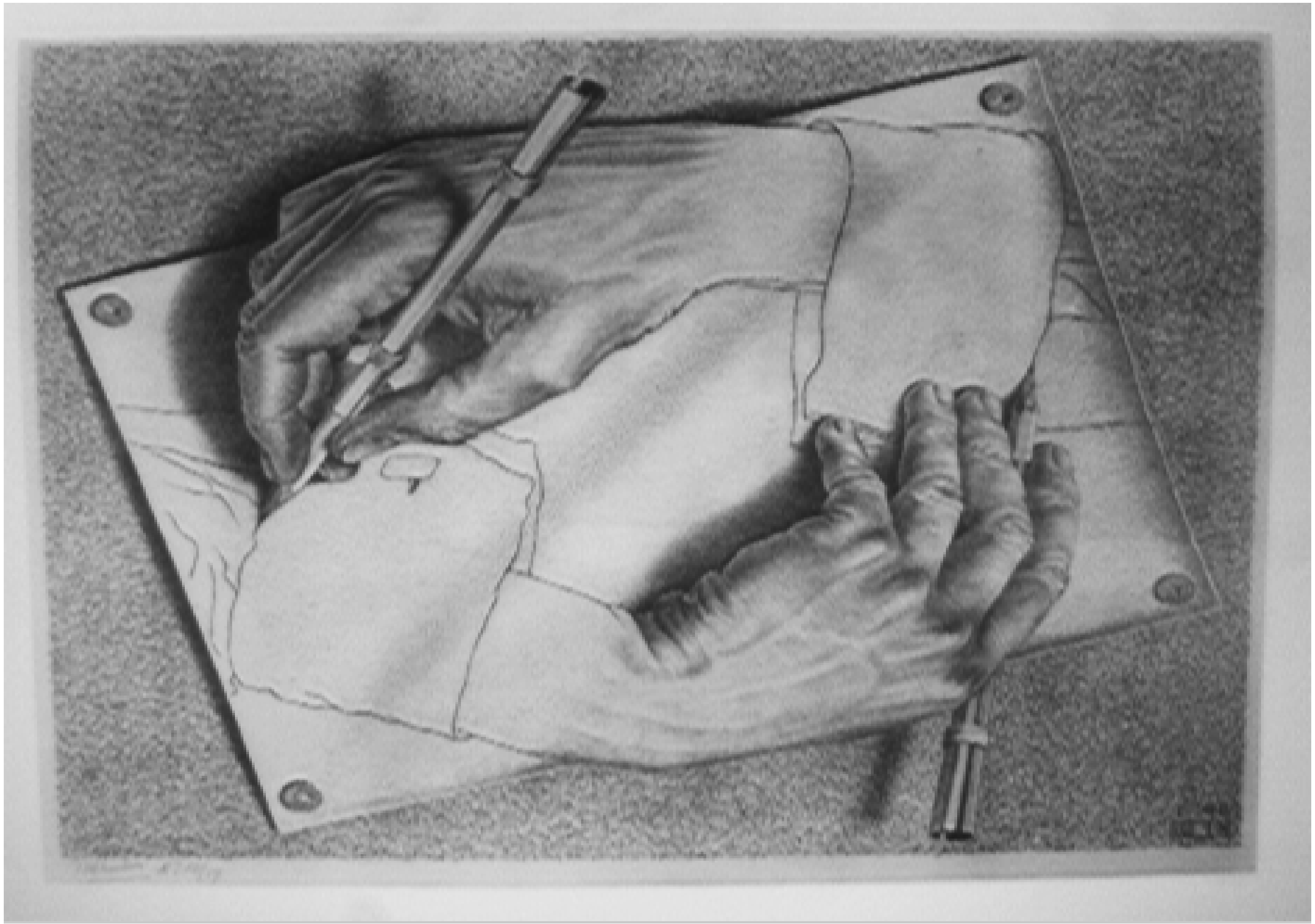
Drawing 2D Primitives

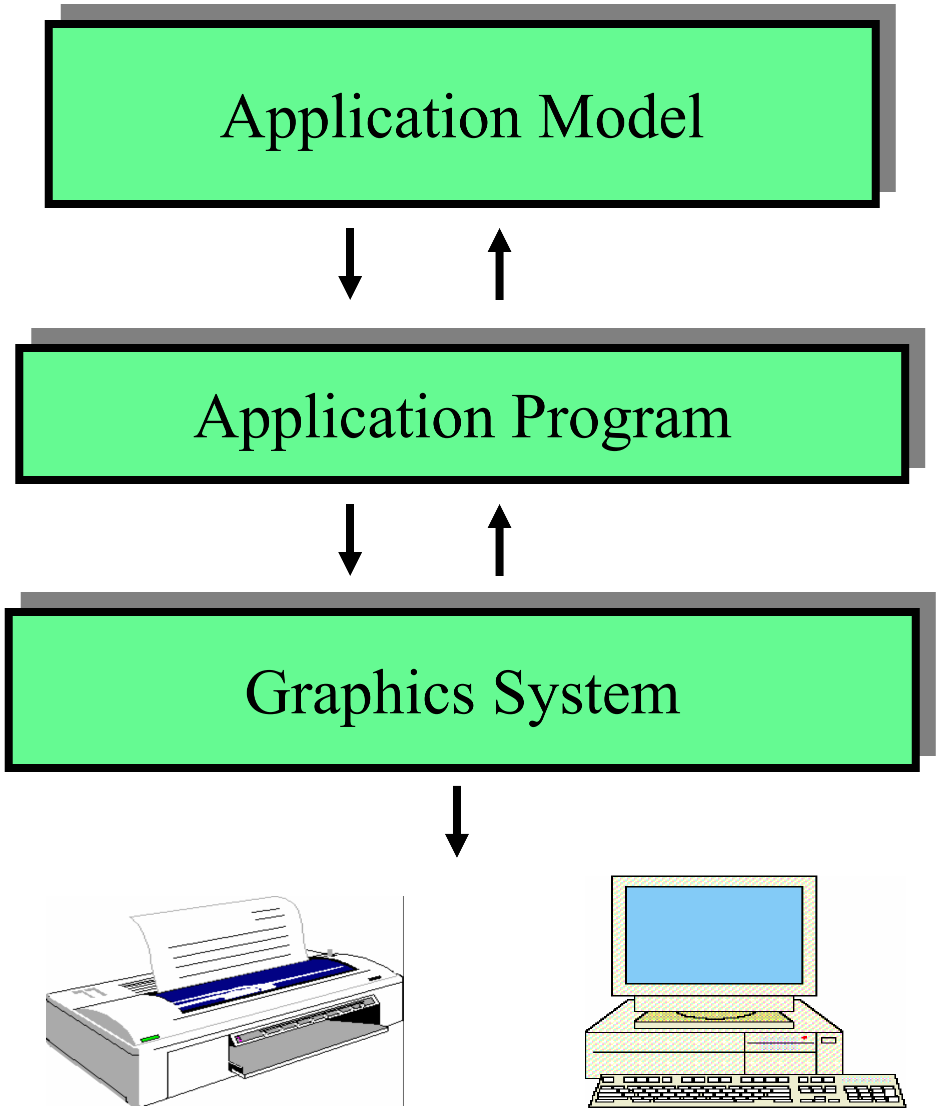
Foley & Van Dam, Chapter 3



Topics

- Interactive Graphic Systems
- Drawing lines
- Drawing circles
- Filling polygons

Interactive Graphic System



Interactive Graphic System

Application Model

–Represents data and objects to be displayed on the output device

Application Program

–Creates, stores into, and retrieves from the *application model*

–Handles user-inputs

–Sends output commands to the *graphics system*:

- *Which* geometric object to view (point, line, circle, polygon)
- *How* to view it (color, line-style, thickness, texture)

Graphics System

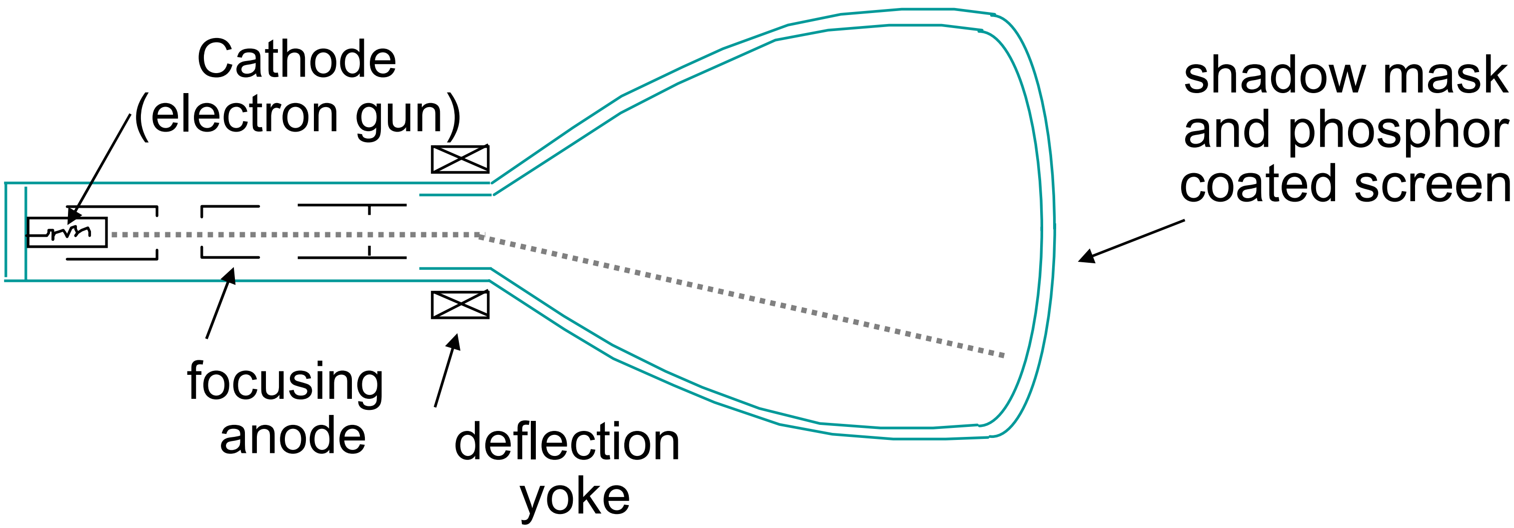
–Intermediates between the *application program* and the *interface hardware:*

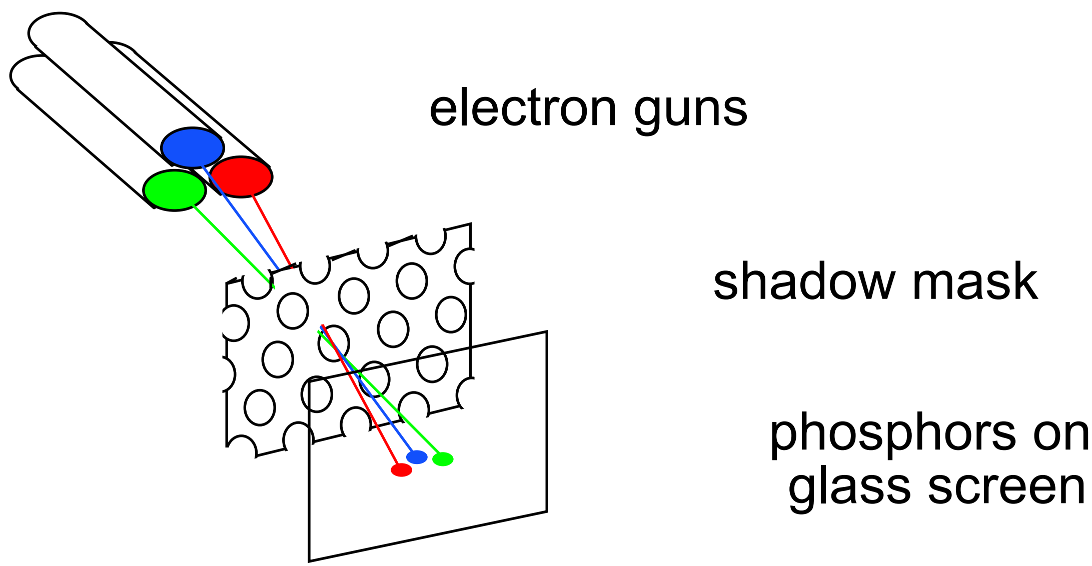
- •Output flow
- •Input flow

–Causes the application program to be *device-independent*.

