Problem 0

I affirm the code of student conduct. I, the student enrolled in this course, am taking this exam by myself without collaborating with or using the help of other persons, external resources, or services.

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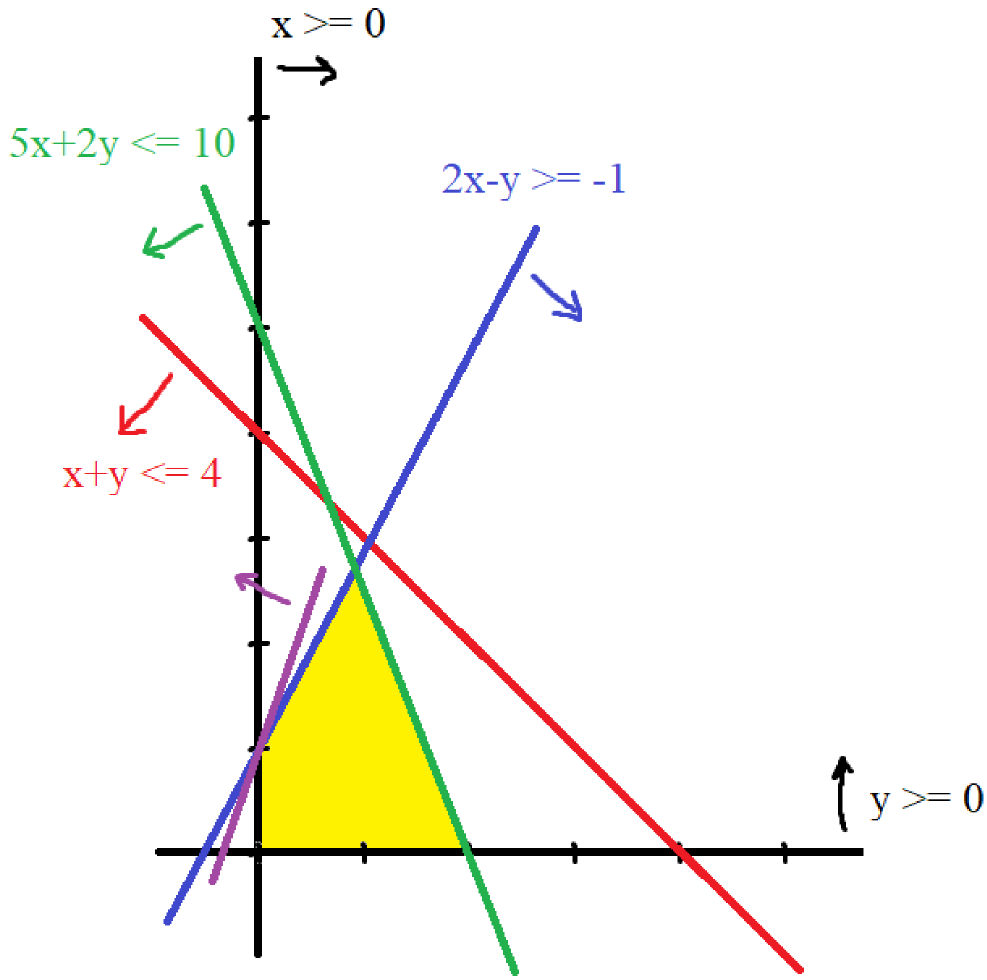
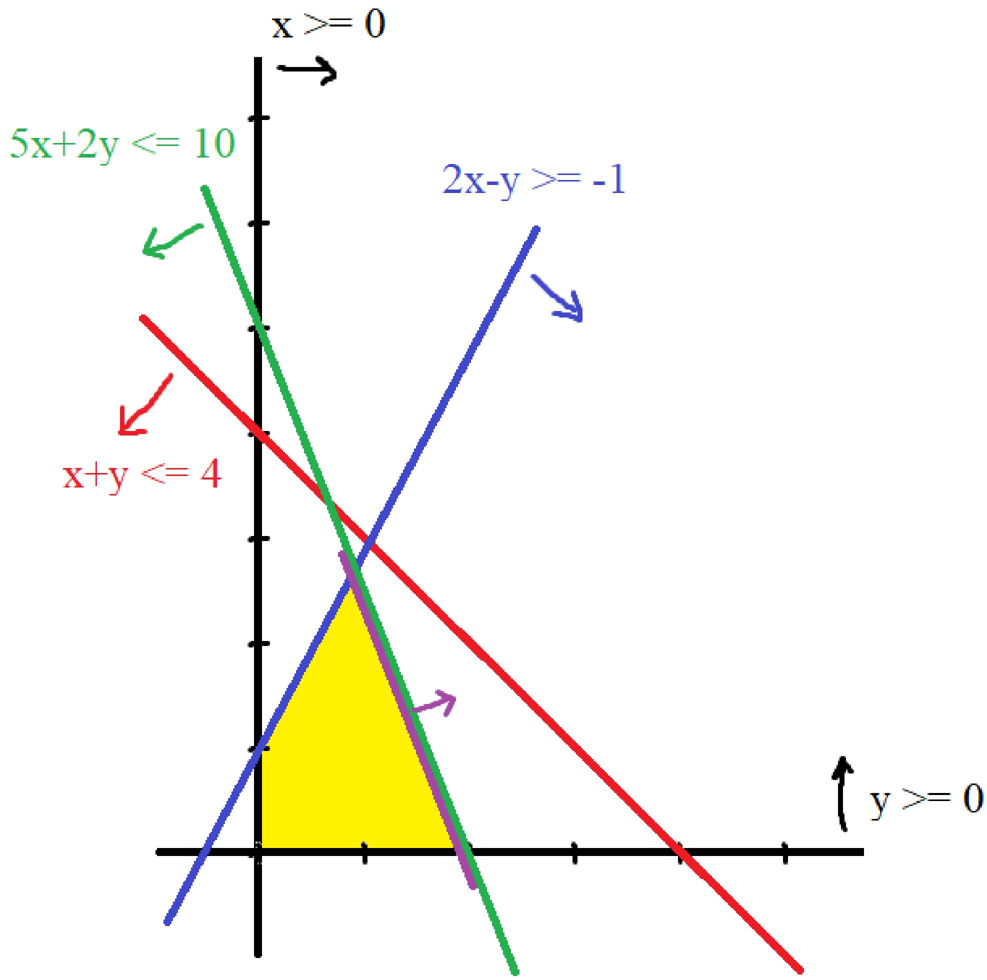
Problem 1 (Graphical Method)

