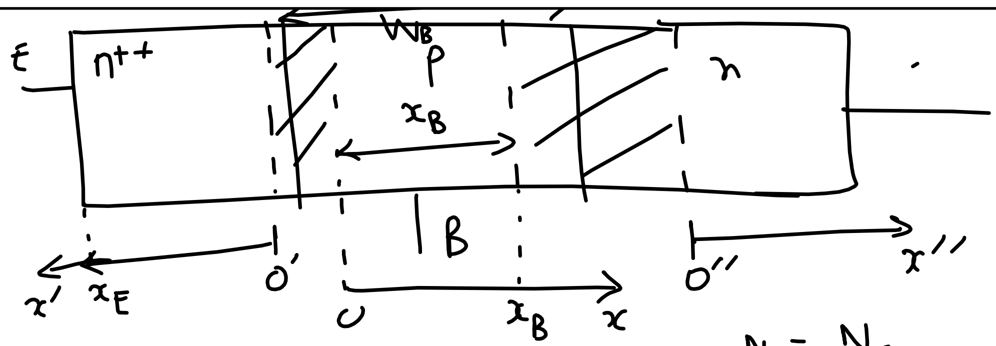


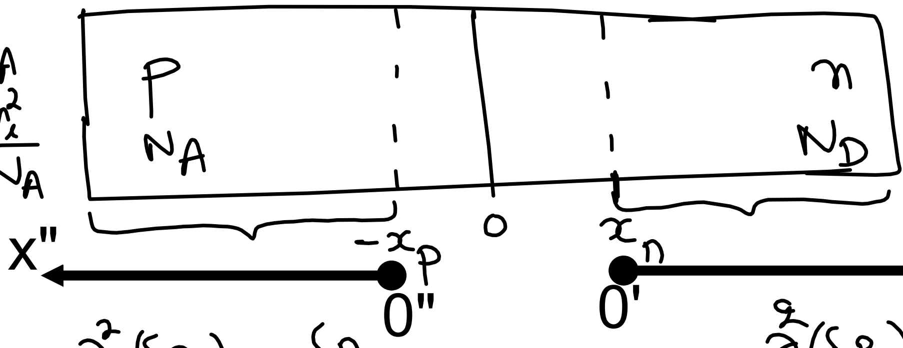

$N_D = N_E$
$L_p = L_E$
$D_p = D_E$
$\tau_p = \tau_E$
$p_{n0} = p_{E0} = \frac{n_i^2}{N_E}$

$N_A = N_B$
$L_n = L_B$
$D_n = D_B$
$\tau_n = \tau_B$
$n_{p0} = n_{B0} = \frac{n_i^2}{N_B}$

$N_D = N_C$
$L_p = L_C$
$D_p = D_C$
$\tau_p = \tau_C$
$p_{n0} = p_{C0} = \frac{n_i^2}{N_C}$

$p_{p_0} \simeq N_A$

$n_{p_0} = \dfrac{n_i^2}{N_A}$




$n_{n0} \simeq N_D$

$p_{n0} = \dfrac{n_i^2}{N_D}$

$$0 = D_n \frac{\partial^2 (\delta n_p)}{\partial x^2} - \frac{\delta n_p}{\tau_n} \qquad 0 = D_p \frac{\partial^2 (\delta p_n)}{\partial x^2} - \frac{\delta p_n}{\tau_p}$$