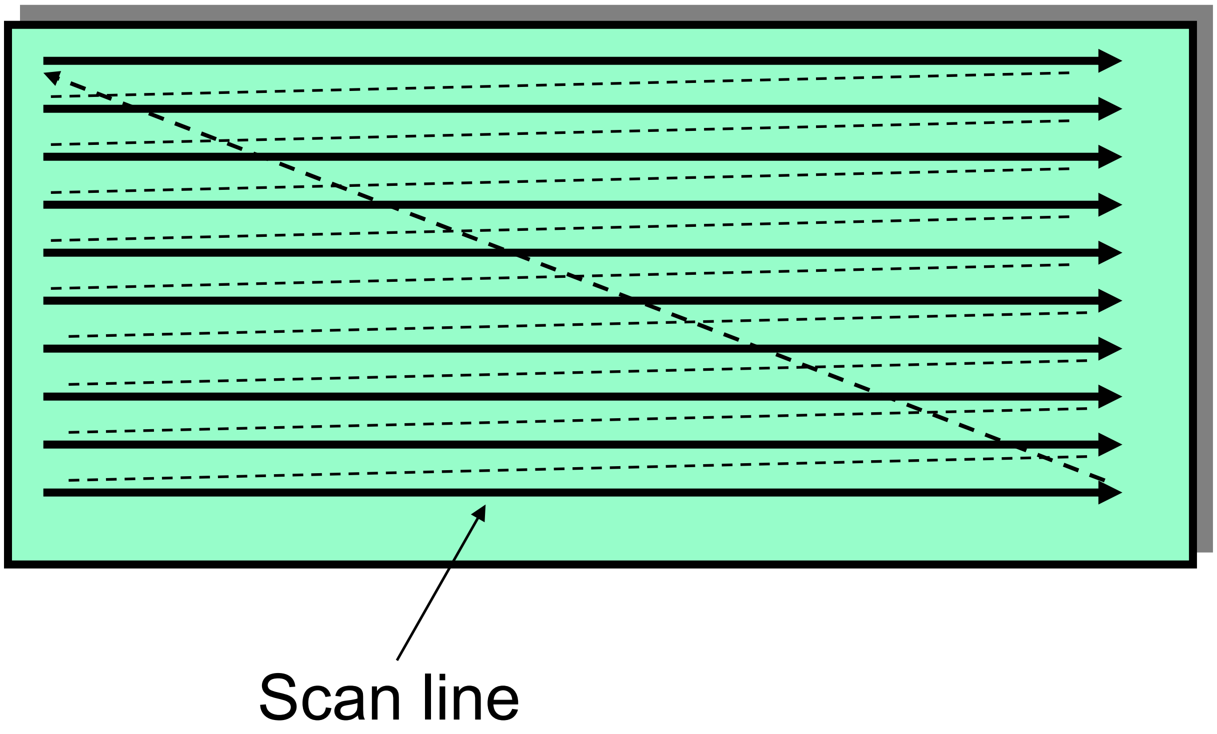
Display Hardware

CRT - Cathode Ray Tube





Raster Scan (CRT)



Display Hardware
FED - Field Emission Display

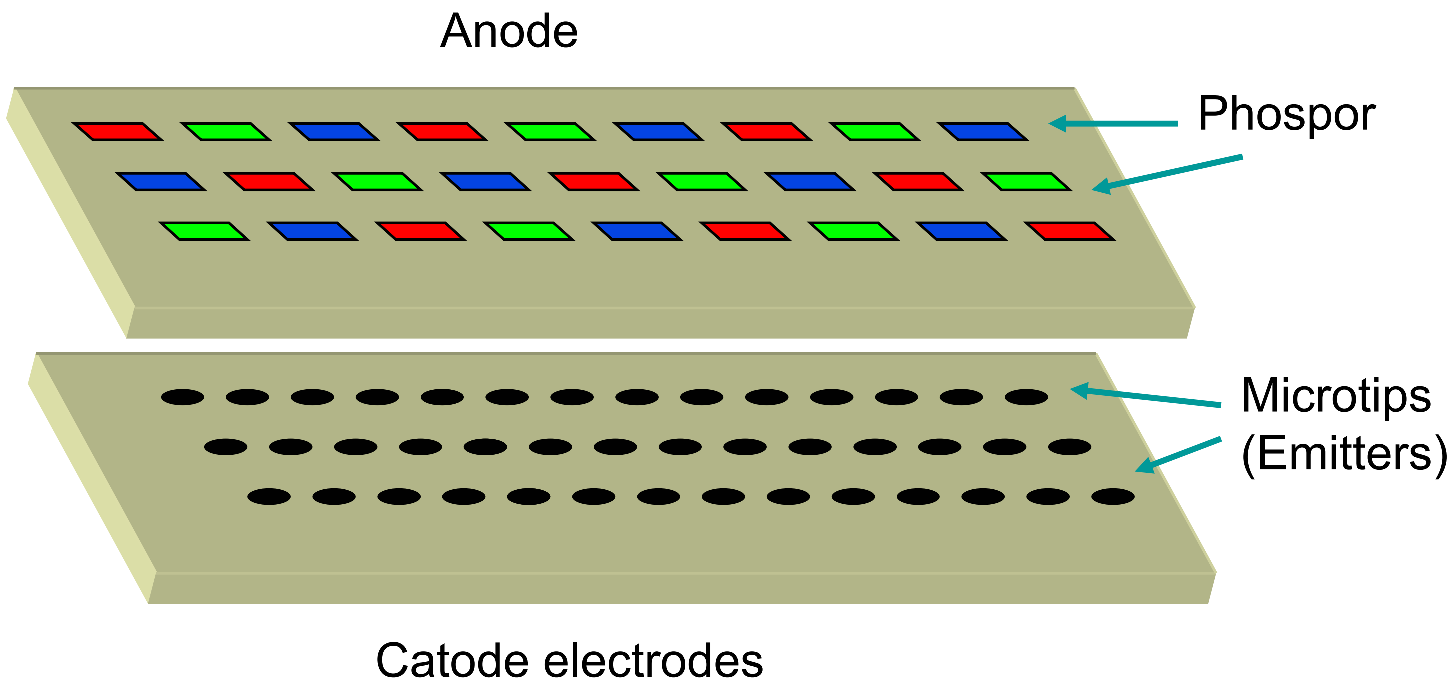
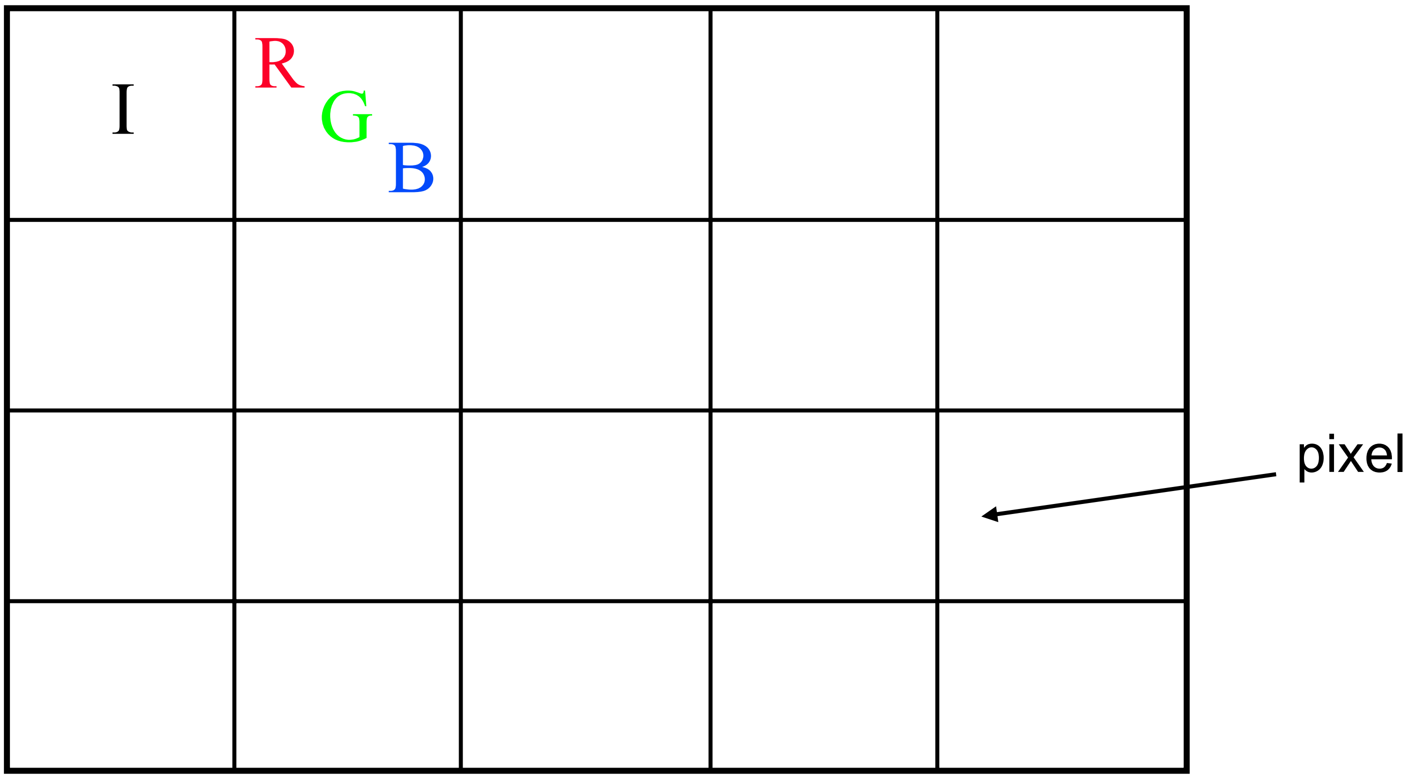
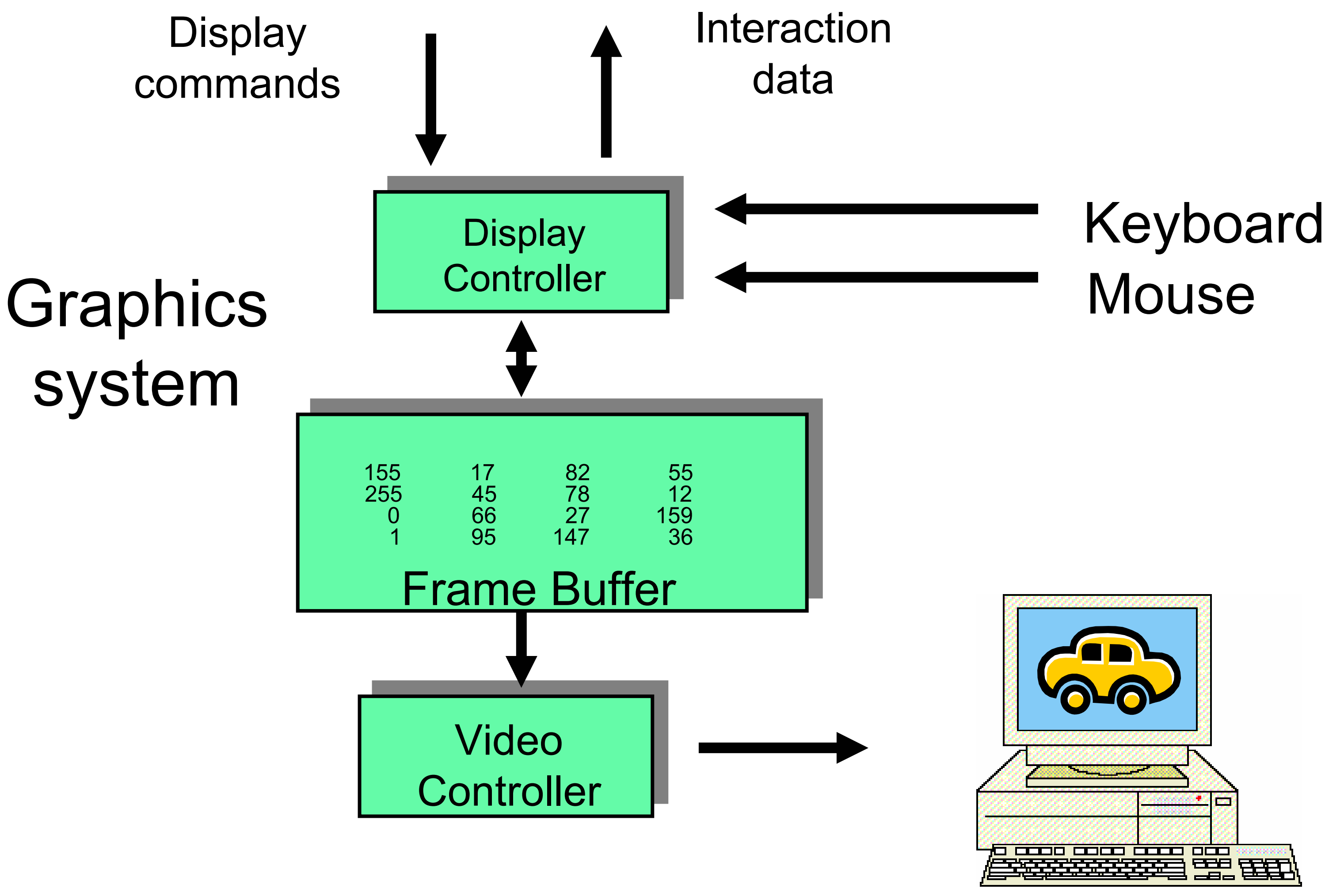


