

1.

1.a. grlex

$$r = x^7 + x^3 - y + 1$$

A handwritten diagram showing the division of a polynomial  $r$  by a monomial  $F_1$ . The divisor  $F_1$  is written as  $x^6 + x^2$ , with terms  $x^6$  and  $x^2$  underlined. The dividend  $r$  is written as  $x^7 + x^3 - y + 1$ , with terms  $x^7$ ,  $x^3$ , and  $-y$  underlined. The division process is shown with a series of subtractions. The first subtraction step shows  $x^7$  minus  $x^6$  (underlined), resulting in  $x$ . The second subtraction step shows  $x^3$  minus  $x^2$  (underlined), resulting in  $x$ . The third subtraction step shows  $-y$  minus  $-y$  (underlined), resulting in  $1$ . The final result is  $x^7 + x^3 - y + 1$ .

$$\begin{array}{r} x^6 + x^2 \\ \underline{-} y + x \\ x^7 + x^3 - y + 1 \\ \hline x^6 + x^2 \\ \underline{-} y + x \\ x^7 + x^3 - y + 1 \\ \hline x^3 + x \\ \underline{-} y + 1 \\ x^7 + x^3 - y + 1 \end{array}$$

$f = (x^6 + x^2)F_1 + (0)F_2 + (x^7 + x^3 - y + 1)$

lex

$$r = 2y^3 - y + 1$$



$$F_1 : x^6 + x^5y + x^4y^2 + x^4 + x^3y + x^2y^2 + 2x^2 + 2xy + 2y^2 + 2$$

$$F_2 : x^6 + x^5y + x^4 + x^3y + 2x^2 + 2xy + 2$$

$$\begin{array}{r} xy^2 - x \\ \hline x - y^3 \end{array} \left( \begin{array}{r} x^7 + x^3y^2 - y + 1 \\ \hline x^7 - x^6y^3 \end{array} \right)$$

$$\begin{array}{r} x^6y^3 + x^3y^2 - y + 1 \\ \hline x^6y^3 - x^6y \end{array}$$

$$\begin{array}{r} x^6y + x^3y^2 - y + 1 \\ \hline x^6y - x^5y^3 \end{array}$$

$$\begin{array}{r} x^5y^4 + x^3y^2 - y + 1 \\ \hline x^5y^4 - x^5y^2 \end{array}$$

$$\begin{array}{r} x^5y^2 + x^3y^2 - y + 1 \\ \hline x^5y^2 - x^4y^3 \end{array}$$

$$\begin{array}{r} x^4y^3 + x^3y^2 - y + 1 \\ \hline x^4y^3 - x^4y \end{array}$$

$$\begin{array}{r} x^4y + x^3y^2 - y + 1 \\ \hline x^4y - x^3y^4 \end{array}$$

$$\begin{array}{r} x^3y^4 + x^3y^2 - y + 1 \\ \hline x^3y^4 - x^3y^2 \end{array}$$

$$\begin{array}{r} 2x^3y^2 - y + 1 \\ \hline 2x^3y^2 - 2x^3 \end{array}$$

$$\begin{array}{r} 2x^3 - y + 1 \\ \hline 2x^3 - 2x^2y^3 \end{array}$$

$$\begin{array}{r} 2x^2y^3 - y + 1 \\ \hline 2x^2y^3 - 2x^2y \end{array}$$

$$\begin{array}{r} 2x^2y - y + 1 \\ \hline \end{array}$$

$$\begin{array}{r} 2x^2y - y + 1 \\ \hline 2x^2y - 2xy \end{array}$$

$$\begin{array}{r} 2xy - y + 1 \\ \hline 2xy - 2x^2 \end{array}$$

$$\begin{array}{r} 2x^2 - y + 1 \\ \hline 2x^2 - 2x \end{array}$$

$$\begin{array}{r} 2x - y + 1 \\ \hline 2x - 2y \end{array}$$

$$\begin{array}{r} r \\ \hline 2y^3 - y + 1 \end{array}$$

$$r = 2y^3 - y + 1$$

1.b. grlex

$$r = x^7 + x^3 - y + 1$$



$F_1$ :

$$F_2: x^6 + x^2$$

$$\begin{array}{r} \overline{x^7y^2 + x^3y^2 - y + 1} \\ \overline{-x^3y^2 - x} \\ \hline x^7y^2 - x^3y^2 - y + 1 \\ \overline{+ x^3y^2 - y + 1} \\ \hline x^7y^2 - y + 1 \end{array}$$

