EE Formula Sheet

Energy

The energy (E) of a photon is given by $E = h\nu$, where h is Planck's constant and ν is the frequency.
The frequency (ν) of a photon is inversely proportional to its wavelength (λ) and can be determined by the equation $\nu = \frac{c}{\lambda}$, where c is the speed of light.

Constants

$$
\begin{aligned}
q &= 1.602 \times 10^{-19}\ \text{C} \\
m_e &= 9.109 \times 10^{-31}\ \text{kg} \\
h &= 6.626 \times 10^{-34}\ \text{J} \cdot \text{s} \\
k &= 1.381 \times 10^{-23}\ \text{J/K} \\
\varepsilon_0 &= 8.854 \times 10^{-14}\ \text{C}^2/(\text{N} \cdot \text{m}^2) \\
c &= 3.00 \times 10^{8}\ \text{m/s} \\
e &\approx 1.6 \times 10^{-19}\ \text{C} \\
kT &\approx 0.026 \text{eVat} T = 300\text{K} \\
h &= 6.62607015 \times 10^{-34} \text{m}^2\text{kg/s} \\
\varepsilon_{ox} &= (3.9)(8.85 \times 10^{-14}) = 3.45 \times 10^{-13} \text{F/m}^2 \\
\varepsilon_s &= (11.7)(8.85 \times 10^{-14}) = 1.04 \times 10^{-12} \text{F/m}^2 \\
E_g &= 1.12\text{V} \\
n_i &= 1.5 \times 10^{10}
\end{aligned}
$$

Formulae

$kT_{temp} = 0.026(\frac{temp}{300})$, kT at a temperature temp
$\sigma = en\mu_n + ep\mu_p$, Conduction
$n_i^2 = n_0 p_0$, concentration at equilibrium
$n_i^2 = n_c n_v e^{-E_g/kT}$,
$n_i^2 \propto T^3 e^{-E_g/kT}$, proportionality ratio
$n_i^2 at500 = (\frac{500}{300})^3 e^{-E_g/kTat500} e^{E_g/kTat300}$, proportional temp
$E = \frac{hc}{\lambda}$, energy of photon
$E_g = E_c - E_v$, Energy band gap
$f(E) = \frac{1}{1+e^{\frac{E-E_f}{kT}}}$, Fermi-Dirac Distribution Function
$n = N_c \cdot e^{-\frac{E_c - E_f}{kT}}$, Electron carrier concentration
$p = N_v \cdot e^{-\frac{E_f - E_v}{kT}}$, Hole carrier concentration
$J_d = q \cdot n \cdot \mu_n \cdot E$, Drift Current
$J_n = q \cdot D_n \cdot \frac{dn}{dx}$, Diffusion Current
$E_g = E_c - E_v$, Energy-Band Gap (Eg)
$\frac{1}{m^*} = \frac{1}{m_l} + \frac{1}{m_t}$, Electron and Hole Effective Mass
$q = 1.602 \times 10^{-19}$ C, Charge of an Electron
$n = N_c \cdot e^{-\frac{E_c - E_f}{kT}}$, Electron Carrier Concentration
$p = N_v \cdot e^{-\frac{E_f - E_v}{kT}}$, Hole Carrier Concentration
$J_n = q \cdot n \cdot \mu_n \cdot E$, Drift Current Density for Electrons
$J_p = q \cdot p \cdot \mu_p \cdot E$, Drift Current Density for Holes
$J_n = q \cdot D_n \cdot \frac{dn}{dx}$, Diffusion Current Density for Electrons
$J_p = q \cdot D_p \cdot \frac{dp}{dx}$, Diffusion Current Density for Holes
$N_c = 2\left(\frac{2\pi m_e kT}{h^2}\right)^{3/2}$, Density of States in the Conduction Band (Nc)
$N_v = 2\left(\frac{2\pi m_h kT}{h^2}\right)^{3/2}$, Density of States in the Valence Band (Nv)
$P_0 = n_i e^{\frac{E_{fi} - E_f}{kT}}$
$P_0 = \frac{N_A - N_D}{2} + \sqrt{(\frac{N_A - N_D}{2})^2 + n_i^2}$
$N_0 = \frac{N_D - N_A}{2} + \sqrt{(\frac{N_D - N_A}{2})^2 + n_i^2}$
$f_F(E) = \frac{1}{1+e^{\frac{E-E_f}{kT}}}$, Fermi-Dirac Distribution Function
$f_F(E) = e^{\frac{-(E-E_f)}{kT}})$, Boltzman Approximation when $E - E_F >> kT$
$\mu_n = \frac{q \cdot \tau_n}{m^*}$, Electron Mobility
$\mu_p = \frac{q \cdot \tau_p}{m^*}$, Hole Mobility
$G = \alpha \cdot I$, Generation Rate of Electron-Hole Pairs
$R = B \cdot np - A \cdot n_i^2$, Recombination Rate
$\frac{\partial n}{\partial t} + \nabla \cdot \mathbf{J}_n = G - R$, Continuity Equation for Electron Current
$\frac{\partial p}{\partial t} - \nabla \cdot \mathbf{J}_p = G - R$, Continuity Equation for Hole Current
$P_0 + N_D = n_0 + N_A$, Charge neutrality
$J_{drf} = en\mu_n E + ep\mu_p E = \sigma E$, Total Drift
$I = AJ_{drf}$, Current
$E = Volt/Len$, Electric Field
$V_{dn} = \mu_n E$, Drift velocity for electrons
$V_{dp} = \mu_p E$, Drift velocity for holes
$n_i^2 = N_c N_v \cdot e^{\frac{-E_g}{kT}}$, intrinsic Fermi energy

