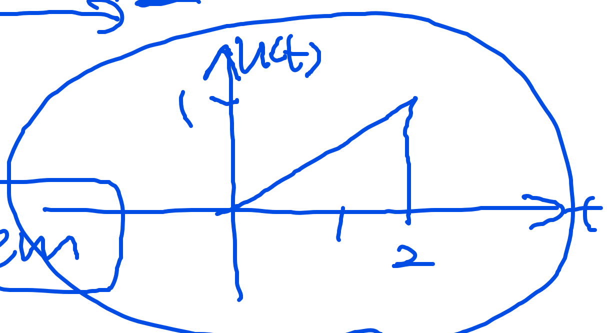
CT Systems with w/ Memory and w/o Memory

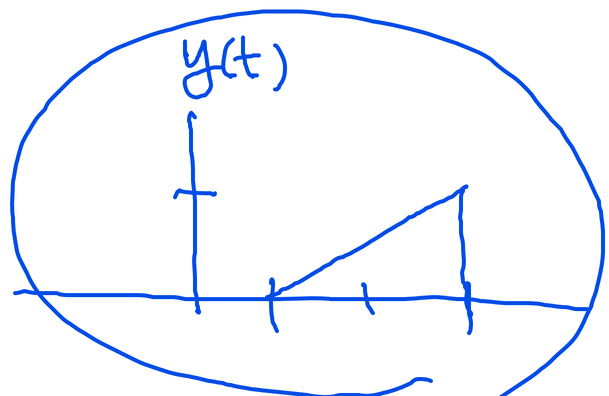
i.e) $y(t) = \underbrace{u(t-1)}_{\text{past value}} + 2u(t) - \underbrace{3u(t+2)}_{\text{future value.}}$

if past value or future value $\neq 0$, then $y(t)$ is a system with memory

i.e) $u(t) \rightarrow \boxed{\text{unit-time delay}} \rightarrow y(t) = u(t-1)$

Causal system

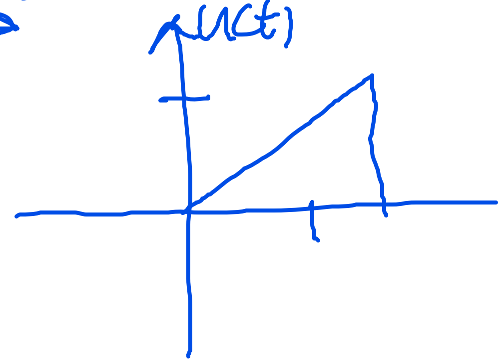


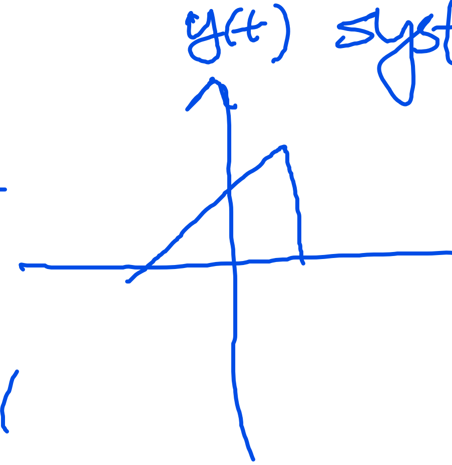


output depends on the past and/or current value.

$$y(t) = \int_{-\infty}^{t} u(\tau)\, d\tau$$

$$u(t) \longrightarrow \boxed{\text{unit-time advance}} \longrightarrow y(t) = u(t+1) \Rightarrow \text{non-causal system}$$





i.e $y(t) = \int_{-\infty}^{t+1} u(\tau)\,d\tau$

↑ non causal!

Examples:

1. $y(t) = 5\,x(t)$ $\Rightarrow$ w/o memory, causal.
2. $y(t) = \sin[x(t)]$ $\Rightarrow$ w/o memory, causal
3. $y(t) = \sin[x(t)] + \sin[x(\underline{t-1})]$ w/ memory, causal.
4. $y(t) = \sin[x(t)] + \sin[x(\underline{t+1})]$ w/ memory, non-causal
5. $y(t) = (\sin t)\,x(t)$ w/o memory, causal.
6. $y(t) = (\sin t)\,x(t-1)$ w/ memory, causal.