$\sqsubseteq$

$$r = x^7 + x^3 - y + 1$$

**lex**

$$r = y^{23} + y^{11} - y + 1$$



$$F_1 : x^6y^2 + x^5y^5 + x^4y^8 + x^3y^{11} + x^2y^{14} + xy^{17} + xy^{20} + xy^{23} + y^{26}$$

$$F_2 =$$

$$\begin{array}{r} x-y^3 \\ \times y^2+x \\ \hline x^2y^2+x^3y^2-y+1 \\ x^2y^2-x^3y^5 \\ \hline x^6y^5+x^3y^2-y+1 \\ x^6y^5-x^5y^8 \\ \hline x^5y^8+x^3y^2-y+1 \\ x^5y^8-x^4y \\ \hline x^4y^6+x^3y^2-y+1 \\ x^4y^6-x^3y^4 \\ \hline x^3y^{14}+x^3y^2-y+1 \\ x^3y^{14}-x^2y^{17} \\ \hline x^2y^{17}+x^2y^{17}-y+1 \\ x^2y^{17}-x^2y^5 \\ \hline x^2y^{17}+x^2y^5-y+1 \\ x^2y^{17}-x^2y^{20} \\ \hline x^2y^{20}+x^2y^8-y+1 \\ x^2y^{20}-xy^8 \end{array}$$

$$\begin{array}{r} xy^{20}+xy^8-y+1 \\ xy^{20}-y^{23} \\ \hline xy^8+y^{23}-y+1 \\ y^{23}+y^{11}-y+1 \end{array}$$

$$r = y^{23} + y^{11} - y + 1$$



**2.**

2.a.  $r = z$



$F_1 : y^2 z^2 + y$   
 $F_2 : y^3 z^2 + y^2 z^3 + y^2 + y z^6 + y z^3 + z^9 + z^6 - z$   
 $F_3 : z^{10} + z^8 + z^6 + z^4 + z^2 + z^6 + z^4 + z^2 + z$

$$\begin{array}{r} x-y^2 \\ \hline y-z \\ \hline z-1 \end{array} \left| \begin{array}{l} x y^2 z^2 + x y - y z \\ \hline x y^2 z^2 - y^4 z^2 \end{array} \right.$$

$$\begin{array}{r} x y + y^4 z^2 - y z \\ \hline x y - y \end{array} \left| \begin{array}{l} y^4 z^2 + y^3 - y z \\ \hline y^4 z^2 - y^3 \end{array} \right.$$

$$\begin{array}{r} y^3 z^3 + y^3 - y z \\ \hline y^3 z^3 - y z^6 \end{array} \left| \begin{array}{l} y^3 + y z^6 - y z \\ \hline y^3 - y z^3 \end{array} \right.$$

$$\begin{array}{r} y^2 z^6 + y^2 z^3 - y z \\ \hline y^2 z^6 - y z \end{array} \left| \begin{array}{l} y^2 z^3 + y z^9 - y z \\ \hline y^2 z^3 - y z^6 \end{array} \right.$$

$$\begin{array}{r} y z^9 + y z^6 - y z \\ \hline y z^9 - z \end{array} \left| \begin{array}{l} y z^6 - y z + z^{12} \\ \hline y z^6 - z^9 \end{array} \right.$$

$$\begin{array}{r} -y z + z^{12} + z^9 \\ \hline -y z + z^4 \end{array} \left| \begin{array}{l} z^{12} + z^9 - z^6 \\ \hline z^{12} - z^{10} \end{array} \right.$$

$$\begin{array}{r} z^{10} + z^9 - z^6 \\ \hline z^{10} - z^4 \end{array} \left| \begin{array}{l} z^9 + z^4 - z^6 \\ \hline \end{array} \right.$$

$\frac{z^9 + z^4 - z^6}{z} \longrightarrow r$



