Transport and Mobility Modelling

PART A

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Q1. Demand modelling

A new policy is proposed in the UK which aims to affect daily commutes (travel-to-work journeys). The policy is defined as follows:

• Discourage long commutes (≥ 10 km, one way), originating in a densely populated area i ($m_i \geq 10{,}000$), by introducing a daily 1 pence per km fee for any such trips.

• Encourage long commutes (≥ 10 km, one way), originating in rural, sparsely populated areas ($m_i < 10{,}000$), by providing a daily subsidy of £2 per trip.

For the purpose of this analysis, you should assume that the UK is made up of a set of zones i, with populations m_i , and pairwise distances $d_{i,j}$ (km) between zones $j \neq i$.

a. Net-positive or Net-negative

The Radiation model is in the form:

$$T_{i,j} = T_i \frac{P_i P_j}{(P_i + S_{i,j})(P_i + P_j + S_{i,j})}$$

T_i,j is the number of trips from i to j, T_i is the number of trips originating in i (a product of employment rate and population of the area), P is the population of the area, and S is the sum of the populations of areas within the circle centred at i extending to j not including the areas i and j.

Start by assessing current situation..

The total number of trips is therefore given by:

$$\sum T_{i,j} = \sum_{j=1}^{n} \sum_{i=1}^{n} T_i \frac{P_i P_j}{(P_i + S_{i,j})(P_i + P_j + S_{i,j})}$$

where

$$S_{i,j} = \sum_{k=1}^{n} P_k | (d_{k,i} \leq d_{i,j}, k \neq i, j, i \neq j)$$

The benefit to the UK treasury occurs for every pairing of origin-destination areas where the origin population is greater than or equal to 10000, and the distance between the pair is greater than 10km. For each of these events the fee is £0.01 multiplied by the distance and the number of trips.

And so the benefit B: (distance in kM)

$$B = \sum_{i=1}^{n} \sum_{j=1}^{n} 0.01 d_{i,j} T_{i,j} | (d_{i,j} \geq 10, P_i \geq 10000, i \neq j)$$

The cost to the UK treasury is £2 multiplied by the number of trips where the distance is greater than 10km and the population of the origin area is less than or equal to 10000.

And so the cost C:

$$C = \sum_{i=1}^{n} \sum_{j=1}^{n} 2 T_{i,j} | (P_i < 10000, d_{i,j} \geq 10)$$

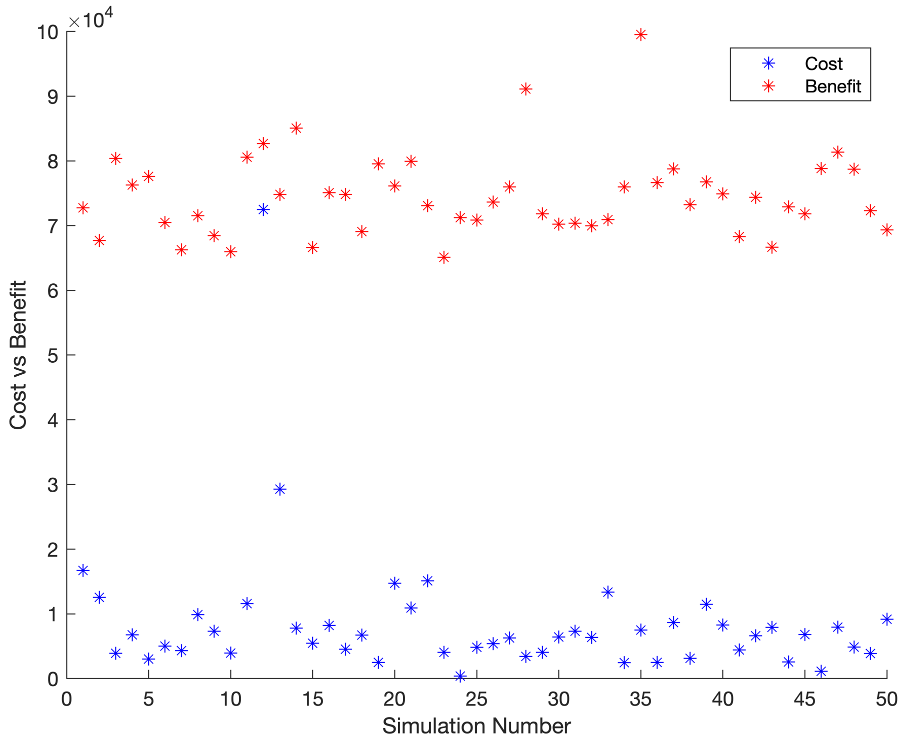
With T_i,j and S_i,j defined as above.

The **net cos**t is therefore B - C, where if this value is positive it is net-positive for the UK treasury.

```
% run simulation n times
n = 50;
figure;
```

```
hold on
cost_v = [];
benefit_v = [];
for sim = 1:n
    % Define parameters
    numAreas = 50;  % Number of towns
    population = randi([1, 50000], 1, numAreas); % Random pop for each area
    distanceMatrix = randi([1, 100], numAreas, numAreas); % Random distance mat

    % Initialize T_ij matrix
    T_ij = zeros(numAreas, numAreas);

    % Calculate S_ij
    S_ij = zeros(numAreas, numAreas);
    for i = 1:numAreas
        for j = 1:numAreas
            if i ~= j
                S_ij(i, j) = sum(population(distanceMatrix(:, i)...
                    <= distanceMatrix(i, j) & (1:numAreas)' ~= i & (1:numAreas)' ~= j)
            end
        end
    end
    cost = 0;
    benefit = 0;
    % Calculate T_ij
    for i = 1:numAreas
        for j = 1:numAreas
            if i ~= j
                numerator = population(i) * population(j);
                denominator = (population(i) + S_ij(i, j)) * (population(i)...
                    + population(j) + S_ij(i, j));
                T_ij(i, j) = population(i) * numerator / denominator;
            end
        end
    end
    for i = 1:numAreas
        for j = 1:numAreas
            if population(i) >= 10000 && distanceMatrix(i, j) >= 10
                % Calculate benefit
                benefit = benefit + 0.01 * T_ij(i, j) * distanceMatrix(i,j);
            end

            if distanceMatrix(i, j) >= 10 && population(i) < 10000
                % Calculate cost
                cost = cost + 2 * T_ij(i, j);
            end
        end
    end

    cost_v(sim) = cost;
```

```
    benefit_v(sim) = benefit;

end
plot(1:sim,cost_v,'*',Color='b',DisplayName='Cost');
plot(1:sim,benefit_v,'*', Color='r', DisplayName='Benefit');
xlabel('Simulation Number')
ylabel('Cost vs Benefit')
legend
```



With this random silulation set-up it appears that the benefit is nearly always > cost and so in this simulation it is net-positive however this is not representative of the UK.

b. Percentage of people affected by the policy

The percentage will be:

$$\frac{\sum T_{i,j}|(P_i \geq 10000, d_{i,j} \geq 10) + T_{i,j}|(P_i < 10000, d_{i,j} \geq 10)}{\sum T_{i,j}|(i \neq j)}$$

This is the sum of all trips greater than 10km from densely populated areas, and the sum of all trips greater than 10km from rural areas, divided by the total number of trips occuring.

However, if considering the sum of all trips from both P >= 10000 and P < 10000, this sum effects people from all population sizes, and the only dependency becomes the trip distance.

$$\frac{\sum T_{i,j}|d_{i,j} \geq 10}{\sum T_{i,j}}$$

(multiplied by 100)

```
% for the example
numer = 0;
totalTrips = sum(sum(T_ij));
for i = 1:numAreas
        for j = 1:numAreas
            if distanceMatrix(i, j) >= 10
                % numerator
                numer = numer + T_ij(i,j);
            end
        end
end

% in this simulation
percentage_effected = (numer/totalTrips) * 100
```

```
percentage_effected = 25.3264
```

c. Six city system

City	x	y	P_i
A	0	0	50000
B	5	5	20000
C	10	0	50000
D	0	-7	20000
E	-7	-3	18000
F	-12	6	15000

i. How many trips originate in A and terminate in C

```
x = [0,5,10,0,-7,-12];
y = [0,5,0,-7,-3,6];
p = [50000, 20000, 50000, 20000, 18000, 15000];

% make a origin - destination distance matrix
d_ij = zeros(6,6);

for i = 1:length(x) % origin
    for j = 1:length(y) % destination
        x_dist = abs(x(i) - x(j));
        y_dist = abs(y(i) - y(j));
```

```
        d_ij(i,j) = sqrt(x_dist^2 + y_dist^2);
    end
end

% trips originating in A
employ = 0.755  % the national employment rate from ONS 2023
```

```
employ = 0.7550
```

```
T_a = p(1) * employ
```

```
T_a = 37750
```

```

% s term
S_a = 0;
for i = 1: length(d_ij)
    if d_ij(i,1) < d_ij(3,1) && i~=1  && i~=3 % exclude start/end
        S_a = S_a + p(i);
    end
end

T_ac = T_a*(p(1)*p(3))/((p(1)+ S_a)*(p(1)+p(3)+S_a));

% trips going from A to C:
round(T_ac)
```

```
ans = 5531
```

ii. What is the ratio of T_ac / T_ca

```
T_c = p(3) * employ
```

```
T_c = 37750
```

```

% s term
S_c = 0;
for i = 1: length(d_ij)
    if d_ij(i,3) < d_ij(1,3) && i~=1 && i~=3 % exclude start/end
        S_c = S_c + p(i);
    end
end
S_c
```

```
S_c = 20000
```

```

T_ca = T_c*(p(1)*p(3))/((p(3)+ S_c)*(p(1)+p(3)+S_c));

% ratio
ratio_ac_ca = T_ac / T_ca
```

```
ratio_ac_ca = 0.4923
```

More trips go from C to A than vice versa.