§H8

Banding: negative voltages shift plot up. For P-Type, $E_F < E_{Fi}$, Accumulation when $V_g < 0$, Depletion when $V_g > 0$.
$\phi_{ms} is the work function$ $\phi_{ms} = \phi'_m - \left[x' + \frac{E_g}{2e} + |\phi_{fp}|\right]$,(V) Aluminum Gate
$\phi_{ms} = -\left(\frac{E_g}{2e} + |\phi_{fp}|\right)$,(V) n* Polysilicon Gate
$\phi_{ms} = \frac{E_g}{2e} - |\phi_{fp}|$,(V) p* Polysilicon Gate
$\phi_{Fp} = -\frac{kT}{e} \ln\left(\frac{N_A}{n_i}\right)$, (V)
$\phi_s = 2|\phi_{Fp}|$, (V) Surface Potential
$x_{dt} = \left[\frac{4\epsilon_s |\phi_{fp}|}{eN_A}\right]^{1/2}$ (cm)
$V_G = V_{ox} + \phi_s + \phi_{ms}$
$V_{FB} = V_{ox} + \phi_{ms}$, at flatband, $V_g \equiv V_{FB}$, $\phi_s = 0$
$V_{FB} = \phi_{ms} - \frac{Q'_{ss}}{C_{ox}}$, from above
$Q'_{SD}(max) = -eN_A x_{dt}$
$Q'_{SS} = C_{ox}(\phi_{ms} - V_{FB})$, (C/cm^2)
$V_{TN} = \frac{|Q'_{SD}(max)|}{C_{ox}} + V_{FB} + 2|\phi_{Fp}|$, threshold voltage
$\gamma = \frac{\sqrt{2e\epsilon_s N_A}}{C_{ox}}$
$V_{TN} = V_{FB} + 2|\phi_{Fp}| + \gamma\sqrt{2|\phi_{Fp}|}$, threshold voltage