Image Representation in Raster Displays



Raster Display



Terminology

Pixel: Picture element.
- Smallest accessible element in picture
- Assume rectangular or circular shape

Aspect Ratio: Ratio between physical dimensions of a pixel (not necessarily 1)

Dynamic Range: The ratio between the minimal (not zero!) and the maximal light intensity a display pixel can emit

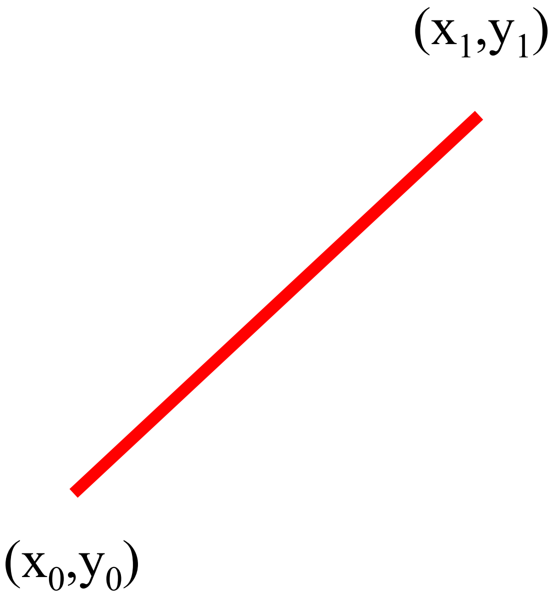
Resolution: The number of distinguishable rows and columns in the device. Measured in:
- Absolute values (1K x 1K) or,
- Density values (300 dpi [=dots per inch])

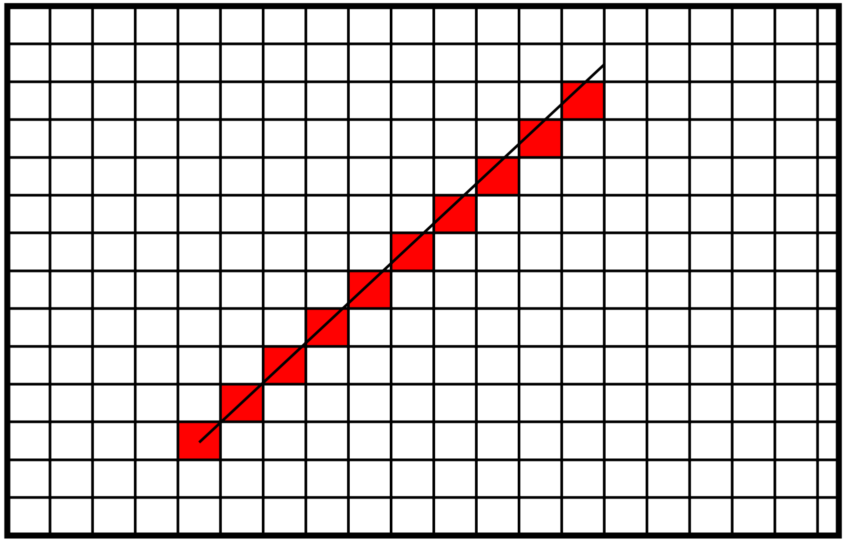
Screen Space: A discrete Cartesian coordinate system of the screen pixels

Object Space: The Cartesian coordinate system of the universe, in which the objects (to be displayed) are embedded

Scan Conversion

The conversion from a geometrical representation of an object to pixels in a raster display





Representations

Implicit formula:

Constraint(s) expressed as

$$f(x,y,..)=0$$

A k-dimensional surface embedded in n-dimensions

$$f_i(x_1,x_2,..,x_n)=0 \quad ; \quad i=1..n-k$$

Explicit formula:

For each x define y as $y=f(x)$

Good only for "functions".

Parametric formula:

Depending on free parameter(s)

$$x=f_x(t)$$
$$y=f_y(t)$$

For k-dimensional surface there are k free parameters

Line in 2 dimensions

- Implicit representation:

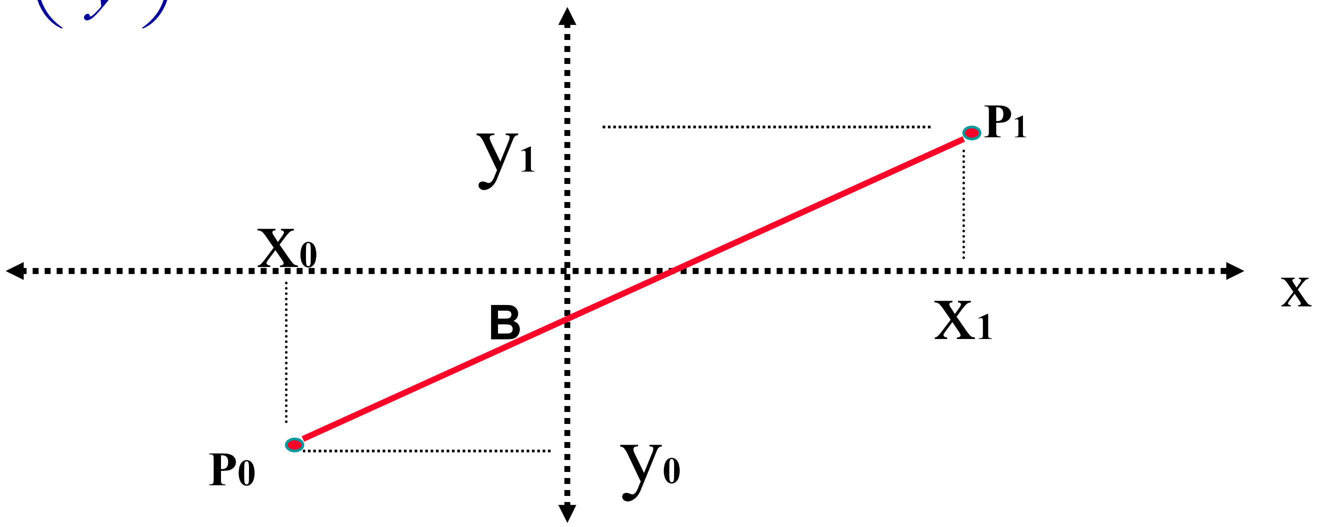
$$\alpha x + \beta y + \gamma = 0$$

- Explicit representation:

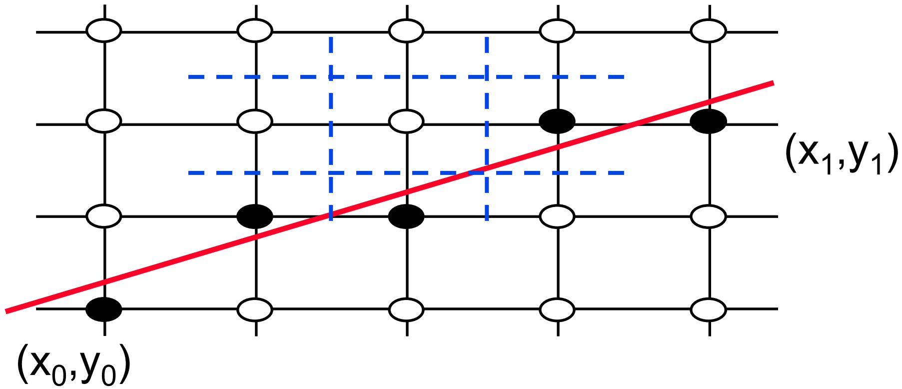
$$y = mx + B \qquad m = \frac{y_1 - y_0}{x_1 - x_0}$$

