Problem 0

I affirm the code of student conduct. I, the student enrolled in this course, am taking this exam by myself without collaborating with or using the help of other persons, external resources, or services.

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Problem 1 (degeneracy)

$$\begin{aligned} s_0 &= 9 \\ s_1 &= 1 \\ s_{01} &= 10 \end{aligned}$$

$$\begin{aligned} c_1 &= -s_1^2 - 2s_1s_{01} - s_{01}^2 - 4s_1 - 4s_{01} - 4 && = -169 \\ c_3 &= s_0s_1 + s_1^2 - s_1s_{01} && = 0 \\ d_3 &= s_0^2 + s_0s_1 - s_0s_{01} && = 0 \\ c_4 &= -3s_1^2 - 6s_1s_{01} - 3s_{01}^2 - 12s_1 - 12s_{01} - 12 && = -507 \end{aligned}$$

EXAM-Dict

$$\begin{array}{rcrrrr} \zeta = & & -169x_1 & +0x_3 & +0w_3 & -507x_4 \\ w_1 = & 5 & -\ 6x_1 & +2x_3 & -\ w_3 & -\ 3x_4 \\ w_2 = & 2 & +\ 4x_1 & -6x_3 & +2w_3 & -\ 2x_4 \\ x_2 = & 0 & +\ 3x_1 & & -\ w_3 & +\ 2x_4 \end{array}$$

i.

$$\text{P-Sol} = \begin{pmatrix} x_1 = 0 \\ \color{red}{x_2 = 0} \\ x_3 = 0 \\ x_4 = 0 \\ \color{red}{w_1 = 5} \\ \color{red}{w_2 = 2} \\ w_3 = 0 \end{pmatrix},$$

$$\text{D-Sol} = \begin{pmatrix} \color{red}{y_1 = -169} \\ y_2 = 0 \\ \color{red}{y_3 = 0} \\ \color{red}{z_1 = 0} \\ z_2 = 0 \\ z_3 = 0 \\ \color{red}{z_4 = -507} \end{pmatrix}$$

ii. Yes, EXAM-Dict is primal degenerate because in the basic solution, the coefficient of the basic variable x_2 is 0.

iii. (iii)

iv. Yes, EXAM-Dict is dual degenerate because in the dual basic solution, the coefficients of the basic variables y_3, z_1 are 0.

v. (v)

Problem 2 (lexicographic simplex method)

$$\begin{array}{rrrrrl}
\text{maximize} & -4x_1 + & x_2 & +5x_3 + & 3x_4 & \\
\text{s.t.} & -6x_1 + & x_2 & -1x_3 - & 2x_4 & \leq 0 \\
& & -\frac{15}{8}x_2 & -6x_3 + & \frac{17}{8}x_4 & \leq 1 \\
& 4x_1 + & 2x_2 & +2x_3 & & \leq 0 \\
& \multicolumn{5}{l}{x_1, x_2, x_3, x_4 \geq 0}
\end{array}$$

$$\Big\downarrow \text{add slack variables}$$

$$\begin{array}{rrrrrrrrl}
\text{maximize} & -4x_1 + & x_2 & +5x_3 + & 3x_4 & & & & \\
\text{s.t.} & -6x_1 + & x_2 & - x_3 - & 2x_4 & +x_5 & & & = 0 \\
& & -\frac{15}{8}x_2 & -6x_3 + & \frac{17}{8}x_4 & & +x_6 & & = 1 \\
& 4x_1 + & 2x_2 & +2x_3 & & & & +x_7 & = 0 \\
& \multicolumn{8}{l}{x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0}
\end{array}$$

$$\Big\downarrow \text{add perturbation}$$

$$\begin{array}{rrrrrrrrl}
\text{maximize} & -4x_1 + & x_2 & +5x_3 + & 3x_4 & & & & \\
\text{s.t.} & -6x_1 + & x_2 & - x_3 - & 2x_4 & +x_5 & & & = \varepsilon_1 \\
& & -\frac{15}{8}x_2 & -6x_3 + & \frac{17}{8}x_4 & & +x_6 & & = 1+\varepsilon_2 \\
& 4x_1 + & 2x_2 & +2x_3 & & & & +x_7 & = \varepsilon_3 \\
& \multicolumn{8}{l}{x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0}
\end{array}$$

Step 1

- Current objective value:

$$\bar{\zeta}(0,0,0,0,\varepsilon,1+\varepsilon_2,\varepsilon_3) = 0$$

- Dictionary:

$$\begin{aligned}
\zeta &= && -4x_1 + x_2 + 5x_3 + 3x_4 \\
x_5 &= \varepsilon_1 && + 6x_1 - x_2 + x_3 + 2x_4 \\
x_6 &= (1+\varepsilon_2) && + \tfrac{15}{8}x_2 + 6x_3 - \tfrac{17}{8}x_4 \\
x_7 &= \varepsilon_3 && - 4x_1 - 2x_2 - 2x_3
\end{aligned}$$

- Entering variable:

  Select x_3 as the entering variable, with the following ratios:

$$\begin{aligned}
x_5 &= \varepsilon_1 + \lambda && \Longrightarrow \lambda \leq \infty \\
x_6 &= (1+\varepsilon_2) + 6\lambda && \Longrightarrow \lambda \leq \infty \\
x_7 &= \varepsilon_3 - 2\lambda && \Longrightarrow \lambda \leq \tfrac{1}{2}\varepsilon_3 && \leftarrow \text{minimum ratio}
\end{aligned}$$

- Pivot:

  Pivot with x_7 as the leaving variable

$$\begin{aligned}
\zeta &= && \tfrac{5}{2}\varepsilon_3 - 14x_1 - 4x_2 - \tfrac{5}{2}x_7 + 3x_4 \\
x_5 &= && (\varepsilon_1 + \tfrac{1}{2}\varepsilon_3) + 4x_1 - 2x_2 - \tfrac{1}{2}x_7 + 2x_4 \\
x_6 &= (1+\varepsilon_2+3\varepsilon_3) && - \tfrac{33}{8}x_2 - 3x_7 - \tfrac{17}{8}x_4 \\
x_3 &= && \tfrac{1}{2}\varepsilon_3 - 2x_1 - x_2 - \tfrac{1}{2}x_7
\end{aligned}$$

Step 2

- Current objective value:

$$\bar{\zeta}(0, 0, \tfrac{1}{2}\varepsilon_3, 0, \varepsilon_1 + \tfrac{1}{2}\varepsilon_3, 1 + \varepsilon_2 + 3\varepsilon_3, 0) = \tfrac{5}{2}\varepsilon_3$$

- Dictionary:

$$\begin{aligned}
\zeta &= && \tfrac{5}{2}\varepsilon_3 - 14x_1 - 4x_2 - \tfrac{5}{2}x_7 + 3x_4 \\
x_5 &= && (\varepsilon_1 + \tfrac{1}{2}\varepsilon_3) + 4x_1 - 2x_2 - \tfrac{1}{2}x_7 + 2x_4 \\
x_6 &= (1+\varepsilon_2+3\varepsilon_3) && - \tfrac{33}{8}x_2 - 3x_7 - \tfrac{17}{8}x_4 \\
x_3 &= && \tfrac{1}{2}\varepsilon_3 - 2x_1 - x_2 - \tfrac{1}{2}x_7
\end{aligned}$$

- Entering variable:

  Select x_4 as the entering variable, with the following ratios:

$$
\begin{array}{lll}
x_5 = (\varepsilon_1 + \frac{1}{2}\varepsilon_3) + 2\lambda & \Longrightarrow \lambda \leq \infty & \\
x_6 = (1 + \varepsilon_2 + 3\varepsilon_3) - \frac{17}{8}\lambda & \Longrightarrow \lambda \leq \frac{8}{17} + \frac{8}{17}\varepsilon_2 + \frac{24}{17}\varepsilon_3 & \leftarrow \text{minim} \\
x_3 = \frac{1}{2}\varepsilon_3 + 0\lambda & \Longrightarrow \lambda \leq \infty &
\end{array}
$$

- Pivot:

  Pivot with x_6 as the leaving variable

$$
\begin{array}{rl}
\zeta = & (\frac{24}{27} + \frac{24}{17}\varepsilon_2 + \frac{229}{34}\varepsilon_3) - 14x_1 - \frac{167}{17}x_2 - \frac{229}{34}x_7 - \frac{24}{17}x_6 \\
x_5 = & (\frac{16}{17} + \varepsilon_1 + \frac{16}{17}\varepsilon_2 + \frac{113}{34}\varepsilon_3) + 4x_1 - \frac{100}{17}x_2 - \frac{113}{34}x_7 - \frac{16}{17}x_6 \\
x_4 = & (\frac{8}{17} + \frac{8}{17}\varepsilon_2 + \frac{24}{17}\varepsilon_3) - \frac{33}{17}x_2 - \frac{24}{17}x_7 - \frac{8}{17}x_6 \\
x_3 = & \frac{1}{2}\varepsilon_3 - 2x_1 - x_2 - \frac{1}{2}x_7
\end{array}
$$

Step 2

Done! Current feasible solution is optimal.

- Current objective value:

$$
\bar{\zeta}\begin{pmatrix}
x_1 = 0 \\
x_2 = 0 \\
x_3 = \frac{1}{2}\varepsilon_3 \\
x_4 = \frac{8}{17} + \frac{8}{17}\varepsilon_2 + \frac{24}{17}\varepsilon_3 \\
x_5 = \frac{16}{17} + \varepsilon_1 + \frac{16}{17}\varepsilon_2 + \frac{113}{34}\varepsilon_3 \\
x_6 = 0 \\
x_7 = 0
\end{pmatrix} = \frac{24}{27} + \frac{24}{17}\varepsilon_2 + \frac{229}{34}\varepsilon_3
$$

  With the perturbation removed, this becomes

$$
\bar{\zeta}(0, 0, 0, \tfrac{8}{17}, \tfrac{16}{17}, 0, 0) = \frac{24}{27}
$$