Q2. Assignment modelling

a. Derive minimisation problems that you would solve to determine the UE and SO assignments

The links in the problem have costs:

$c_1(x_1) = a + x_1$ and $c_2(x_2) = 1 + bx_2$

Subject to:

$x_1 + x_2 = d$

$x_1, x_2 \geq 0$

User Equilibrium occurs when the costs for either route become equal:

$a + x_1 = 1 + \mathrm{bx}_2$

$x_1 + x_2 = d$

Solved such that:

$$x_1 = \frac{-a + \mathrm{bd} + 1}{b + 1}, x_2 = \frac{a + d - 1}{b + 1}$$

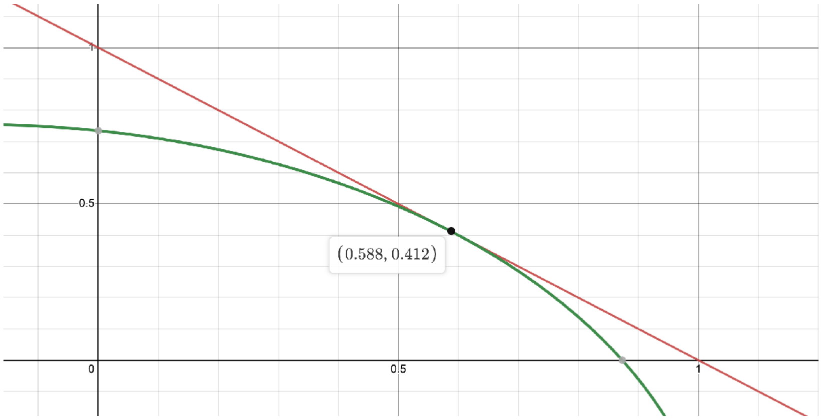
Whereas the **system optimal** solution considers when the total system travel time is minimised, and so the total travel time is the sum of the route costs multiplied by the respective flows:

Minimise:

$f = \mathrm{ax}_1 + x_1^2 + x_2 + \mathrm{bx}_2^2$

$x_1 + x_2 = d$

This can be solved graphically (or using optimisation software), using f as an objective function, finding the minimum value of f that meets the demand constraint (plotting x_2 against x_1, f in green, constraint in red, for a=0.4, b=0.7, d= 1):




```
% define minimisation problem for User Equilibrium and System Optimal
a_var = 0.4;
b_var = 0.7;
d_var = 1;

H = [1, 0; 0, b_var]; % x^2 coefficients
f = [a_var; 1]; % x coefficients
Aeq = [1, 1];
beq = d_var;
options = optimoptions('quadprog', 'Display', 'off');

x_ue = UE_sol(H, f, Aeq, beq, options)
```

```
x_ue = 2×1
    0.7647
    0.2353
```

```
x_so = SO_sol(H, f, Aeq, beq, options) % the quadratic coefficients *2
```

```
x_so = 2×1
    0.5882
    0.4118
```

b. Price of Anarchy (PoA)

The PoA is the ratio of User Equilibrium Cost over the System Optimal Cost

In this case:

$$\text{POA} = \frac{\sum_i x_i^{\text{UE}} c_i}{\sum_i x_i^{\text{SO}} c_i} = \frac{x_1^{\text{UE}} c_1\left(x_1^{\text{UE}}\right) + x_2^{\text{UE}} c_2\left(x_2^{\text{UE}}\right)}{x_1^{\text{SO}} c_1\left(x_2^{\text{SO}}\right) + x_1^{\text{SO}} c_2\left(x_2^{\text{SO}}\right)}$$

c. Varying demand for fixed a and b values

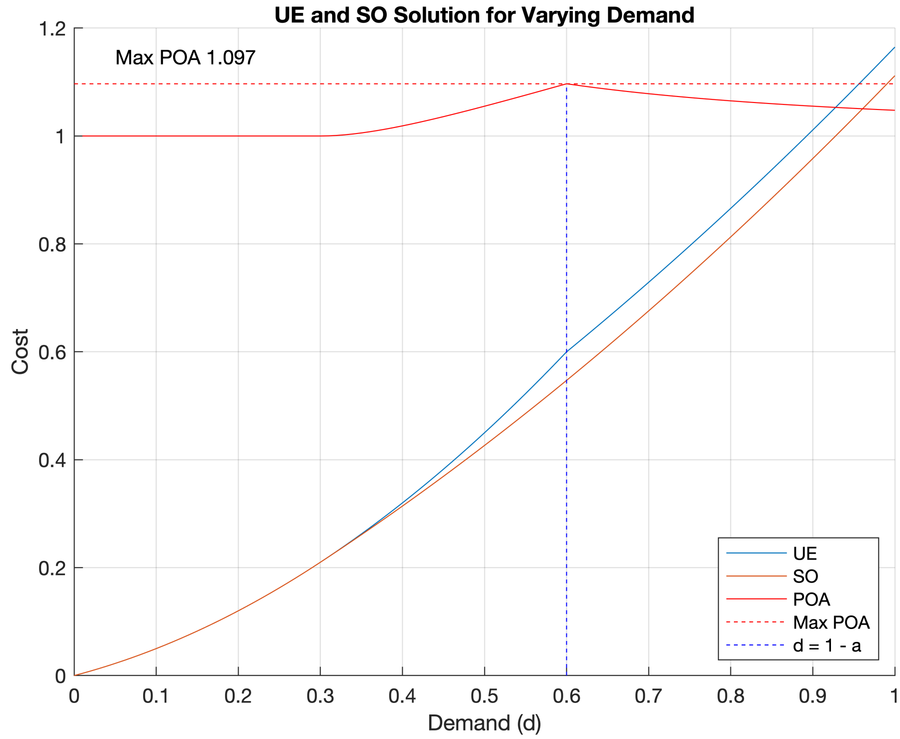
```
clf;
% Initialize matrices to store solutions
d_values = 0:0.01:1; % up to d=1 as thats the interesting bit
x_ue_vals = zeros(length(d_values), 2);
```

```
x_so_vals = zeros(length(d_values), 2);

cost_ue = zeros(length(d_values),1);
cost_so = zeros(length(d_values),1);

% Solve the UE problem for each demand value
for i = 1:length(d_values)
    d = d_values(i);

% Define matrix H, f, and equality constraint
    H = [1, 0; 0, b_var];
    f = [a_var; 1];
    Aeq = [1, 1];
    beq = d;

% Solve the quadratic programming problem using quadprog
    x_ue = quadprog(H, f, [], [], Aeq, beq,[0,0],[],[], options);
    x_so = quadprog(2*H, f, [],[], Aeq, beq,[0,0],[],[], options);

% Store the solution
    x_ue_vals(i, :) = x_ue';
    x_so_vals(i, :) = x_so';

    cost_ue(i) = x_ue_vals(i,1)*(a_var + x_ue_vals(i,1)) + x_ue_vals(i,2)...
        *(1+b_var*x_ue_vals(i,2));
    cost_so(i) = x_so_vals(i,1)*(a_var + x_so_vals(i,1)) + x_so_vals(i,2)...
        *(1+b_var*x_so_vals(i,2));
end

% POA
poa = cost_ue./cost_so;
% find max POA
max_poa = max(poa)
```

```
max_poa = 1.0967
```

```
% Plot the results
figure;
hold on
plot(d_values, cost_ue, '-', 'DisplayName', 'UE');
plot(d_values, cost_so, '-', 'DisplayName', 'SO');
plot(d_values, poa, 'r-', 'DisplayName', 'POA');
% plot horizontal line for max poa
plot([0,max(d_values)],[max_poa, max_poa], 'r--', 'DisplayName','Max POA')
% plot where d=1-a
plot([1-a_var,1-a_var],[0,max_poa],'b--', 'DisplayName','d = 1 - a')

legend(Location="southeast");
xlabel('Demand (d)');
ylabel('Cost');
title('UE and SO Solution for Varying Demand');
```

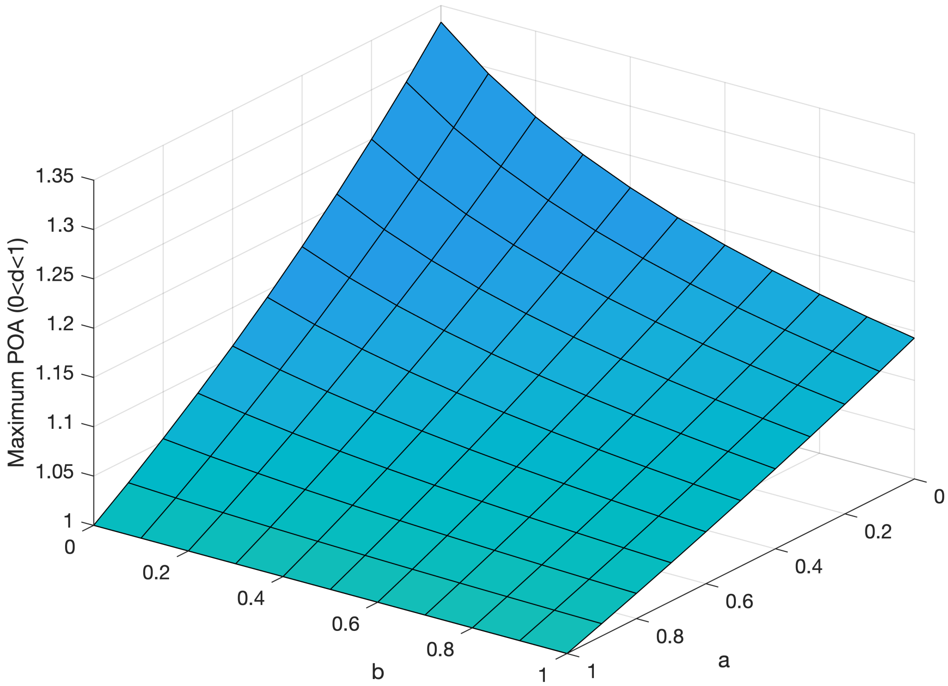
```
text(0.05,max_poa+0.05,sprintf('Max POA %.3f', max_poa));
grid on;
```



The range of d chosen above has been decided by considering the original link cost functions, and the point of maximum POA. For 0 < a,b < 1 the maximum POA will always occur at d = 1 - a (for *a* within the bounds specified) and so d is not shown above 1 as the POA remains slowly tapering off from there onwards. As b tends to 1, and a tends to 1, the POA tends to 1.

d. Varying a and b and calculating the maximum POA for any value of d.

```
figure;
% plot the function
x = 0:0.1:1;
y = 0:0.1:1;
[X,Y] = meshgrid(x,y);
Z = zeros(size([X,Y]));
for i = 1:length(x)
    for ii = 1:length(y)
        Z(i,ii) = POA_ab([X(i,ii),Y(i,ii)]);
    end
end
surfl(X,Y,Z(:,1:length(x)))
xlabel('a')
ylabel('b')
zlabel('Maximum POA (0<d<1)')

view([126.273 32.113]) % to make it clearer
```