- Parametric representation:

$$P = \begin{pmatrix} x \\ y \end{pmatrix} \qquad P = P_0 + (P_1 - P_0)\,t \qquad t \in [0..1]$$



Scan Conversion - Lines



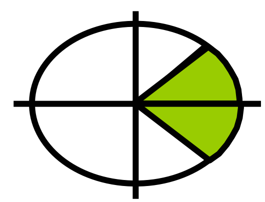
$$y = mx + B$$

$$\text{slope} = m = \frac{y_1 - y_0}{x_1 - x_0}$$

$$\text{offset} = B = y_1 - mx_1$$

Assume $|m| \leq 1$

Assume $x_0 \leq x_1$



<u>Basic Algorithm</u>

```
For x = x0 to x1
      y = mx + B
      PlotPixel(x,round(y))
end;
```

For each iteration: 1 float multiplication, 1 addition, 1 round

Scan Conversion - Lines

Incremental Algorithm

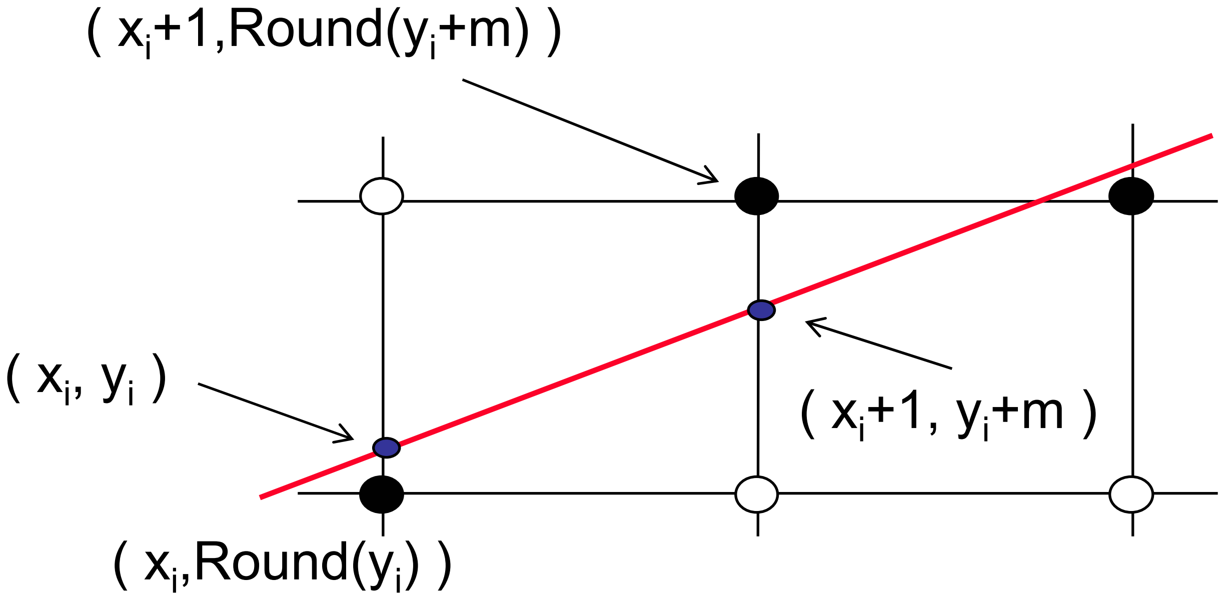
$$y_{i+1} = mx_{i+1} + B = m(x_i + \Delta x) + B = y_i + m\Delta x$$

if $\Delta x = 1$ then $\boxed{y_{i+1} = y_i + m}$

<u>Algorithm</u>

```
y=y0
For x = x0  to  x1
      PlotPixel(x,round(y))
      y =  y  +  m
end;
```

Scan Conversion - Lines



Pseudo Code for Basic Line Drawing

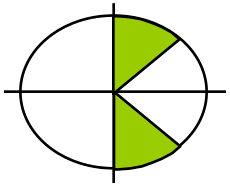
Assume $x_1 > x_0$ and line slope absolute value is ≤ 1

```
Line(x0,y0,x1,y1)
begin
      float dx, dy, x, y, slope;
      dx := x1-x0;
      dy := y1-y0;
      slope := dy/dx;
      y := y0;
      for x:=x0 to x1 do
      begin
            PlotPixel( x,Round(y) );
            y := y+slope;
      end;
end;
```

Basic Line Drawing

Symmetric Cases:

$|m| \geq 1$



$x = x_0$

For $y = y_0$ to y_1

PlotPixel(round(x),y)

$x = x + 1/m$

end;

For each iteration:

- 1 addition, 1 rounding

Drawbacks:

- Accumulated error
- Floating point arithmetic
- Round operations

Special Cases:

$m = \pm 1$ (diagonals)

$m = 0, \infty$ (horizontal, vertical)

Symmetric Cases:

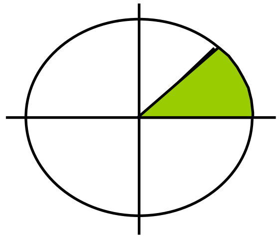
if $x_0 > x_1$ for $|m| \leq 1$ or $y_0 > y_1$ for $|m| \geq 1$

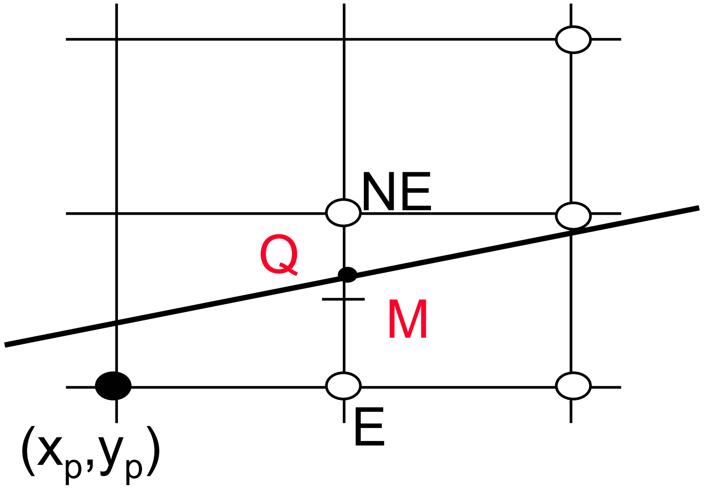
swap($(x_0,y_0),(x_1,y_1)$)

Midpoint (Bresenham) Line Drawing

Assumptions:

- $x_0 < x_1$, $y_0 < y_1$
- $0 < \text{slope} < 1$





Given (x_p,y_p), the next pixel is E = $(x_p +1,y_p)$ or NE = (x_p+1,y_p+1)

Bresenham: sign(M-Q) determines NE or E

$$M = (x_p +1,y_p +1/2)$$

Midpoint (Bresenham) Line Drawing

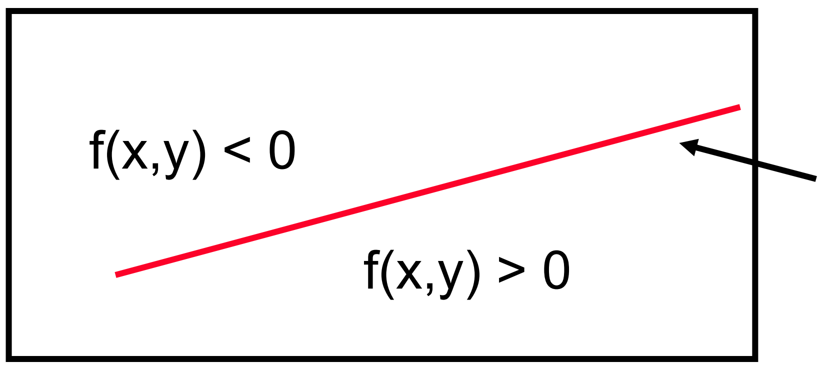
$$\boxed{y = \frac{dy}{dx}x + B}$$

Implicit form of a line:

$$\boxed{f(x,y) = ax + by + c = 0}$$

$$f(x,y) = dy\,x - dx\,y + B\,dx = 0$$

(a>0)



f(x,y) = 0

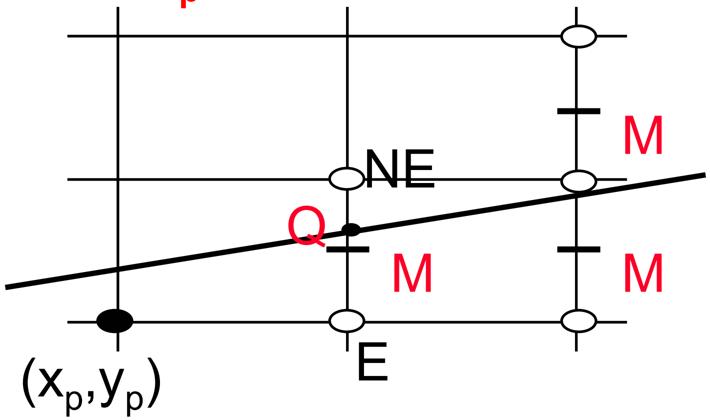
Decision Variable :

$$d = f(M) = f(x_p+1, y_p+1/2) = a(x_p+1) + b(y_p+1/2) + c$$

- choose NE if $d > 0$
- choose E if $d \leq 0$

Midpoint (Bresenham) Line Drawing

What happens at $x_p + 2$?



