# 1.

The zero polynomial is a specific element of $k[x]$ where all the coefficients are $0$. A zero function is not a plain polynomial, but a function that evaluates to $0$ everywhere.

Let $k = \{0, 1\}$. Then $p = x^2 - x \in k[x]$ is *not* the zero polynomial. Now consider the function $f : k \to k$ defined by evaluating the polynomial $p$. Then $f(x) = 0$ for all $x \in k$, and $f$ is the zero function.

## 2.

```
> mygcd := proc(a, b)
    if a < b then return mygcd(b, a); end if;
    if b = 0 then return a; end if;
    return mygcd(a - b, b);
  end proc;
> mygcd3 := proc(a, b, c)
    mygcd(mygcd(a, b), c);
  end proc;
```

```
> mygcd3(42615, 834510, 1830)
                                   15                                  (11)
```

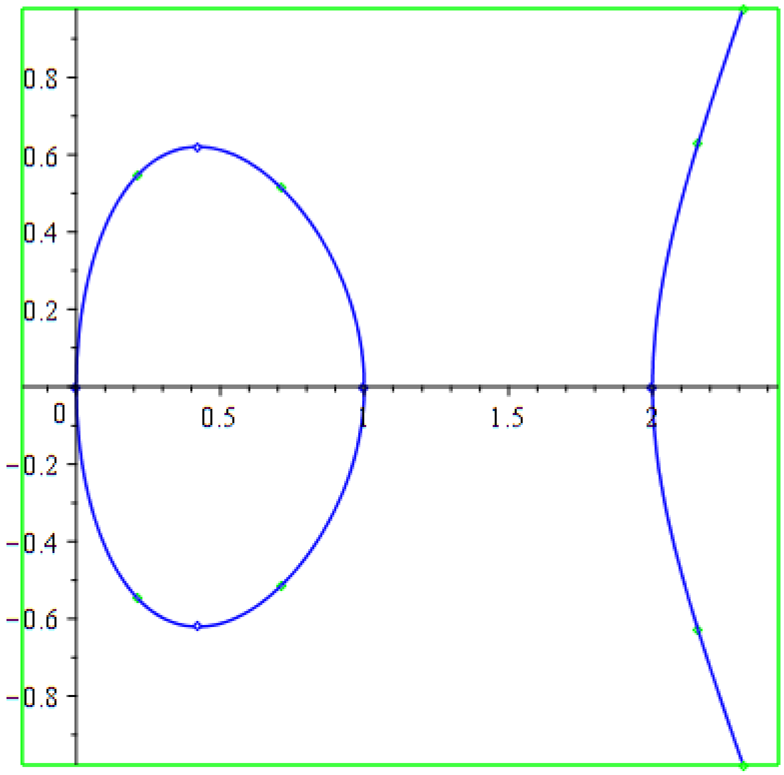
## 3.

When a polynomial $f$ is divided by $x - a$, the remainder is a constant $c$. If $a$ is a root, plugging it into $f = q(x - a) + c$ implies $c = 0$. Let $a_i$ for $1 \leq i \leq n$ be the $n$ roots of $f$. Then by dividing by every $x - a_i$,

$$f = q \cdot \prod_{i=1}^{n} (x - a_i)$$

where $q$ is a polynomial of degree $\deg(f) - n$. It follows that $n \leq \deg(f)$.

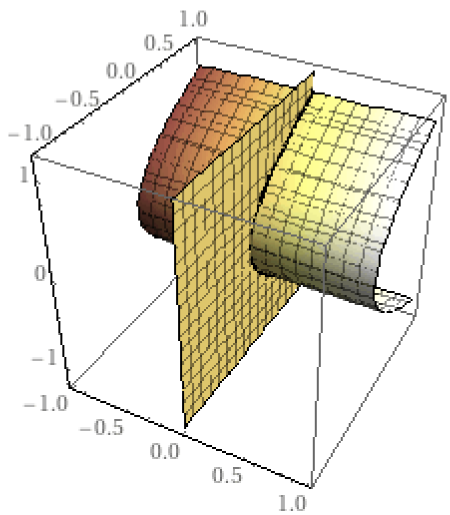
# 4.




