Exercise 8

1 Explicit MPC

Compute the explicit MPC controller for the following

$$\begin{aligned} u*(x) = \text{argmin} \quad & 2u^\top u + (x+u)^\top H(x+u) \\ \text{s.t.} \quad & u \in [-1, 1] \end{aligned}$$

where H is is the solution to the discrete-time algebraic Riccati equation for $A = 1, B = 1, Q, = 1$ and $R = 2$.

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Compute H from the algebraic Riccati equation

$$\begin{aligned} H &= Q + A^\top HA - A^\top HB(R + B^\top HB)^{-1}B^\top HA \\ H &= 1 + H - \frac{H^2}{H+2} \\ 0 &= H^2 - H - 2 = (H-2)(H+1) \end{aligned}$$

$H = 2 > 0$ is the solution, so the parametric optimization problem is now

$$\begin{aligned} u*(x) = \text{argmin} \quad & 2u^\top u + 2(x+u)^\top (x+u) \\ \text{s.t.} \quad & u \in [-1, 1] \end{aligned}$$

We solve for the case with no constraints by setting the derivative to zero:

$$4u^* + 4(x + u^*) = 0 \Rightarrow u = -\frac{x}{2}$$

This is the control law for $u \in [-1, 1]$. The control law will saturate if u is outside this range.

$$u^*(x) = \begin{cases} -1 \quad \text{if} \quad x \in (2, \infty) \\ -\frac{x}{2} \quad \text{if} \quad x \in [-2, 2] \\ 1 \quad \text{if} \quad x \in (-\infty, -2) \end{cases}$$