By inspection it is clear that the maximum POA occurs when both a and b are 0, with the minimum occurring when a is 1. The maximum value of POA is 1.332. a and b both impact this value however only when a is not 1. When lim(a =1 and b=1) the costs for the routes are identical and so the UE and SO solutions will be the same. When lim(a=0 and b=0), the route costs will always remain as x_1 vs 1.

```
% find minimum
fun = @(x)(-POA_ab(x));
nvars = 2; % a and b
lb = [0,0];
ub = [1,1];
options = optimoptions('particleswarm','SwarmSize',10,MaxIterations=10);
[x, fval] = particleswarm(fun,nvars,lb,ub,options);
```

```
Optimization ended: number of iterations exceeded OPTIONS.MaxIterations.
```

```
% at maximum POA
a = x(1)
```

```
a = 0
```

```
b = x(2)
```

```
b = 0
```

```
Max_POA = -fval
```

```
Max_POA = 1.3332
```

Q3. Microscoping Modelling

a. Analytical expression of maximum flow

$$v(\rho) = v_0\left(1 - \frac{\rho}{\rho_{\max}}\right)^2$$

The flow for any velocity is given as

$$f = v\rho$$

$$f = v_0\rho - \frac{2v_0\rho^2}{\rho_{\max}} + \frac{v_0\rho^3}{\rho^2{}_{\max}}$$

$$\frac{\mathrm{d}}{\mathrm{d}\rho} f = v_0 - \frac{4v_0\rho}{\rho_{\max}} + \frac{3v_0\rho^2}{\rho^2{}_{\max}}$$

The maximum will occur when

$$\frac{d}{\mathrm{d}\rho} f = 0$$

This occurs when (solving for density)

$\rho = \frac{\rho_{\max}}{3}$ or when its equal to maximum density, but this means the flow has stopped.

and so the maximum flow is

$$f_{\max} = \frac{v\rho_{\max}}{3}$$

b. Theoretical lower bound of exits N

Theoretical maximum velocity:

$$v\left(\rho = \frac{\rho_{\max}}{3}\right) = \frac{4}{9}v_0$$

So the maximum flow rate of people:

$$f_{\max} = \frac{4v_0\rho_{\max}}{27}\ [\text{people}\ m^{-1}s^{-1}]$$

Evacuation area:

$$\text{area} = 10N\ [m]$$

Evacuation time:

time = 300s

Lower bound of N:

$$\frac{60000}{f_{\max}} = 300 \times 10\, N$$

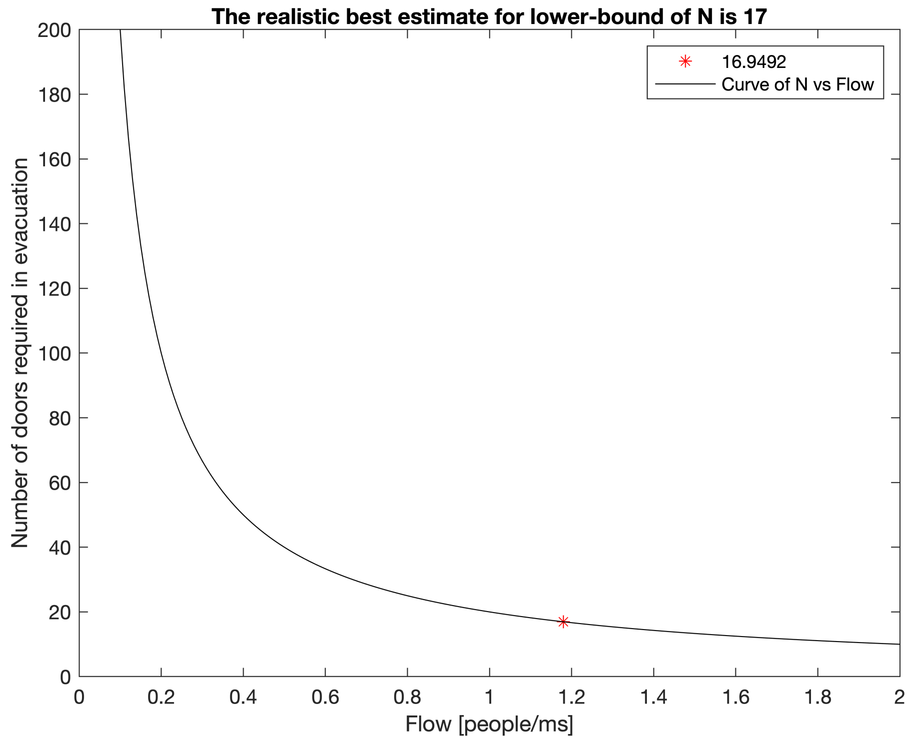
$$N_{LB} = \frac{135}{v_0 \rho_{\max}} = \frac{135}{\text{flow}}$$

Plot flow against number of doors needed

```
clf;
% plot flow against number of doors
flow = 0.1:0.01:2;
N = 60000./(300*10*flow);
% realistic max flow would be 70.8 pers/m min (measured in concert)
% https://www.sciencedirect.com/science/article/pii/S0379711220300448
max_flow = 70.8/60
```

```
max_flow = 1.1800
```

```
plot(max_flow, 60000/(300*10*max_flow), 'r*', DisplayName=num2str(60000/(300*10*max_fl
hold on;
plot(flow,N,'k-', DisplayName='Curve of N vs Flow');
xlabel('Flow [people/ms]');
ylabel('Number of doors required in evacuation');
legend()
title('The realistic best estimate for lower-bound of N is 17')
hold off;
```



c. Numerical value

```
N = round(135 / (1.3*8))
```

```
N = 13
```

d. Detailed microscopic model

Social force model definition:

$$\frac{\mathrm{d}}{\mathrm{d}t} v_\alpha(t) = f_\alpha(t) + \text{noise}$$

$$f_\alpha(t) = \frac{1}{\tau_\alpha}\left(v_{0_\alpha} - v_\alpha\right) + \sum_{\beta(\neq\alpha)} f_{\alpha\beta}(t) + \sum_i f_{\alpha i}(t)$$

The equation above shows that the force on a pedestrian is equal to the sum of the acceleration to desired velocity, the sum of forces from other pedestrians, and the sum of forces from boundaries.

- **Each pedestrian is a fan of one of the two teams currently playing. Assume a random allocation of team affinity at the start of the simulation.**

At the start of the simulation, each pedestrian is randomly assigned to one of the two teams (Team A or Team B). Let T_α denote the assignment of pedestrian α.

$$T_\alpha = 1 \text{ or } 0$$
$$\text{with :}$$
$$P(X = X) = \ 0.5$$

- **A pedestrian's desired walking speed will be 50% higher than normal if he/she is currently in a location where there is a majority of fans supporting the other team (within a 10 m radius around the person).**

So at each timestep, the following is calculated:

$$v_{0_\alpha} = v_{0_\alpha} \times (1 + 0.5 \times M)$$

Where M is calculated as followe:

$$M(\alpha) = \frac{\sum_{\beta(\neq\alpha)} 1 \mid (d_{\alpha\beta} < 10 \;\&\; T_\beta \neq T_\alpha)}{\sum_{\beta(\neq\alpha)} 1 \mid (d_{\alpha\beta} < 10 \;\&\; T_\beta = T_\alpha)} \;,\; d_{\alpha\beta} = \text{distance between supporters}$$

the piecewise function:

$$M = 1 \; \{M(\alpha) > 1\}$$
$$M = 0 \; \{M(\alpha) \leq 1\}$$

(Giving the ratio of supporters of the other team vs supporters of own team and using that as the 1 or 0 piecewise function.)

- **A pedestrian α will be subject to a strong repulsive social force from other pedestrians (β ≠ α) that are (i) within a 5 m radius around the person; and (ii) supporting the other team; and (iii) are located within the 180 ◦ semi-circle in front of α (i.e., in the direction of motion of α).**

$f_{\alpha\beta} = w(\theta_{\alpha\beta})g(d_{\alpha\beta})$

The repulsive force is composed of an angular and distance component. The distance component is modelled as follows:

$$g(d_{\alpha\beta}) \ = A \times \exp\left(\frac{R - d_{\alpha\beta}}{B}\right)\widehat{d_{\alpha\beta}}$$

$F_{\alpha\beta}$ is the force of repulsion from pedestrian α to β, acting along the direction of the normalised vector between pedestrians $\hat{d}$.

A is parameter constant that will determine the strength of the repulsive force

B is a parameter constant determining how the repulsive force decays with distance between pedestrians. With a small value of B, e.g. 0.1, the force will be very high within 5m and decay very quickly as the distance increases further.

The force is only applied if the distance $d_{\alpha\beta} < R$ is less than the radius (5m)

The angular component can be modelled as follows:

$$w(\theta_{\alpha\beta}) = \lambda + (1 - \lambda)\frac{1 - \cos(\theta_{\alpha\beta})}{2}$$

If $\lambda = 1$, the force is equal for all values of θ, if $\lambda = 0$, the force is only active for $\frac{\pi}{2} \leq \theta \leq 3\frac{\pi}{2}$

For the purpose of the model, it can be either modelled by using a value of $\lambda = 0$, and this will give a cardiod-like shape in the forward direction.

or a piecewise function defined as:

$$w(\theta_{\alpha\beta}) \ = 1, \quad \frac{\pi}{2} \leq \theta \leq 3\frac{\pi}{2}$$

$$w(\theta_{\alpha\beta}) \ = 0, \quad \text{else}$$

Calculating the angle, where v is the velocity vector of the pedestrians. A head-on course results in an angle of pi - the point where a repulsive force is required.

