LESSON 3

UNIFORM LANE ELECTROMAGNETIC WAVES (UPEMWs)

I. WHAT YOU WILL LEARN IN LESSON 3:

In this lesson, you will learn about UPEMWs propagating in a lossless infinite medium which may be vacuum or a dielectric (or insulating) medium. A UPEMW actually consists of two UPWs, one representing the electric field component of the wave and the other representing the magnetic field component of the wave. The properties of UPEMWs are given to you in this lesson without proof. The proof of these properties will be given in a later lesson when Maxwell Equations representing the basic foundational equations of all electromagnetic fields are discussed.

II. PROPERTIES OF UPEMWs:

UPEMWs are vector wavefunctions which are a variant of the scalar wavefunctions discussed in Lessons 1 and 2. All that this means is that the electric and magnetic wavefunctions have a direction in space in addition to being functions of space and time as discussed in the case of scalar wavefunctions. A simple example of the expressions for electric and magnetic field in a UPEMW propagating in the z direction is given below in Eq. (1):

$$\mathbf{E}(z,t) = \mathbf{i_x}\, E_0 \cos(\omega t - kz) \qquad (1a)$$

$$\mathbf{H}(z,t) = \mathbf{i_y}\, H_0 \cos(\omega t - kz) \qquad (1b)$$

where $\mathbf{i_x}$ and $\mathbf{i_y}$ are unit vectors in the x and y directions respectively and E_0 and H_0 are amplitude constants. The unit vectors in the directions of electric and magnetic fields will generally be called $\mathbf{i_E}$ and $\mathbf{i_H}$. The values of $\mathbf{i_E}$ and $\mathbf{i_H}$ Eq. (1) are thus $\mathbf{i_E} = \mathbf{i_x}$ and $\mathbf{i_H} = \mathbf{i_y}$.

Eq. (1) may more generally be written as

$$\mathbf{E}(z,t) = \mathbf{i_E}\, E_0 \cos(\omega t - \mathbf{k.r}) \qquad (2a)$$

$$\mathbf{H}(z,t) = \mathbf{i_y}\, H_0 \cos(\omega t - \mathbf{k.r}) \qquad (2b)$$

where $\mathbf{k}$ in the present simple example is $\mathbf{k} = k\mathbf{i_z}$ but in a more general case may be expressed as $\mathbf{k} = k\mathbf{i_K}$. Summarizing, when dealing with UPEMWs, one has to contend with the following three things:

- three unit vectors $\mathbf{i_E}$, $\mathbf{i_H}$ and $\mathbf{i_K}$
- two amplitude quantities E_0 and H_0, and
- and the same space-time factor $\cos(\omega t - \mathbf{k.r})$.

UPEMW PROPERTY # 1:

The space-time factor $\cos(\omega t - \mathbf{k.r})$ is the same for both the electric and magnetic field functions, and has the same interpretation as in Lesson 2. In the context of UPEMWs, it should be evident that the electric and magnetic fields are in phase or synchronism in space and time as shown in Fig. 1. Thus, if we take a snapshot of the variation of $E_x(z, t=t_0)$ and $H_y(z, t=t_0)$ with z at a fixed time $t = t_0$, we find that the electric field and magnetic field profiles are in phase, i.e., the maxima points for E_x and H_y profiles are at the same points along the z axis, the minima points are at the same points along the z axis, and so on.

