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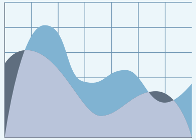
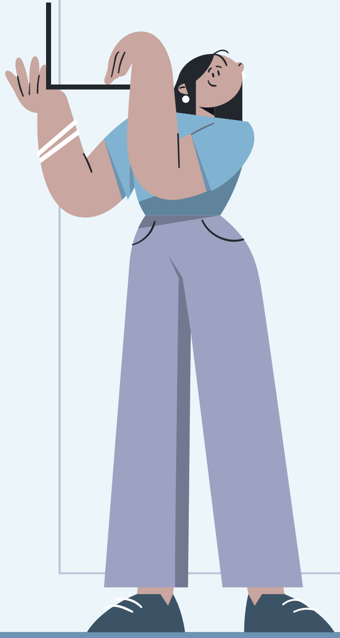


Structural Equation Modeling: CFA

Speaker Series

Tessa Correig, Tomas Lock, Sofia Serantes, Vittorio Fialdini, JP, Pilar Guerrero.



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Introduction to SEM

Structural Equation Modeling (SEM)

- SEM is a statistical model that seeks to explain the relationships among multiple variables.
- It helps us to understand how different variables are related to each other.

Imagine we have a bunch of variables that we think are connected somehow.

- SEM uses a set of equations to show these connections between what we're trying to explain (dependent variables) and the things we think are causing the explanation. It is kind of like solving a puzzle.

Structural Equation Modeling (SEM)

Is a mix of:

Factor Analysis

Focuses on finding hidden factors.

Multiple Regression Analysis

Empathizes on looking at relationships.

Constructs and latent factors

- Some variables and theoretical concepts we care about are not directly visible, these are called **constructs** and **latent factors** consecutively.
- Constructs are the theoretical ideas or concepts researchers are interested in, while latent factors are the statistical tools used to represent these constructs in quantitative models.
- We use multiple observed variables that are believed to be indicative of or associated with these hidden factors we want to understand.

Characteristics

- It does more than traditional regression models.
- It looks at many relationships at once.
- Defines a theoretical model to explain the entire set of relationships
- Over-identifying assumptions (meaning variables are explained by a unique set of variables that does not include all possible relationships)

Other Multivariate techniques

MANOVA and multiple regression. SEM is similar to multiple regression, where relationships for each endogenous construct are expressed in regression equations.

The measurement model in SEM also appears similar to exploratory factor analysis. However, SEM is confirmatory, meaning relationships are specified in advance, unlike the exploratory nature of factor analysis.

SEM allows the use of multiple measures for a construct, addressing and correcting for measurement error in the estimation process.

This enhances the accuracy of relationships between constructs.

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Theory and Structure of SEM

Role of Theory in SEM

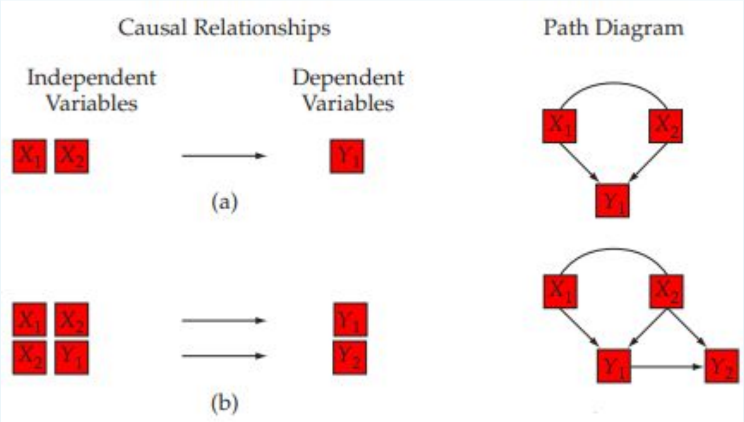
Specifying Relationships

Establishing Causation

Developing modeling strategy

Specifying Relationships

Specifying relationships involves determining the connections between observed variables (indicators) and latent constructs (factors or variables that are not directly observable but inferred from observed variables).



