Problem 1

There are four decision variables, $\mathbf{x} = (x_1, x_2, x_3, x_4)$, where x_i represents the amount invested into the i'th bond. I assume that investments must be made in whole dollar amounts, so $\mathbf{x} \in \mathbb{Z}^4$. Let $E, W, D \in \mathbb{R}^4$ represent the expected returns, worst-case returns, and durations of the four bonds. Then the objective function we want to maximize is

$$f(\mathbf{x}) = \sum_{i=1}^{4} x_i E_i = x_1 E_1 + x_2 E_2 + x_3 E_3 + x_4 E_4$$

subject to the following constraints:

Constraint 0 (realistic)

$$\begin{aligned} x_1, x_2, x_3, x_4 &\geq 0, \\ x_1 + x_2 + x_3 + x_4 &\leq 1\,000\,000. \end{aligned}$$

Constraint 1 (worst-case)

$$\sum_{i=1}^{4} x_i W_i \geq 8\% \times 1\,000\,000.$$

Constraint 2 (duration)

$$\frac{1}{1\,000\,000} \sum_{i=1}^{4} x_i D_i \leq 6.$$

Constraint 3 (diversification)

$$x_1, x_2, x_3, x_4 \leq 40\% \times 1\,000\,000.$$

The objective function and all constraints are linear in x_1, x_2, x_3, x_4.

Problem 2

Let x_i^j where $i \in I = \{A, B, C\}$ and $j \in J = \{1, 2, 3, 4\}$, represent the number of hours the manager assigns designer i to work on project j. This models the manager's decision as the matrix

$$\mathbf{x} = (x_i^j) = \begin{pmatrix} x_A^1 & x_A^2 & x_A^3 & x_A^4 \\ x_B^1 & x_B^2 & x_B^3 & x_B^4 \\ x_C^1 & x_C^2 & x_C^3 & x_C^4 \end{pmatrix}, \quad \text{where } x_i^j \in \mathbb{R}.$$

(Note that the manager may choose fractional hours). Let s_i^j be similarly defined as the manager's scoring of designer i's capability to work on project j, and let r^j where $j \in J$ be the number of required hours for project j. Then the objective function we wish to maximize is

$$f(\mathbf{x}) = \sum_{\substack{i \in I, \\ j \in J}} x_i^j s_i^j$$

subject to the following constraints:

$$\begin{aligned} 0 \leq \sum_{j \in J} x_i^j \leq 80, &\qquad \text{a designer can only work for 0 to 80 hours total;} \\ \sum_{i \in I} x_i^j \geq r^j, &\qquad \text{each project must meet the minimum required hours.} \end{aligned}$$