Multi-agent Cooperative Patrolling of Designated Points on Graphs

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1 The problem

In *patrolling* problems, one or more agents move around a given terrain to defend or supervise it by visiting designated places in the terrain with sufficient frequency [2].

Here we consider the problem where the terrain is represented by an undirected graph with specified edge lengths, on which agents move with speed 1 or less. Each vertex has an *idle time*, and is said to be *guarded* by a set of moving agents if it is visited at least once in every time period of that length. We are interested in deciding whether there is a schedule for a given number of agents to guard all vertices.

Since the problem for general graphs is NP-hard even for a single agent, we consider the following special graphs: *lines* (paths), *stars* and *unit-length cliques*. A star is a graph where there is a special vertex called the *centre*, which does not need to be guarded, and all other vertices are adjacent only to the centre. A unit-length clique is a complete graph with all edges equal in length. Since all that matters for patrolling is the time it takes to travel between each pair of nodes, a unit-length clique can be regarded as a special case of a star with all edges equal in length.

2 Previous work with no cooperation

Coene et al. [1] studied a slightly different version of the problem which require that each vertex be guarded already by one agent alone. In this setting, they showed polynomial time algorithms and NP-hardness results for some graph classes. We remove this requirement and study the problem where a vertex can be guarded by several agents in cooperation.

The two versions of the problem come to the same thing when there is only one agent. For this setting, Coene et al. [1] gave polynomial time algorithms for trees with uniform idle times and for lines, and proved that the problem is NP-hard for stars with arbitrary idle times. Thus, our study of the cooperating agents will focus on the cases where the problem is not already known to be NP-hard for single agent, that is, for (1) lines, (2) unit-length cliques, (3) stars with uniform idle times.

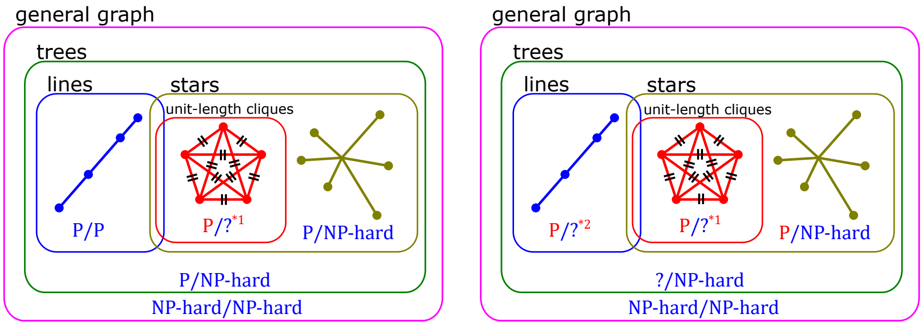

Figure 1: Classes of graphs and the complexity of the patrolling problem. The left figure is for a single agent and the right figure is for multiple agents in cooperation. The left side of the slash represents the case of uniform idle times, and the right side represents the case of arbitrary idle times.

3 Uniform idle time

We first considered the case where all vertices have the same idle time. We proved that our problem can be solved in polynomial time both for lines and for stars, because in these cases the situation is simplified in the following way: For lines, we showed that if patrolling is possible at all, then there is a patrolling schedule where each vertex is guarded by one agent alone. For stars, there is a schedule where the only form of cooperation is for several agents to periodically visit a subset of vertices in the same order at a certain time interval.

For these graphs, we can also efficiently solve the optimization problem where each vertex has a profit and we want to maximize the sum of the profit of the guarded vertices.

4 Arbitrary idle times

On the other hand, because it was difficult to determine the complexity for distinct idle times, we considered another variant of the problem where, instead of idle times, the exact times at which each point must be visited is specified.

For this setting, we found an algorithm that greedily determines the motions of the agents in lines (Figure 1-*2). Furthermore, we showed the NP-hardness of the optimization problem mentioned at the end of section 3 for unit-length cliques (Figure 1-*1) by a reduction from the maximum independent set problem.

References

[1] S. Coene, F. C. R. Spieksma, and G. J. Woeginger. Charlemagne's challenge: the periodic latency problem. *Operations research*, 59(3), pp. 674–683, 2011.

[2] J. Czyzowicz, L. Gąsieniec, A. Kosowski, and E. Kranakis. Boundary patrolling by mobile agents with distinct maximal speeds. In *Proc. 19th Annual European Symposium on Algorithms* (ESA), LNCS 6942, pp. 701–712, 2011.