- The relationships of a model can be depicted through a path diagram, indicating how variables influence each other.
- In example (a) there is a simple three-construct model, where X1 and X2 are used to predict Y1. We can show this model with a single equation with Y1 as a function of X1 and X2.
- In example (b) we add a second endogenous construct in the form of Y2, we want to know the effects of X1 and X2 on Y1 but we simultaneously consider the effects of X2 and Y1 on Y2. In this case Y1 is simultaneously a dependent and an independent variable

Establishing Causation

Theory plays a crucial role in guiding the specification of causal relationships between variables within the SEM framework. Simply testing a SEM model and analyzing its result cannot establish causality, theoretical support is needed to support a cause-and-effect relationship.

For example, consider the question: Do employees' feelings about their supervisors cause job satisfaction? A theoretical justification for causation could exist, such as increased familiarity with supervisor approaches leading to improved understanding, reactions, and subsequently, increased job satisfaction. Theoretical reasoning supports the idea that more favorable feelings about supervisors cause higher job satisfaction.

Developing Modeling Strategy

Developing a modeling strategy involves making decisions about model specification, selecting variables to include, how they are related, and how the model will be tested and validated. There are three main types of modeling strategy: Confirmatory modeling, Competing modeling and Model development.

Confirmatory Modeling:

- Involves specifying a theoretical model with defined relationships and non-relationships
- SEM assesses how well this proposed model fits the observed data
- Opposite of exploratory methods like stepwise regression
- Finding acceptable fit provides support for the proposed model, but there may exist other models with similar fits

Competing Modeling:

- Focuses on comparing a theoretical estimated model with alternative theories by assessing relative fit.
- Strongest test involves comparing and testing competing models representing different, plausible theories.
- Aims to identify which theory or model fits the data better, moving closer to comparing competing theories.

Model Development:

- Theory provides a starting point, but the goal is to refine the model based on empirical support from SEM insights.
- Caution required to ensure the final model isn't overly tailored to the specific sample and maintains theoretical support.
- Models developed empirically should be validated with an independent sample to ensure generalizability.

A Simple Example of SEM

Let's consider the following situation in developing a model for a company, they would like to answer two key research questions are: (1) what factors influence job satisfaction and (2) is job satisfaction related to employees' likelihood of looking for another job (i.e., quitting their present job)?

Dependent Variables (Endogenous)

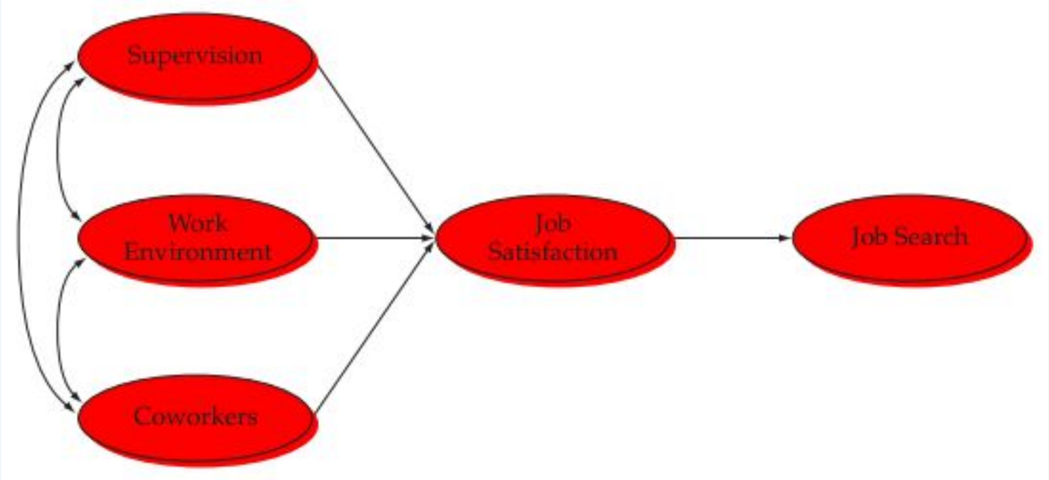
- Job Satisfaction
- Job Search

Independent Variables (Exogenous)

- Supervision
- Work Environment
- Coworkers

The supervision, work environment, and coworker constructs are identified as exogenous variables because they are not predicted by constructs within the model. Job search is clearly an endogenous variable because it is represented as a dependent variable. Job satisfaction is dependent on the supervision, work environment and coworker constructs but it is also an independent variable towards job search.