Ideal C-V Characteristics

$C'_{(acc)} = C_{ox}$, Accumulation
$C'_{(depl)} = C_{ox}, Depletion$
$C_{total} \equiv C_{ox}$, at low frequency
$C_{total} \equiv C_{ox}||C'_{SD}$, at high frequency
$C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}}$, Oxide Capacitance
$C_{min} = \frac{\varepsilon_{ox}}{t_{ox} + (\frac{\varepsilon_{ox}}{\varepsilon_s}) \cdot X_{dt}}$,
$L_D = \sqrt{\frac{kT}{e} \cdot \frac{e_S}{e_{N_A}}}$,
$C_{FB} = \frac{\varepsilon_{ox}}{t_{ox} + (\frac{\varepsilon_{ox}}{\varepsilon_s}) \cdot L_D}$,
When $V_{GD} = V_{TN}$, pinchoff occurs.
$V_{DS}(sat) = V_{GS} - V_{TN}$
$I_D = \frac{1}{2}\mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})^2$, Saturation
$I_D = \frac{W \mu_n C_{ox}}{2L} \left[2(V_{GS} - V_{TH}) - V_{DS}^2\right]$, Linear

PMOS

$$\text{Linear} \begin{cases} V_{SG} > -V_{TP} \\ 0 \leq V_{SD} \leq V_{SD}^{(sat)} \end{cases}$$

$$\text{Saturation} \begin{cases} V_{SG} > -V_{TP} \\ V_{SD} \geqslant V_{SD}^{(sat)} \end{cases}$$

$$V_{DS}^{(sat)} = V_{GS} - V_{TN} \longrightarrow V_{SD}^{(sat)} = V_{SG} + V_{TP}$$

Summary
Ideal Long-Channel MOSFET

NMOS	PMOS
Transition point $V_{DS}(\text{sat}) = V_{GS} - V_{TN}$	**Transition point** $V_{SD}(\text{sat}) = V_{SG} + V_{TP}$
Nonsaturation bias $[V_{DS} \leq V_{DS}(\text{sat})]$; $I_D = K_n\left[2(V_{GS} - V_{TN})V_{DS} - V_{DS}^2\right]$	**Nonsaturation bias** $[V_{SD} \leq V_{SD}(\text{sat})]$; $I_D = K_p\left[2(V_{SG} + V_{TP})V_{SD} - V_{SD}^2\right]$
Saturation bias $[V_{DS} \geq V_{DS}(\text{sat})]$; $I_D = K_n(V_{GS} - V_{TN})^2$	**Saturation bias** $[V_{SD} \geq V_{SD}(\text{sat})]$; $I_D = K_p(V_{SG} + V_{TP})^2$

BJT

Operating regions

- (BE Junction) Forward $V_{BE} > 0$
- (BE Junction) Reverse $V_{BE} < 0$
- (CB Junction) Forward $V_{CB} < 0$
- (CB Junction) Reverse $V_{CB} > 0$
- Cutoff: BE reverse, CB reverse
- Saturation: BE forward, CB forward
- Active: BE forward, CB reverse
- Inverse Active: BE reverse, CB forward

Depletion or enhancement
Depletion mode when $V_{TN} < 0$, enhancement mode when $V_{TN} > 0$
A mosfet is in Saturation when $V_{GS} > V_{TH}$ and $V_{DS} > V_{DS}(sat)$
$V_{bi}(BE) = \frac{kT}{e} \ln \frac{N_B N_E}{n_i^2}$

Implantation

Add acceptor atoms to increase V_{TN}
$\Delta V_{TN} = \frac{eD_I}{C_{ox}}$, number of atoms per cm sq

Notes

Common Source: Input connected to gate, output connected to drain.
Common Drain (Source Follower): Input connected to gate, output connected to source.
Common Gate: Input connected to source, output connected to drain.
When $N_A >> N_D$, the semiconductor is p-type.
When $N_D >> N_A$, the semiconductor is n-type.

