

01-06-18

① Plot an image in Python
see notebook

② Convolution

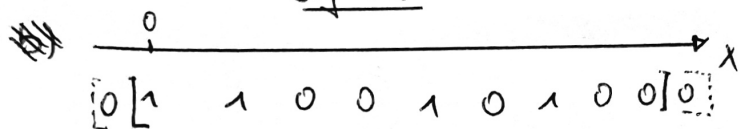
a) Commutative property

$$(h * f)[x] = \sum_{i=0}^{N-1} h_i f_{x-i} = \sum_{i=x}^{N-1+x} h_{i-x} f_{-i} = \sum_{i=x-(N-1)}^x h_{i-x+(N-1)} f_{-i+(N-1)}$$

$$\stackrel{\text{p.b.c}}{=} \sum_{i=0}^{N-1} h_{-(i-x)} f_{-(-i)} = \sum_{i=0}^{N-1} h_{x-i} f_i = (f * h)[x]$$

b) Boundary conditions

signal



$\square \hat{=}$ signal
 $\square \hat{=}$ signal + zero padding

\downarrow
result

"valid"

$$\begin{bmatrix} 1 & 1 & -1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

zero-padding

$$\begin{bmatrix} -1 & 1 & 1 & -1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

c) Implementation

see notebook

d) Separable kernels

naive: $9 \times 25 \times 25$ kernel $\hat{=}$ $\begin{cases} 9 \cdot 25 \cdot 25 & \text{Multiplications} \\ 9 \cdot 25 \cdot 25 - 1 & \text{Additions} \end{cases}$

image pts
times 100,800,800

$$\hat{=} \begin{cases} 36 \cdot 10^{10} & \text{Multiplications} \\ 35,9 \cdot 10^{10} & \text{Additions} \end{cases}$$

separable: $9 \times 1 \times 1 * 1 \times 25 \times 1 * 1 \times 1 \times 25$ kernel

$$\hat{=} \begin{cases} 9 + 25 + 25 & \text{Multiplications} \\ (9-1) + (25-1) + (25-1) & \text{Additions} \end{cases} \times \begin{cases} 100,800,800 & \text{image pts.} \end{cases} \hat{=} \begin{cases} 3,8 \cdot 10^9 & \text{Mult.} \\ 3,6 \cdot 10^9 & \text{Add.} \end{cases}$$

③, ④, ⑤ see notebook