A Simple Example of SEM



This path diagram represents the relationship between all of our constructs.

Although multicollinearity wasn't mentioned by the research questions, relationships amongst exogenous constructs are generally assumed.

A Simple Example of SEM

SEM uses a covariance structure analysis, and as a result it focuses on explaining covariation among all measured variables, which together form the covariance matrix.

To understand how data are input into SEM, think of the covariance matrix among the five variables. the number of unique values in the matrix is the five diagonal values (variances) plus the 10 unique off-diagonals (covariances), for a total of 15.

	S	WE	CW	SAT	SRCH
	Var (S)				
Observed	Cov (S,WE)	Var (WE)			
Covariance	Cov (S,CW)	Cov (WE,CW)	Var (CW)		
	Cov (S,SAT)	Cov (WE,SAT)	Cov (CW,SAT)	Var (SAT)	
	Cov (S,SRCH)	Cov (WE,SRCH)	Cov (CW,SRCH)	Cov (SAT,SRCH)	Var (SRCH)

A Simple Example of SEM

The covariance matrix is created for estimated results and observed results, both are then compared to form the residual covariance matrix.

	Supervision	Work Environment	Coworkers	Job Satisfaction	Job Search
(A) Observed Covariance Matrix: (S)	*Var (SP)*	—	—	—	—
	.20	*Var (WE)*	—	—	—
	.20	.15	*Var (CW)*	—	—
	.20	.30	.50	*Var (JS)*	—
	–.05	.25	.40	.50	*Var(JS)*
(B) Estimated Covariance Matrix: (Σ)	—	—	—	—	—
	.20	—	—	—	—
	.20	.15	—	—	—
	.20	.30	.50	—	—
	.10	.15	.25	.50	—
(C) Residuals: Observed Minus Estimated Covariances					
Supervision	—	—	—	—	—
Work Environment	.00	—	—	—	—
Coworkers	.00	.00	—	—	—
Job Satisfaction	.00	.00	.00	—	—
Job Search	–.15	.10	.15	.00	—

A Simple Example of SEM

The covariance matrix is created for estimated results and observed results, both are then compared to form the residual covariance matrix.

The residual table shows that three residuals are not zero (-.15, .10, .15)

These findings indicate that the SEM model does not perfectly explain the covariance between these constructs.

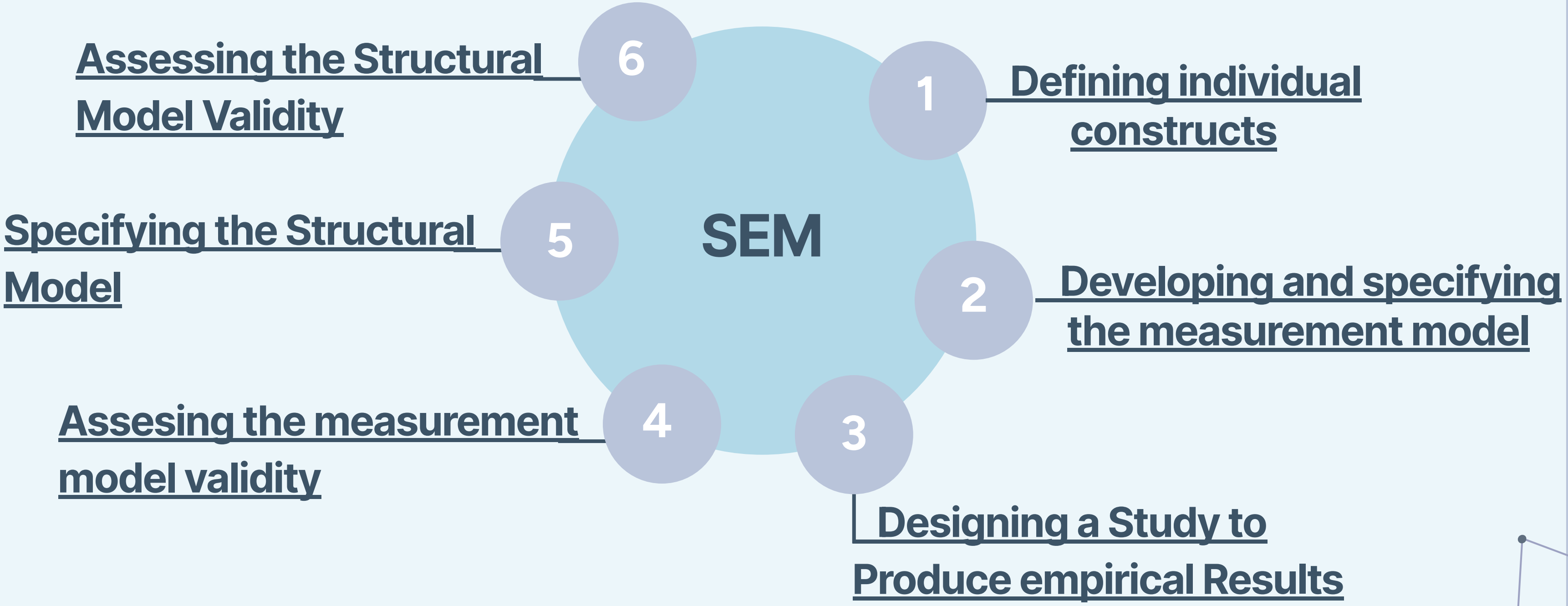
It could also suggest that the researcher’s theory is inadequate in explaining Job Search, but we need additional information before rejecting the proposed theory