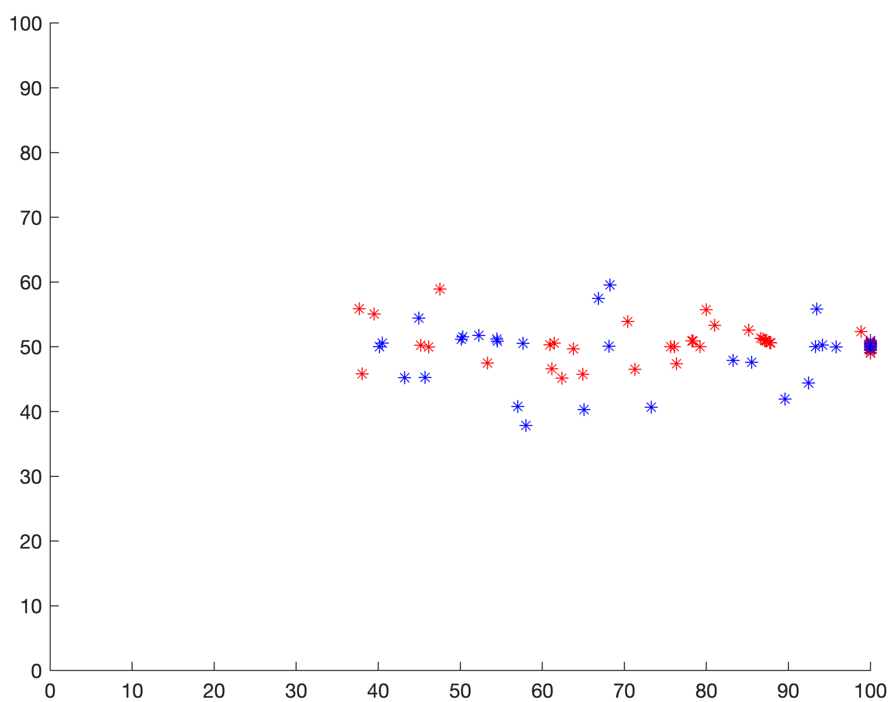
$$\cos(\theta_{\alpha\,\beta}) = \frac{v_\alpha \cdot v_\beta}{|v_\alpha|\,|v_\beta|}$$

Example code

```
% building a microscopic model of fans
% v(ρ) = v0(1 − ρ/ρmax)2
% define space
size_x = 100;
size_y = 100;
x = 0:0.1:size_x;
y = 0:0.1:size_y;
N = 100;
v0 = 1;
v_max = 1;
rho_max = 8;

% generate N pedestrians (x,y,vx0,vy0, Vx, Vy)
pedestrians = zeros(N,6);
% set v0 for all pedestrians - they are all trying to leave via the right
% had side middle
pedestrians(:,3) = 0;
pedestrians(:,4) = 0;

p_colours = zeros(N,3);

% init loop
for alpha = 1:N
%     place the pedestrian in a random location within boundary
    x_p = rand(1)*size_x;
    y_p = rand(1)*size_y;
    pedestrians(alpha,1)=x_p;
    pedestrians(alpha,2)=y_p;

%   assign to team randomly
%   initially
    if randi(2)>1
%         plot(x_p,y_p, '*', Color='b');
        p_colours(alpha,:) = [0,0,1];
    else
%         plot(x_p,y_p, '*', Color='r');
        p_colours(alpha,:) = [1,0,0];
    end
end
% timestepping
steps = 50;
figure;
for t = 1:steps
    clf
    for alpha = 1:N
        density_count = 0;

        % update v0
        [pedestrians(alpha,3), pedestrians(alpha,4)] =...
            v0_fun(pedestrians(alpha,1), pedestrians(alpha,2),size_x,size_y/2);
```

```
    % pedestrian v0 is 50% higher if majority other team within 10m
    radius = 10;
    for beta = 1:N
        if alpha ~= beta && p_colours(alpha) ~= p_colours(beta)
            if distance(pedestrians(alpha,:),pedestrians(beta,:)) < radius
                pedestrians(alpha,3:4) = pedestrians(alpha,3:4) * 1.5;
                density_count = density_count+1;
            end
        end
    end

    % repulsive force
    radius2 = 5;
    A = 10;
    B = 0.1;
    for beta = 1:N
        if alpha ~= beta && p_colours(alpha) ~= p_colours(beta)
            if distance(pedestrians(alpha,:),pedestrians(beta,:)) < radius2
            % Calculate the normalized vector between pedestrians
            normalized_vector=(pedestrians(alpha,1:2)-pedestrians(beta,...
                1:2))/distance(pedestrians(alpha,:),pedestrians(beta,:));

            % Calculate the angle between the velocity and the normalized vector
            angle = acos(dot(pedestrians(alpha,3:4), normalized_vector) /...
                norm(pedestrians(alpha,3:4)));

            % Repulsive force components
            distance_force = A * exp(-B * distance(pedestrians(alpha,:),...
                pedestrians(beta,:))) * normalized_vector;
            angular_force = 1 * angle * normalized_vector;

            % Total repulsive force
            repulsive_force = distance_force + angular_force;

            % Update the velocity based on the repulsive force
            pedestrians(alpha, 3:4) = pedestrians(alpha, 3:4)...
                + repulsive_force;
            end
        end
    end

    rho = rho_fun(density_count, radius);
    pedestrians(alpha, 5:6) = vel_fun(pedestrians(alpha,3:4), rho, rho_max);
    vel = sqrt(pedestrians(alpha,5)^2 + pedestrians(alpha,6)^2);
    if vel > v_max
        pedestrians(alpha,5:6) = pedestrians(alpha,5:6) * v_max/vel;
    end

    % update location (red travel right, blue travel left) with
```

```
        % deletion for dissapearance
        new_x = pedestrians(alpha,5) + pedestrians(alpha,1);
        new_y = pedestrians(alpha,6) + pedestrians(alpha,2);
        test_x = new_x - size_x;
        test_y = new_y - size_y;
        if test_x > 0
            pedestrians(alpha,1) = size_x;
        else
            pedestrians(alpha,1) = new_x;
        end
        if test_y > 0
            pedestrians(alpha,2) = size_y;
        else
            pedestrians(alpha,2) = new_y;
        end

        hold on
        plot(pedestrians(alpha,1),pedestrians(alpha,2),...
            '*', 'Color', p_colours(alpha,:));
        xlim([0 100]);
        ylim([0 100]);

    end
    pause(0.01)
    hold off% Pause for 0.01 s
end
```



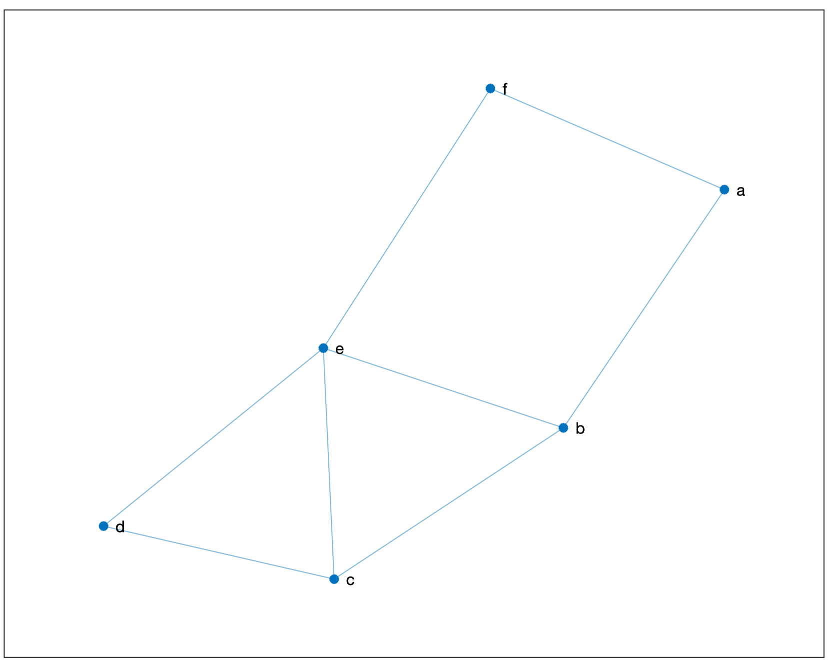
Animation of two teams leaving through a single narrow door on the right hand side.

Q4. Data-driven modelling

a. Centrality measures

i. Direct demand model

```
% The nodes are defined clockwise from top-left
clf;
% Create a graph
nodes = {'a','b','c','d','e','f'};
edges = {'a','b'; 'b','c'; 'c','d'; 'd','e'; 'e','f'; 'f','a'; 'c','e'; 'b','e'};
G = graph(edges(:, 1), edges(:, 2));
% Compute centralities
betweenness = centrality(G, 'betweenness');
degree = centrality(G, 'degree');
closeness = centrality(G, 'closeness');

% Known real values
real_val = [7,4.5,3,6,5.5,5.5];

% Plot the graph
plot(G, 'NodeLabel', G.Nodes.Name);
```