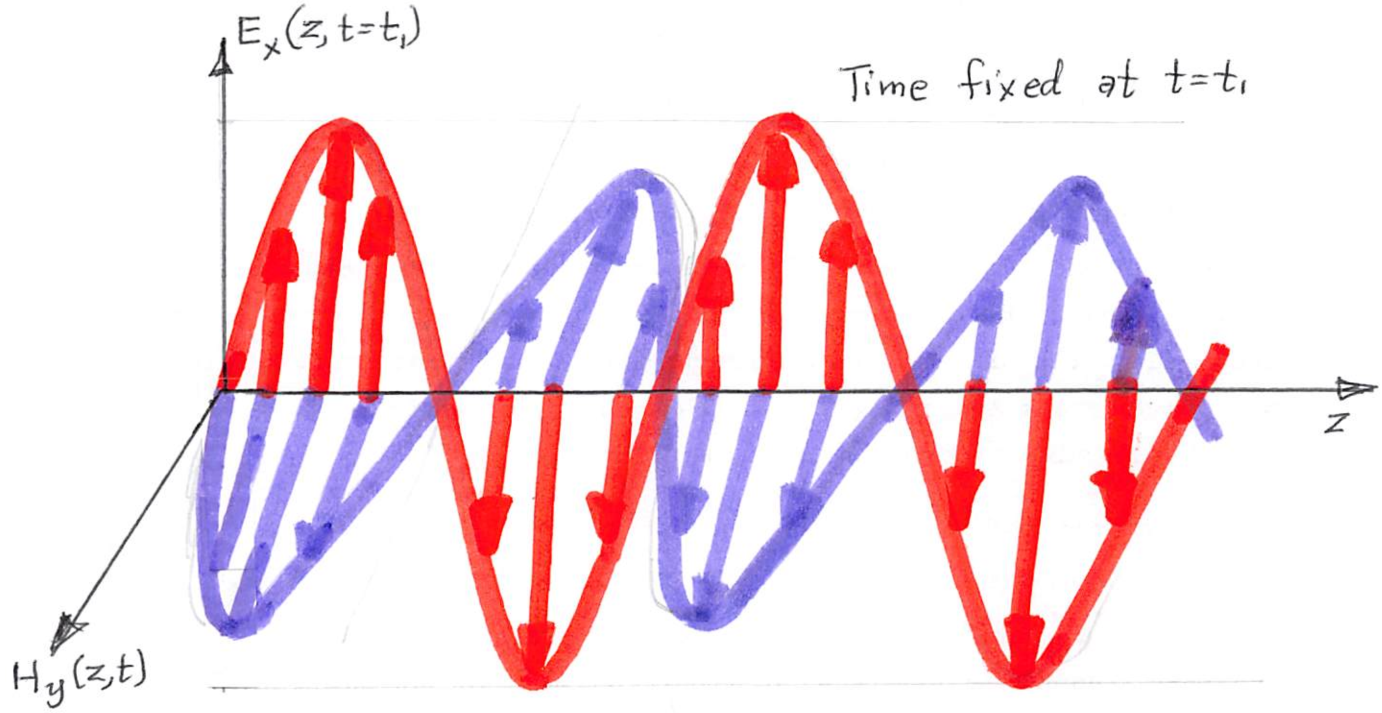

Fig. 1. The variation of E_x and H_y with z for a fixed value of t.

UPEMW PROPERTY # 2:

The three unit vectors $\mathbf{i_E}$, $\mathbf{i_H}$ and $\mathbf{i_K}$ form a **right-handed triad** of unit vectors, i.e., the three unit vectors are mutually orthogonal in a right-handed sense. If any two of these vectors are known, the third one may be found by using one of the following three equations:

$$\mathbf{i_E} \times \mathbf{i_H} = \mathbf{i_K} \tag{3a}$$

$$\mathbf{i_H} \times \mathbf{i_K} = \mathbf{i_E} \tag{3b}$$

$$\mathbf{i_K} \times \mathbf{i_E} = \mathbf{i_H} \tag{3c}$$

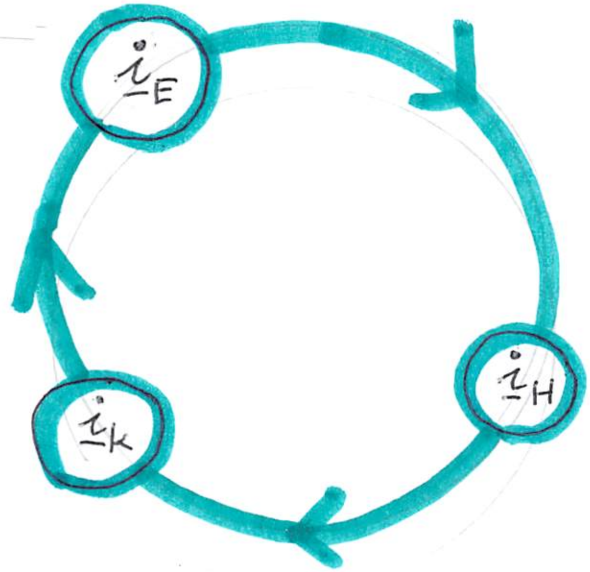

Fig. 2. A mnemonic for finding an unknown unit vector if the other two are known.