	Supervision	Work Environment	Coworkers	Job Satisfaction	Job Search
(A) Observed Covariance Matrix: (S)	*Var (SP)*	—	—	—	—
	.20	*Var (WE)*	—	—	—
	.20	.15	*Var (CW)*	—	—
	.20	.30	.50	*Var (JS)*	—
	−.05	.25	.40	.50	*Var(JS)*
(B) Estimated Covariance Matrix: (Σ)	—	—	—	—	—
	.20	—	—	—	—
	.20	.15	—	—	—
	.20	.30	.50	—	—
	.10	.15	.25	.50	—
(C) Residuals: Observed Minus Estimated Covariances					
Supervision	—	—	—	—	—
Work Environment	.00	—	—	—	—
Coworkers	.00	.00	—	—	—
Job Satisfaction	.00	.00	.00	—	—
Job Search	−.15	.10	.15	.00	—

03

Six Stages of SEM

What are the six stages?



Stage 1: Defining Individual Constructs

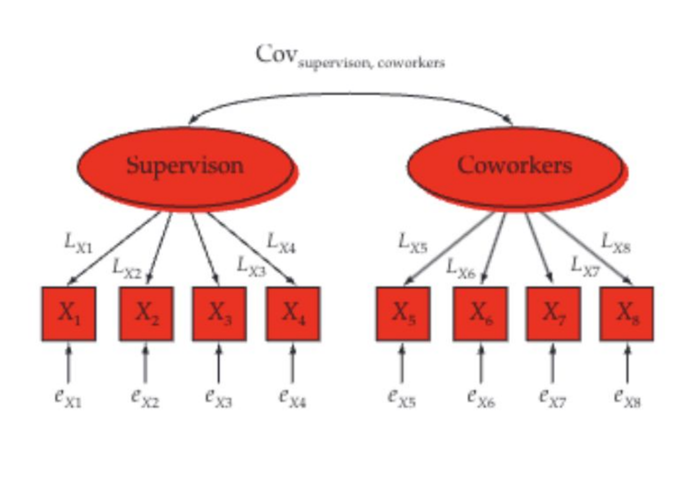
1. The process starts by having a good definition of the constructs (what you are trying to measure)

2. The operationalization is typically based on two approaches:
Scales from Prior Research: Using existing scales from literature
New Scale Development: Necessary when studying new or under-researched areas

3. Pretesting: uses a sample resembling the target group to assess item suitability, important when applying scales in unique contexts or beyond their typical use. **Items are empirically evaluated like in the final analysis**, enabling the **improvement or removal of underperforming** items.