If E was chosen at $x_p + 1$

$$M = (x_p + 2, y_p + 1/2)$$

$$d_{new} = f(x_p + 2, y_p + 1/2) = a(x_p + 2) + b(y_p + 1/2) + c$$

$$d_{old} = f(x_p + 1, y_p + 1/2) = a(x_p + 1) + b(y_p + 1/2) + c$$

$$d_{new} = d_{old} + a = d_{old} + dy$$

$$\boxed{d_{new} = d_{old} + \Delta_E}$$

If NE was chosen at $x_p + 1$

$$M = (x_p + 2, y_p + 3/2)$$

$$d_{new} = f(x_p + 2, y_p + 3/2) = a(x_p + 2) + b(y_p + 3/2) + c$$

$$d_{old} = f(x_p + 1, y_p + 1/2) = a(x_p + 1) + b(y_p + 1/2) + c$$

$$d_{new} = d_{old} + a + b = d_{old} + dy - dx$$

$$\boxed{d_{new} = d_{old} + \Delta_{NE}}$$

Midpoint (Bresenham) Line Drawing

Initialization:

First point = (x_0,y_0), first MidPoint = $(x_0+1,y_0+1/2)$

$$
\begin{aligned}
d_{start} = f(x_0+1,y_0+1/2) &= a(x_0+1) + b(y_0+1/2) + c \\
&= ax_0 + by_0 + c + a + b/2 \\
&= f(x_0,y_0) + a + b/2 = a + b/2
\end{aligned}
$$

$$\boxed{d_{start} = dy - dx/2}$$

Enhancement:

To eliminate fractions, define:

$$f(x,y) = 2(ax + by + c) = 0$$

$$\boxed{d_{start} = 2dy - dx}$$

$$\Delta_E = 2dy$$

$$\Delta_{NE} = 2(dy-dx)$$

Midpoint (Bresenham) Line Drawing

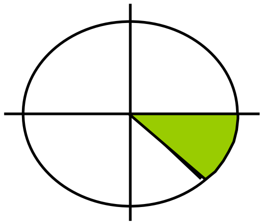
- The sign of $f(x_0+1, y_0+1/2)$ indicates whether to move East or North-*East*
- At the beginning $d=f(x_0+1, y_0+1/2)=2dy-dx$
- The increment in d (after this step) is:

  If we moved East: $\Delta_E=2dy$

  If we moved North-East: $\Delta_{NE}=2dy-2dx$

Comments:

- Integer arithmetic (dx and dy are integers)
- One addition for each iteration
- No accumulated errors
- By symmetry, we deal with $0>slope>-1$



Pseudo Code for Midpoint Line Drawing

```
Line(x_0,y_0,x_1,y_1)
begin
    int dx, dy, x, y, d, Δ_E, Δ_NE ;
    x:= x_0;    y=y_0;
    dx := x_1-x_0;    dy := y_1-y_0;
    d := 2*dy-dx;
    Δ_E := 2*dy;    Δ_NE := 2*(dy-dx);
    PlotPixel(x,y);
    while(x < x_1) do
        if (d < 0) then
          d:=d+ Δ_E;
          x:=x+1;
        end;
        else
          d:=d+ Δ_NE;
          x:=x+1;
          y:=y+1;
        end;
        PlotPixel(x,y);
    end;
end;
```

Assume $x_1 > x_0$ and $0 <$ slope ≤ 1

Scan Conversion - Circles

Implicit representation (centered at the origin, radius R):

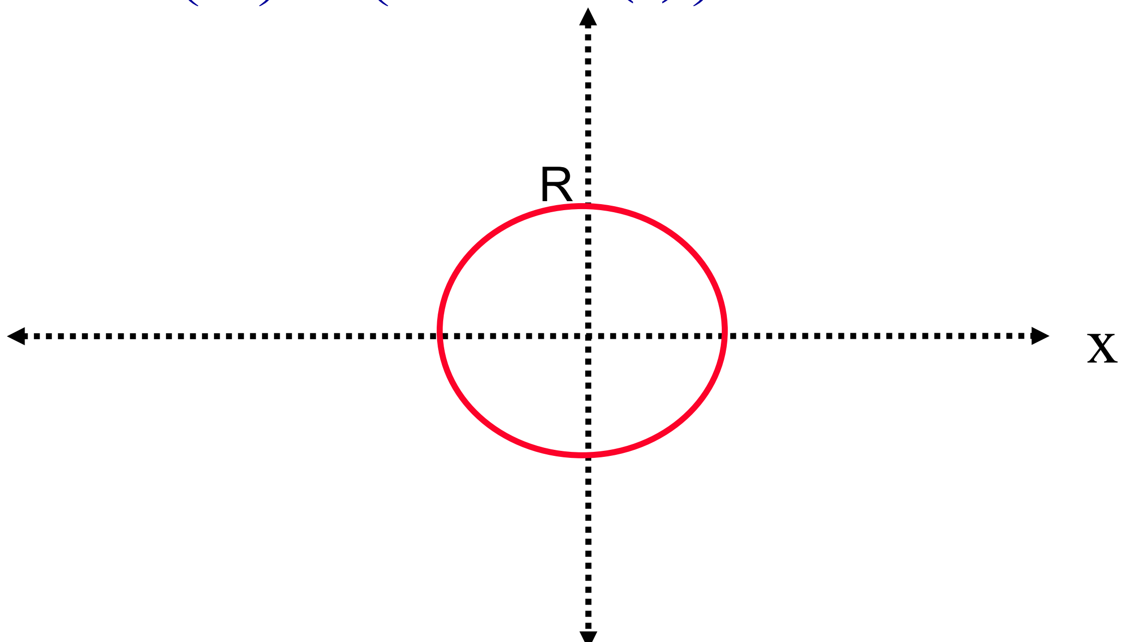
$$x^2 + y^2 - R^2 = 0$$

Explicit representation:

$$y = \pm\sqrt{R^2 - x^2}$$

Parametric representation:

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} R\cos(t) \\ R\sin(t) \end{pmatrix} \qquad t \in [0..2\pi]$$



Scan Conversion - Circles

Basic Algorithm

```
For x = -R  to  R
      y =  sqrt(R²-x²)
      PlotPixel(x,round(y))
      PlotPixel(x,-round(y))
 end;
```

Comments:

- Square-root operations are expensive
- Floating point arithmetic
- Large gap for x values close to R

Scan Conversion - Circles

Exploiting Eight-Way Symmetry

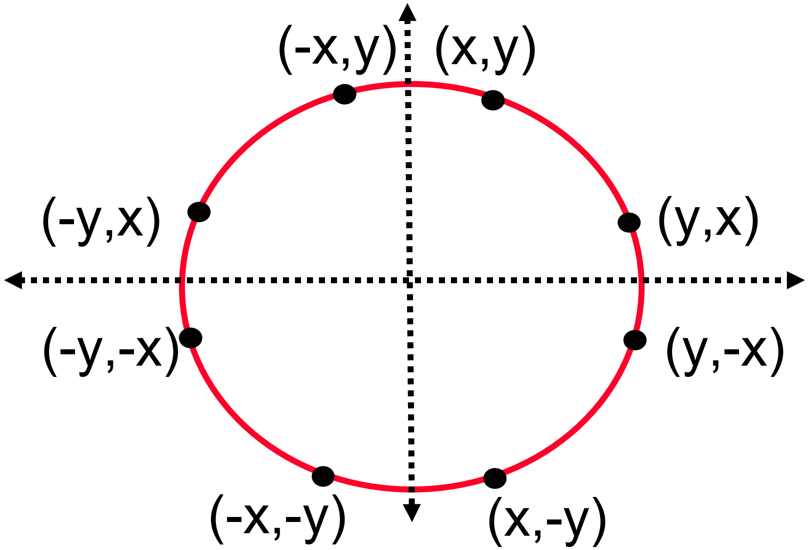
For a circle centered at the origin:

If (x,y) is on the circle then

(y,x) (y,-x) (x,-y) (-x,-y) (-y,-x) (-y,x) (-x,y)

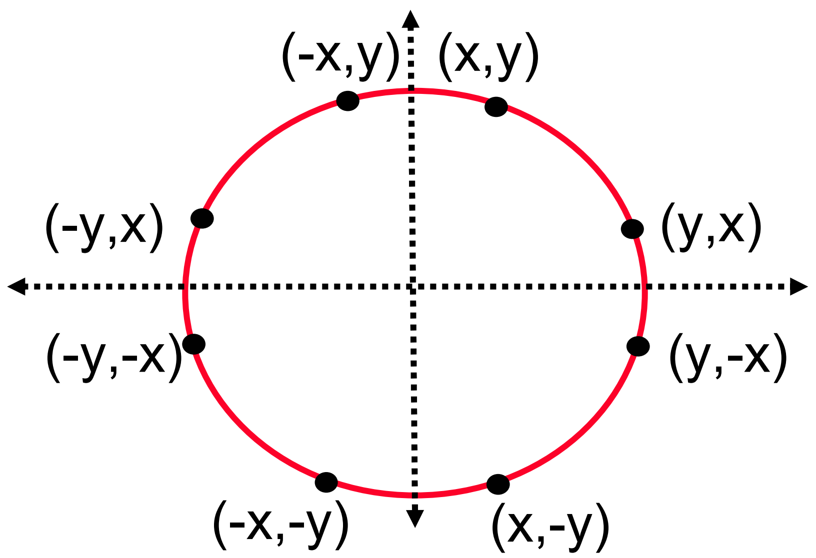
are on the circle as well.

Therefore we need to compute only one octant (45°) segment.



Scan Conversion - Circles

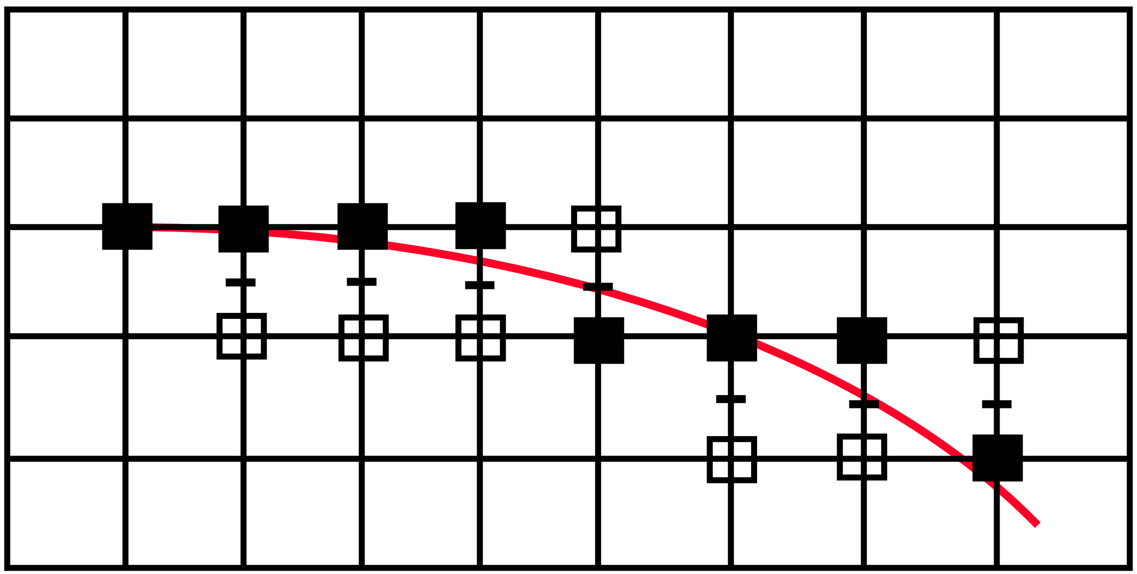
```
CirclePoints(x, y)
begin
    PlotPixel(x,y);
    PlotPixel(y,x);
    PlotPixel(y,-x);
    PlotPixel(x,-y);
    PlotPixel(-x,-y);
    PlotPixel(-y,-x);
    PlotPixel(-y,x);
    PlotPixel(-x,y);
end;
```



Circle Midpoint (for one octant)

(The circle is located at (0,0) with radius R)

