Alda UB

04

(1)

asymptotische Komplexität

z.z. $f_i \in O(f_{i+1})$

allg. Beweisprinzip: 1. vollst. Induktion.

Bew.: • $\exists c, x_0$

• $x \Rightarrow x+1$

2. $\exists c: \lim_{x\to\infty} \frac{f_i(x)}{c \cdot f_{i+1}(x)} \leq 1$

a)

z.z. $(x+20)\log_{10}(x+20) \in O(x \log_2 x)$

Bew. $\circledast := \lim_{x\to\infty} \frac{(x+20)\log_{10}(x+20)}{x \log_2(x)} = \frac{\ln(2)}{\ln(10)} \lim_{x\to\infty} \frac{(x+20)\ln(x+20)}{x \ln(x)} =: \frac{\ln(2)}{\ln(10)} \lim_{x\to\infty} \frac{f(x)}{g(x)}$

(1)
$$\left\{\begin{array}{l} \cdot\ f, g \text{ bestimmt div} \\ \cdot\ f'(x) = \ln(x+20) + (x+20)\cdot\frac{1}{x+20} = \ln(x+20)+1 \\ \cdot\ g'(x) = \ln(x) + 1 \quad \text{insb.} \neq 0 \end{array}\right\} \ldots$$

(2)
$$\left\{\begin{array}{l} \cdot\ f''(x) = \frac{1}{x+20} \\ \cdot\ g''(x) = \frac{1}{x} \quad \neq 0 \end{array}\right\} \lim_{x\to\infty} \frac{f''(x)}{g''(x)} = \frac{x}{x+20} =: \lim_{x\to\infty} \frac{\alpha(x)}{\beta(x)}$$

(3)
$$\left\{\begin{array}{l} \cdot\ \alpha'(x) = 1 \\ \cdot\ \beta'(x) = 1 \quad \neq 0 \end{array}\right\} \lim_{x\to\infty} \frac{\alpha'(x)}{\beta'(x)} = 1 \overset{(3)}{=} \lim_{x\to\infty} \frac{\alpha(x)}{\beta(x)} = \lim_{x\to\infty} \frac{f''(x)}{g''(x)}$$

$$= \ldots = \lim \frac{f(x)}{g(x)}$$

ln mon. steig.

$\Rightarrow \circledast < 1$

$\square$

b)

$$2^{\sqrt{\log_2(x)}} < \sqrt{x} + \log_2(x) < x \log_2(x) < x^3 + 12x^2 + 200x + 999 < 3^x < x^x$$

$$\Leftrightarrow \quad f_E < f_B < f_D < f_F < f_A < f_C$$

Beweise:

$f_E < f_B$: fehlt

$$f_B < f_D: \quad \lim_{x \to \infty} \frac{\sqrt{x} + \log_2(x)}{x \log_2(x)} \overset{\text{l'Hosp.}}{=} \ln(2) \frac{1/(2\sqrt{x}) + 1/(\ln(2) \cdot x)}{\ln(x) + 1} \longrightarrow 0$$

$$f_D < f_F: \quad \frac{x \log_2(x)}{x^3 + 12x^2 + 200x + 999} \xrightarrow{\text{l'Hosp.}} \frac{1}{\ln(2)} \frac{\ln(x) + 1}{3x^2 + 24x + 200} \longrightarrow \frac{1/x}{6x + 24} \longrightarrow 0$$

$$f_F < f_A: \quad \frac{x^3 + 12x^2 + 200x + 999}{e^{\ln(3) \cdot x}} \longrightarrow \frac{3x^2 + 24x + 200}{\ln(3)\, e^{\ln(3) x}} \longrightarrow \frac{6x + 24}{\ln(3)^2\, e^{\ln(3) x}} \longrightarrow \frac{6}{\ln(3)^3\, 3^x} \longrightarrow 0$$

$$\frac{x \ln 3}{x \ln x} \longrightarrow \frac{\ln 3}{\ln x} \to 0$$

$$\Rightarrow \quad \to -\infty$$

$$f_A < f_C: \quad \frac{e^{x \ln 3}}{e^{x \ln x}} = e^{\overbrace{x(\ln 3 - \ln(x))}^{\to -\infty}} \longrightarrow 0$$

gilt $\exists c: \lim_{N \to \infty} \frac{f(N)}{c\, g(N)} \leq 1 \Rightarrow \exists c: f(N) \leq c\, g(N) \Leftarrow \underbrace{\lim_{N \to \infty} \frac{f(N)}{g(N)} = 0}_{\text{auch asymptotisch ?}}$?