Stage 2: Developing and specifying the measurement model

In this stage the main objective of the researcher is to specify the measurement model by accurately defining the latent constructs and then assigning observable indicators to these constructs.



When creating the measurement model:

- Can we empirically support the validity and unidimensionality of the constructs?
- How many indicators should be used for each construct? What are the trade-offs for increasing or decreasing the number of indicators?
- Should the measures be considered as portraying the constructs (meaning that they describe the construct) or seen as explaining the construct (such that we combine indicators into an index)?

ATTENTION PLEASE!

Stage 3: Designing a study to produce empirical evidence

Now the researcher must reflect upon the study that was created and asses its adequacy.

Sample Size matters:

Depending on the study being performed it needs to follow the minimum SS requirements:

Handling of missing data:

Is the missing data sufficient and nonrandom so as to cause problems in estimation or interpretation?

If missing data must be remedied, what is the best approach?

Issues with model estimation:

Model Structure

Parameters

Estimation techniques

Computer programs

Stage 4: Assessing the measurement model validity

During this stage the model is tested through the Goodness of fit

Basic Goodness of Fit:

How well the theoretical model (what the researchers thought up) matches the real-world data (what was actually observed). The closer the estimated data from the model is to the observed data, the better the fit.

Fit Indices: Other methods to measure how well your model fits the data

Absolute fit: How well the model fits the data on its own

Incremental fit: Compares the model to a baseline (null model)

Parsimony Fit: Rewards simpler models

GOF guidelines:

No Absolute Cut-offs

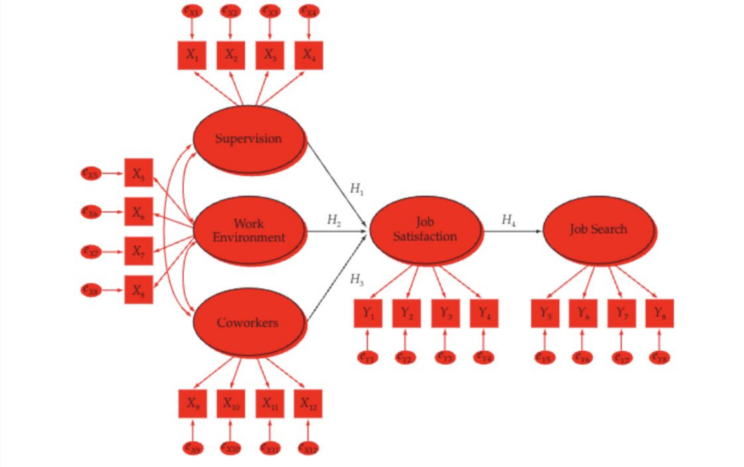
Use Multiple Indices

Model Comparison

Theory Testing vs. Model Fit

Stage 5: Specifying the Structural Model

In this stage the main objective is converting measurement model to structural model



Specified Hypothesized Structural Relationships and Measurement Specification

When creating structural model:

Specifying Hypothesized Relationships: Define the theoretical pathways between constructs, such as 'Supervision' influencing 'Job Satisfaction'.

Hypothesis Formulation: Enumerate each hypothesized relationship (H1, H2, etc.) to facilitate clear and testable predictions within the structural model.

Empirical Verification: Undertake the empirical testing of these hypotheses to assess the validity of the structural relationships posited in the model.

Stage 6: Assessing the Structural Model Validity

Theoretical Pathways: Establish and articulate the expected influences between constructs (e.g., 'Supervision' → 'Job Satisfaction').
Visual Modeling: Deploy a path diagram to visually represent each hypothesized structural relationship.

Hypotheses Enumeration: Assign a unique identifier (H1, H2, etc.) to each predicted relationship for clarity and reference.

Data-Driven Analysis: Conduct statistical tests to compare the theoretical predictions against real-world data to affirm or refute the hypothesized model structure.

04

CFA applied to SEM

What is CFA

CFA is a way of testing how well a prespecified measurement theory composed of measured variables and factors fits reality as captured by data.

We must specify:

- ➔ Number of factors in a set of variables.
- ➔ Which factor each variable will load on before results can be computed.

This statistical technique does not assign variables to factors.

