SAMPLE PROBLEM SET 1

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[1] A uniform plane electromagnetic wave, propagating along the +z direction in a dielectric medium with a phase velocity $V_{ph} = 1.5 \times 10^8\ m/s$, is linearly polarized along x. Given that the wave frequency is 1 GHz and that the wave electric field has a peak value of 10^{-3} V/m at $t=0$ and z=0.1m, obtain

i) the instantaneous expression for the wave electric field for any t and z; and
ii) the locations where the wave electric field is a positive maximum when $t=10^{-9}s$.

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[2] Obtain the wavelength of uniform plane EM wave in vacuum for the following frequencies:

i) 60Hz (power frequency), ii) 535-1605 kHz (AM radio) iii) 88-106MHz (FM radio) iv)4-8 GHz (C Band, used in satellite communication), v) 7.5×10^{14} - 1.75×10^{15} Hz (visible light), vi) 10^{18} Hz (typical frequency for x-rays).

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[3] As an inverse to problem [2], find the frequencies of uniform plane electromagnetic waves in vacuum for the following wavelengths:

i) 1 km; ii) 1m; iii) 1cm, iv) 1mm, (v) 1 μm; vi) 1nm; v)1 Angstrom (=10^{-10}m or 0.lnm).

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[4] The electric field vector of a uniform plane EM wave propagating in vacuum is given by the phasor expression

$$\vec{E}(z) = -j2\, e^{jkz}\, \mathbf{i}_x + 4e^{jkz}\, \mathbf{i}_y$$

i) Write down the instantaneous field expression for the electric field

ii) What is the locus of the tip of the instantaneous electric field vector E(z,t) in the xy plane, i.e., what is the wave polarization?

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[5] The instantaneous expression for the electric field in a uniform plane electromagnetic wave propagating in a dielectric medium is

$$\mathbf{E}(z,t) = \mathbf{i}_x \cos[2\pi.10^9 t - (200\pi/3)z - 60^0],\ \text{mV/m}.$$

Find:

1) the frequency of the wave in Hz
2) the wavelength of the wave in m
3) phase velocity Vph
4) relative refractive index n_r of the medium; and
5) the polarization of the wave

SOLUTION TO SAMPLE PROBLEM SET 1

[1] $E_x = E_0 \cos(\omega t - kz + \phi)$

i) $f = 1\text{ GHz} = 10^9\text{ Hz} \rightarrow \omega = 2\pi f = 2\pi \cdot 10^9\text{ Hz}$

$v_{ph} = \frac{\omega}{k}$, $\therefore k = \frac{\omega}{v_{ph}} = \frac{2\pi \cdot 10^9\text{ rad/s}}{1.5 \times 10^8\text{ m/s}} = \frac{20\pi}{1.5}\ m^{-1} = \frac{40\pi}{3}\ m^{-1}$

Also $E_0 = 10^{-3}$ V/m (given)

The only unknown now is ϕ which is found using the initial condition

$E_x(\frac{1}{10}m, 0) = E_0 \rightarrow 1 = \cos(-k \cdot \frac{1}{10} + \phi)$

Lowest soln is $-\frac{k}{10} + \phi = 0 \rightarrow \phi = \frac{k}{10} = \frac{40\pi}{3} \cdot \frac{1}{10} = \frac{4\pi}{3}$

$\therefore$ Desired soln is $E_x(z,t) = 10^{-3} \cos\left[2\pi \cdot 10^9 t - \frac{40\pi z}{3} + \frac{4\pi}{3}\right]$ V/m <u>Answer</u>

ii) $E_x(z, 10^{-9}s) = 10^{-3} \cos\left[2\pi - \frac{40\pi z}{3} + \frac{4\pi}{3}\right] = 10^{-3} \cos\left[\frac{10\pi}{3} - \frac{40\pi z}{3}\right]$ V/m

The cosine has a maximum positive value when

$$\frac{10\pi}{3} - \frac{40\pi z}{3} = 0, 2\pi, 4\pi, \ldots$$

or $\frac{40z}{3} = \frac{10}{3}, \frac{10}{3} \mp 2, \frac{10}{3} \mp 4, \ldots$

yielding

$$z = \frac{1}{4}, \frac{1}{10}, \frac{2}{5}, -\frac{1}{20}, \frac{11}{60}, \ldots \text{ m}$$ <u>Answer</u>

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[2] $f\lambda = c \rightarrow f = 60\text{ Hz}, \lambda = \frac{3\times10^8}{60} = 5\times10^6$ m

$f = 535-1605\text{ kHz} \rightarrow \lambda = 1.87\times10^2 \rightarrow 5.6\times10^2$ m

$f = 88-106\text{ MHz} \rightarrow \lambda = 2.83 \rightarrow 3.41$ m

$f = 4-8\text{ GHz} \rightarrow \lambda = 3.75\times10^{-2} \rightarrow 7.50\times10^{-2}$ m