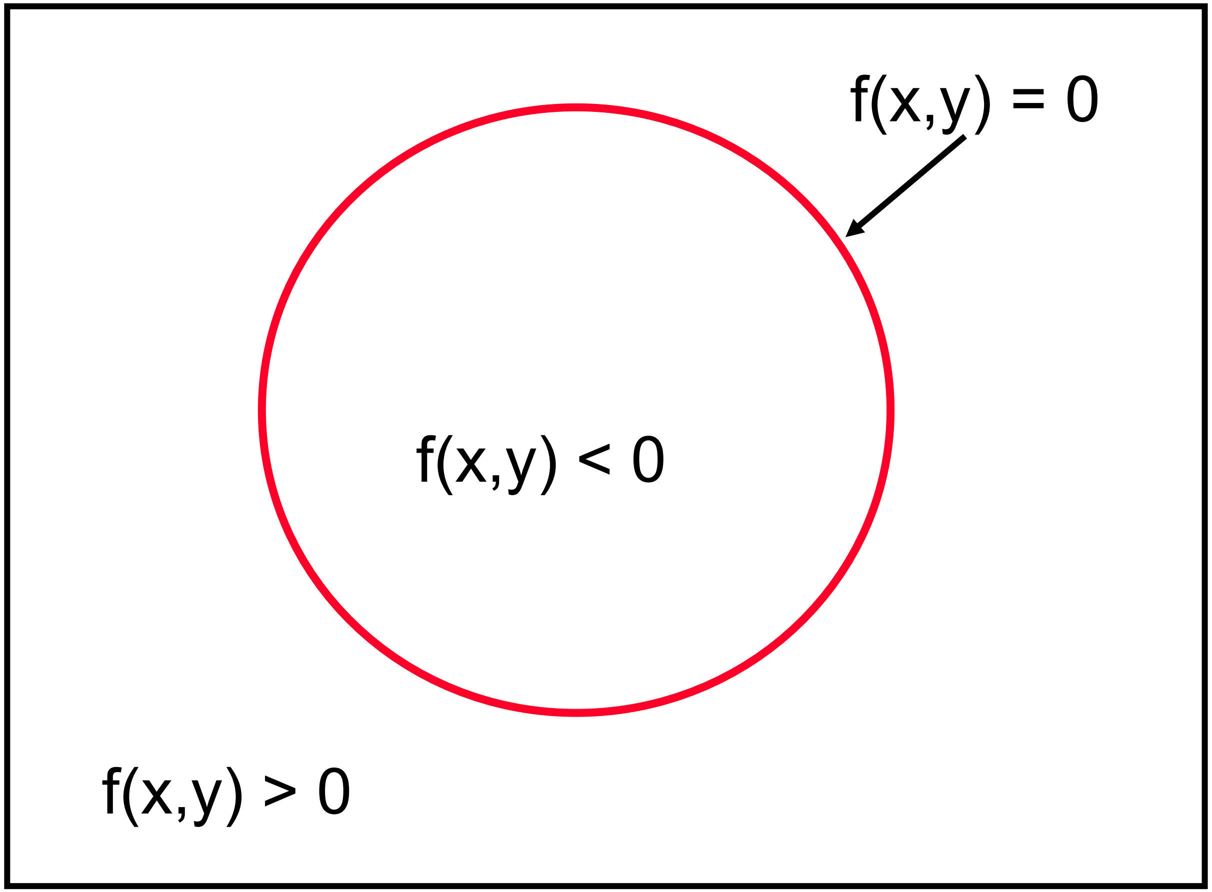
- We start from $(x_0,y_0)=(0,R)$
- One can move either East or South-East
- Again, d(x,y) will be a threshold criteria at the midpoint



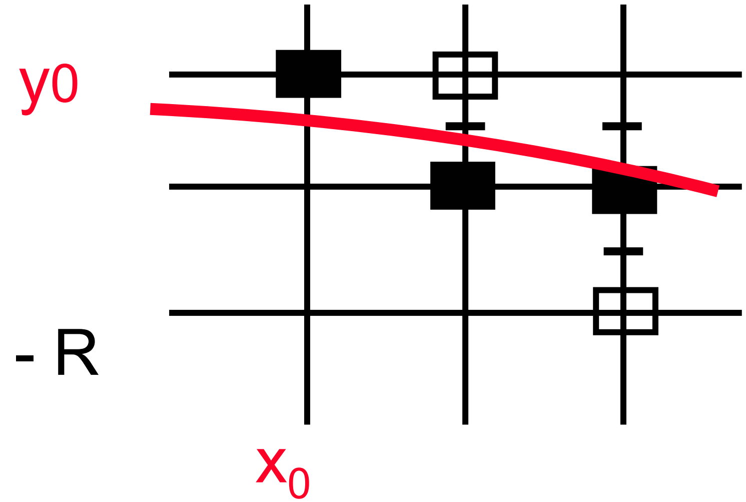
Circle Midpoint (for one octant)

Threshold Criteria:

$$d(x,y)=f(x,y) = x^2 + y^2 - R^2 = 0$$



Circle Midpoint (for one octant)



- At the beginning

$$d_{start} = d(x_0+1, y_0-1/2)$$
$$= d(1, R-1/2) = 5/4 - R$$

- If d<0 we move East:

$$\Delta_E = d(x_0+2, y_0-1/2) - d(x_0+1, y_0-1/2) = 2x_0+3$$

- if d>0 we move South-East:

$$\Delta_{SE} = d(x_0+2, y_0-3/2) - d(x_0+1, y_0-1/2) = 2(x_0-y_0)+5$$

- Δ_E and Δ_{SE} are not constant anymore
- Since d is incremented by integer values, we can use

$$d_{start} = 1-R$$

yielding an integer algorithm. This has no affect on the threshold criteria.

Pseudo Code for Circle Midpoint

```
MidpointCircle (R)
begin
    int x, y, d;
    x := 0;
    y :=R;
    d := 5.0/4.0-R;
    CirclePoints(x,y);
    while ( y>x ) do
        if ( d<0 ) then          /* East */
            d := d+2x+3;
            x := x+1;
        end;
        else                     /* South East */
            d := d+2(x-y)+5;
            x := x+1;
            y := y-1;
        end;
        CirclePoints( x,y );     /* Mirror to create the other seven octants */
end;
```

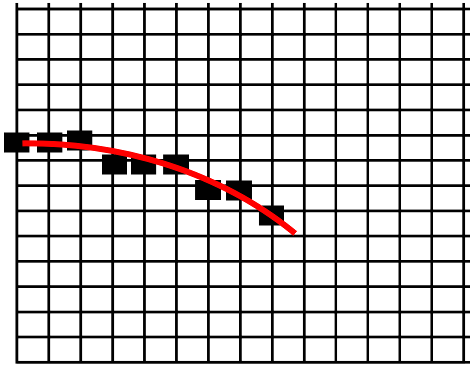
Pseudo Code for Circle Midpoint

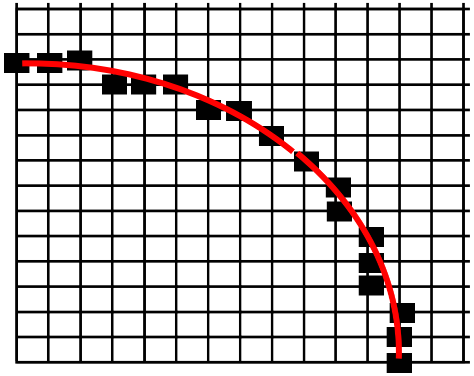
```
MidpointCircle (R)
begin
    int x, y, d;
    x := 0;
    y :=R;
    d := 1-R;                   /* originally  d := 5.0/4.0 - R  */
    CirclePoints(x,y);
    while ( y>x ) do
        if ( d<0 ) then         /* East */
            d := d+2x+3;        /* Multiplication! */
            x := x+1;
       end;
       else                     /* South East */
           d := d+2(x-y)+5;     /* Multiplication! */
           x := x+1;
           y := y-1;
      end;
      CirclePoints( x,y );      /* Mirror to create the other seven octants */
end;
```

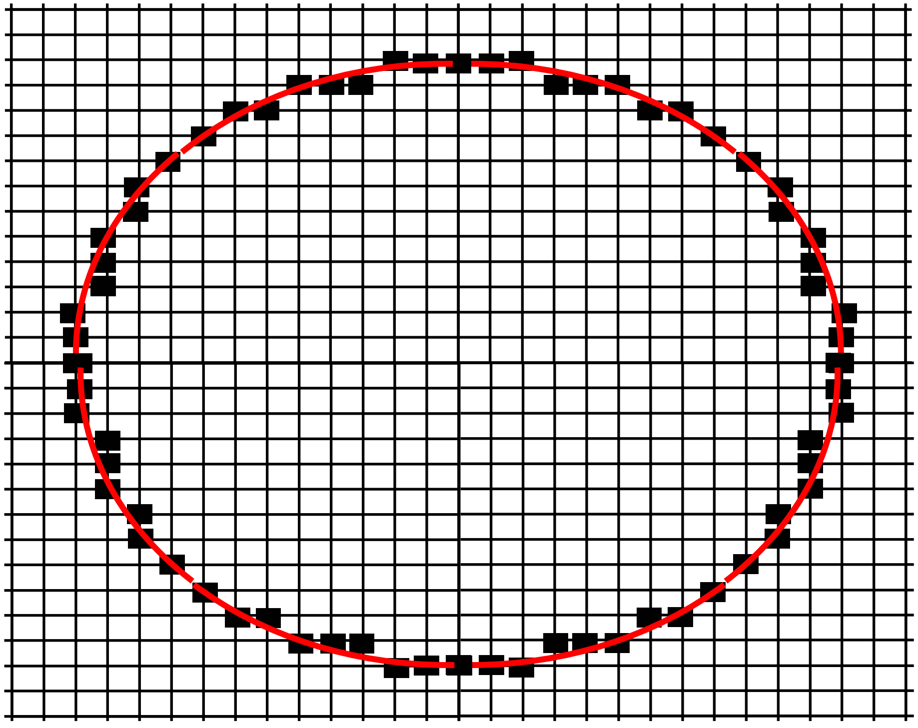
Pseudo Code for Circle Midpoint

```
MidpointCircle (R)
begin
    int x, y, d;
    x := 0;
    y :=R;
    d := 1-R;                    /* originally  d := 5.0/4.0 - R  */
    ΔE = 3;
    ΔSE = -2R+5;
    CirclePoints(x,y);
    while ( y>x ) do
        if ( d<0 ) then          /* East */
            d := d+ΔE;           /* See Foley & van Dam pg. 87 */
            ΔE := ΔE+2;
            ΔSE := ΔSE+2;
            x := x+1;
        end;
        else                     /* South East */
            d := d+ΔSE;          /* See Foley & van Dam pg. 87 */
            ΔE := ΔE+2;
            ΔSE := ΔSE+4;
            x := x+1;
            y := y-1;
        end;
        CirclePoints( x,y );     /* Mirror to create the other seven octants */
end;
```

