07_CV7_UB

Notation: States of each variable encoded in one hot encoding

1) a)

$m_{\psi_5 \to x_5}(x_5) = [5,3,1]$

$m_{x_5 \to \psi_{2,5}}(x_5) = [5,3,1]$

$m_{\psi_{2,5} \to x_2}(x_2) = [5,4]$ $\quad \underset{x_5}{\operatorname{argmin}} = \left[\begin{bmatrix}0\\0\\1\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}\right]$

$m_{\psi_1 \to x_1}(x_1) = [11,3]$

$m_{\psi_{1,2} \to x_1}(x_1) = [21,24]$ $\quad \underset{x_2}{\operatorname{argmin}} = \left[\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}\right]$

$m_{x_2 \to \psi_{1,2}}(x_2) = [18,21]$

$m_{\psi_2 \to x_2}(x_2) = [6,3]$

$m_{\psi_{2,3} \to x_2}(x_2) = [7,14]$ $\quad \underset{x_3}{\operatorname{argmin}} = \left[\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}1\\0\end{bmatrix}\right]$

$m_{x_3 \to \psi_{2,3}}(x_3) = [4,13]$

$m_{\psi_3 \to x_3}(x_3) = [1,10]$

$m_{\psi_{3,4} \to x_3}(x_3) = [3,3]$ $\quad \underset{x_4}{\operatorname{argmin}} = \left[\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}\right]$

$m_{x_4 \to \psi_{3,4}}(x_4) = [0,0]$

$\boxed{\text{accumulated cost root: } [32,27]}$

b) Backpropagation:

$[32,27] \Rightarrow x_1 = \begin{bmatrix}0\\1\end{bmatrix} \Rightarrow x_2 = \begin{bmatrix}0\\1\end{bmatrix} \Rightarrow x_5 = \begin{bmatrix}0\\0\\1\end{bmatrix}$

$\Rightarrow x_3 = \begin{bmatrix}1\\0\end{bmatrix} \Rightarrow x_4 = \begin{bmatrix}1\\0\end{bmatrix}$

c) see notebook

d) (a) yes ; (b) no

(a) works, because its still an undirected, acyclic graph.

(b) suppose

$\psi_{2,4}$	x_4	
x_2	0	99
	99	0

→ Backprop. leads to $x_4 = \begin{bmatrix}0\\1\end{bmatrix}$ and $x_4 = \begin{bmatrix}1\\0\end{bmatrix}$ ↯

3

Second Order Markovian Trellis Systems

Energy $\mathcal{E}_2(z) = \sum_i \psi_i^{(1)}(z_i) + \psi_i^{(2)}(z_{i-1}, z_i) + \psi_i^{(3)}(z_{i-2}, z_{i-1}, z_i)$

with $\mathcal{E}_2 \in \mathbb{R}$ scalar and $z_i \in \{0,1\}^{1\times K}$ 1D vector

$$\Longrightarrow \begin{cases} \psi_i^{(1)} \hat{=} \text{ 1D - Tensor of length } K \\ \psi_i^{(2)} \hat{=} \text{ 2D - Tensor of length } K \times K \\ \psi_i^{(3)} \hat{=} \text{ 3D - Tensor of length } K \times K \times K \end{cases}$$

Visualisation of energy for $i=3$, $K=2$:

$$\mathcal{E}_2(z) = [\psi_1] \cdot z_1 + [\psi_2] z_2 + [\psi_3] z_3 + z_1 \left[\, \psi_{1,2} \,\right] z_2$$

$$+ z_2 \left[\, \psi_{2,3} \,\right] z_3 + $$

Now reorder information according to Tip $\tilde{z}_i = z_{i-1} \left[\, 1 \ \ \ddots \,\right]$ are hot encoded

$$\Longrightarrow \tilde{\psi}_i^{(1)}(\tilde{z}_i) = \psi_{i-1}^{(1)} \cdot \tilde{z}_i \, \psi_i^{(2)} \cdot (\psi_i^{(1)})^T$$

$$1 \times K \cdot K \times K \cdot K \times K \cdot K \times 1 = 1 \times 1$$

$$\Longrightarrow \tilde{\psi}_i^{(2)}(\tilde{z}_{i-1}, \tilde{z}_i) = \psi_i^{(3)}\left(\{1\}^{1\times K} \tilde{z}_{i-1}, \{1\}^{1\times K} \tilde{z}_i, \tilde{z}_i \{1\}^{K \times 1}\right)$$

⇝ 3D-Tensor is reused just the correct "old arguments" are projected out of the "new arguments"