Transistor formulas

$I_C = \beta \cdot I_B$, Conduction Parameter
$I_B = \frac{I_E}{\beta+1}$,
$\alpha = \frac{I_C}{I_E}$, Current Ratio
$I_C = I_E - I_B$, Kirchhoff's Current Law
$V_{CE} = V_{BE} + V_{CB}$, Voltage Relationships
$I_C = I_{C0}\left(e^{\frac{V_{BE}}{V_T}} - 1\right)$, BJT Current Equation
$I = I_0 \cdot \left(e^{\frac{V}{n \cdot V_T}} - 1\right)$, Schottky Diode Equation
$I_D = \frac{1}{2}\mu_n C_{ox} \frac{W}{L}\left(V_{GS} - V_{TH}\right)^2$, MOSFET Drain Current Equation
$I_D = \mu_n C_{ox} \frac{W}{L}\left[(V_{GS} - V_{TH})V_{DS} - \frac{V_{DS}^2}{2}\right]$, MOSFET Drain Current Equation (Triode Region)
$g_m = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D}$, Transconductance Parameter
$A_v = -g_m \cdot R_D$, Voltage Gain Formula

EE General Formulae

$rms = \frac{1}{\sqrt{2}}$,
$V = I \cdot R$, Ohm's law.
$P = V \cdot I$, DC Power.
$P = V \cdot I \cdot \cos(\theta)$, AC power.
$E = P \cdot t$, Energy.
$C = \frac{Q}{V}$, Capacitance.
$V = L \cdot \frac{di}{dt}$, Inductance.
$\tau = R \cdot C$, Time constant to reach 63.2% of capacitors final voltage.
$\tau = \frac{L}{R}$, Time constant to reach 63.2% of inductors final value.
$\frac{N_1}{N_2} = \frac{V_1}{V_2}$, Transformer turns ratio.
$V_{\text{peak}} = \sqrt{2} \cdot V_{\text{rms}}$, Peak AC Voltage.
$V_{\text{rms}} = \frac{V_{\text{peak}}}{\sqrt{2}}$, RMS AC Voltage.
$V_{\text{avg}} = \frac{1}{T}\int_0^T V(t)\,dt$, RMS AC Voltage.
$V_{\text{out}} = V_{\text{in}} \cdot \frac{R_2}{R_1+R_2}$, voltage divider.
$R_{\text{eq}} = R_1 + R_2 + \ldots + R_n$, series resistors.
$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}$, Parallel resistors.
$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots + \frac{1}{C_n}$, Series capacitors.
$C_{\text{eq}} = C_1 + C_2 + \ldots + C_n$, parallel capacitors.

Basic integration Rules

$\int kf(u)du = k\int f(u)du + C$,
$\int [f(u) \pm g(u)]du = \int f(u)du \pm \int g(u)du$, $\int du = u + C$,
$\int u^n du = \frac{u^{n+1}}{n+1} + C, n \neq -1$, $\int \frac{du}{u} = \ln|u| + C$,
$\int \frac{u}{du} = \frac{u^2}{2} + C$, $\int e^u du = e^u + C$,$\int e^{4u} = \frac{e^{4u}}{4} + C$,
$\int a^u du = \left(\frac{1}{\ln a}\right) a^u + C$,

Some Integrals

$\int \sin u du = -\cos u + C$, $\int \cos u du = \sin u + C$,
$\int \tan u du = -\ln|\cos u| + C$, $\int \cot u du = \ln|\sin u| + C$,
$\int \sec u du = \ln|\sec u + \tan u| + C$,
$\int \csc u du = -\ln|\csc u + \cot u| + C$, $\int \sec^2 u du = \tan u + C$,
$\int \csc^2 u du = -\cot u + C$, $\int \sec u \tan u du = \sec u + C$,
$\int \csc u \cot u du = -\csc u + C$, $\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a} + C$,
$\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a} + C$, $\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\text{arcsec}\frac{|u|}{a} + C$,
$\int \sin 3x = -\frac{1}{3}\cos 3x$, $\int e^{-4x} = \frac{e^{-4x}}{-4}$
$\int kdx = kx + C$, $\int xdx = \frac{1}{2}x^2 + C$, $\int x^2dx = \frac{1}{3}x^3 + C$,
$\int \frac{1}{x}dx = \ln|x| + C$, $\int e^x dx = e^x + C$,$\int k^u du = \frac{k^u}{\ln u} + C$,
$\int \ln x dx = x\ln x - x + C$, $\int \cos x dx = \sin x + C$,
$\int \sin x dx = -\cos x + C$, $\int \sec^2 x dx = \tan x + C$,
$\int x^n dx = \frac{x^{n+1}}{n+1} + C$, $\int \tan x = -\ln(\cos x) + C$,