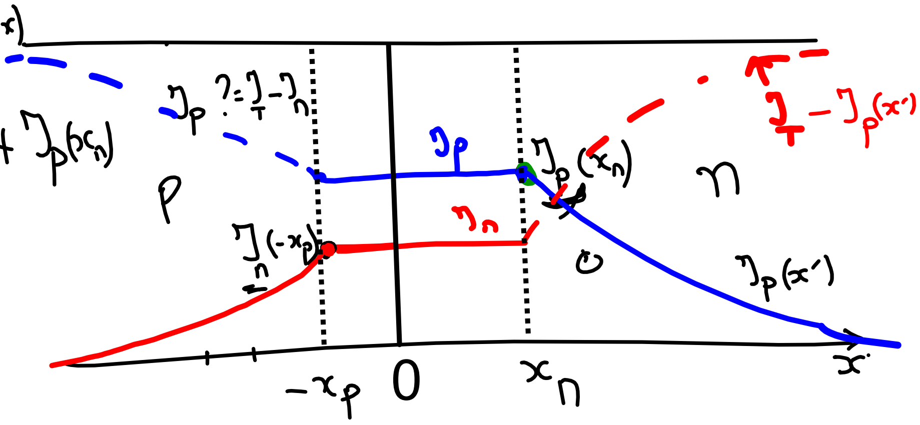
$$\delta n_p(x'') = n_{po}\left(e^{eV_a/kT} - 1\right) e^{-x''/L_n} \qquad \delta p_n(x') = p_{no}\left(e^{\frac{eV_a}{kT}} - 1\right) e^{-x'/L_p}$$

$$J_n(x'') = -e\frac{D_n}{L_n} n_{po}\left(e^{\frac{eV_a}{kT}} - 1\right) e^{-x''/L_n} \qquad J_p(x') = \frac{eD_p}{L_p} p_{no}\left(e^{\frac{eV_a}{kT}} - 1\right) e^{-x'/L_p}$$

$$J_T = \underbrace{\left[\frac{eD_p}{L_p} p_{no} + \frac{eD_n}{L_n} n_{p_0}\right]}_{J_s}\left(e^{eV_a/kT} - 1\right)$$

$J_T = J_n(x) + J_p(x)$

$J_T = J_n(-x_p) + J_p(x_n)$

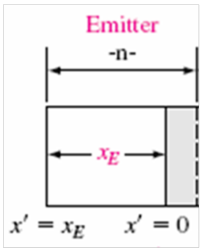


$$EB \begin{cases} J_E = J_{p_E}(0') + J_{nE}(0) \\ J_E = \pm e D_E \left.\frac{\partial(\delta p_E)}{\partial x}\right|_{x'=0} + e D_B \left.\frac{\partial(\delta n_B)}{\partial x}\right|_{x=0} \end{cases}$$

$$BC \begin{cases} J_C = J_{nC}(x_B) + J_{pC}(0'') \\ \quad = e D_B \left.\frac{d(\delta n_B)}{dx}\right|_{x_B} - e D_C \left.\frac{d(\delta p_C)}{\partial x}\right|_{x''=0} \end{cases}$$

$$J_B = J_E - J_C$$

## Emitter Region




**1) Continuity equation:** $$\frac{\partial^2(\delta p_E(x'))}{\partial x'^2} - \frac{\delta p_E(x')}{L_B^2} = 0$$

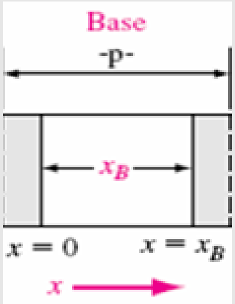
General solution: $$\delta p_E(x') = Ae^{\frac{x'}{L_E}} + Be^{\frac{-x'}{L_E}}$$

**2) Boundary conditions:**

$$\delta p_E(0') = p_{E0}\left[e^{\left(\frac{eV_{BE}}{kT}\right)} - 1\right]$$

$$\delta p_E(x_E) = 0$$

# Base Region




**1) Continuity equation:** $$\frac{\partial^2[\delta n_B(x)]}{\partial x^2} - \frac{\delta n_B(x)}{L_B^2} = 0$$

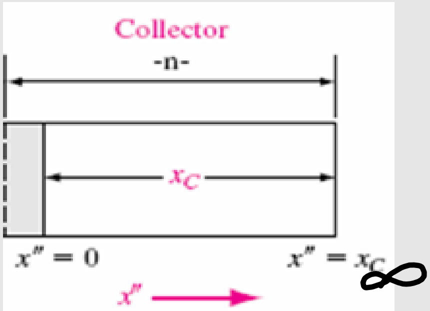
General solution: $$\delta n_B(x) = Ce^{\frac{x}{L_B}} + De^{\frac{-x}{L_B}}$$

