$$\boxed{\begin{aligned} V_{TN} &= \frac{|Q'_{SD}(max)|}{C_{ox}} + V_{FB} + 2|\phi_{Fp}| \\ V_{TN} &= V_{FB} + 2|\phi_{Fp}| + \gamma\sqrt{2|\phi_{Fp}|} \end{aligned}}$$

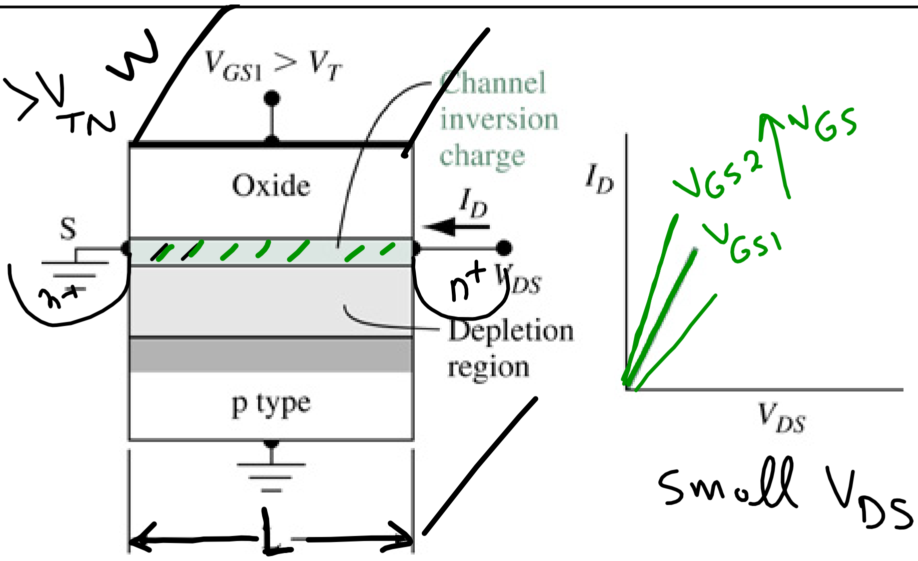
$$V_{FB} = \phi_{ms} - \frac{Q'_{ss}}{C_{ox}} \quad ; \quad C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} \quad (F/cm^2)$$

$$Q'_{SD}(max) = -eN_A x_{dT} \quad ; \quad x_{dT} = \left(\frac{4\epsilon_s|\phi_{Fp}|}{eN_A}\right)^{1/2}$$

$$\phi_{Fp} = -\frac{kT}{e}\ln\left(\frac{N_A}{n_i}\right)$$

$$\frac{|Q'_{SD}(max)|}{C_{ox}} = \frac{eN_A\left(\frac{2\cdot 2\epsilon_s |\phi_{Fp}|}{eN_A}\right)^{1/2}}{C_{ox}} = \underbrace{\frac{\sqrt{2e\epsilon_s N_A}}{C_{ox}}}_{\gamma}\sqrt{2|\phi_{Fp}|} = \gamma\sqrt{2|\phi_{Fp}|}$$

