26/02/2014

Problem solving seminar
Test 2

Instructions

1. Work independently
2. Time: **50 minutes**
3. Books, notes, and calculators **are not allowed**
4. Please write down your name in **capital** letters, please write down your **e-mail** as well

Good luck!

Questions

1. If you are interested in taking part in the final test and have any *strong* objections agains either Saturday, March 8^{th}, or Friday 14^{th}, please let me know.

2. Prove that for integers $n \geq k \geq 1$ the number

$$\frac{\gcd(n,k)}{n}\binom{n}{k}$$

is an integer.

3. Consider the following game played by two players, Alice and Bob. Alice starts and players alternate turns until one of them wins or loses. If a player cannot make a move then they *lose*.

Start with two piles containing m and n coins respectively. On a player's turn they *must* remove all the coins from one pile and then they *must* divide the other pile into two new *non-empty* piles. Who has a winning strategy when $m = 2014$ and $n = 2014$?