Linearity of Memoryless Systems

A memoryless system is linear if

① $u_1(t) \longrightarrow y_1(t)$

$u_2(t) \longrightarrow y_2(t)$

$\Longrightarrow u_1(t) + u_2(t) \longrightarrow y_1(t) + y_2(t)$

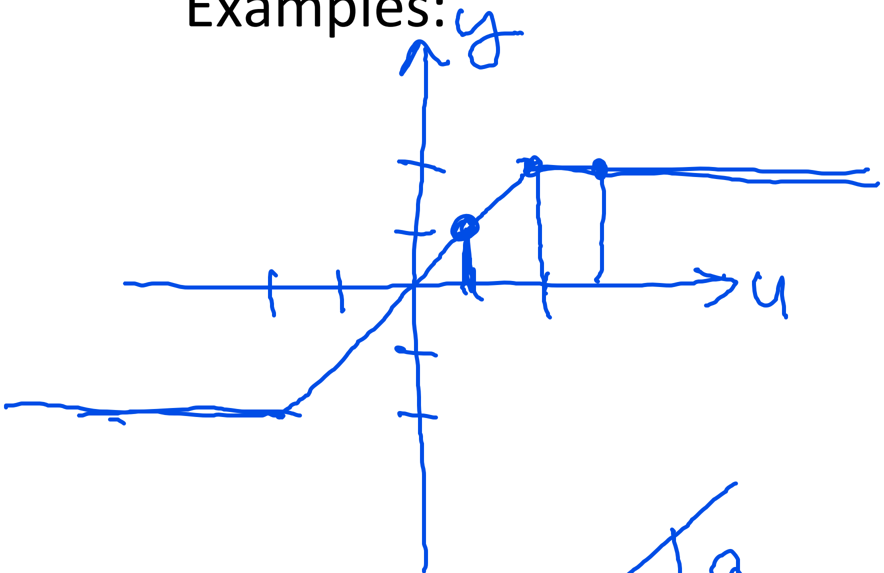
(additivity property)

and

② $\alpha u_1(t) \longrightarrow \alpha y_1(t)$ (homogeneity property)

otherwise, a system is non-linear

Examples:

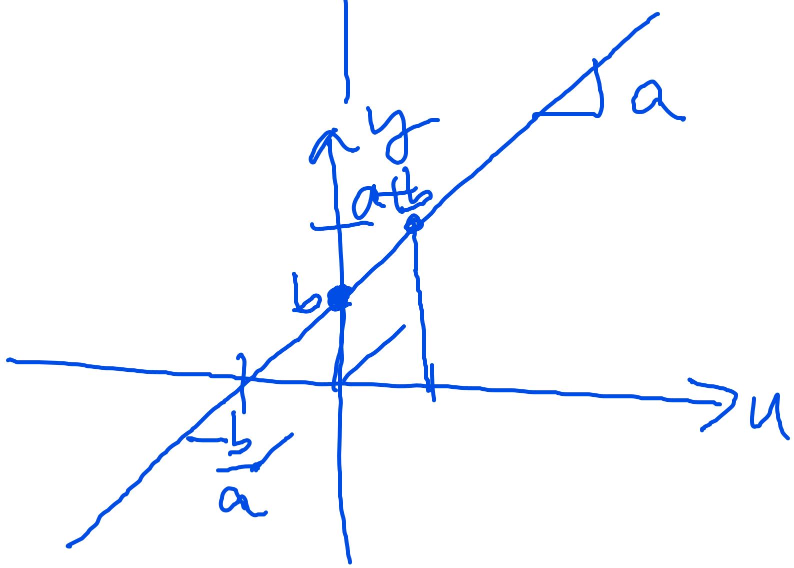


let $u_1 = 1 \longrightarrow y_1 = 1$

$u_2 = 2 \longrightarrow y_2 = 2$

but $u_1 + u_2 = 3 \longrightarrow 2 \neq \underline{y_1 + y_2}$

∴ non linear



$u = 0 \longrightarrow y = b$

$u = 1 \longrightarrow y = a + b$

$\underline{0+1} \longrightarrow \underline{a+2b} \neq \underline{a+b}$

∴ non linear

Note: system is linear if

$y(t) = a u(t) \Rightarrow$ goes through the origin

Note: Additivity implies homogeneity but not conversely

consider $y(t) = \cos[u(t)]$ satisfy homogeneity

but $\cos[u_1(t) + u_2(t)] \neq \cos[u_1(t)] + \cos(u_2(t))$

not linear.