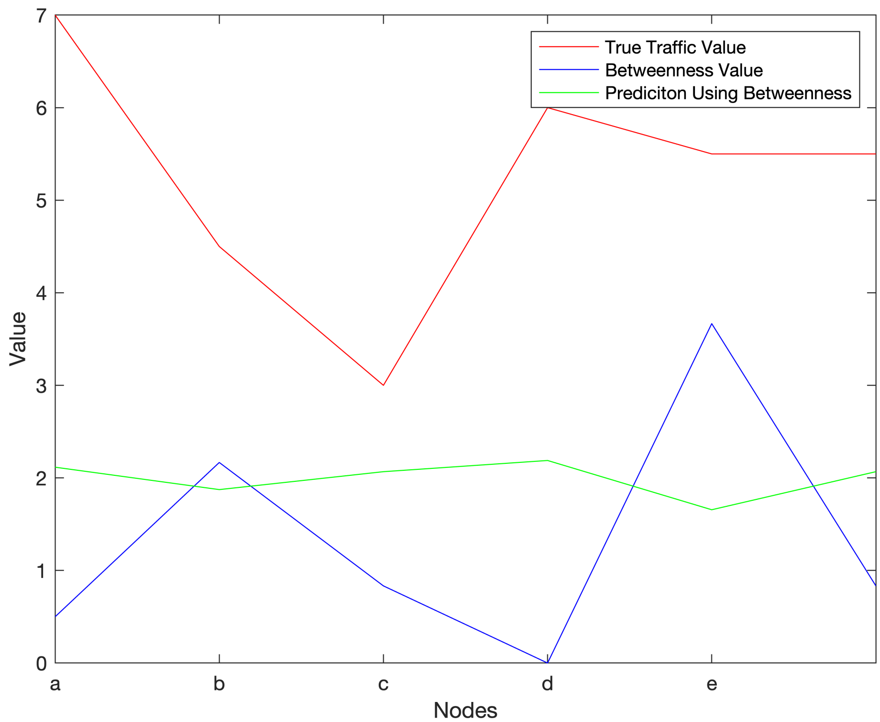

```
% use a regression model
R = fitlm(real_val(1:5)',betweenness(1:5)')
```

```
R =
Linear regression model:
    y ~ 1 + x1

Estimated Coefficients:
                   Estimate      SE        tStat      pValue
                   ________    _______    ________    _______

    (Intercept)      2.1882     2.9873     0.73249    0.51691
    x1             -0.14516    0.55569    -0.26123    0.81082


Number of observations: 5, Error degrees of freedom: 3
Root Mean Squared Error: 1.69
R-squared: 0.0222,  Adjusted R-Squared: -0.304
F-statistic vs. constant model: 0.0682, p-value = 0.811
```

```
predictions = 2.1882 - 0.14516.*(betweenness);
percent_err = (real_val(6) - predictions(6))*100/ real_val(6)
```

```
percent_err = 62.4139
```

```
% plot the real values and betweenness
plot(1:6,real_val, 'Color','r', DisplayName='True Traffic Value')
hold on
plot(1:6,betweenness,'Color','b',DisplayName='Betweenness Value')
plot(1:6,predictions,'Color','g',DisplayName='Prediciton Using Betweenness')
xticks(1:5)
xticklabels(nodes)
xlabel('Nodes')
ylabel('Value')
legend
```



As shown in the graph, the betweenness value does not correlate very well with the true amount of traffic measured, with a RMSE = 1.69, implying a weak correlation, and a direct prediction using the betweeness value gives an 62% error.

ii. Alternative model

Assume all lengths in graph = 1, using the inverse distance weighting

$$\rho_o = \frac{\sum_i \frac{\rho_i}{d_{\mathrm{oi}}}}{\sum_i \frac{1}{d_{\mathrm{oi}}}}$$

```
% for node f:
distances_f = [1,2,2,2,1]; % the distances of nodes from f (clockwise)
numerator = sum(real_val(1:5)./distances_f);
denominator = sum(ones(1,5)./distances_f);
prediction = numerator/denominator
```

```
prediction = 5.5000
```

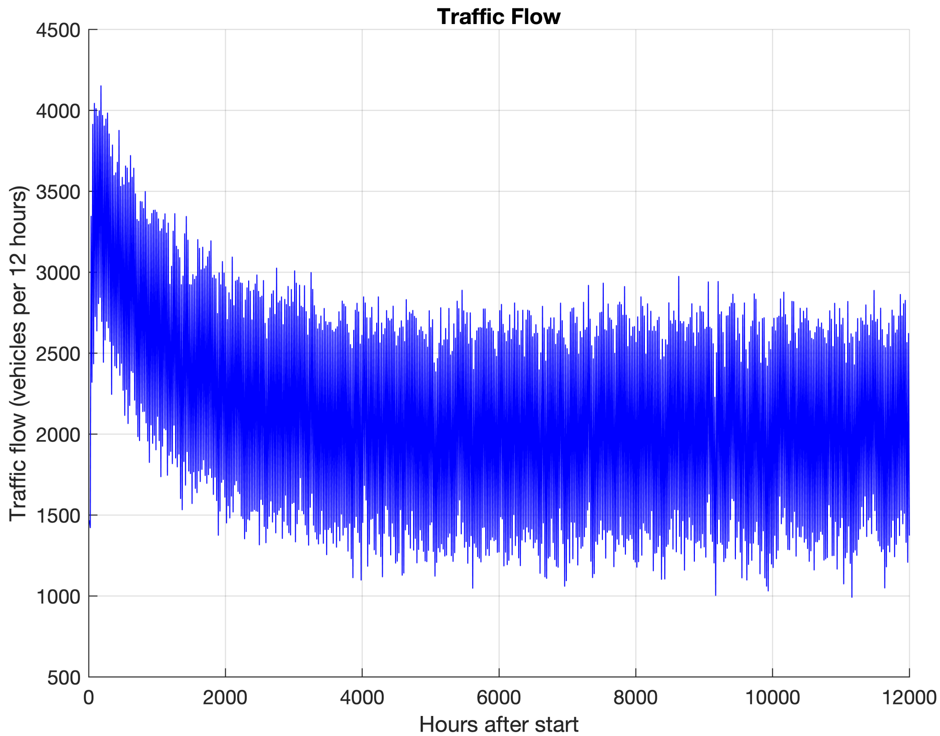
```
error = prediction - real_val(6)
```

```
error = 0
```

b. Bristol road time series

i. Dynamics description

```
% define constants
alpha = 0.99;
beta = 0.5;
s = 4;
sigma = 0.1;

% starting conditions
Y_0 = 0;
a_0 = 0.75;
c_0 = 0.5;
% plot Y as a function of t
figure
hold on
timesteps = 1:1000;
vals = zeros(length(timesteps),3);
for t = timesteps
    [Y_1, a_1, c_1] = Y_t(alpha, beta, s, sigma, a_0, Y_0, c_0);
    Y_0 = Y_1;
    a_0 = a_1;
    c_0 = c_1;
    vals(t,:) = [Y_0, a_0,c_0];
end
hours = timesteps * 12;
vehicles = vals(:,1) * 1000;

plot(hours,vehicles, '-', Color='b');
xlabel('Hours after start')
ylabel('Traffic flow (vehicles per 12 hours)')
title('Traffic Flow');
grid on;
hold off;
```



This shows the traffic flow on the road in bristol. The traffic flow increases rapidly at the beginning up to a maximum of around 4000 vehicles / 12 hours and slowly decreases to a level of around 2000 vehicles / 12 hours at which it stays relatively constant into the future. The time series is noisy with the flow constantly varying.

Influence of each term:

a(t): Autoregressive component, this component is influenced by its past value, and therefore suggests a slow decay of this component and a relationship to its previous state.

b(t): This component references the previous value of traffic (Y) and so adds a consistent decay into the model through a lag component, where it references the previous time step.

c(t): Seasonal component, due to the use of the cosine term, it creates periodic behaviour in the model. In this case it is likely used to model increases in traffic during half of the day. (it must be slowly increasing to account for the decay due to a and b, for the model to stabilise as it does.)

d(t): Noise component, adds the randomness associated with traffic data, attempting to account for short term fluctuations not described in the components above, for example changes in traffic due to weather, accidents or events.

ii. Discuss usefulness of model

- When looking at the model for a smaller number of time steps it becomes clear that there is a seasonal term increasing during the night and decreasing during the day which would likely be incorrect, unless this road has bars, nightclubs and restaraunts exclusively. A model of a general street would likely shift this term by 1 timestep (12 hours).

- It is unlikely that a true road would have its greatest usage during the first timesteps of its operation, and so it is more likely that the stable section of the time series (after approximately 300 timesteps) would be used to model a general road. This is unless this road provides some form of improvement to peoples journeys (while its new) which then decreases as the road is used more, this could be due to road surface quality deteriorating for example.
- The model should be validated against real traffic data from the Bristol road to assess its performance and identify areas for improvement - although the parameters have been estimated based on data, a side by side comparison could allow for greater analysis and better fine tuning. Additionally, other components could be introduced to improve accuracy.
- Another point of improvement would be to include a further seasonal term for weekends, likely reducing the flow if its a commuter heavy road.

```
% calculate the drift and diffusion of the data
% Fit a linear regression to estimate drift
X = [ones(length(timesteps), 1), hours'];
coefficients = X \ vehicles;
drift = coefficients(2)
```

```
drift = -0.0552
```

The drift component shows the average change in traffic value over time, it would likely be zero if only looking at the long term of this model, and would be highly negative in the short term.

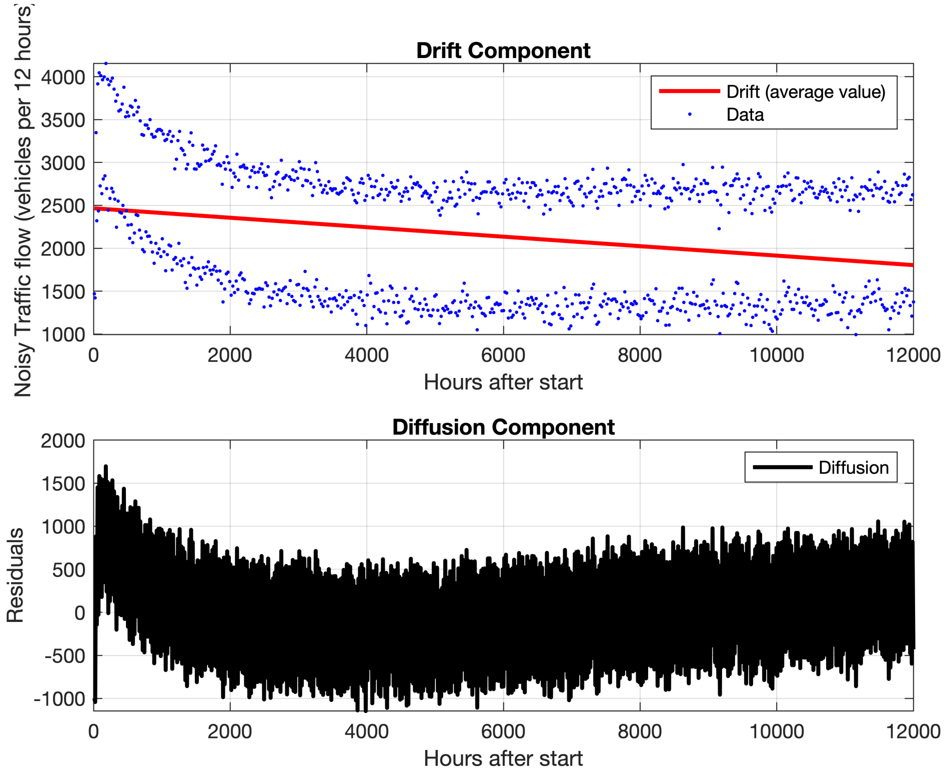
```
% Calculate residuals to estimate diffusion
residuals = vehicles - X * coefficients;
diffusion = std(residuals)
```

```
diffusion = 712.6575
```

This diffusion term represents the sum of the average daily fluctuations and noise to show how much the model varies timestep to timestep - i.e. every 6 hours.

```
% Plot drift and diffusion components
figure;
subplot(2, 1, 1);
plot(hours, drift * hours + coefficients(1), 'r-', 'LineWidth', 2);
hold on;
scatter(hours, vehicles, 'b.');
xlabel('Hours after start');
ylabel('Noisy Traffic flow (vehicles per 12 hours)');
title('Drift Component');
legend('Drift (average value)','Data')
grid on;

subplot(2, 1, 2);
plot(hours, residuals, 'k-', 'LineWidth', 2);
```

```
xlabel('Hours after start');
ylabel('Residuals');
title('Diffusion Component');
legend('Diffusion')
grid on;
```



Drift: Represents the average rate of change of the traffic flow over time. This indicates that the traffic is gradually decreasing over time but only from the initial few hours. The drift term would likely be zero if only considering the long term model as the model finds a stable point after approximately 300 timesteps.