It is important to note that the measurement theory should specify independence for all error variance terms.

SEM and CFA

CFA is fundamental for SEM because, CFA will be used to test a measurement theory. Then, this measurement theory may then be combined with a structural theory to fully specify a SEM model.

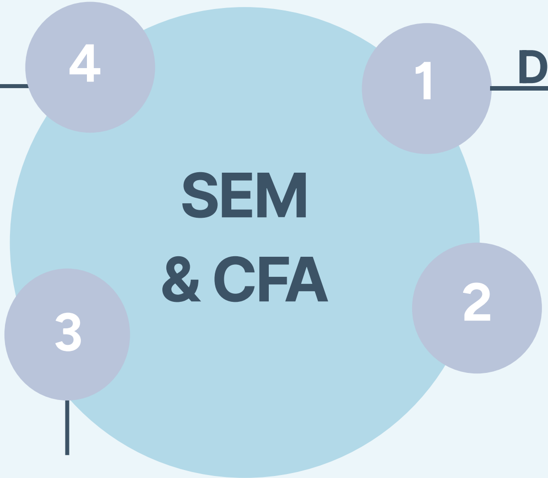
Measurement Theory: A measurement theory specifies precisely how measured variables logically and systematically represent constructs involved in a theoretical model.

05

4 SEM Stages

For testing Measurement theory Validation with CFA

What are the six stages?



1. Defining individual constructs
2. Developing the overall measurement model
3. Designing a Study to Produce empirical Results
4. Assesing the measurement model validity

Stage 1: Defining the individual Constructs

The process begins by defining all constructs that will comprise the measurement model.

If the researcher has experience with measuring one of these constructs, then perhaps some scale that was previously used can be applied again.

When a previously applied scale is not available, the researcher may use psychometric principles and the steps of scale development to produce a measure.

Psychometric principles:

1. Only the loadings theoretically linking a measured item to its corresponding latent factor are freely estimated.
2. All other possible loadings areas sum to be equal to zero.
3. No covariance exists among the residuals.

It is essential that researchers consider:

- ➔ The operational requirements.
- ➔ The construct validity of the newly designed scale.

Stage 2: Developing the overall Measurement Model

Unidimensionality

Congeneric Measurement Model

Items per Construct

Reflective VS Formative Measurements

Stage 2: Developing the overall Measurement Model

Unidimensionality

- Unidimensional measures mean that a set of measured variables (indicators) can be explained by only one underlying construct.
- There are two types of relationships among variables that impact unidimensionality:
  - When researchers allow a single measured variable to be caused by more than one construct.
  - The covariance among error variance terms of two measured variables.
    - There are two types of covariance between error variance terms exist:
      - Within-construct error covariance
      - Between-construct error covariance

Stage 2: Developing the overall Measurement Model

Congeneric Measurement Model

- SEM terminology often states that a measurement model is constrained meaning that it refers specifically to the set of fixed parameter estimates.
- When a measurement model also hypotheses no covariance between or within construct error variances, meaning they are all fixed at zero, the measurement model is said to be congeneric.
- **Congeneric measurement models** are considered to be sufficiently constrained to represent good psychometric properties.

Stage 2: Developing the overall Measurement Model

Items per Construct

Formula for identification

$$1/2[p(p+1)]$$

where p = the number of measured variables.

Levels of identification

Underidentified Model
Just-Identified Model
Overidentified Model

Model Fit

Chi square
CFI

At least 3 or 4 items per construct

Stage 2: Developing the overall Measurement Model

Reflective vs Formative Measurements

Reflective

View latent constructs as the driving force behind the measured variables.

The model is grounded in classical test theory.

The latent construct is influencing or causing the observed variables.

Formative Measurements

Measured variables, instead of being influenced by a latent construct.

The idea is that each measured variable is a cause of the index score.

Stage 3: Designing a Study to Produce Empirical Results

In this stage we design a study that will produce confirmatory results.