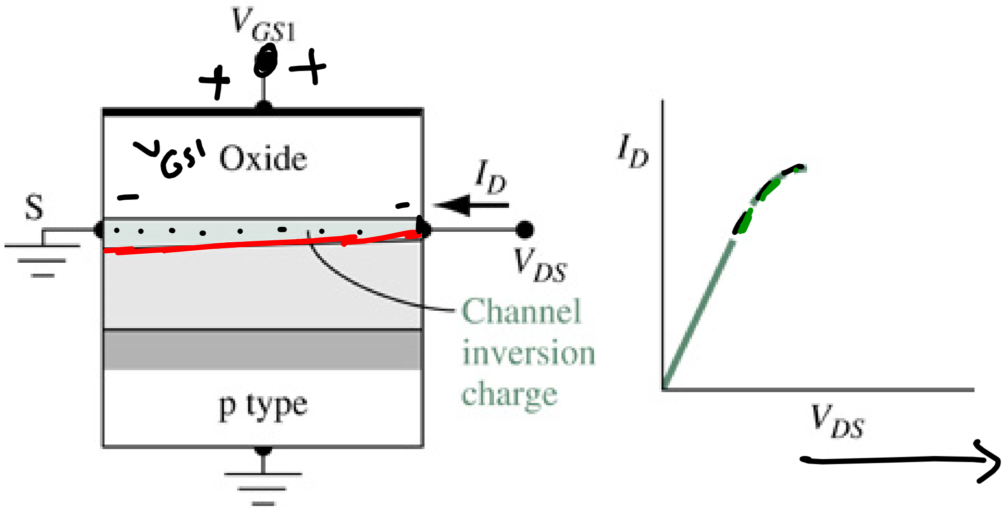
$V_{GS} = V_{GS1} > V_{TN}$



(a)

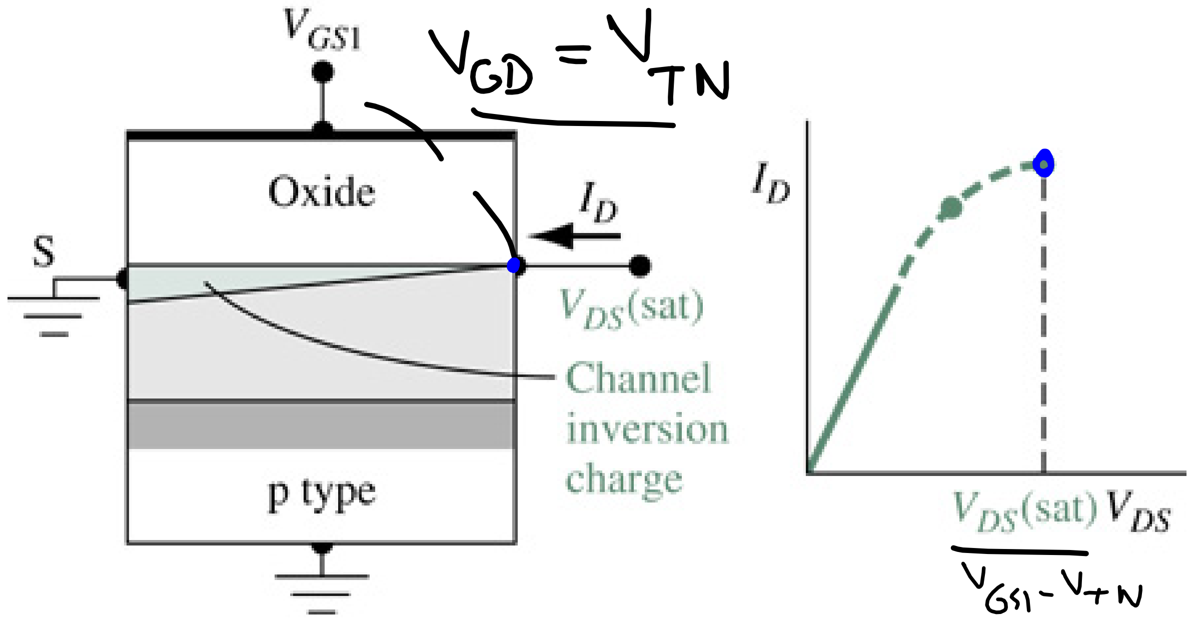
Small V_{DS}

$J = en v$


V_{GS1}
+ +
V_{GS1} Oxide
S
I_D
V_{DS}
Channel inversion charge
p type
I_D
V_{DS}

(b)

$V_{GD} = V_{TN}$



(c)