$f = 7.5\times10^{14} - 1.75\times10^{15}\text{ Hz} \rightarrow \lambda = 1.71\times10^{-7} \rightarrow 4.00\times10^{-7}$ m

$f = 10^{18} \rightarrow \lambda = 3\times10^{-10}$ m

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[3] $f\lambda = c \rightarrow \lambda = 1\text{ km}, f = \frac{3\times10^8\text{ m/s}}{1\times10^3\text{ m}} \rightarrow 3\times10^5$ Hz

$\lambda = 1\text{m}, f = 3\times10^8$ Hz

$\lambda = 1\text{cm}, f = 3\times10^{11}$ Hz

$\lambda = 1\text{mm}, f = 3\times10^{14}$ Hz

$\lambda = 10^{-6}\text{m} \rightarrow f = 3\times10^{18}$ Hz

$\lambda = 10^{-9}\text{m} \rightarrow f = 3\times10^{21}$ Hz

$\lambda = 10^{-10}\text{m} \rightarrow f = 3\times10^{22}$ Hz

[4]. $\underline{\bar{E}}(z) = -j2\, e^{jkz}\, \underline{i}_x^o + 4 e^{jkz}\, \underline{i}_y^o$

i) $\underline{E}(z,t) = Re\left[\underline{\bar{E}}(z)\, e^{j\omega t}\right]$

$= 2 \sin(\omega t + kz)\, \underline{i}_x^o + 4 \cos(\omega t + kz)\, \underline{i}_y^o$ Answer

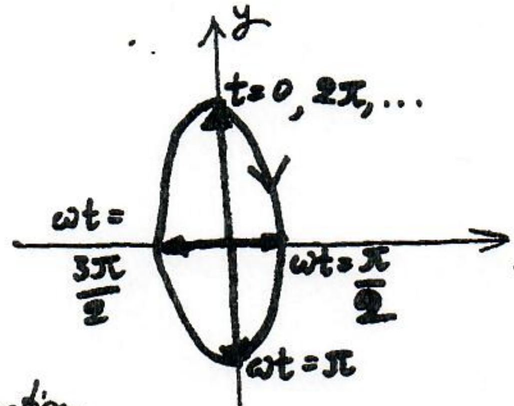
ii) Take $\underline{E}(0,t) = 2 \sin \omega t\, \underline{i}_x^o + 4 \cos \omega t\, \underline{i}_y^o$

$\underline{E}(0,0) = \underline{i}_y^o\, 4$

$\underline{E}(0,\frac{T}{4}) = \underline{i}_x^o 2$

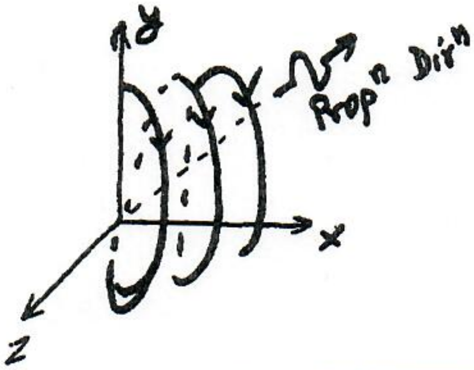
$\underline{E}(0,\frac{T}{2}) = -\underline{i}_y^o\, 4$

$\underline{E}(0,\frac{3T}{4}) = -\underline{i}_x^o\, 2$



Signal is right elliptically polarized about the $-z$ direction of propagation

Answer



[5] $\underline{E}(z,t) = \underline{i}_x^o\, E_x(z,t)$ where

$E_x(z,t) = E_o \cos(\omega t - kz - 60^o)\ mV/m$

where $\omega = 2\pi . 10^9$ rad/s, $k = \frac{200\pi}{3}\ \frac{rad}{m}$, $E_o = 1\ \frac{mV}{m}$

Answer

i) $f = \omega/2\pi = 10^9\ Hz = 1\ GHz$

ii) $k = 2\pi/\lambda \rightarrow \lambda = 2\pi/k = \cancel{2\pi} . \frac{3}{\cancel{200\pi}}\ m = 0.03\, m$ Answer

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iii) $v_{ph} = \frac{\omega}{k} = \frac{2\pi . 10^9\ rad/s}{\frac{200\pi}{3}\ rad/m} = \frac{3 \times 10^9}{100}\ m/s = 3 \times 10^7\ m/s$ Answer

iv) n_r = (vel of EMW in vacuum)/(vel of EMW in dielectric), i.e.,

n_r is "a velocity reduction factor" relative to vel of EMW in vacuum

i.e., $\therefore$ vel of EMW in dielectric = $\frac{\text{vel of EMW in vacuum}}{n_r}$

$\therefore n_r = \frac{3 \times 10^8\ m/s}{3 \times 10^7\ m/s} = 10$ Answer

v) The UPEMW is linearly polarized along x Answer