**2) Boundary conditions:**

$$\delta n_B(0) = n_{B0}\left[e^{\left(\frac{eV_{BE}}{kT}\right)} - 1\right]$$

$$\delta n_B(x_B) = n_{B0}\left(e^{\frac{eV_{BC}}{kT}} - 1\right)$$

# Collector Region




**1) Continuity equation:** $\dfrac{\partial^2(\delta p_C(x''))}{\partial x''^2} - \dfrac{\delta p_C(x'')}{L_C^2} = 0$

General solution: $\delta p_C(x'') = Ee^{\frac{x''}{L_C}} + Fe^{\frac{-x''}{L_C}}$

**2) Boundary conditions:**

$$\delta p_C(0'') = p_{C0}\left(e^{\frac{eV_{BC}}{kT}} - 1\right)$$

$$\delta p_C(\infty) = 0$$

## Summary

$$J_{pE}(0') = -\frac{eD_E p_{E0}}{L_E}\left[e^{\frac{eV_{BE}}{kT}} - 1\right]\coth\left(\frac{x_E}{L_E}\right)$$

$$J_{nE}(0) = -\frac{eD_B n_{B0}}{L_B}\left\{\frac{\left[e^{\frac{eV_{BE}}{kT}} - 1\right]}{\tanh\left(\frac{x_B}{L_B}\right)} - \frac{\left(e^{\frac{eV_{BC}}{kT}} - 1\right)}{\sinh\left(\frac{x_B}{L_B}\right)}\right\}$$

$$J_{nC}(x_B) = -\frac{eD_B n_{B0}}{L_B}\left\{\frac{\left[e^{\frac{eV_{BE}}{kT}} - 1\right]}{\sinh\left(\frac{x_B}{L_B}\right)} - \left(e^{\frac{eV_{BC}}{kT}} - 1\right)\coth\left(\frac{x_B}{L_B}\right)\right\}$$

$$J_{pC}(0'') = eD_C \frac{p_{C0}}{L_C}\left(e^{\frac{eV_{BC}}{kT}} - 1\right)$$

$$J_E = J_{pE}(0') + J_{nE}(0)$$

$$J_E = -\frac{eD_E p_{E0}}{L_E}\left[e^{\frac{eV_{BE}}{kT}} - 1\right]\coth\left(\frac{x_E}{L_E}\right) - \frac{eD_B n_{B0}}{L_B}\left\{\frac{\left[e^{\frac{eV_{BE}}{kT}} - 1\right]}{\tanh\left(\frac{x_B}{L_B}\right)} - \frac{\left(e^{\frac{eV_{BC}}{kT}} - 1\right)}{\sinh\left(\frac{x_B}{L_B}\right)}\right\}$$

$$J_C = J_{nC}(x_B) + J_{pC}(0'')$$

$$J_C = -\frac{eD_B n_{B0}}{L_B}\left\{\frac{\left[e^{\frac{eV_{BE}}{kT}} - 1\right]}{\sinh\left(\frac{x_B}{L_B}\right)} - \left(e^{\frac{eV_{BC}}{kT}} - 1\right)\coth\left(\frac{x_B}{L_B}\right)\right\} + eD_C \frac{p_{C0}}{L_C}\left(e^{\frac{eV_{BC}}{kT}} - 1\right)$$