Diffusion: Magnitude of randomness. This would generally represents the random fluctuations or uncertainties in the traffic flow, however for this time-series is shows the variation due to the seasonal term. This shows that initially there is a lot of randomness and a lot of daily variation in the flow, which then quickly drops and then gradually increases. These fluctuations tend to show that for long term prediction the randomness is increasing, this would likely be inaccurate as the usage of a road would tend to stabilise.

All Functions

```
function sol = UE_sol(H, f, Aeq, beq, options)
    sol = quadprog(H, f, [], [], Aeq, beq, [0,0],[],[], options);
end

function sol = SO_sol(H,f, Aeq, beq, options)
    sol = quadprog(2*H, f, [], [], Aeq, beq, [0,0],[],[], options);
end

function max_poa = POA_ab(x)
```

```
    options = optimoptions('quadprog', 'Display', 'off');

    % unpackage a and b
    a_var = x(1);
    b_var = x(2);

    % Initialize matrices to store solutions
    d_values = 0:0.1:1;
    x_ue_vals = zeros(length(d_values), 2);
    x_so_vals = zeros(length(d_values), 2);

    cost_ue = zeros(length(d_values),1);
    cost_so = zeros(length(d_values),1);

    % Solve the UE problem for each demand value
    for i = 1:length(d_values)
        d = d_values(i);

    % Define matrix H, f, and equality constraint
        H = [1, 0; 0, b_var];
        f = [a_var; 1];
        Aeq = [1, 1];
        beq = d;

    % Solve the quadratic programming problem using quadprog
        x_ue = quadprog(H, f, [], [], Aeq, beq,[0,0],[],[], options);
        x_so = quadprog(2*H, f, [],[], Aeq, beq,[0,0],[],[], options);
        % H*2 in this case is the Beckmann Formulation

    % Store the solution
        x_ue_vals(i, :) = x_ue';
        x_so_vals(i, :) = x_so';

        cost_ue(i) = x_ue_vals(i,1)*(a_var + x_ue_vals(i,1))...
            + x_ue_vals(i,2)*(1+b_var*x_ue_vals(i,2));
        cost_so(i) = x_so_vals(i,1)*(a_var + x_so_vals(i,1))...
            + x_so_vals(i,2)*(1+b_var*x_so_vals(i,2));
    end

    % POA
    poa = cost_ue./cost_so;
    % find max POA
    max_poa = max(poa);
end

function velocity = vel_fun(v0, rho, rho_max)
       velocity = v0*(1-rho/rho_max)^2;
end

function density = rho_fun(N, radius)
```

```
        density = N/(pi*radius^2);
end

function [v0x,v0y] = v0_fun(x,y, xt,yt)
    % calculate angle
    v0x = (xt - x);
    v0y = (yt - y);
    v0x = v0x/sqrt(v0x^2 + v0y^2);
    v0y = v0y/sqrt(v0x^2 + v0y^2);

end

function dist = distance(alpha, beta)
    x1 = alpha(1); y1 = alpha(2);
    x2 = beta(1); y2 = beta(2);
    dist = sqrt((x2-x1)^2 + (y2-y1)^2);
end


function [Y_1, a_1, c_1] = Y_t(alpha, beta, s, sigma, prev_a,prev_y, prev_c)

    a_1 = alpha * prev_a;
    b_1 = beta * prev_y;

    c = 0;
    for j = 1:(s/2)
        c = c + cos(2*pi*j*prev_c/s);
    end
    c_1 = c;

    d_1 = normrnd(0,sigma);

    Y_1 = a_1 + b_1 + c_1 + d_1;

end
```

PART B: Essay

UEFA Euro 2028 Transport Plan

Phil Blecher

December 2023

1 Introduction

The 2028 UEFA European Football Championship, Euro 2028, will be hosted by England, Scotland, Wales, Northern Ireland, and the Republic of Ireland. The tournament will bring together 24 teams from qualified countries, (with up to two automatic qualifications, should the hosts fail to reach this stage), and over 51 games to decide the winner. The matches will be played in the 10 host stadiums, distributed around 9 host cities around the British Isles. The aims of this transport plan are designed to support the main legacy of the entire tournament: making football accessible, delivering tangible long-term benefits for society, and economic benefits to the host cities [1]. Additionally, fan experience and value for money will be greatly shaped by the transportation plan and such form part of the key considerations. These last two aims go hand in hand with limiting the environmental impact of such a tournament, as making travel easier and cheaper generally correlates with optimised travel itineraries and improved transport connections. The location of stadiums is shown in figure 1 (all figures are shown in the appendix).

2 Game Organisation

A starting point for addressing the aims of providing excellent fan experience, value for money and accessibility is by first visualising and then optimising the expected movement of fans around the host countries. The tournament is expected to sell up to 3 million tickets [2] to a varied audience consisting of visitors from around the British Isles and all over Europe, those viewing a single match and those viewing multiple, and those following their team or staying at a single location. To best cater to such a varied set of fan requirements, the following initial decisions have been made for this transport plan (note that the full transport plan will consider all games, while only the group stage is considered in this report):

- **The movement of teams will be minimised.** Although this will correlate with a reduction in overall travel during the tournament, therefore reducing the environmental impact, this decision is primarily made to enhance the fan experience. With European teams moving around the host countries less, the fans choosing to follow them will spend less time and money during their trip than they would if the games were played at randomly distributed locations.
- **The total number of tickets for the group stages will be maximised.** By ensuring that the sum of games at each stadium and the respective stadiums' capacities is maximised the number of tickets available for the group stages will also be increased. This will result in more fans being able to watch a game and result in greater general accessibility to the tournament. As this is a secondary objective, its impact on the total objective function will be weighted lower than the previous objective.
- **The movement of the host team(s) will be maximised.** Fans of the host countries already in the British Isles will be distributed all around. By ensuring that the host teams play at a range of locations, the overall travel of home team fans will likely be reduced as a greater proportion of the fans will go to 1 match only due to the high demand and cost for tickets at such tournaments.
- **The first game of the tournament will be played by the host team at Wembley.** As the largest and most well-known stadium of the tournament, Wembley will host the opening game [2].

2.1 Method

The 36 games that will be played as part of the group stage of the tournament, need to be distributed between the 10 stadiums over the first couple of weeks of the tournament. This stage offers the opportunity for optimisation as per the decisions outlined above. The problem has been set up in the form of a global optimisation where the arrangement of stadiums for the games is the solution variable. The constraints include the points mentioned above as well as more trivial features of this model: all stadiums must host at least one game during this stage, all games must be played and the distances between stadiums are set as geographical distances (between the points shown in figure 1), as it is likely that greater distances will result in greater travel times. This last assumption is not entirely accurate however it will still return reasonable results as the travel times between stadiums will correlate with the geographical distances.

Upon set up of the problem, a global optimisation algorithm has been used to determine an optimal set of the placings for the games for the tournament. A genetic algorithm (GA) [3] from the MATLAB Optimisation Toolbox [4] has been used due to its simple implementation and the stochastic nature of the problem with a very large set of solutions (10^{36}) [3]. A baseline set of placings has been set randomly as a control solution for later comparison. The best estimate solution for the global optimum is produced by running the optimisation, demonstrating a potential game-to-stadium assignment and the resultant total and individual team travel distance. Figure 2 shows the distribution of games to stadiums for both random assignment and an optimised assignment. The distances covered by the teams individually (and combined) are only representative of the small proportion of fans that choose to travel alongside their chosen team. This figure is therefore a representation of the overall efficiency of the team journeys around the British Isles. From this optimisation, a few key observations can be made:

1. The total distance has been reduced (and therefore the efficiency of the stadium assignment has increased).
2. The distance of the home team is relatively high, see figure 6.
3. The stadium use distribution has changed. The distribution is now more uneven as the travel between the most distant stadiums is reduced, this is shown in figure 2.
4. The most common stadium-to-stadium team journey combinations have been identified as shown in figure 3.
5. Team 1 plays the first game at Wembley as set out in the constraints, see figures 4 and 5.

During the process of setting up this optimisation, another constraint was added for a more realistic result: the maximum number of games at a single stadium has been limited to 6 for this stage of the tournament. The group stage games will take place over approximately 2 weeks, and to play more than 6 matches at a single location in that time frame would cause significant difficulties for the organisers. This changed the results significantly as in an ideal optimisation many games would be played at a single stadium meaning teams do not have to travel as much, but this would come at the cost of fan experience and number of fans descending onto a single location. The most common journeys, as seen in figure 3 can then be used as a starting point for consideration of transport link improvements.

2.2 Acting on Findings

The reduction in total distance travelled will have the impacts mentioned in the section above. Although the geographical distance does not necessarily equate to cost, time or emissions, this reduction will correlate with a reduction in those measures. So a reduction in total distance will nevertheless benefit teams and fans at the tournament. It is important to note that the stadiums will vary in capacity so maximising the total number of tickets sold in the group stages can also be optimised with a problem that considers the capacity of each stadium and the maximisation of the total value as the objective. For this transport plan, this objective is secondary but has been included in the formulation of the problem. As a result, it is more likely that larger stadiums that are closer together are chosen for consecutive games.

