Problem 1

$$\begin{array}{ll} \text{Maximize} & 4x_1 + 6x_2 - 5x_3 \\ \\ \text{such that} & \begin{array}{rrrl} 5x_1 & -3x_2 & -2x_3 & \leq 10 \\ 2x_1 & +3x_2 & -3x_3 & \leq 10 \\ 2x_1 & +x_2 & +2x_3 & \leq 7 \end{array} \end{array}$$

Add slack variables w_1, w_2, w_3 and put into standard equation form. (All variables shown have a nonnegativity restriction).

$$\begin{array}{ll} \text{Maximize} & 4x_1 + 6x_2 - 5x_3 \\ \\ \text{such that} & \begin{array}{rrrrrrl} 5x_1 & -3x_2 & -2x_3 & +w_1 & & & = 10 \\ 2x_1 & +3x_2 & -3x_3 & & +w_2 & & = 10 \\ 2x_1 & +x_2 & +2x_3 & & & +w_3 & = 7 \end{array} \end{array}$$

Note that the slack variables are primal feasible. Create the dictionary for the basis of non-basic variables $\{w_1, w_2, w_3\}$.

$$\begin{array}{ll} \text{Maximize} & 4x_1 + 6x_2 - 5x_3 \\ \\ \text{such that} & \begin{array}{rcrrrr} w_1 & = & 10 & -5x_1 & +3x_2 & +2x_3 \\ w_2 & = & 10 & -2x_1 & -3x_2 & +3x_3 \\ w_3 & = & 7 & -2x_1 & -x_2 & -2x_3 \end{array} \end{array}$$

Select x_1 as the entering variable. It has the following ratios:

$$\begin{array}{lll} w_1 = 10 - 5\lambda & \implies \lambda \leq 2 & \leftarrow \text{minimum ratio} \\ w_2 = 10 - 2\lambda & \implies \lambda \leq 5 & \\ w_3 = 7 - 2\lambda & \implies \lambda \leq \dfrac{7}{2} & \end{array}$$

With w_1 leaving and x_1 entering, rewrite the dictionary for the new basis of non-basic variables $\{x_1, w_2, w_3\}$.

$$\begin{array}{ll} \text{Maximize} & 4\left(2 + \frac{3}{5}x_2 + \frac{2}{5}x_3 - \frac{1}{5}w_1\right) + 6x_2 - 5x_3 \\ \\ \text{such that} & \begin{array}{rcrlrr} x_1 & = & \multicolumn{4}{l}{2 + \frac{3}{5}x_2 + \frac{2}{5}x_3 - \frac{1}{5}w_1} \\ w_2 & = & 10 & -2\left(2 + \frac{3}{5}x_2 + \frac{2}{5}x_3 - \frac{1}{5}w_1\right) & -3x_2 & +3x_3 \\ w_3 & = & 7 & -2\left(2 + \frac{3}{5}x_2 + \frac{2}{5}x_3 - \frac{1}{5}w_1\right) & -x_2 & -2x_3 \end{array} \end{array}$$

$$\Downarrow$$

$$\begin{array}{ll} \text{Maximize} & 8 + \frac{42}{5}x_2 - \frac{17}{5}x_3 - \frac{4}{5}w_1 \\ \\ \text{such that} & \begin{array}{rcrrrr} x_1 & = & 2 & +\frac{3}{5}x_2 & +\frac{2}{5}x_3 & -\frac{1}{5}w_1 \\ w_2 & = & 6 & -\frac{21}{5}x_2 & +\frac{11}{5}x_3 & +\frac{2}{5}w_1 \\ w_3 & = & 3 & -\frac{11}{5}x_2 & -\frac{14}{5}x_3 & +\frac{2}{5}w_1 \end{array} \end{array}$$

Select x_2 as the entering variable with ratios:

$$\begin{aligned}
x_1 &= 2 + \frac{3}{5}\lambda && \implies \lambda \le \infty \\
w_2 &= 6 - \frac{21}{5}\lambda && \implies \lambda \le \frac{10}{7} \\
w_3 &= 3 - \frac{11}{5}\lambda && \implies \lambda \le \frac{15}{11} && \leftarrow \text{minimum ratio}
\end{aligned}$$

With w_3 leaving and x_2 entering, rewrite the dictionary for the new basis of non-basic variables $\{x_1, w_2, x_2\}$.