$J_B = J_E - J_C$

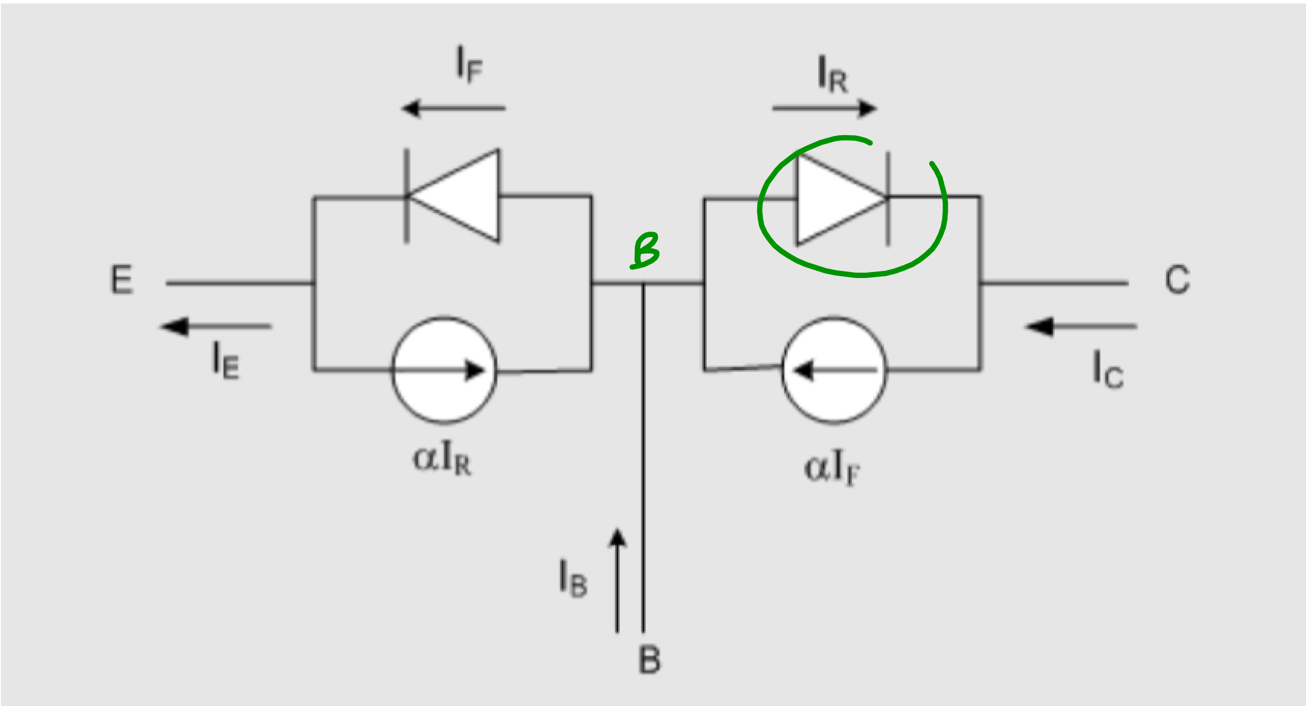
$$J_E \approx -\frac{eD_E p_{E0}}{x_E}\left[e^{\frac{eV_{BE}}{kT}}-1\right]-\frac{eD_B n_{B0}}{x_B}\left[e^{\frac{eV_{BE}}{kT}}-1\right]+\frac{eD_B n_{B0}}{x_B}\left(e^{\frac{eV_{BC}}{kT}}-1\right)$$

$$J_E = \underbrace{-e\left[\frac{D_E p_{E0}}{x_E}+\frac{D_B n_{B0}}{x_B}\right]\left[e^{\frac{eV_{BE}}{kT}}-1\right]}+\underbrace{\frac{eD_B n_{B0}}{x_B}\left(e^{\frac{eV_{BC}}{kT}}-1\right)}$$