Ex): what about

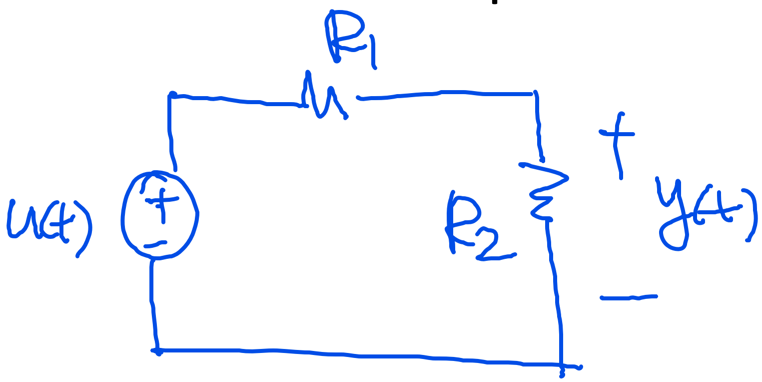
$y(t) = (\cos 20t)\, u(t)$ (modulation)

since $\cos 20t[\alpha_1 u_1(t) + \alpha_2 u_2(t)] =$

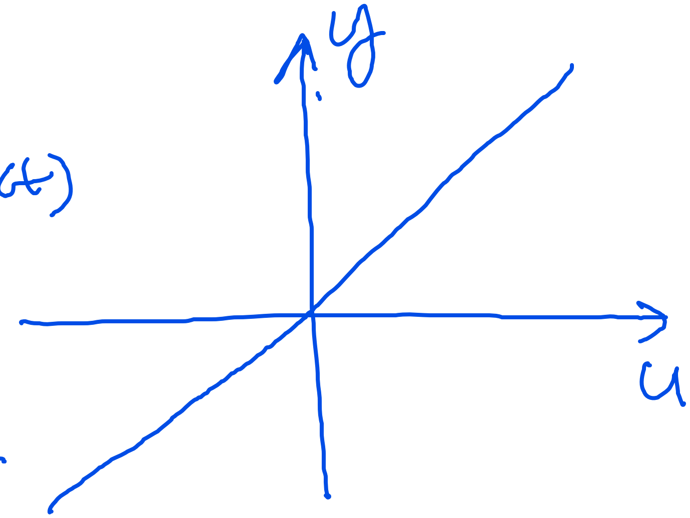
$\alpha_1 \cos 20t[u_1(t)] + \alpha_2 \cos 20t[u_2(t)]$

$\therefore$ linear

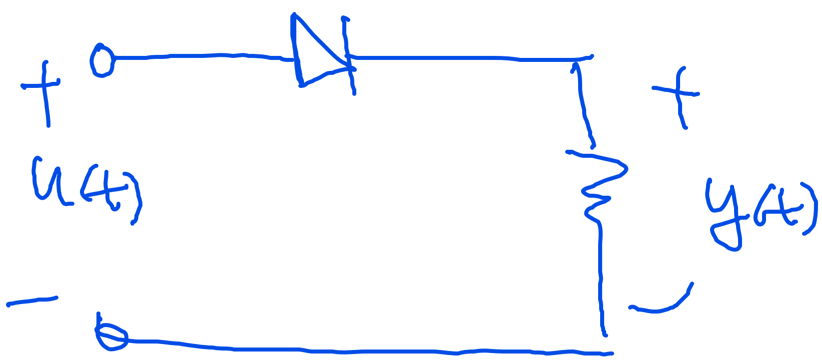
Circuits Examples:

