5/02/2014

Problem solving seminar
Test 1

Instructions

1. Work independently
2. Time: **50 minutes**
3. Books, notes, and calculators **are not allowed**
4. Please write down your solutions for Problem 1 & 2 on **individual** sheets
5. Please write down your name in **capital** letters, please write down your **e-mail** as well

Good luck!
Tomasz Tkocz

Problems

1. Let $P : \mathbb{C} \longrightarrow \mathbb{C}$ be a polynomial with integer coefficients. Suppose that $|P(z)| \leq 2$ for every $z \in \mathbb{C}$ with $|z| = 1$. Prove that P has at most 4 nonzero coefficients.

2. A standard parabola is the graph of a quadratic polynomial $y = x^2 + ax + b$ with leading coefficient 1. By its vertex we mean the point $(-a/2, -a^2/4 + b)$. Three standard parabolas with vertices V_1, V_2, V_3 intersect pairwise at points A_1, A_2, A_3. Let $A \mapsto s(A)$ be the reflection of the plane with respect to the x-axis.

Prove that standard parabolas with vertices $s(A_1)$, $s(A_2)$, $s(A_3)$ intersect pairwise at the points $s(V_1)$, $s(V_2)$, $s(V_3)$.