Pinchoff

At pinchoff, $V_{GD} = V_{TN}$, $V_{DS} \equiv V_{DS}(sat)$

$V_{SD} = V_S - V_D$

$= -(V_D - V_S)$

$= -V_{DS}$

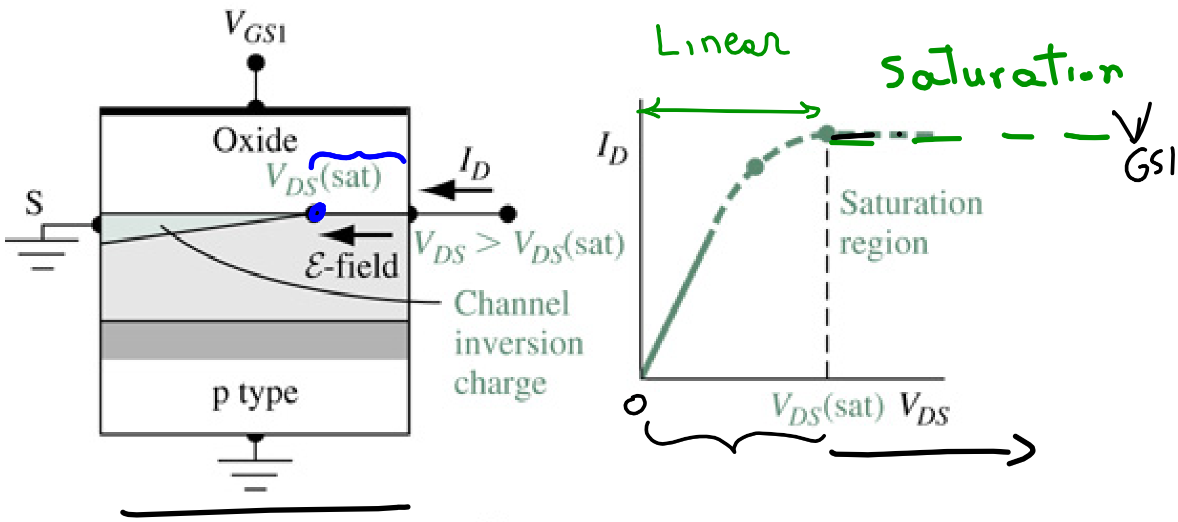
$V_{GD} = V_{TN}$

$V_{GS} + V_{SD} = V_{TN}$