Measurement Scales

SEM and Sampling

Specifying the Model

Issues in Identification

Problems in Estimation

Stage 3: Designing a Study to Produce Empirical Results

Measurement Scales

- CFA models typically contain reflective indicators measured with an ordinal or better measurement scale.
- All the indicators for a construct are not required to be of the same scale type, and different scale values do not have to be normalized prior to using SEM.
- **Normalization** → recommended prior to estimating the model as it can make interpreting coefficients and response values easier.
- Typical survey research data suitable for a CFA model using SEM as it has few restrictions.

Stage 3: Designing a Study to Produce Empirical Results

SEM and Sampling

- CFA requires the use of multiple samples, particularly when the measurement model includes new scales.
- An initial sample can be examined with EFA and the results used for further purification.
- Then an additional sample(s) should be drawn to perform the CFA.

Stage 3: Designing a Study to Produce Empirical Results

Specifying the Model

- In CFA the researcher specifies the indicators associated with each construct and the correlations between constructs.
- “Setting the scale” of a latent factor → must be done for both exogenous and endogenous constructs in one of two ways:
  - Fix one of the factor loadings on each construct to a specific value (1 is typically used)
  - Fix the value of the variance of the construct (again 1 is typically used)

Stage 3: Designing a Study to Produce Empirical Results

Issues in Identification

Recognizing Identification Problems

- Symptoms of Identification problems:
  - Inability of the program to invert the information matrix
  - Unreasonable or impossible estimates
  - Models that result in unstable parameter estimates in the presence of small changes.

Sources and Remedies of Identification Problems

- Incorrect Indicator Specification
  - Indicators are not properly linked to constructs.
- "Setting the Scale" of a Construct
  - not "setting the scale" of each construct.
- Too Few Degrees of Freedom
  - Small samples increases this kind of problems

Stage 3: Designing a Study to Produce Empirical Results

Problems in Estimation

Illogical Standardized Parameters

- Occurs when correlation estimates between constructs exceed |1.0| or standardized path coefficients exceed |1.0|.
- Possible causes:
  - Identification problems
  - Data issues
  - Poorly specified constructs

Heywood Cases

- A SEM solution that produces an error variance estimate of less than zero.
- Problematic in CFA models with small samples.
- SEM program may still produce a solution in which the model does not fully converge.

Stage 4: Assessing the Measurement Model Validity

In this stage we compare the model's result with the real data by using goodness of fit.

Path Estimates

Most fundamental assessments of construct validity which involves the measurement relationships between variables.

Construct Validity

Deals with the accuracy of the measurement. Is the extent to which a set of measured items accurately reflect the theoretical latent constructs they are designed to measure.

Model Diagnostics

Stage in which additional diagnostic information that may suggest modifications for either addressing unresolved problems or improving the model's validity are provided.

Stage 4: Assessing the Measurement Model Validity

Path Estimates

Size and Statistical Significance

Rules of thumb: standardized indicator loadings should be at least .5 and ideally .7 or higher.

High loadings confirm that the indicators are strongly related to their associated constructs and are one indication of construct validity.

Low loadings suggest that a variable is a candidate for deletion from the model.

Identifying Problems

Make sure loadings make sense.

Items with the same valence (e.g., positive or negative wording) should produce the same sign.

Loadings above 1.0 or below -1.0 are out of the feasible range and are an important indicator of a problem with the model.

Stage 4: Assessing the Measurement Model Validity

Construct Validity

Factor Loadings

High loadings on a factor indicate that they converge on a common point,
The square of a standardized factor loading represents how much variation in an item is explained by the latent factor and is termed the variance extracted of the item.

Average Variance Extracted

$$AVE = \frac{\sum_{i=1}^{n} L_i^2}{n}$$

Construct Reliability

$$CR = \frac{\left(\sum_{i=1}^{n} L_i\right)^2}{\left(\sum_{i=1}^{n} L_i\right)^2 + \left(\sum_{i=1}^{n} e_i\right)}$$

Stage 4: Assessing the Measurement Model Validity

Model Diagnostics

Standardized Residuals

The raw residuals divided by the standard error of the residual.

Modification Indices

A modification index is calculated for every possible relationship that is not estimated in a model.

Specification Searches

An empirical trial-and-error approach that uses model diagnostics to suggest changes in the model

Caveats in Model Respecification

When more than 20% of the measured variables are dropped a new dataset should be used.