$$\begin{array}{ll}
\text{Maximize} & 8 + \frac{42}{5}\left(\frac{15}{11} - \frac{14}{11}x_3 + \frac{2}{11}w_1 - \frac{5}{11}w_3\right) - \frac{17}{5}x_3 - \frac{4}{5}w_1 \\
\\
\text{such that} & \begin{array}{rcllll}
x_1 &=& 2 & +\frac{3}{5}\left(\frac{15}{11} - \frac{14}{11}x_3 + \frac{2}{11}w_1 - \frac{5}{11}w_3\right) & +\frac{2}{5}x_3 & -\frac{1}{5}w_1 \\
w_2 &=& 6 & -\frac{21}{5}\left(\frac{15}{11} - \frac{14}{11}x_3 + \frac{2}{11}w_1 - \frac{5}{11}w_3\right) & +\frac{11}{5}x_3 & +\frac{2}{5}w_1 \\
x_2 &=& \multicolumn{4}{l}{\frac{15}{11} - \frac{14}{11}x_3 + \frac{2}{11}w_1 - \frac{5}{11}w_3}
\end{array}
\end{array}$$

$$\Downarrow$$

$$\begin{array}{ll}
\text{Maximize} & \frac{214}{11} - \frac{155}{11}x_3 + \frac{8}{11}w_1 - \frac{42}{11}w_3 \\
\\
\text{such that} & \begin{array}{rclccc}
x_1 &=& \frac{31}{11} & -\frac{3}{11}w_3 & -\frac{4}{11}x_3 & -\frac{1}{11}w_1 \\
w_2 &=& \frac{3}{11} & +\frac{21}{11}w_3 & +\frac{83}{11}x_3 & -\frac{4}{11}w_1 \\
x_2 &=& \frac{15}{11} & -\frac{5}{11}w_3 & -\frac{14}{11}x_3 & +\frac{2}{11}w_1
\end{array}
\end{array}$$

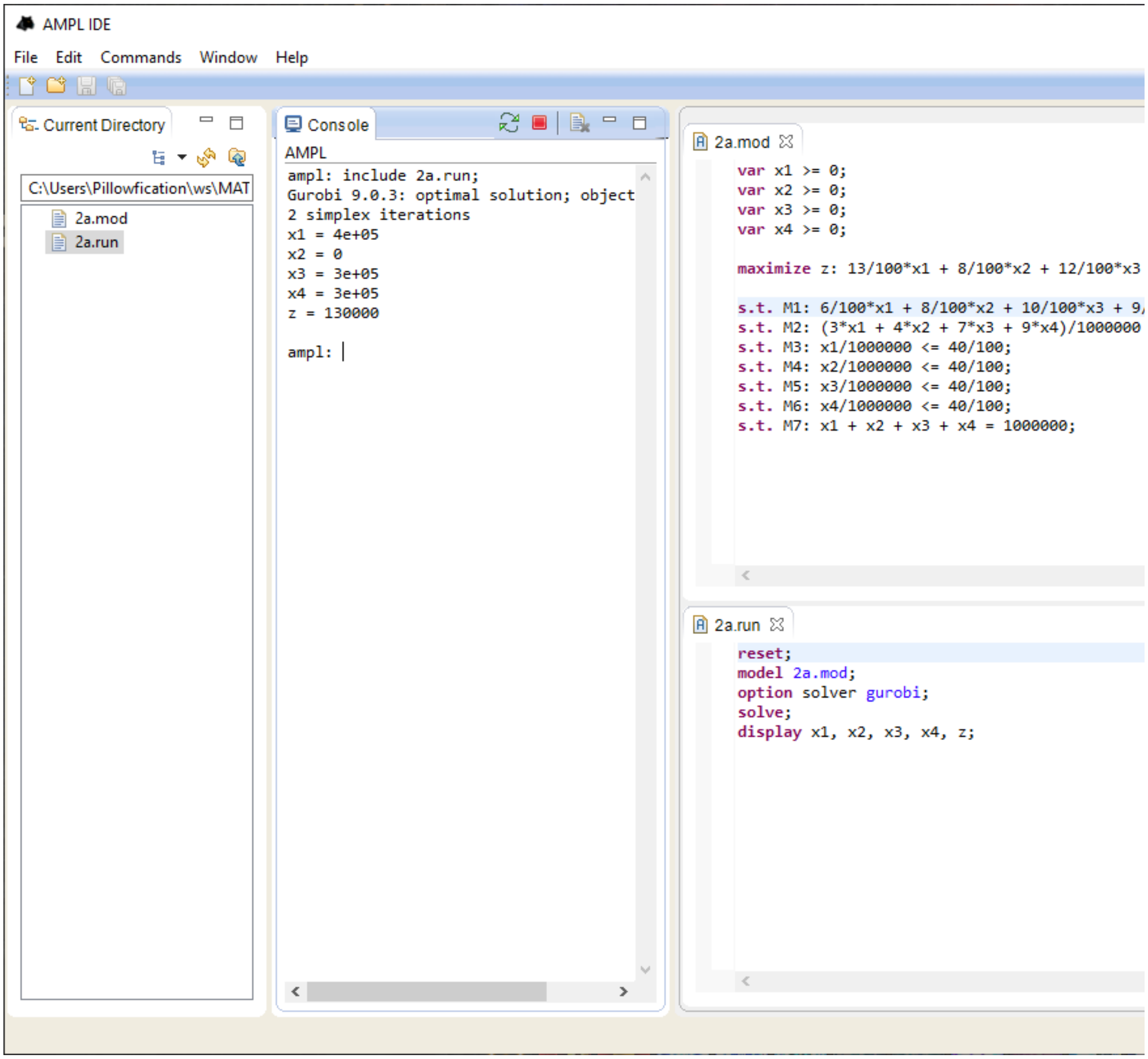
Select w_1 as the entering variable with ratios:

$$\begin{aligned}
x_1 &= \frac{31}{11} - \frac{1}{11}\lambda && \implies \lambda \le 31 \\
w_2 &= \frac{3}{11} - \frac{4}{11}\lambda && \implies \lambda \le \frac{3}{4} && \leftarrow \text{minimum ratio} \\
x_2 &= \frac{15}{11} + \frac{2}{11}\lambda && \implies \lambda \le \infty
\end{aligned}$$

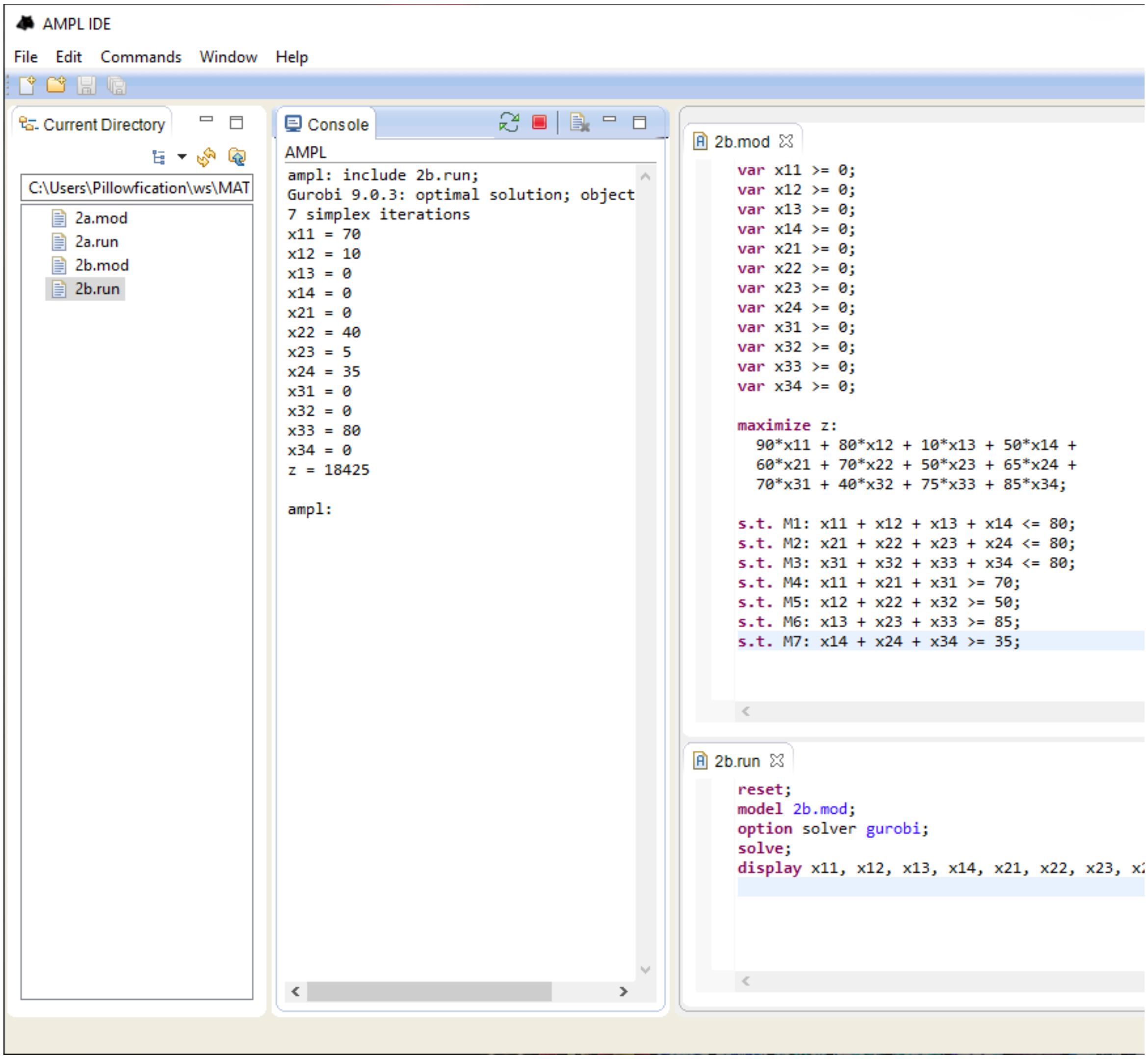
With w_2 leaving and w_1 entering, rewrite the dictionary for the new basis of non-basic variables $\{x_1, w_1, x_2\}$.