$V_{GS} - V_{DS} = V_{TN}$

$V_{GS} - V_{DS}(sat) = V_{TN}$

$$\boxed{V_{DS}(sat) = V_{GS} - V_{TN}}$$

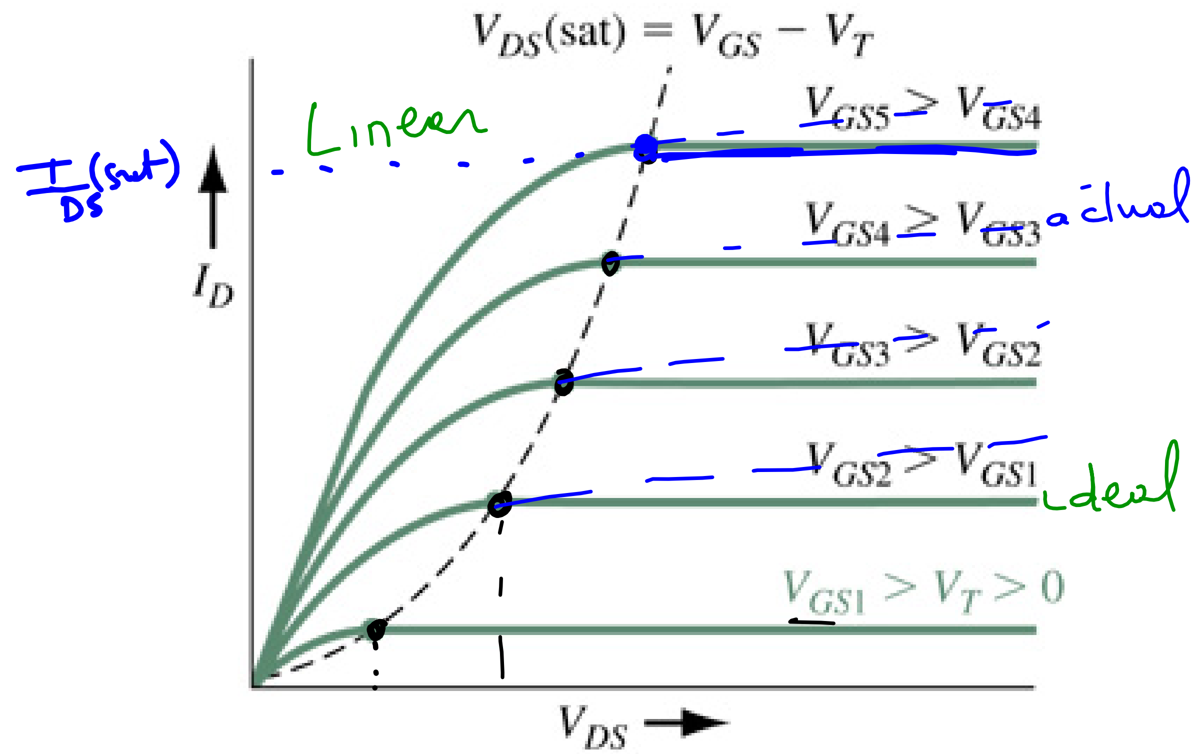


L

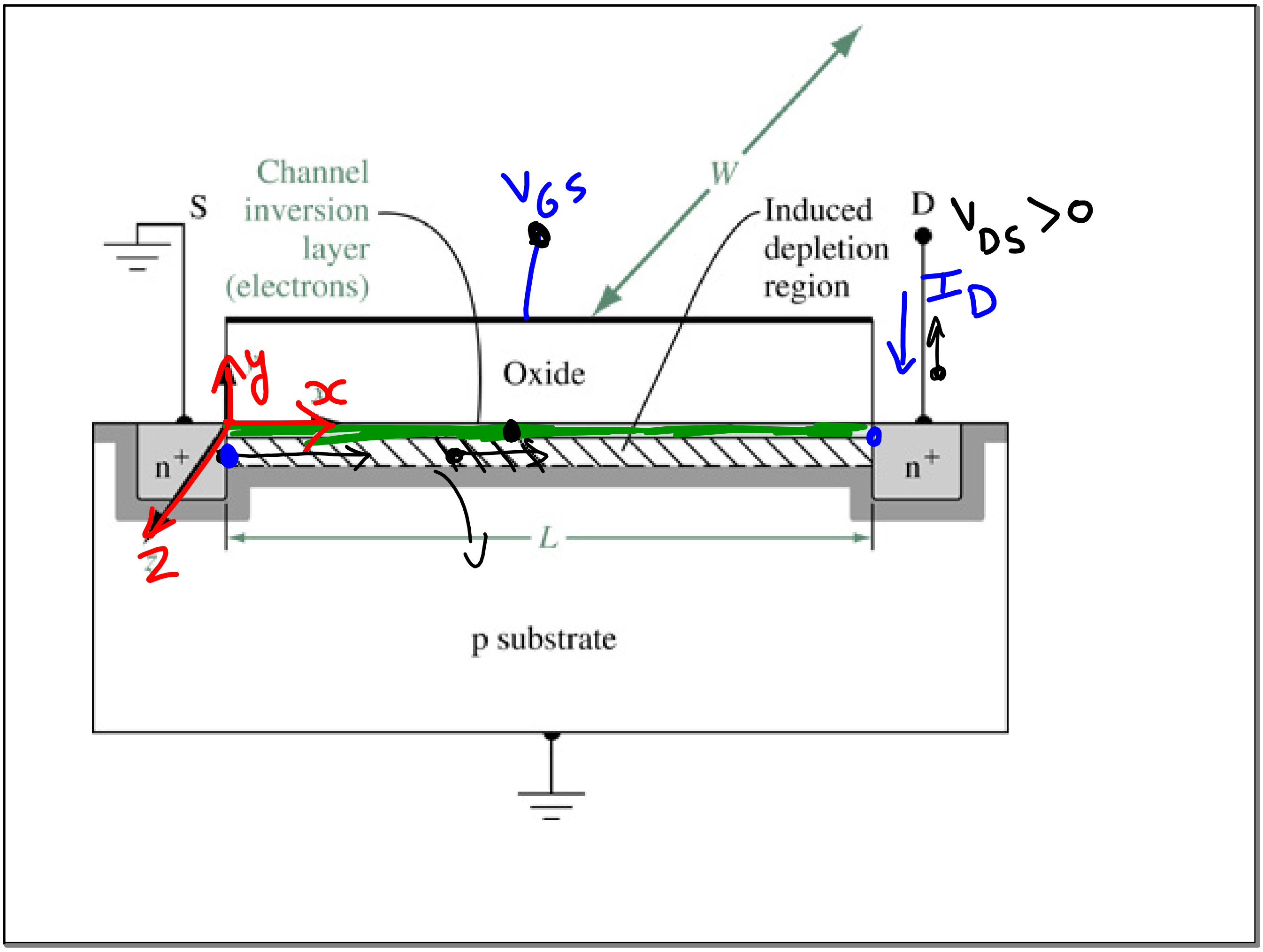
(d)