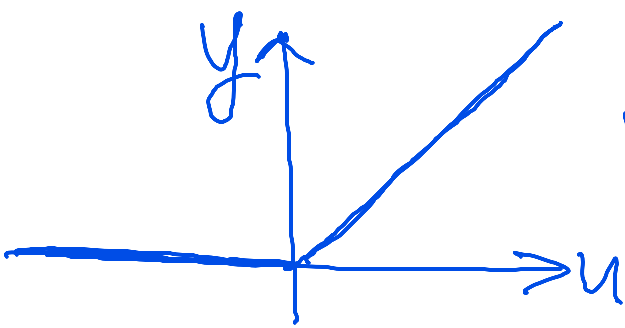


$$y(t) = \frac{R_2}{R_1+R_2}\, u(t)$$



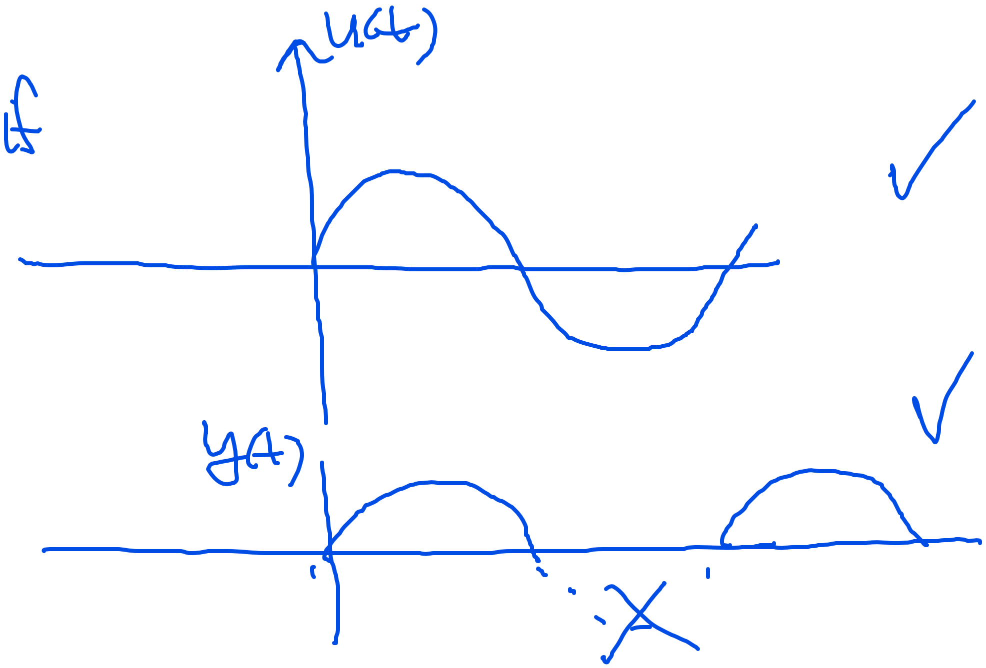
∴ linear





✓

If



✓

✓

∴ Additivity works but homogeneity fails

∴ non-linear

Examples:

① Consider a system $y(t) = \exp[x(t)]$

$x_1(t) = g(t) \longrightarrow y_1(t) = \exp(g(t))$

$x_2(t) = kg(t) \longrightarrow y_2(t) = \exp(kg(t)) = [\exp(g(t))]^k$

since $ky_1(t) = k\exp[g(t)] \neq y_2(t)$

$\therefore$ non-linear

② $y(t) = u^2(t)$

$y_1(t) = u_1^2(t)$

$y_2(t) = u_2^2(t)$

$\Longrightarrow y(t) = y_1(t) + y_2(t) = u_1^2(t) + u_2^2(t)$

$\neq [u_1(t) + u_2(t)]^2$

$\therefore$ non-linear

③ $y(t) = t^2 u(t)$

$y_1(t) = t^2 u_1(t)$

$y_2(t) = t^2 u_2(t)$

$\Longrightarrow t^2[u_1(t) + u_2(t)] = t^2[u_1(t) + u_2(t)]$

$\therefore$ linear

Linearity of Systems with Memory

the response of systems depends on

① input ② states (memory or initial conditions)

$$\left.\begin{array}{l} x_i(t_0) \\ u_i(t), \quad t \geq t_0 \end{array}\right\} \longrightarrow y_i(t), \quad t \geq 0$$

System is linear if

$$\left.\begin{array}{l} \boxed{x_1(t_0) + x_2(t_0)} \\ u_1(t) + u_2(t), \quad t \geq t_0 \end{array}\right\} \longrightarrow y_1(t) + y_2(t), \quad t \geq 0$$