The most common links, as seen in figure 3, are between stadiums 1 and 3 (Wembley and Tottenham), 2 and 7 (Cardiff and Birmingham), and 4 and 5: the Etihad Stadium and Everton. This is likely as both stadiums in the pairings have a reasonably high capacities (e.g. 61000 and 53000 for 4 and 5) and are located geographically close. Running the optimisation several times shows that the optimal configuration of stadiums tends to vary although these stadium-to-stadium journey combinations appear almost every time. For the best fan experience, the transport links on the most common routes should be improved as well as the accommodation availability at each location. Using these findings and objectives as a starting point, this transport plan will now assess each link on a smaller scale, comparing the potential routes and costs between stadiums.

Overall, this transport plan proposes that an optimisation methodology should be employed to determine the game locations for the group stage, round of 16, and finals games during the tournament. The optimisation shown above only considers the group stage as a demonstration. This improved game layout proposal will reduce travel during the tournament (reducing costs to fans and environmental impacts), improve the accessibility of home nation games as home fans will not need to travel as far, and allow for deeper insight into which improvements to the transport network will be required in the future.

3 Individual Links

The travel times are approximations, sourced from Google Maps [5].

3.0.1 The link between Wembley and the Tottenham Hotspur Stadium

Transport options for teams and fans travelling between these two stadiums include the following options:

- A drive between the two stadiums should in theory take only 1h, however, driving in London can be costly, frustrating and most importantly unsustainable for large amounts of people and as such this option should be avoided.
- A multimodal public transport option will take between 1h 10mins to 1h 30mins depending on the route chosen with a variety of tube line options available. This route specifically has been optimised as large events at both of these stadiums are common.

3.0.2 The link between Manchester and Liverpool

Transport options for teams and fans travelling between these two cities include the following options:

- Multiple driving options varying in distance between 37.7 miles to 45.5 miles and taking up to 1h 10mins.
- A multimodal public transport option takes approximately 1h 40 to 2h which requires at least 3 distinct trains, a bus, and walking.

3.0.3 The link between Cardiff and Birmingham

Transport options for teams and fans travelling between these two cities include the following options:

- Multiple driving options varying in distance between 117 miles to 124 miles and taking up to 2h 15mins.
- A multimodal public transport option taking approximately 2h 30 to 3h which requires at least 1 train and a bus to get to the stadium.

3.1 Link-Route Analysis

Looking more closely at the link between Manchester and Liverpool the following simple graph can be created, see figure 7, the cost functions C_1, C_2 and flows x_1, x_2 are shown. The cost functions for this set-up are modelled as so (with costs considered constant for public transport apart from busses).

Route 1: 2 busses to get to Manchester Victoria, train to Lime Street Liverpool, train to Sandhills. Traffic will impact the bus times and so impact the total travel time. The costs are given in minutes per individual journey as this will likely most impact fan experience.

$$C_1 = 107 + 0.05x_1 \tag{1}$$

Route 2: Drive directly (ignoring parking requirements) via M62 will be modelled with a linear term for queuing. This term will represent the time increase due to larger traffic flows.

$$C_2 = 60 + 0.2x_2 \tag{2}$$

The costs have been determined arbitrarily to show that traffic will impact the driving time more than the public transport, these values are not realistic and are only used to give an estimate of the effect of traffic. Solving the user equilibrium (UE) problem for this network in terms of demand d gives the values of:

$$x_1 = -188 + 0.8d \tag{3}$$

$$x_2 = 188 + 0.2d \tag{4}$$

With the ratio of public transport to private journeys R is given by the following equation:

$$R = 4 - \frac{4700}{x + 940} \tag{5}$$

This means that for this journey the user equilibrium will tend towards a resultant ratio of 4 times the flow using public transport as driving (for a condition where the impact of traffic on driving is 4 times the impact on public transport). This calculation can be conducted for all pairings of locations. For the greatest proportion of fans to happily switch to public transport (thereby reducing environmental impact and providing a greater economic return for the transport industry) the factor by which traffic affects public transport travel times, should be reduced. The value of 4 in the example above is a ratio of the linear cost terms relating each of the travel times to the congestion from the added flow. The greater this ratio value, the more people will choose public transport over private vehicles. This can be increased by either slowing down car traffic (which would have a negative response from the public) or by reducing the influence of congestion on public transport.

3.2 Recommended Policies

In the real world, the bulk of the added time cost to public transport will be in the form of either busses and coaches caught in traffic or delays and cancellations to trains. To tackle these two problems, this transport plan proposes the following solutions:

1. An increase in bus, coach and multiple occupancy vehicle lanes on suburban entry routes.
2. An increased availability of trains between critical locations, such as Manchester and Liverpool, for the duration of the tournament.

In addition, this transport plan will propose two more policies for greater impact:

1. Promotion of each city as a visitor destination.
2. * Promotion of novel modes of transportation for premium experiences (see section 5).

** Note that this forms a potential aim of the transport plan that will rely on technological development and other factors outside the influence of this plan. See section 5.*

The first solution, an increase in bus lanes and similar route improvements has already been implemented in many large cities around the British Isles. However, the expansion of such a policy to the final sections of motorways will further reduce the congestion on intercity coach and bus travel meaning that low-cost coaches will become a more attractive travel method for all fans during the tournament. This policy would also permanently improve coach travel in the future, encouraging more travellers around the UK and Ireland to switch from private vehicles to coaches for longer journeys. This is in line with the Euro 2028 aim of football for the future. Increasing the use of intercity coach travel will help to reduce overall travel costs and therefore make it more accessible to more people.

Increasing the number of trains running during off-peak times between key tournament stadium pairings will reduce the risk of delays to rail travel. This policy was implemented in London for the 2012 Olympic Games [6] and played a key role in allowing the games to be visited almost entirely by public transport. The key routes where this policy should be implemented are listed in section 3.0. By reducing delays, the general public will have more trust in rail services and therefore will consider using them for more journeys. This will in turn increase revenue for those operating allowing the firms to reduce costs for customers. This, however, will require cooperation from all rail transport providers, as a uniform decrease in rail fares will encourage more people to use public transport.

Furthermore, by promoting each host city as a tourist destination, thereby investing in tourist experience in each local council, the effect of the layout of games (with teams travelling less) will be compounded with the desire of fans to explore destinations. Visiting fans from various countries will be able to set up bases at individual locations (at least during the group stages) that allow fans to go to as many games as possible while only having to pay for accommodation in a single location.

4 Monitoring Success and Data Gathering

Utilising data-driven modelling, the implementation of this transport plan can be measured during the group stages of the tournament and therefore the optimisation methodology adapted for later games. A key part of the plan considers the distances and number of journeys between stadiums. A general method of validating this model could consider collecting journey location data from ticket holders. This would involve asking purchasers their accommodation locations for each game ticket. These data could then be used to scale the number of journeys to stadiums and their respective starting locations. These numbers would influence the implementation of policy 2 from section 3.2, as the links with greater demands would have a greater requirement for train increases. Additionally, data gathered on ticket holders would in general provide the host nations with a better prediction of fan flows throughout the tournament and therefore allow them to plan any increases in services.

5 Novel Modes of Transport

As mentioned in the recommended policies novel modes of transportation will be promoted for the demonstration of technical advancement, premium fan experiences, and for the development of these modes in the host countries. The UK is currently leading the way in developing airships, with Hybrid Air Vehicles (HAV) aiming to demonstrate short commercial routes by 2026 [7]. The vehicle developed by HAV will be able to transport up to 100 passengers [8] making it ideal for both premium travel and potentially football team travel. This technology will begin to be employed in a commercial sense by first connecting remote Scottish isles. With the requirement of a successful proof of concept, this transport plan recommends that airships such as these should be used to provide an alternative transport method for fans wanting a unique experience or for teams and fans who would otherwise travel by private jet. The benefits of such decisions would include a reduction in environmental impact from private jets, an improved travel experience as these airships will be designed for luxury travel, and most importantly promotion of this technology to other countries and investors.
The routes that would be targeted for airship use would be the short air journeys required by teams to get from Great Britain to Northern Ireland or the Republic of Ireland. With a journey distance of only 210-230km from the central Liverpool Everton Stadium to the Dublin or Belfast stadiums, the cross Irish Sea route would be made much more efficient by airships. The current options include slow and expensive ferries or cheap flights from locations all over the UK. Neither of these options would provide the best fan or team travel experience due to either the long total travel time or long travel to airports and as a result provide an opportunity to airship manufacturers. The short geographical distance between locations will mean that even a slower transport method is viable. Increased investment in airships will form a critical part of the worldwide push for sustainable aviation, and the showcase of technology from host countries will provide greater outside investment and interest from European countries.

6 Plan Summary

This transport plan aims to meet the objectives of the tournament as a whole, as well as the independent objectives for transport during the event, by:

- **Making football accessible.** Reducing the travel required by fans means less money and time are required to view multiple games, this means that more people can access the tournament first-hand.
- **Providing long-term benefits for society and economy.** Encouraging public transportation and touristic development of cities will improve the quality of life of regular commuters and increase tourism around the British Isles.
- **Improving fan experience, value for money and sustainability of the tournament.** Alongside reducing travel and accommodation costs by reducing fan movement during the tournament, fans from Europe will have a better experience staying in a single city, with each host city becoming a visitor destination (through investment in tourist experiences).

Finally, this tournament will aim to provide 80% of fans with the opportunity to travel via public transport [2] making this tournament more sustainable than previous years (compared to 60% public transport usage at Euro 2016 [9]). It will also promote the development of novel transport options and therefore be a positive step towards better aviation.

References

[1] UEFA. UEFA Euro 2028 in the UK and Republic of Ireland. *uefa.com*, 2023.

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[3] Melanie Mitchell. *An introduction to genetic algorithms.* Complex adaptive systems. The MIT Press, Cambridge, MA, US, 1996.

[4] The MathWorks Inc. Matlab version: 9.13.0 (r2022b), 2022.

[5] Google. Google Maps, 2023.

[6] Olympic Delivery Authority. London 2012 Transport Plan (2nd Edition). Technical report, Gov.uk, London, 2011.

[7] Alan Dron. Airship Resurgence. *The Aeronautical Society*, 2022.

[8] Giant airships could connect remote Scottish islands with 'lifeline' routes. *Professional Engineering, IMechE*, 2022.