Linear : $\begin{cases} V_{GS} > V_{TN} \\ 0 \leq V_{DS} \leq V_{DS}(sat) \end{cases}$

Saturation $\begin{cases} V_{GS} > V_{TN} \\ V_{DS} > V_{DS}(sat) \end{cases}$



Ideal I-V characteristics for long channel MOSFET


Channel inversion layer (electrons)
S
W
Induced depletion region
D
Oxide
n+
n+
L
p substrate
VGS
VDS > 0
ID
y
x
z

$$J_x = \sigma \mathcal{E}_x \; ; \; \sigma = e n \mu_n$$

$$J_x = e n \mu_n \mathcal{E}_x$$

$$I_x = \int_z \int_y J_x \, dy \, dz = \int_z \int_y e n \mu_n \mathcal{E}_x \, dy \, dz$$

$$\varphi_n' \equiv -\int_y e n \, dy$$ inversion layer charge per cm^2

$$I_x = -W \mu_n \varphi_n' \mathcal{E}_x \; ; \; I_x \text{ is constant}$$

$$\mathcal{E}_x = -\frac{dV_x}{dx}$$

$$Q_n' = -C_{ox}\left[\underbrace{(V_{GS}-V_x)} - V_{TN}\right]$$

$$I_x = -W\mu_n C_{ox}\left[(V_{GS}-V_x)-V_{TN}\right]\frac{dV_x}{dx}$$

$$\int_0^L I_x\,dx = -W\mu_n C_{ox}\int_0^{V_{DS}}\left[(V_{GS}-V_x)-V_{TN}\right]dV_x$$

$$I_D = -I_x$$

$$-I_D L = -W\mu_n C_{ox}\left[(V_{GS}-V_{TN})V_x - \frac{V_x^2}{2}\right]_0^{V_{DS}}$$

$$\boxed{I_D = \frac{W\mu_n C_{ox}}{2L}\left[\underbrace{2(V_{GS}-V_{TN})V_{DS}} - \underbrace{V_{DS}^2}\right]}$$

Line

In saturation, $I_D = I_D(\text{sat})$

set $V_{DS} = V_{DS}(\text{sat}) = V_{GS} - V_{TN}$

$$\boxed{I_D(\text{sat}) = \frac{W \mu_n C_{ox}}{2L}\left[V_{GS} - V_{TN}\right]^2}$$ saturation

$K_n \equiv \dfrac{W \mu_n C_{ox}}{2L}$: conduction parameter

$k_n' \equiv \mu_n C_{ox}$: process conduction parameter

$$K_n = k_n'\left(\frac{W}{2L}\right)$$

NMOS $\longrightarrow$ PMOS

V_{GS}	$-V_{GS} = V_{SG}$
V_{DS}	$-V_{DS} = V_{SD}$
V_{TN}	$-V_{TP}$
n	p
p	n

PMOS

Linear
$$\begin{cases} V_{SG} > -V_{TP} \\ 0 \leq V_{SD} \leq V_{SD}(sat) \end{cases}$$

Saturation
$$\begin{cases} V_{SG} > -V_{TP} \\ V_{SD} \geq V_{SD}(sat) \end{cases}$$