additivity

and

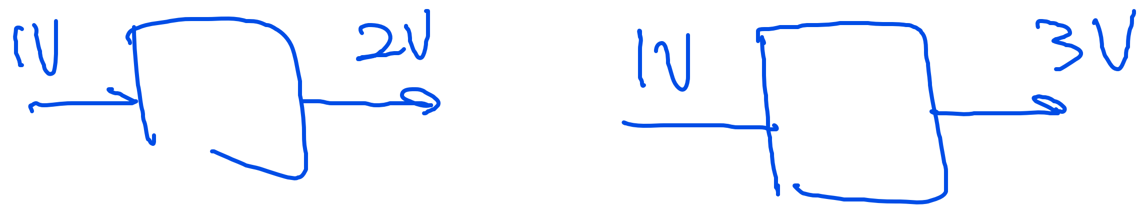
$$\left.\begin{array}{l} \boxed{\alpha x_1(t_0)} \\ \alpha u_1(t), \quad t \geq t_0 \end{array}\right\} \longrightarrow \alpha y_1(t), \quad t \geq 0$$

homogeneity

Total response = zero-input response (due to x_i) + zero-state response (due to u_i)

Time Invariance

- If characteristics of a system do not change with the time $\Rightarrow$ time invariant.

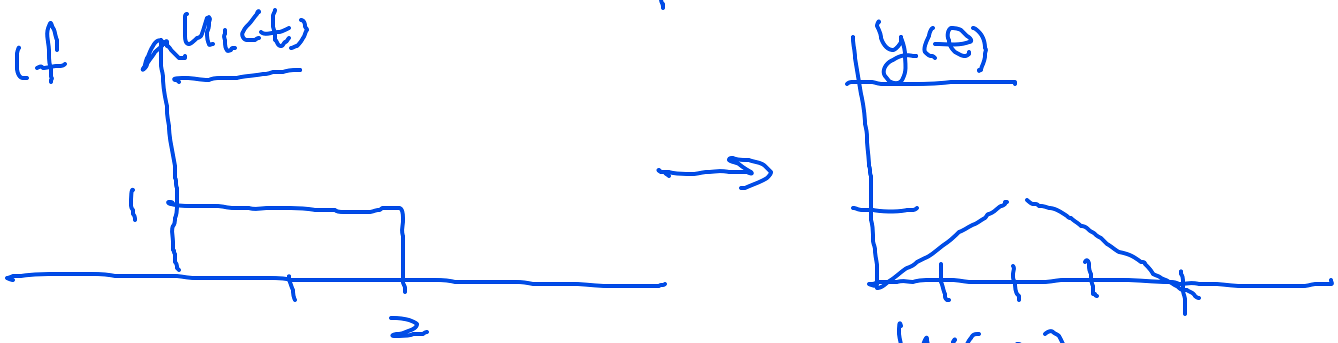


$$\left.\begin{array}{l} x(t_0) \\ u(t),\ t \ge t_0 \end{array}\right\} \rightarrow y(t),\ t \ge t_0$$

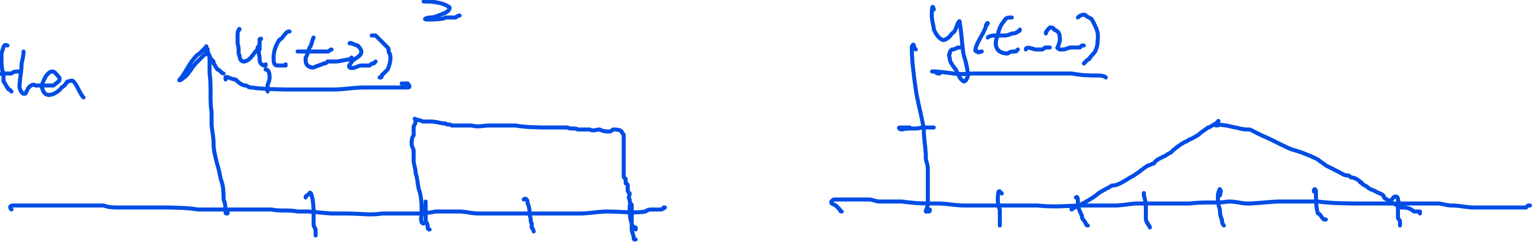
$$\Rightarrow \left.\begin{array}{l} x(t_0 + t_1) \\ u(t - t_1),\ t \ge t_0 + t_1 \end{array}\right\} \rightarrow y(t - t_1)$$

Consider a LTI system (Linear Time Invariant)