[9] Ben Avison. How transport flowed at UEFA EURO 2016. *Hostcity.com*, 2016.

Appendix

Plots Discussed in the Report

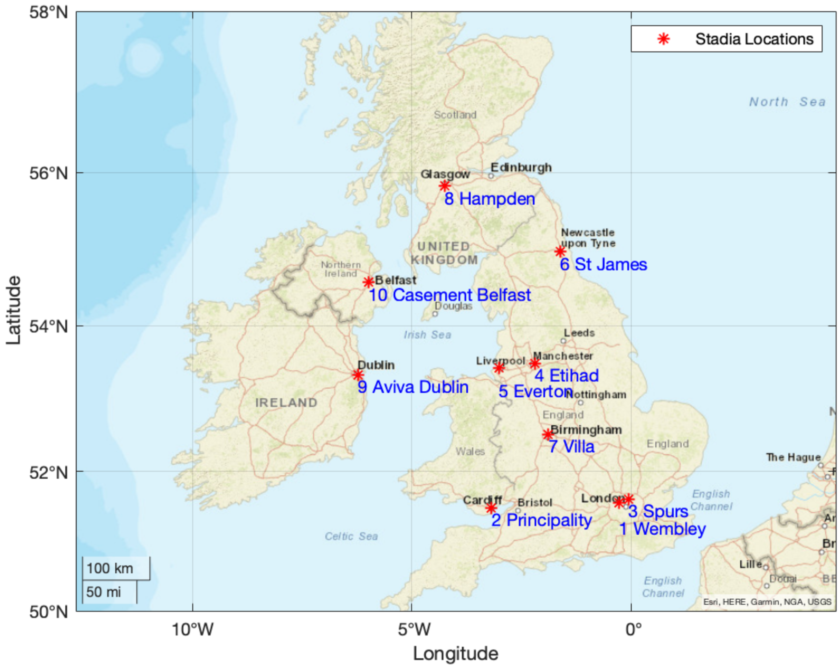

Figure 1: Map of the British Isles and stadiums for Euro 2028.

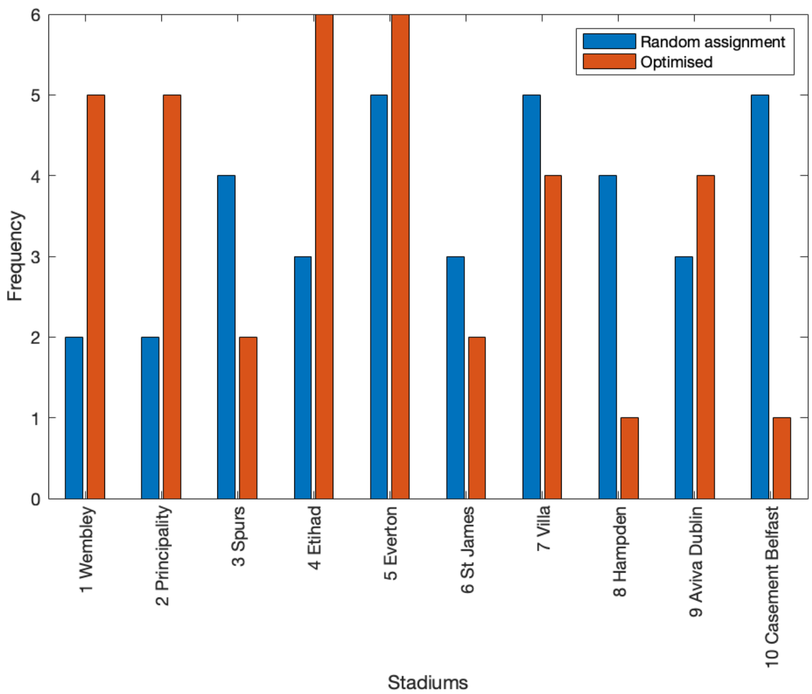

Figure 2: The distribution of games in the group stages.

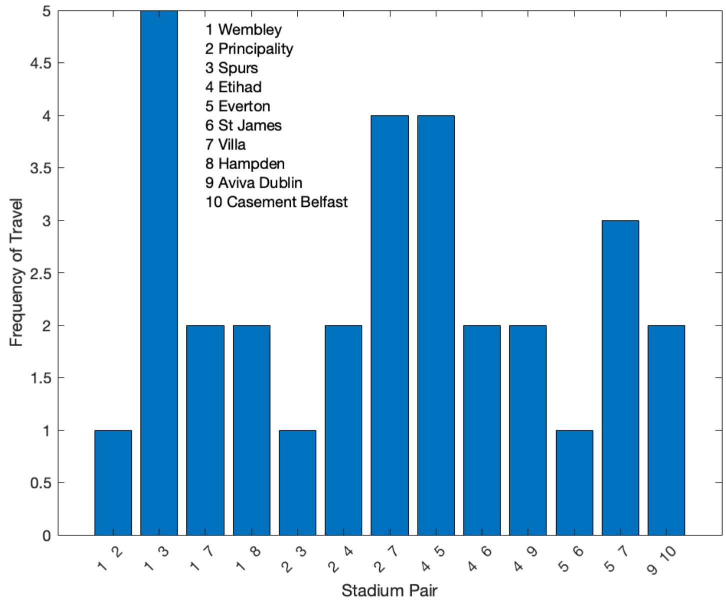

Figure 3: The most common consecutive stadium-to-stadium journeys in the group stage as determined by the stadium-game pairing optimisation.

```
Optimization terminated: maximum number of generations exceeded.
x = 1×36
```

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1	9	2	3	3	1	1	1	3	1	1	3	7	4	

Figure 4: Solution to the optimisation problem, showing which number game (x-axis) should be played where (row 1). Note game one is played at stadium one - Wembley.

```
games'
```

```
ans = 2×36
```

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1	1	1	2	2	3	5	5	5	6	6	7	9	9	
2	2	3	4	3	4	4	6	7	8	7	8	8	10	11	

Figure 5: Matrix showing which teams (rows 1 and 2) play each game (x-axis). Note first game contains team 1 - the home team.

```
indi_dists'
```

```
ans = 1×24
```

	1	2	3	4	5	6	7	8	9
1	740.9417	0	0	15.6859	0	0	0	15.6859	164.93

Figure 6: The individual teams' distances travelled during the group stage (note the high value for team 1).

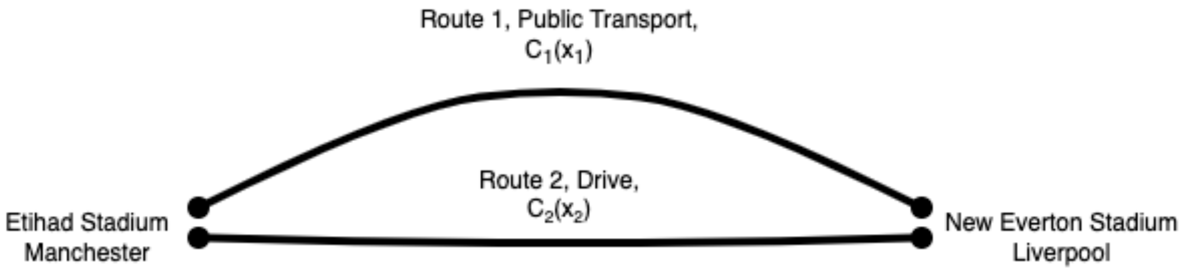

Figure 7: A simplified visualisation of the transport routes between Manchester and Liverpool.

Code Snippet of the Objective Function for Optimisation Discussed in the Report

```
% objective function where x is just the location for each game
fun = @(x) calculateTotalDistance(x, n_teams, games,...
    distances_km, capacities);

function objective = calculateTotalDistance(x, n_teams, games,...
    distances_km, capacities)

    % Calculate total capacity using vectorized operations
    total_capacity = sum(capacities(x));

    % Normalize total capacity
    norm_total_capacity = total_capacity / 36;

    % Calculate the total distance based on the binary vector x
    % indicating which games are played at each stadium

    n_games = length(games);
    movement_matrix = zeros(3,n_games);
    for game = 1:n_games
        % assign a random stadium to each game
        movement_matrix(1,game) = round(x(game));
        % place the teams that would play there
        movement_matrix(2,game) = games(game,1);
        movement_matrix(3,game) = games(game,2);
    end

    % Calculate the total distance moved for each team
    prev_locations = zeros(1,n_teams);
    total_teams_dist = 0;
    indi_team_dist = zeros(n_teams,1);

    home_team_a = 1;
    home_team_b = 1;

    for game = 1:n_games
    % check what location the team was at
        % team A
        prev_i_a = prev_locations(movement_matrix(2,game));
        current_i_a = movement_matrix(1,game);
        % team B
        prev_i_b = prev_locations(movement_matrix(3,game));
        current_i_b = movement_matrix(1,game);

        % make england (team 1) visit more stadiums
        % if team==1
        if movement_matrix(2,game) == 1
            home_team_a = -1;
        elseif movement_matrix(3,game) == 1
            home_team_b = -1;
        else
            home_team_a = 1;
            home_team_b = 1;
        end

        if prev_i_a ~= current_i_a
            % calculate the distance moved
```

```
            if prev_i_a ~= 0
                dist_moved = distances_km(prev_i_a, current_i_a);
            else
                dist_moved = 0;
            end
            % update previous loc
            prev_locations(movement_matrix(2,game)) = current_i_a;
            total_teams_dist = total_teams_dist + home_team_a*dist_moved;
            indi_team_dist(movement_matrix(2,game)) =...
                indi_team_dist(movement_matrix(2,game)) + dist_moved;
        end

        % team B
        if prev_i_b ~= current_i_b
            % calculate the distance moved

            if prev_i_b ~= 0
                dist_moved = distances_km(prev_i_b, current_i_b);
            else
                dist_moved = 0;
            end
            % update previous loc
            prev_locations(movement_matrix(3,game)) = current_i_b;
            total_teams_dist = total_teams_dist + home_team_b*dist_moved;
            indi_team_dist(movement_matrix(3,game)) =...
                indi_team_dist(movement_matrix(3,game)) + dist_moved;
        end
    end

    % Return the total distance as the objective value
    total_distance = total_teams_dist;
    objective = total_distance - norm_total_capacity/20;
end
```