Problem 2

(a)


AMPL IDE
File Edit Commands Window Help
Current Directory
C:\Users\Pillowfication\ws\MAT
2a.mod
2a.run
Console
AMPL
ampl: include 2a.run;
Gurobi 9.0.3: optimal solution; object
2 simplex iterations
x1 = 4e+05
x2 = 0
x3 = 3e+05
x4 = 3e+05
z = 130000
ampl:
2a.mod
var x1 >= 0;
var x2 >= 0;
var x3 >= 0;
var x4 >= 0;
maximize z: 13/100*x1 + 8/100*x2 + 12/100*x3
s.t. M1: 6/100*x1 + 8/100*x2 + 10/100*x3 + 9/
s.t. M2: (3*x1 + 4*x2 + 7*x3 + 9*x4)/1000000
s.t. M3: x1/1000000 <= 40/100;
s.t. M4: x2/1000000 <= 40/100;
s.t. M5: x3/1000000 <= 40/100;
s.t. M6: x4/1000000 <= 40/100;
s.t. M7: x1 + x2 + x3 + x4 = 1000000;
2a.run
reset;
model 2a.mod;
option solver gurobi;
solve;
display x1, x2, x3, x4, z;

(b)


AMPL IDE
File Edit Commands Window Help
Current Directory
C:\Users\Pillowfication\ws\MAT
2a.mod
2a.run
2b.mod
2b.run
Console
AMPL
ampl: include 2b.run;
Gurobi 9.0.3: optimal solution; object
7 simplex iterations
x11 = 70
x12 = 10
x13 = 0
x14 = 0
x21 = 0
x22 = 40
x23 = 5
x24 = 35
x31 = 0
x32 = 0
x33 = 80
x34 = 0
z = 18425
ampl:
2b.mod
var x11 >= 0;
var x12 >= 0;
var x13 >= 0;
var x14 >= 0;
var x21 >= 0;
var x22 >= 0;
var x23 >= 0;
var x24 >= 0;
var x31 >= 0;
var x32 >= 0;
var x33 >= 0;
var x34 >= 0;
maximize z:
90*x11 + 80*x12 + 10*x13 + 50*x14 +
60*x21 + 70*x22 + 50*x23 + 65*x24 +
70*x31 + 40*x32 + 75*x33 + 85*x34;
s.t. M1: x11 + x12 + x13 + x14 <= 80;
s.t. M2: x21 + x22 + x23 + x24 <= 80;
s.t. M3: x31 + x32 + x33 + x34 <= 80;
s.t. M4: x11 + x21 + x31 >= 70;
s.t. M5: x12 + x22 + x32 >= 50;
s.t. M6: x13 + x23 + x33 >= 85;
s.t. M7: x14 + x24 + x34 >= 35;
2b.run
reset;
model 2b.mod;
option solver gurobi;
solve;
display x11, x12, x13, x14, x21, x22, x23,