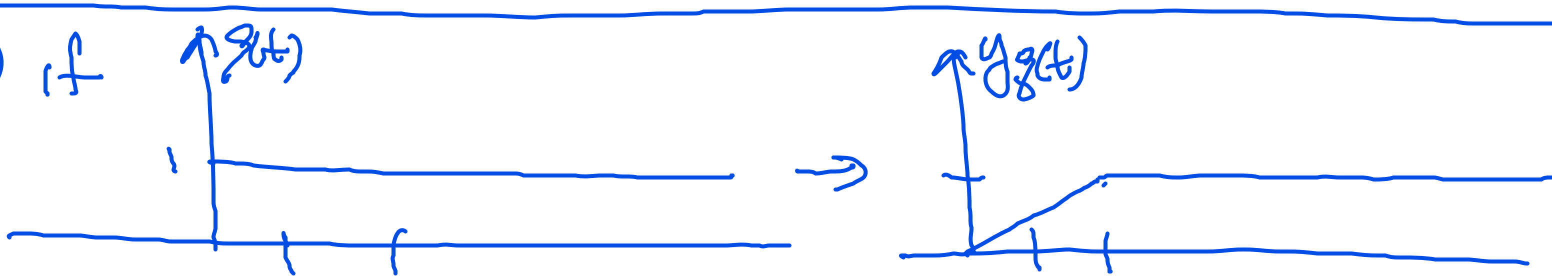
① if

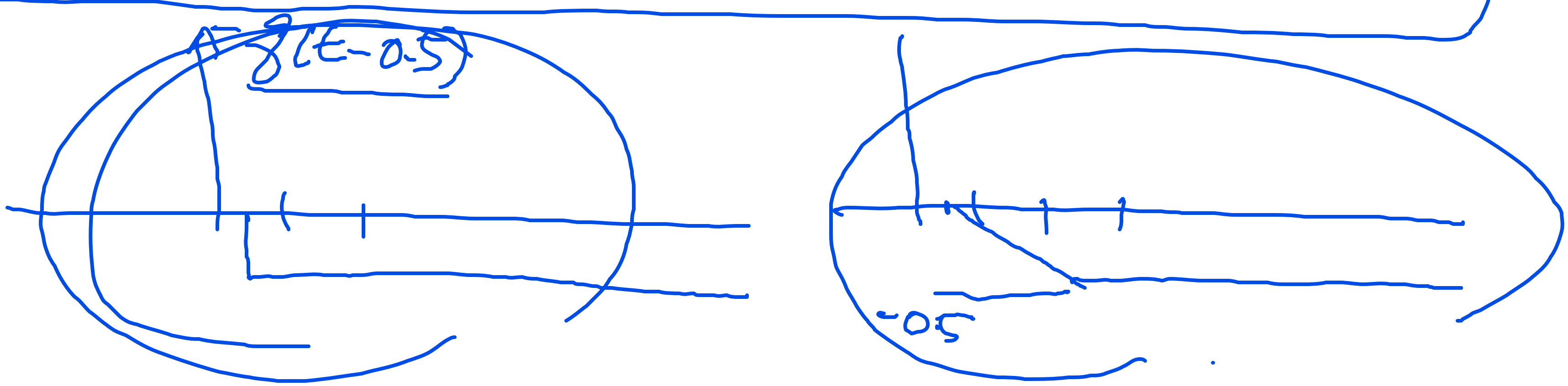


then



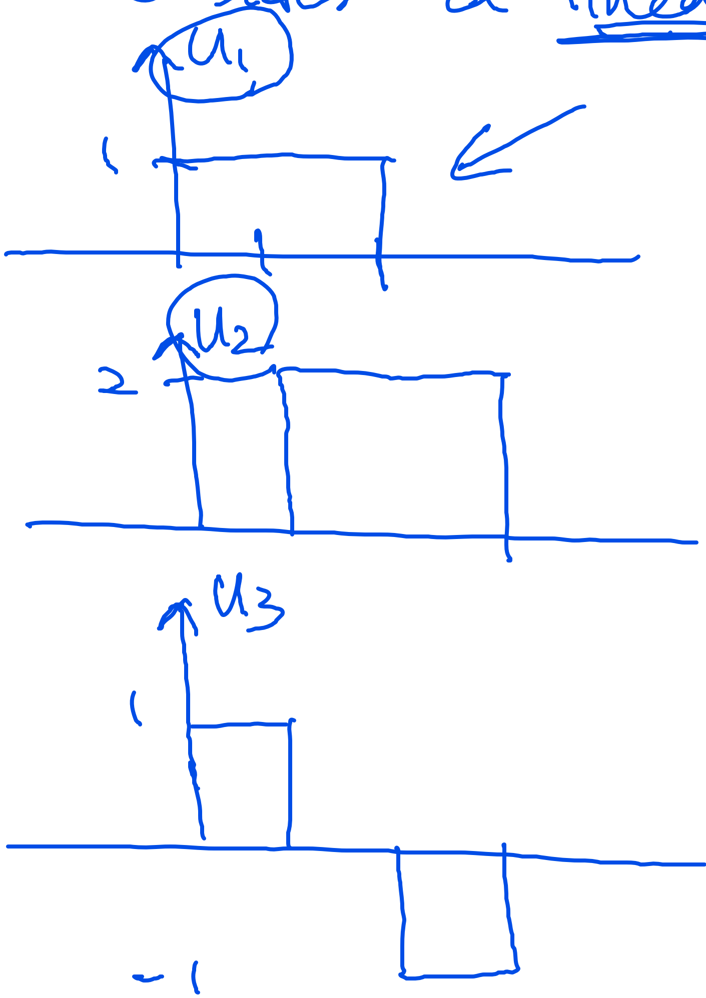
② if





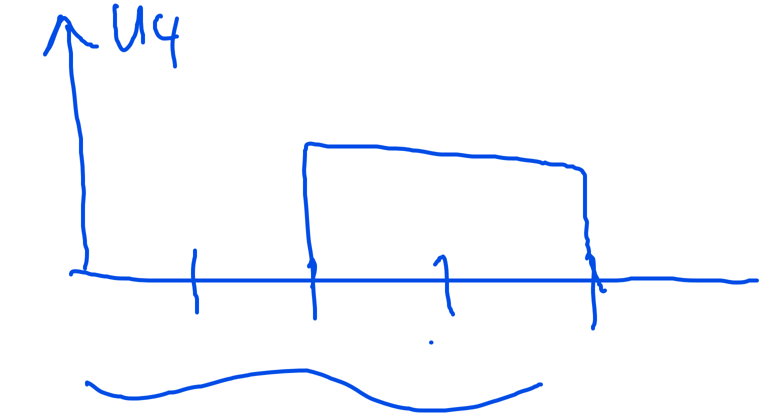
Linear Time Invariance

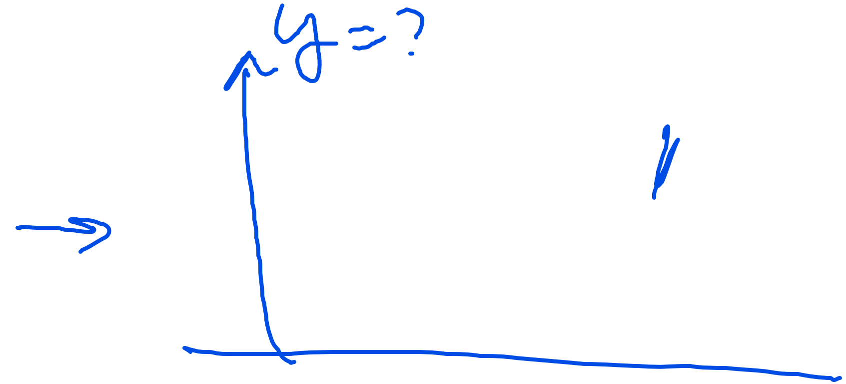
Consider a linear system (not necessarily time invariant)



$\Rightarrow$ $\underline{u_1 \to \frac{1}{2} u_2}$ $\underline{y_2 = y_1 - \frac{1}{2} y_2}$

But





output can be obtained by $y_1(t-2)$ off the system is time invariant

Consider $y(t) = \int_{t_0}^{t} u(\tau)\,d\tau + y(t_0)$

$$y_1(t) = \int_{t_0}^{t} \underline{u_1}(\tau)\,d\tau + y_1(t_0)$$

$$y_2(t) = \int_{t_0}^{t} u_2(\tau)\,d\tau + y_2(t_0)$$

$$y_1(t) + y_2(t) = \int_{t_0}^{t} [u_1(\tau) + u_2(\tau)]\,d\tau + y_1(t_0) + y_2(t_0)$$

$$= \int_{t_0}^{t} [u_1(\tau) + u_2(\tau)]\,d\tau + y(t_0)$$

$\Rightarrow$ <u>linear</u>