

# Graphs III

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## Kruskal's Algorithm

- MST by merging smaller trees into bigger ones
- Graph  $F$  = set of trees (each vertex = separate tree)
- Set  $S$  = set of edges
- While  $S$  not empty and  $F$  not spanning:
  - o Remove minimum  $e$  from  $S$
  - o If edge connects two trees, add to  $F$

## DisjointSet ADT

- Very good for finding and union
- Keep each subtree as a set of vertices

## Quick-Find

- Check if componentId (group) is the same
- find =  $O(1)$ , union =  $O(n)$

## Quick-Union

- Record parent (root parent = itself)
- Find: compare root ids by traversing upwards
- Union: update parent
- Unbalanced =  $O(n)$  for both

## Balancing Trees (Weighted Union)

- Instead of using the height of tree to balance, use size
- Max height =  $O(\log n)$
- Tree only increases in height when the merging trees have the same height
- Find and Union are now  $O(\log n)$

## Path Compression

- After finding root, set parent of each traversed node to the root
- Alternative: make roots point to grandparent
- Weighted union + path compression = flat trees, almost linear operations

## Kruskal's Algorithm - Improved

- Sort edges, iterate through edges in order =  $O(E \log E)$ 
  - o  $O(E \log E) = O(E \log V^2) = O(E \log V)$
- For  $E$  edges, find/ union =  $O(E\alpha)$