Circle Midpoint







Polygon Fill

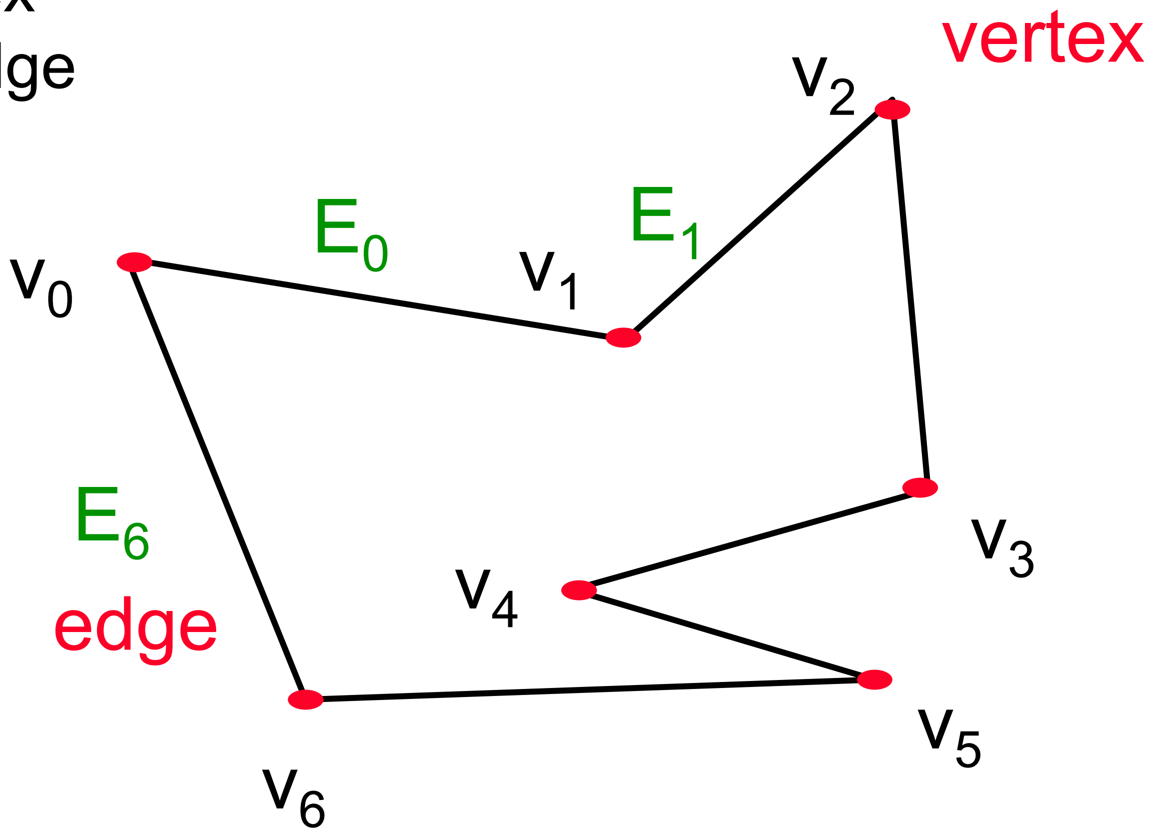
Representation:

Polygon = V_0, V_1, V_2, .. V_n

$V_i=(x_i,y_i)$ - vertex

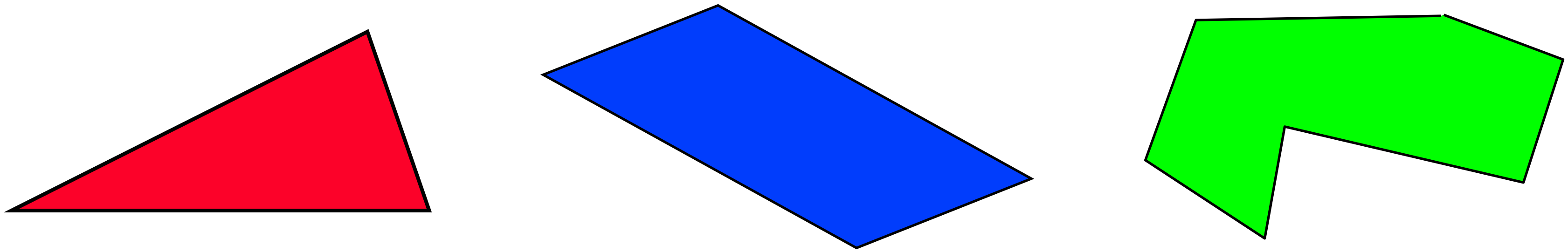
$E_i=(v_i,v_i+1)$ - edge

$E_n=(v_n,v_0)$

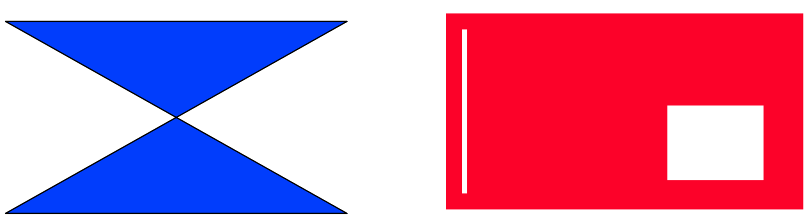


Scan Conversion - Polygon Fill

Problem: Given a closed 2D polygon, fill its interior with a specified color, on a graphics display



Assumption: Polygon is simple, i.e. no self intersections, and simply connected, i.e. without holes

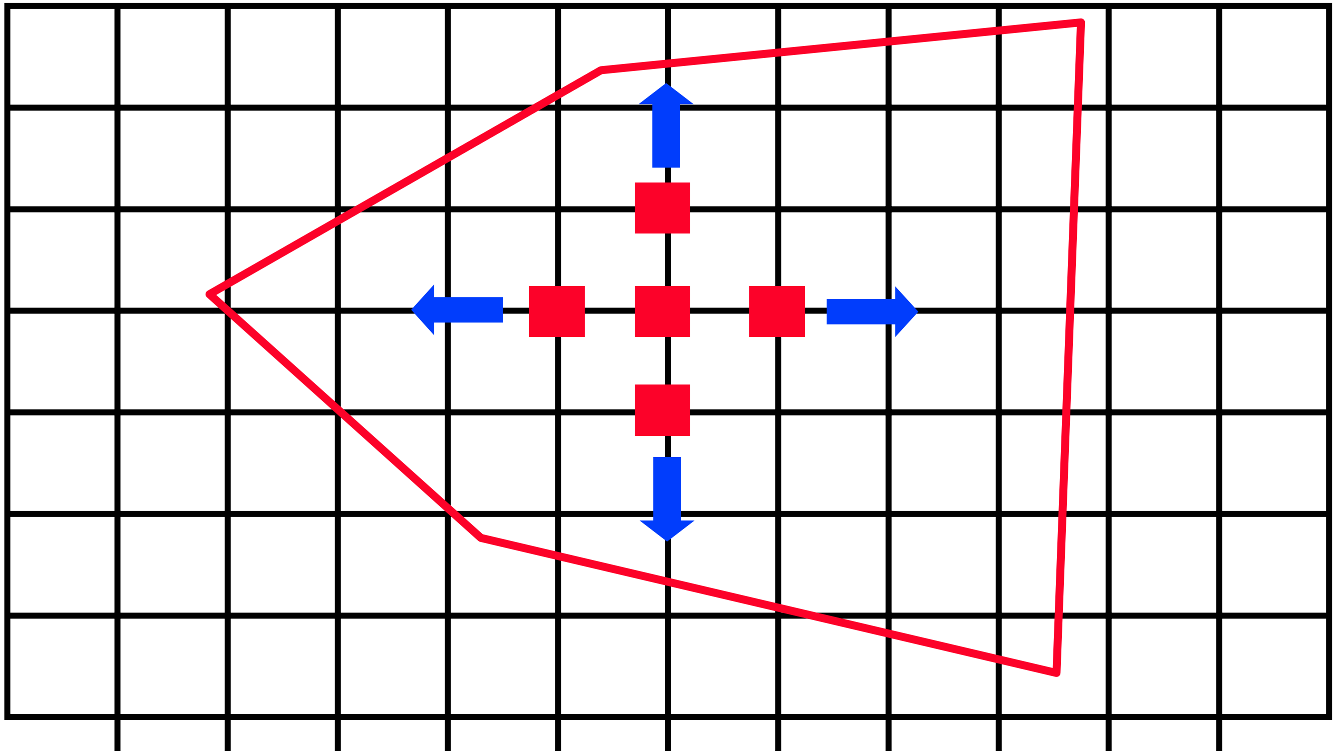


Solutions:

- Flood fill
- Scan Conversion

Flood Fill Algorithm

- Let P be a polygon with n vertices, v_0 .. v_{n-1}
- Denote $v_n=v_0$
- Let c be a color to paint P
- Let p=(x,y) be a point in P

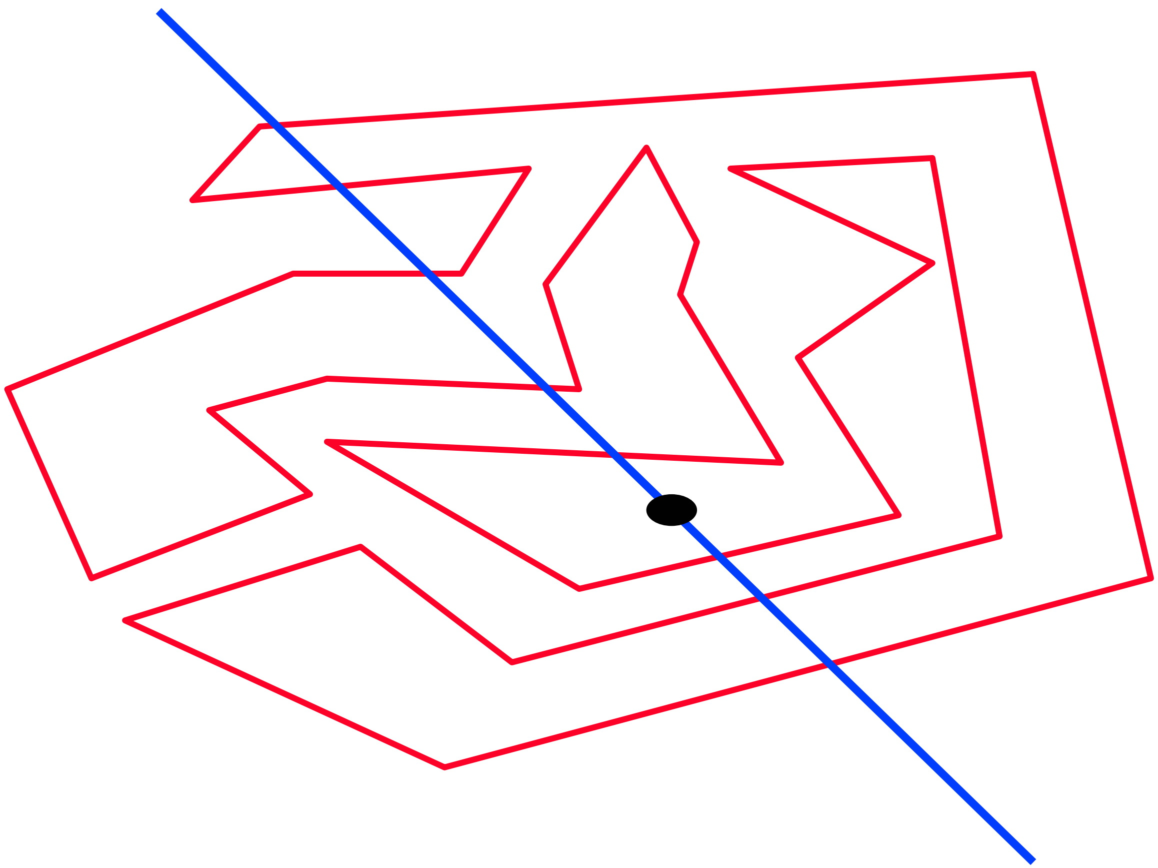


Flood Fill Algorithm

```
FloodFill(P,x,y,c)
if  (OnBoundary(x,y,P) or Colored (x,y,c))
   then return;
else begin
   PlotPixel(x,y,c);
   FloodFill(P,x+1,y,c);
   FloodFill(P,x,y+1,c);
   FloodFill(P,x,y-1,c);
   FloodFill(P,x-1,y,c);
end;
```

Slow algorithm due to recursion, needs initial point

Fill Polygon



Question: How do we know if a given point is inside or outside a polygon?