This variety has symmetry about the $x$-axis.

# 5.

The polynomial $xz^2 - xy = x(z^2 - y)$ is zero when $x = 0$ or $y = z^2$. This gives us the $yz$-plane, and a $y = z^2$ parabola on that plane that stretches out infinitely perpendicular to it.

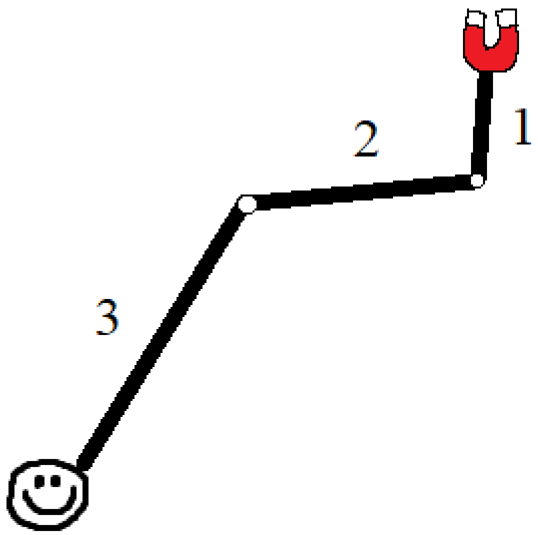



(Plotted with WolframAlpha)

6.

## 7.

(a)




(b) 3 variables will be required to describe the orientations of each of the 3 arms which are independent of each other.

(c) c

(d) The variety would be 3-dimensional, since it requires 3 variables to describe.

(e) The maximum distance the arm can extend from the origin is $3 + 2 + 1 = 6$, which means $u^2 + v^2 \leq 6^2$.

(f) Yes. The arm can take on the radius $r = 0$ by folding the two smaller arms back on the longest arm. It can also take on $r = 6$ by fully extending the arm. Thus all values $0 \leq r \leq 6$ can be achieved, then the arm simply needs to pivot about the origin into place.

# 8.

Putting the system into reduced row-echelon form gives us

$$\begin{array}{rrrrl} x & & +4z & -3w & = 5, \\ & y & -3z & +2w & = -3, \end{array}$$

which gives the parameterization

$$\begin{aligned} x &= 5 - 4s + 3t, \\ y &= -3 + 3s - 2t, \\ z &= s, \\ w &= t, \end{aligned}$$

for all $s, t \in \mathbb{R}$.

# 9.

Let $x, y$ be parameterized by $s \in \mathbb{R}$ such that

$$\begin{aligned} x &= s, \\ y &= f(s). \end{aligned}$$

**10.**

**11.**

## 12.

(a) Solving for $y$ in the 2nd equation gives

$$y = \frac{1}{x}.$$

Plugging it into the 1st equation gives

$$x^2 + \frac{1}{x^2} - 1 = 0.$$

## 13.

Suppose $f_i \in I$. Let $p \in \langle f_1, ..., f_s \rangle$. This means there exist polynomials $g_i \in k[x_1, ..., x_n]$ such that $p = g_1 f_1 + \cdots g_s f_s$. By closure of an ideal, $p \in I$ and $\langle f_1, ..., f_s \rangle \subseteq I$.

Suppose $\langle f_1, ..., f_s \rangle \subseteq I$. Then $f_i \in \langle f_1, ..., f_s \rangle$ and $f_i \in I$.

**14.**

> $gcd\left(gcd\left(x^3 + x^2 - 4\,x - 4,\, x^3 - x^2 - 4\,x + 4\right),\, x^3 - 2\,x^2 - x + 2\right)$

$$x - 2 \tag{8}$$

# 15.

Let the 3 polynomials be $p, q, r$. Since $\gcd(p, q, r) = x - 2$, there exist polynomials $a, b, c$ such that

$$ap + bq + cr = x - 2.$$

Multiplying both sides by $x + 2$ gives us

$$a(x+2) \cdot p + b(x+2) \cdot q + c(x+2) \cdot r = x^2 - 4,$$

which means $x^2 - 4$ is in the ideal.