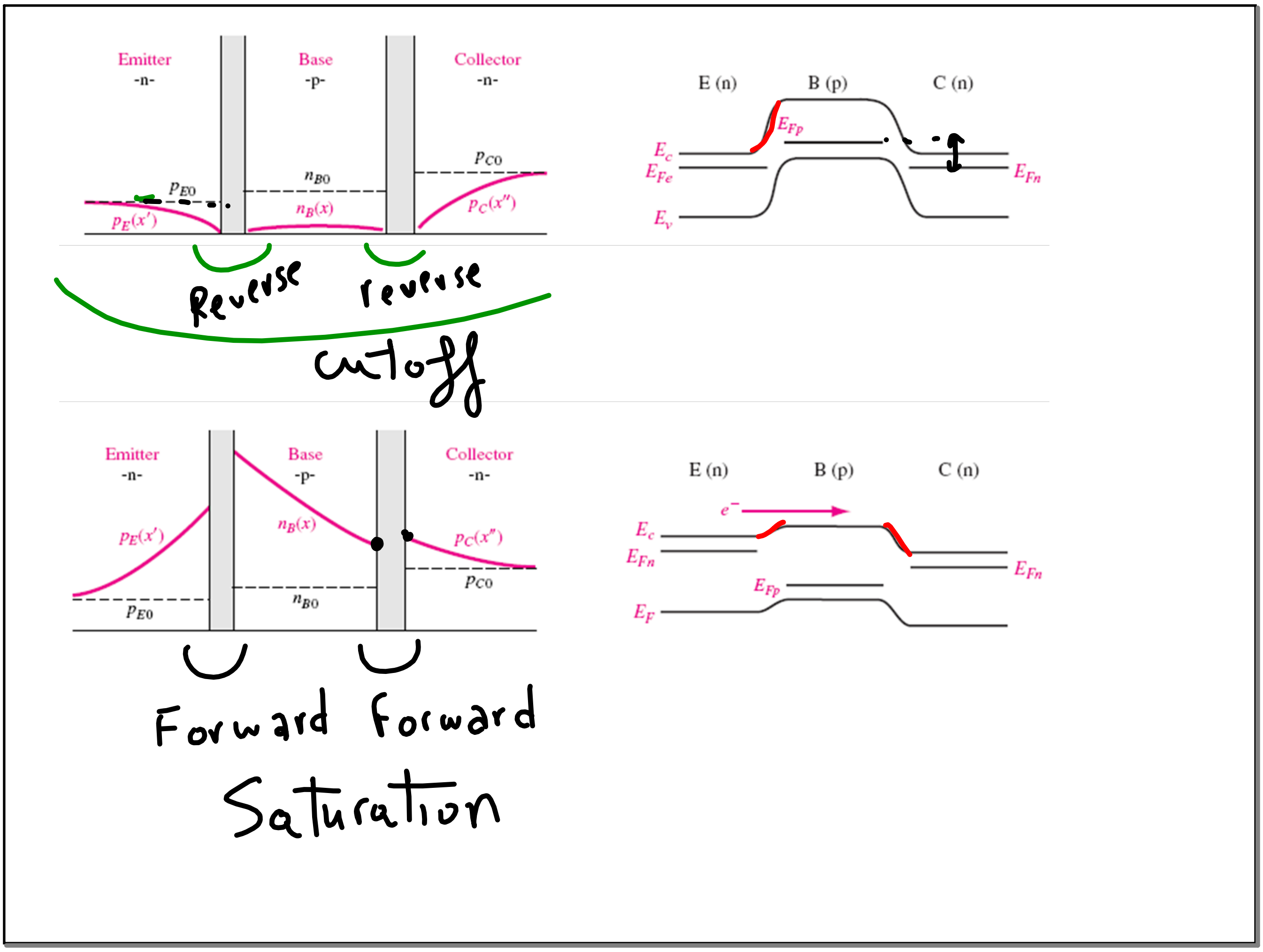
$$J_C \approx -\frac{eD_B n_{B0}}{x_B}\left\{\left[e^{\frac{eV_{BE}}{kT}}-1\right]-\left(e^{\frac{eV_{BC}}{kT}}-1\right)\right\}+eD_C\frac{p_{C0}}{L_C}\left(e^{\frac{eV_{BC}}{kT}}-1\right)$$

