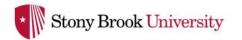
EEO 306

Faculty: Vibha Mane



Name:	
Student ID:	

## Sample Final Total Score: 36 points All problems carry equal weight of 6 points each Please show your work and justify your answers

1. In a game of Blackjack, the tens and face cards (10's, Jacks, Queens and Kings) count as 10 points, and Aces count as either 1 or 11 points. A blackjack occurs if the sum of the two cards is 21 (counting Ace as 11 points). A player is dealt two cards. What is the probability that the player has Blackjack?

- 2. Let X be a Poisson random variable with parameter  $\lambda$ .
  - (a) What is the probability that X is even?
  - (b) Simplify the above expression (X is even), by utilizing the following expansions for  $e^{\lambda}$  and  $e^{-\lambda}$ :

$$e^{\lambda} = 1 + \lambda + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} + \frac{\lambda^4}{4!} \dots$$

and

$$e^{-\lambda} = 1 - \lambda + \frac{\lambda^2}{2!} - \frac{\lambda^3}{3!} + \frac{\lambda^4}{4!} \dots$$

3. A plane is missing, and it is assumed that it is equally likely to have gone down in any of three possible regions. Let  $1-\alpha_i$  denote the probability that the plane will be found upon a search of the *i*th region when the plane is, in fact, in that region, i=1,2,3. (The constants  $\alpha_i$  are called overlook probabilities because they represent the probabilities of overlooking the plane.) What is the conditional probability that the plane is in the *i*th region, given that a search of region 1 is unsuccessful?

4. Random variables  $X_1$  and  $X_2$  have the joint PMF  $p_{X_1,X_2}(x_1,x_2)$  given by the following table:

$p_{X_1,X_2}(x_1,x_2)$	$x_2 = -1$	$x_2 = 0$	$x_2 = 1$
$x_1 = -1$	0	0	1/3
$x_1 = 0$	0	1/3	0
$x_1 = 1$	1/3	0	0

- (a) Compute the marginal PMF  $p_{X_1}(x_1)$ .
- (b) Compute the marginal PMF  $p_{X_2}(x_2)$ .
- (c) Compute the probability  $P(X_1 < X_2)$ .
- (d) Are  $X_1$  and  $X_2$  independent?

- 5. (a) An Urn contains N white balls and M black balls. Draw a ball **with replacement** until a black ball is selected. What is the probability that **exactly** k draws are needed?
  - (b) What is the expected number of draws needed to observe a black ball?

6. Sketch the ensemble, that is, realizations of the random process

$$X(t) = A\cos(2\pi \mathbf{f}t),$$

where f is a uniform random variable  $\mathcal{U}(10,20)$ . That is, f is uniformly distributed in the range [10, 20] Hz and A=5 is a constant.

Blank Sheet