Scan Conversion – Basic Algorithm

- Let P be a polygon with n vertices, v_0 .. v_{n-1}
- Denote $v_n = v_0$
- Let c be a color to paint P

ScanConvert (P,c)

For $j:=0$ *to ScreenYMax do*

  I *:= points of intersection of edges*

    from P with line $y=j$*;*

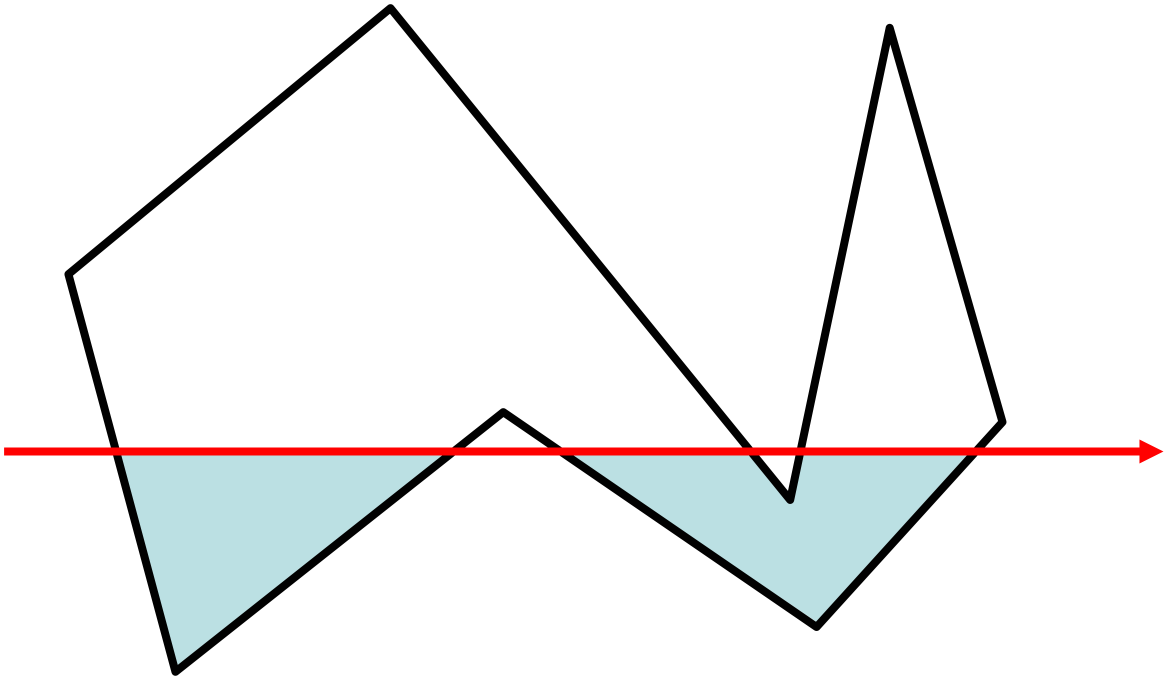
  Sort I *in increasing* x *order and fill*

    with color c *alternating segments;*

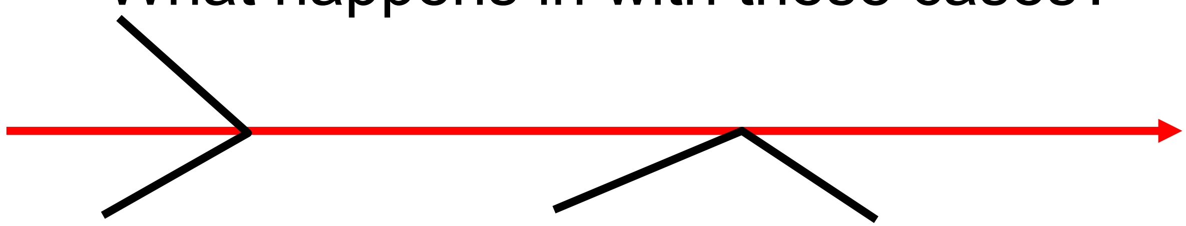
end;

Question: How do we find the intersecting edges?

Scan Conversion – Fill Polygon

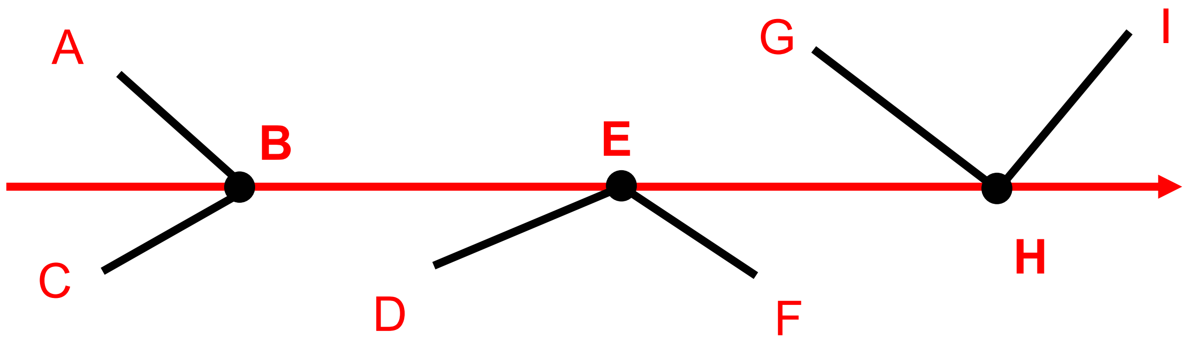


What happens in with these cases?



Scan Conversion – Fill Polygon

Intersections at pixel coordinates



Rule: In the odd/even count, we count y_{min} vertices of an edge, but not y_{max} vertices

Vertex B is counted once because y_{min} of (A,B)
Vertex E is not counted because y_{max} of both (D, E) and (E, F)
Vertex H is counted twice because y_{min} of both (G, H) and (H, I)

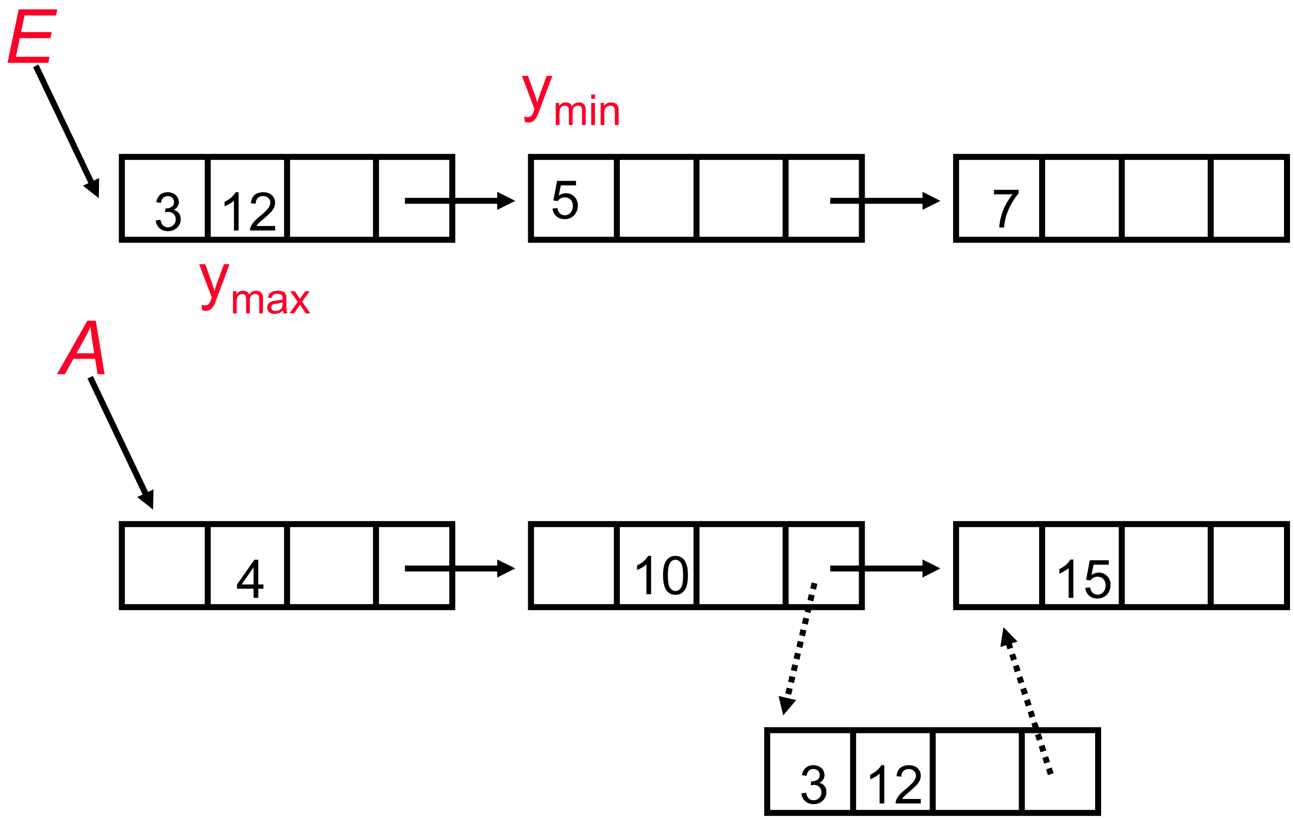
Fill Polygon – Optimized Algorithm

Uses a list of “active” edges A (edges currently intersecting the scan line)

```
ScanConvert(P,c)
Sort all edges E={Ej} in increasing MinY(Ej ) order.
A := ∅;
For  k :=0  to ScreenYMax do
    For each  Ej ∈ E,
        if  MinY(Ej)≤ k   A := A ∪ Ej ;  E=E- Ej
    For each  Ej ∈ A,
       if  MaxY(Ej)≤ k   A := A – Ej
    I:=Points of  intersection of members
       from A with line y=k;
    Sort I in increasing x order and draw
      with color c alternating segments;
end;
```

Fill Polygon – Optimized Algorithm

Implementation with linked lists



Flood Fill vs. Scan Conversion

Flood Fill	Scan Conversion
•Very simple. •Requires a seed point •Requires large stack size •Common in paint packages	•More complex •No seed point is required •Requires small stack size •Used in image rendering