The symmetry implied in Eq. (3) is summarized in Fig. 2 which provides a good mnemonic for Eq. (3), i.e., for finding the direction of the unknown unit vector from the two known unit vectors. The unknown unit vector is simply the cross product of the two known unit vectors taken in the sequence in which they appear in Fig. 2 as one follows the arrow in Fig. 2.

A manual or graphical determination of the cross product of any two of the three unit vectors is readily implemented using the right-hand rule (see Fig. 3). In finding the cross product $\mathbf{i_A} \times \mathbf{i_B}$. of known unit vectors $\mathbf{i_A}$ and $\mathbf{i_B}$, simply curl the fingers of the right hand other than the thumb in the direction in which $\mathbf{i_A}$ must be rotated in order to make it coincide with $\mathbf{i_B}$, then the thumb points in the direction of the unit vector representing the cross product.

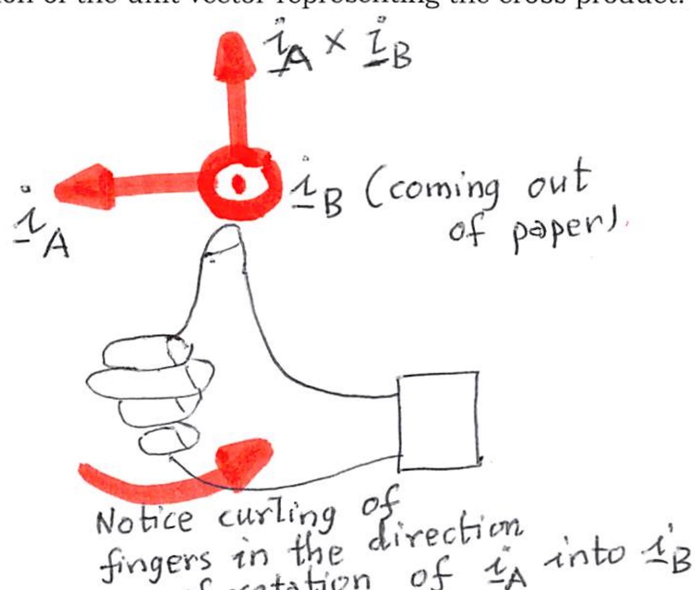

Fig. 3. Finding the cross product of two unit vectors using the right-hand rule.

UPEMW PROPERTY # 3:

The amplitudes of the electric and magnetic field, i.e., E_0 and H_0, respectively, are not independent. If one knows one, then the other one may be found from the formula

$$E_0/H_0 = \eta \tag{4}$$

where η is the so-called characteristic or intrinsic impedance of the medium and given by the formula

$$\eta = (\mu_0\mu_r/\varepsilon_0\,\varepsilon_r)^{1/2} \tag{5}$$

In Eq. (5), $\mu_0 = 4\pi.10^{-7}$ H/m (permeability of free space), μ_r is the relative permeability of the medium with $\mu_r = 1$ for a nonmagnetic medium, $\varepsilon_0 = 8.854.10^{-12}$ F/m = $(1/36\pi).10^{9}$ F/m (permittivity of free space) and ε_r is the relative permittivity or dielectric constant of the medium. The characteristic impedance of free space (or vacuum) is readily seen to be

$$\eta_0 = 377\ \Omega \text{ (or } 120\pi\ \Omega) \tag{6}$$

PHASE VELOCITY OF UPEMWs:

The phase velocity of UPEMWs is

$$v_p = c/n_r \tag{7}$$

where n_r is the so-called relative refractive index of the medium and may be defined as the velocity reduction factor for UPEMW propagation in the medium relative to the velocity $c = 3.10^8$ m/s of UPEMW in vacuum. The formula for n_r in terms of the material properties of the medium is

$$n_r = (\mu_r\,\varepsilon_r)^{1/2} \tag{8}$$