(1)

d) $G = (V, E)$ with matrix of edge weights X

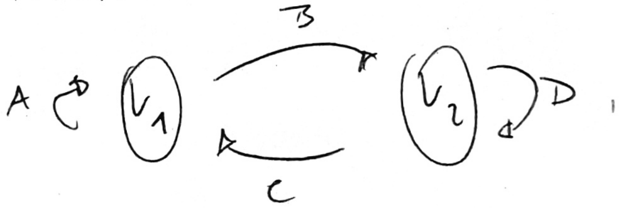
Suppose $X \in M(n \times n) \Longrightarrow |V| = n$. Order and label nodes from now on with labels from 0 to $n-1$.

$x_{ij} = e(i,j)$

Matrix multiplication (min, plus) updates the shortest path by one "Hop" (one edge), whereas for arbitrary matrix α, α_{ij} the (*) shortest path from node i to node j is.

Suppose $X = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$ $X^* := \begin{pmatrix} E & F \\ G & H \end{pmatrix}$ and $A \in M(l \times l)$.

Define $V_1 = \{0, \dots, l\}$, $V_2 = \{l+1, \dots, n-1\}$. Then the matrices can be summarized in this schematic



because A only contains edges from and to V_1, B from V_1 to V_2, ...

E contains shortest paths to and from V_1.

$\alpha := B D^* C$ contains the shortest paths from and to nodes from V_1 over V_2 by "hopping" once to and once from V_2. In comparison A contains direct weights to and from V_1. By taking $B := A \wedge \alpha$ the cheapest "one-hop" path inside V_1 is chosen.

Because E only contains the shortest paths to and from V_1 it suffices to iterate over B by taking B^* to get E.

(*) length