Integration by Parts

$$\int udv = uv - \int vdu$$

Some Identities

$\sin 2x = 2\sin x\cos x$

Pythagorean:

$\sin^2 x + \cos^2 x = 1$, $1 + \tan^2 x = \sec^2 x$, $1 + cot^2 x = \csc^2 x$

Reciprocal:

$\sin x = \frac{1}{\csc x}$, $\cos x = \frac{1}{\sec x}$, $\tan x = \frac{\sin x}{\cos x} = \frac{1}{\cot x}$
$\csc x = \frac{1}{\sin x}$, $\sec x = \frac{1}{\cos x}$, $\cot x = \frac{\cos x}{\sin x} = \frac{1}{\tan x}$

Half Angle:

$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$, $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$

Additional Notes:

$\ln(x * y) = \ln(x) + \ln(y)$, $\ln(x/y) = \ln(x) - \ln(y)$
$\ln x^a = a\ln x$, $\tan\theta = \frac{\sin\theta}{\cos\theta}$
$ax^2 + bx + c = 0$, $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$
$\ln a = c \equiv e^c = a$
$\sqrt[n]{a} = a^{\frac{1}{n}}$, $a^{-n} = \frac{1}{a^n}$, $\sqrt[n]{a^m} = a^{\frac{m}{n}}$, $a^0 = 1$, $(a^m)^n = a^{mn}$,
$a^m * a^n = a^{m+n}$, $\frac{a^m}{a^n} = a^{m-n}$, Rewrite $\sqrt{5x}$ as $\sqrt{5}\sqrt{x}$,

Some Derivatives:

$\frac{d}{du}\sin u = (\cos u)u'$, $\frac{d}{du}\cos u = -(\sin u)u'$,
$\frac{d}{du}\tan u = (\sec^2 u)u'$, $\frac{d}{du}\cot u = -(\csc^2 u)u'$,
$\frac{d}{du}\sec u = (\sec u\tan u)u'$, $\frac{d}{du}\csc u = -(\csc u\cot u)u'$,
$\frac{d}{du}\arcsin u = \frac{u'}{\sqrt{1-u^2}}$, $\frac{d}{du}\arccos u = \frac{-u'}{\sqrt{1-u^2}}$,
$\frac{d}{du}\arctan u = \frac{u'}{1+u^2}$, $\frac{d}{du}\text{arccot } u = \frac{-u'}{1+u^2}$,
$\frac{d}{du}\text{arcsec } u = \frac{u'}{|u|\sqrt{u^2-1}}$, $\frac{d}{du}\text{arccsc } u = \frac{-u'}{|u|\sqrt{u^2-1}}$
$\frac{d}{du}[\ln u] = \frac{1}{u}u'$, $\frac{d}{dx}[e^{-x}] = -e^{-x}$, $e^{\ln a} = a$
$\frac{d}{du}[\sqrt{u}] = \frac{u'}{2\sqrt{u}}$, $e^{3x} = 3e^{3x}$, $\frac{d}{dx}[x] = 1$, $\frac{d}{dx}[c] = 0$,
$\frac{d}{du}[\frac{1}{u}] = \frac{1}{u^2}$, $\frac{du}{u} = \ln|u|$,