aforementioned "strange loops" rather than some keen biological or neuro-electro-chemical process. Such a view, creates hope for real artificial, electromechanical consciousness as there is no anthropocentrism involved in the creation of these strange loops- just that human evolution has done a good job of fashioning it. Consciousness only requires a media capable of supporting such strange, self-referential, loops.

Delving deeper here, we receive a soft-introduction to Russel and Whitehead's famous *Principia Mathematica* which aimed to solve the conundrum of the self-referentiality of the foundations of mathematics. We find later that

Kurt Gödel not only completely and provably rejects this idea but also shatters Hilbert's dreams of converting mathematics into mere "symbol shunting." Although, this idea seems universally accepted now, that Mathematics is semantically meaningful even outside of its application, this was not always the case and many mathematicians (the giant Hilbert himself included) were of the opinion that math was a mere plaything for humans- a toy exercise in logic and ideas or how Hilbert himself described "poetry made up of ideas, not words."

Gödel achieves this by employing a particular mapping scheme from metamathematical statements to those of mathematical number theory via his Gödel Numbering- that is all statements about mathematics are in fact, mathematical statements in themselves. Hofstadter notes that this is in fact very similar to the leap between inanimate to animate and in general is recursive- mathematical statements peer into themselves by their very nature. An important thing to note here is the central incompleteness of mathematics as demonstrated by the sentence G: "G is not provable inside *PM*" which is a *true* statement about mathematics but is still unable to be proven. That is, truth and provability are two totally different concepts. This also asserts that there is some causal power to meaning. This is a nonintuitive statement. We would assume that we can arbitrarily apply any meaning to any rule-bound string of symbols and it would not cause any thing. That is, a rule-bound string of symbols is internally complete [1] or that given our set of rules and symbols we can form all sentences, yet clearly adding meaning to a sentence (consider sentence G) causally implies the incompleteness of our rule-bound string of symbols.[2]

The preface further clarifies the nature of consciousness in that it is not a binary phenomenon, in fact in can be said that everything is conscious- it is a but a matter of how conscious. The "pattern called "I" can shove around inanimate particles in the brain no less than inanimate particles can shove around patterns." It remains to be said what such a view of identity and consciousness says of freedom of will, freedom of action, and freedom of thought. This is later explored in Hofstadter and Daniel Dennet's book *The Mind's I*.

[1] In mathematical logic and metalogic, a formal system is called complete with respect to a particular property if every formula having the property can be derived using that system, i.e. is one of its theorems; otherwise the system is said to be incomplete.

[2] Examples of rule-bound string of symbols include mathematics or really any formal language- better discussion is to be seen in Formal Languages and Automata

# Chapter 2

# Introduction: A Musico-Logical Offering

# Chapter 3

# The MU-Puzzle

# Chapter 4

# Meaning and Form in Mathematics

# Chapter 5

# Figure and Ground

# Chapter 6

# Consistency, Completeness, and Geometry

# Chapter 7

# Recursive Structures and Processes

# Chapter 8

# The Location of Meaning

# Chapter 9

# The Propositional Calculus

# Chapter 10

# Typographical Number Theory

# Chapter 11

# Mumon and Gödel

# Chapter 12

# Levels of Description and Computer Systems

# Chapter 13

# Brains and Thoughts

# Chapter 14

# Minds and Thoughts

# Chapter 15

# BlooP and FlooP and GlooP

# Chapter 16

# On Formally Undecidable Propositions of TNT and Related Systems

# Chapter 17

# Jumping Out of the System

# Chapter 18

# Self-Ref and Self-Rep

# Chapter 19

# Church, Turing, Tarski, and Others

# Chapter 20

# Artificial Intelligence: Retrospects

# Chapter 21

# Artificial Intelligence: Prospects

# Chapter 22

# Strange Loops, or Tangled Hierarchies