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Problem solving seminar
Number Theory

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Homework

1. Let x, y, and z be integers such that $S = x^4 + y^4 + z^4$ is divisible by 29. Show that $29^4 \mid S$.

2. Find the number of positive integers x satisfying the following two conditions: $x < 10^{2014}$ and $10^{2014} \mid x^2 - x$.

3. Show that for each positive integer n,

$$n! = \prod_{i=1}^{n} \operatorname{lcm}\left\{1, 2, \ldots, \left[\frac{n}{i}\right]\right\}.$$