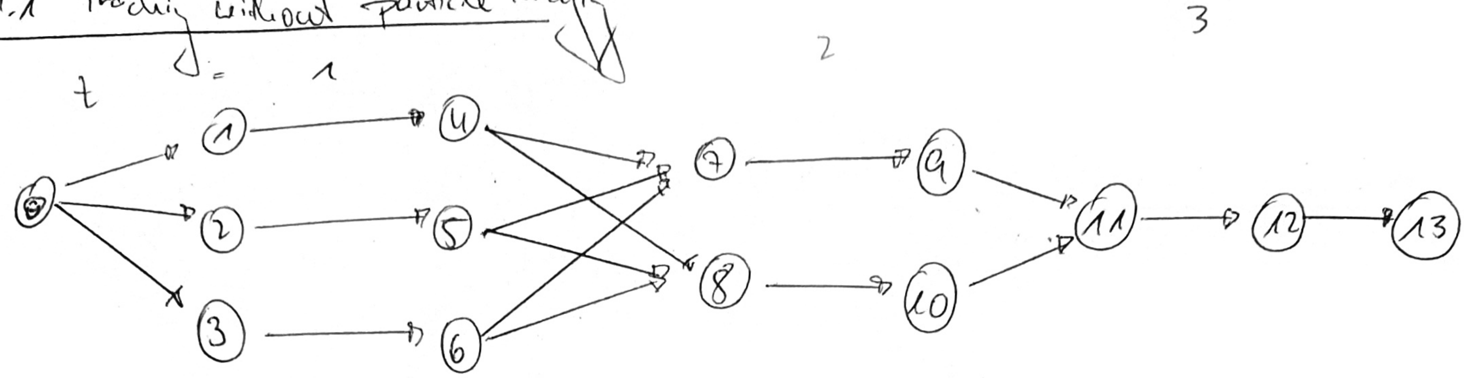
## 1.1 Tracking without particle merging




a) Consider 0 to be start and 13 target-node.

Costs:

- $c_{01} = c_{02} = c_{03} = c_{12,13} = 0$ (No cost for (dis)appearing)
- $c_{14} = -10$ $\quad c_{7,9} = -13$
  $c_{25} = -20$ $\quad c_{8,10} = -8$
  $c_{36} = -18$ $\quad c_{11,12} = -9$
- $c_{47} = (2-0)^2 + (1-0)^2 = 5$
  $c_{48} = (2-1)^2 + (1-6)^2 = 26$
  $c_{57} = (0-0)^2 + (4-0)^2 = 16$
  $c_{58} = (0-1)^2 + (4-6)^2 = 5$
  $c_{67} = (5-0)^2 + (4-0)^2 = 41$
  $c_{68} = (5-1)^2 + (4-6)^2 = 20$

$c_{9,11} = (0-4)^2 + (0-1)^2 = 17$
$c_{10,11} = (1-4)^2 + (6-1)^2 = 34$

capacity: $\mu_e = 1 \quad \forall e \in E$

b)
$$\min_{x} \sum_{e \in E} x_e c_e$$
$$\text{s.t.} \quad \sum_{u:(u,v)\in E} x_{(u,v)} - \sum_{w:(v,w)\in E} x_{(v,w)} = 0 \quad \forall v \in V$$
$$x_e \leq \mu_e$$
$$x_e \in \{0,1\}$$

for $G = (V, E)$ as sketched in a).

c) current assumptions: $N_{tot} = 1$, because there is only one observation for $t=3$

⊕

appearance at $t>0$: $N_{tot} = 1$, same reason. Disappearance not allowed yet.

⊕

dissapperance after $t=2$: $N_{tot} = 3$, the two obs. at $t=2$ have to originate from $t=1$ because none are allowed to dissapear. After that both obs. 4,5 now considered particles could dissappear and obs. 6 could appear

or

disappearance after $t=1$: $N_{tot} = 6$ All obs. represent particles appearing / dissappearing at given frame.

d) In general if $N_{tot} = 1$ holds, otherwise the second shortest path might reuse edges from the shortest path, which contradicts the capacity condition.

e) Yes, if flow over all edges is set to zero, otherwise conservation of mass will prevent $N_{tot} = 0$.

f) choose: Aeq ⟶ Node-arc incidence Matrix without start/target

beq ⟶ 0

Aeeq ⟶ 1

beeq ⟶ 1

For more details see notebook.

g) + h) see notebook

## 1.2 Tracking with particle merging and/or division

a)

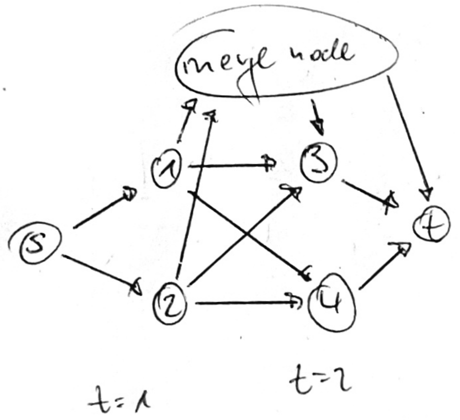
Fig. 2a – If split node is handled like a 'normal' node a splitting process would violate the 'conservation of mass'-constraint.

Fig. 2b – Problem of Fig. 2a would not occur BUT split node could be used as 'normal' node not modelling a splitting process with only one incoming and outgoing edge used.

b)

Flow coupling constraint for (Split node, 3) and (Split node, 4)

c)




with flow coupling constraint for (1, merge node), (2, merge node)

Inverse to split process. The violation of the 'conservation of mass'-constraint this time is avoided by enabling the 'superfluous' particle to be modelled by an edge directly to the target. In the top-example 1,2 merge to 3.

d) The ~~Ex~~ LP from 1.1 can be extended to fit the flow coupling constraint by adding the columns for edges that include a split/merge node in the node-arc incidence matrix. This added entry represents one 'big edge' that ensures if this entry is chosen in the edge-space during optimisation that all edges involved in the splitting/merging process are being accounted for. This now enables columns with more than two non-zero entries, but the rest of the LP is identical to 1.1 f).

Take the model from c). Node-arc incidence matrix:

$$
\begin{array}{c|ccccccccc}
 & (s1) & (s2) & (13) & (14) & (23) & (24) & (m) & (3t) & (4t) \\
\hline
s & -1 & -1 & & & & & & & \\
1 & 1 & & -1 & -1 & & & -1 & & \\
2 & & 1 & & & -1 & -1 & -1 & & \\
3 & & & 1 & & 1 & & 1 & -1 & \\
t & & & & & & & 1 & 1 & 1 \\
4 & & & & 1 & & 1 & & & -1
\end{array}
$$

Edges

Vertices

((m) represents the entry for the 'merge' edges)

The indicated sub-matrix has a determinant of ~~two~~ 2. Meaning the incidence- or equivalently constraint-matrix is not totally unimodular. Leaving the LP to be an Integer-Linear Program, because the feasible region is not set to have integral vertices only.

e)

$$
\begin{array}{lccccccccc}
 & s1 & s2 & 13 & 14 & 23 & 24 & (m) & 3t & 4t \\
\text{integral solution: } c = & [\,0 & 0 & 1 & 1 & 2 & 2 & \infty & 3 & 4\,] \\
\text{no —"— : } c = & [\,0 & 0 & \infty & \infty & \infty & \infty & 1 & 1 & 2\,]
\end{array}
$$

f) method already chosen to formulate d)

solution not implemented.