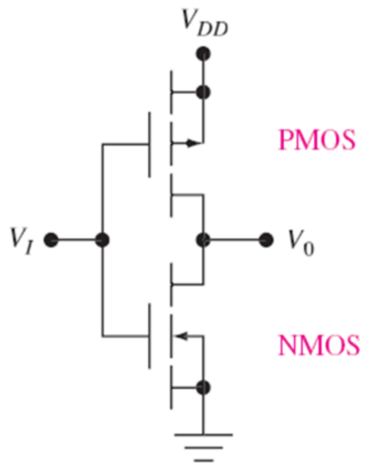
$$V_{DS}(sat) = V_{GS} - V_{TN} \longrightarrow V_{SD}(sat) = V_{SG} + V_{TP}$$

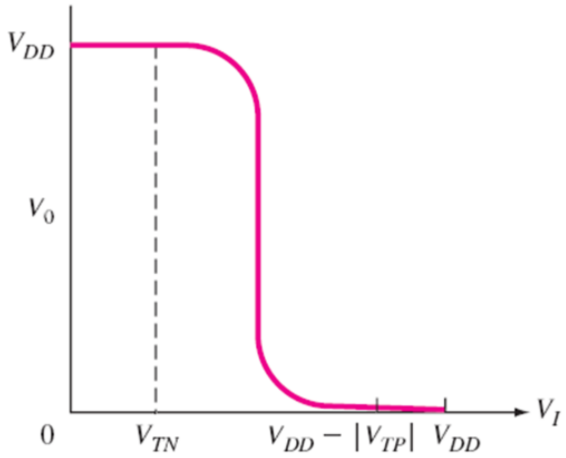
Summary
Ideal Long-Channel MOSFET

NMOS	PMOS
Transition point	**Transition point**
$V_{DS}(\text{sat}) = V_{GS} - V_{TN}$	$V_{SD}(\text{sat}) = V_{SG} + V_{TP}$
Nonsaturation bias	**Nonsaturation bias**
$[V_{DS} \le V_{DS}(\text{sat})]$;	$[V_{SD} \le V_{SD}(\text{sat})]$;
$I_D = K_n\left[2(V_{GS} - V_{TN})V_{DS} - V_{DS}^2\right]$	$I_D = K_p\left[2(V_{SG} + V_{TP})V_{SD} - V_{SD}^2\right]$
Saturation bias	**Saturation bias**
$[V_{DS} \ge V_{DS}(\text{sat})]$;	$[V_{SD} \ge V_{SD}(\text{sat})]$;
$I_D = K_n(V_{GS} - V_{TN})^2$	$I_D = K_p(V_{SG} + V_{TP})^2$

4

Application: CMOS Inverter





For $V_{GS} < V_{TN}$, NMOS IS OFF, PMOS IS ON

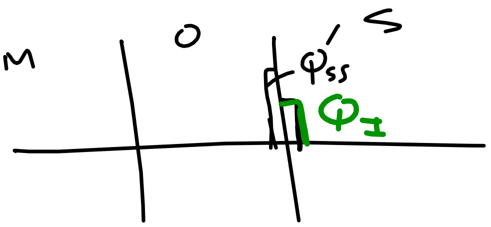
For $V_{GS} > V_{DD} - |V_{TP}|$, NMOS IS ON, PMOS IS OFF

5

$$V_{TN} = \frac{|Q'_{SD}(max)|}{C_{ox}} + V_{FB} + 2|\phi_{Fp}|$$

$$V_{TN} = \frac{eN_A x_{dT}}{C_{ox}} + \left(\phi_{ms} - \frac{Q'_{ss}}{C_{ox}}\right) + 2|\phi_{Fp}|$$

V_T Adjustment by ion implantation



D_I : ion implantation dose $\frac{\#\text{ atoms}}{cm^2}$

$$\varphi_I = \begin{cases} +e D_I & \text{donors} \\ -e D_I & \text{acceptors} \end{cases}$$

$$\Delta V_{TN} = \begin{cases} -\frac{e D_I}{C_{ox}} & \text{donors} \\ +\frac{e D_I}{C_{ox}} & \text{acceptors} \end{cases}$$