(i)



The 5 constraints are graphed in Red, Green, Blue, and Black with their half-plane regions indicated by a small arrow. The intersection of all these regions is the Gold convex polygon. The Purple line indicates the slope of where the objective function remains constant, and its arrow indicates the direction in which the objective function increases. Thus the maximum value occurs at $(0, 1)$.

(ii)



The graph is the same as in (i) but with the new objective function. Since the Purple line coincides with an edge of the feasible region, all points on this edge equally maximize the objective function. Three such points are $(2, 0)$, $(\frac{3}{2}, \frac{5}{4})$, $(1, \frac{5}{2})$.

Problem 2 (LP Modeling)

The decision variables are $A, B \in \mathbb{N}$ indicating the number of A and B sandwiches to be made. A and B can be any natural number (including 0) subject to the following constraints:

Neither type of sandwhich can be more than 3 times the other.

$$\begin{aligned} A &\le 3B, \\ B &\le 3A, \end{aligned}$$

We cannot exceed the available number of carrots and bread slices.

$$\begin{aligned} 0.5A + 2B &\le 1234, \\ 2A + 3B &\le 2200, \end{aligned}$$

We cannot exceed the available number of labor hours.

$$0.1A + 0.25B \le 417 \times 2.$$

The objective function to maximize is

$$f(A, B) = \underbrace{8.5A + 13B}_{\text{revenue}} - \underbrace{\overbrace{\left\lceil \frac{0.1A + 0.25B}{2} \right\rceil}^{\text{shifts required}} \times 34.67}_{\text{operating costs}}.$$

Note that this function is not linear. It can be made linear by ignoring the ceiling function, in which case the new solutions may be suboptimal but close enough.

Problem 3 (Theory)

(i) Let ρ_i denote the i'th row. Then since $\rho_4 = 0$ and $\rho_3 = 2\rho_1 + 100\rho_2$, the set $\{\rho_1, \rho_2\}$ of cardinality 2 is a candidate for a maximal linearly independent set, and thus the matrix's rank must be at most 2.

(ii) Let $s = (1, 0) \in S \cup T$ and $t = (0, -1) \in S \cup T$. The segment joining s and t can be parametrized by $f(x) = (1 - x)s + xt$ for $0 \le x \le 1$. But the point $f(0.5) = (0.5, -0.5)$ is in neither S nor T, so $S \cup T$ is not a convex set.

(iii) By introducing the slack variable x_3, the problem becomes

$$\text{Maximize} \quad x_1$$

$$\text{such that} \quad \begin{aligned} x_1 - x_2 + x_3 &= 7, \\ x_1, x_2, x_3 &\ge 0. \end{aligned}$$

The dictionary corresponding to the basis consisting of the variable x_3 is

$$x_3 = 7 - x_1 + x_2.$$