$$J_C \approx \underbrace{-\frac{eD_B n_{B0}}{x_B}\left[e^{\frac{eV_{BE}}{kT}}-1\right]}+\underbrace{e\left\{D_B\frac{n_{B0}}{x_B}+D_C\frac{p_{C0}}{L_C}\right\}\left(e^{\frac{eV_{BC}}{kT}}-1\right)}$$

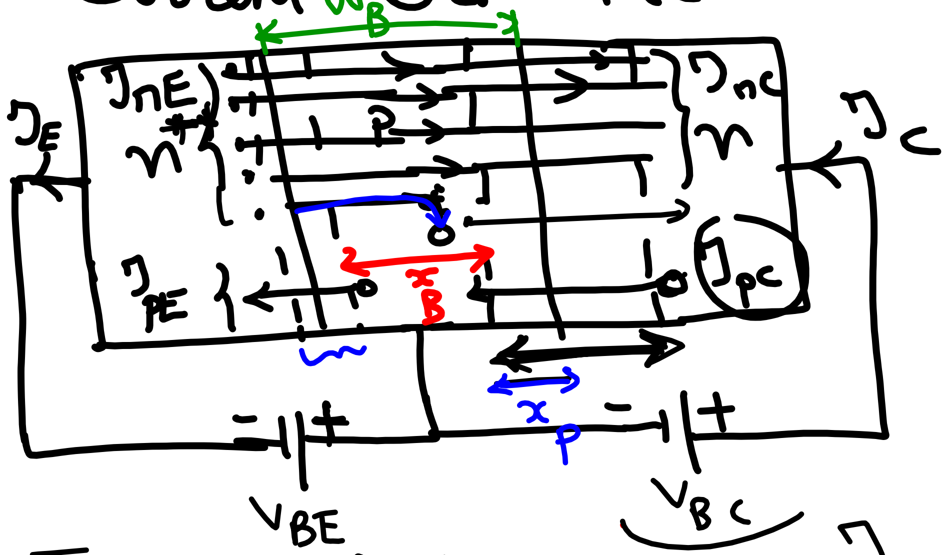
$$J_B = J_E - J_C$$




Ebers-Moll Equivalent Circuit Model for npn BJT


Emitter
-n-
Base
-p-
Collector
-n-
$p_{E0}$
$n_{B0}$
$p_{C0}$
$p_E(x')$
$n_B(x)$
$p_C(x'')$
Reverse
reverse
cutoff
E (n)
B (p)
C (n)
$E_c$
$E_{Fe}$
$E_v$
$E_{Fp}$
$E_{Fn}$
Emitter
-n-
Base
-p-
Collector
-n-
$p_E(x')$
$n_B(x)$
$p_C(x'')$
$p_{E0}$
$n_{B0}$
$p_{C0}$
Forward
Forward
Saturation
E (n)
B (p)
C (n)
$e^-$
$E_c$
$E_{Fn}$
$E_{Fp}$
$E_F$
$E_{Fn}$

# Current Gain Factors




Base Transport factor $\alpha_T \equiv \dfrac{J_{nc}}{J_{nE}} \lesssim 1$

Emitter injection Efficiency $\gamma \equiv \dfrac{J_{nE}}{J_{nE} + J_{pE}} \lll 1$

DC Common base current gain

$$\alpha \equiv \frac{I_{nC}}{I_E} \Rightarrow I_{nC} = \alpha I_E$$

$$I_C = I_{nC} + I_{pC} = \alpha I_E + I_{pC}$$

$$I_C \equiv \alpha I_E + I_{CBO}$$

common base leakage current

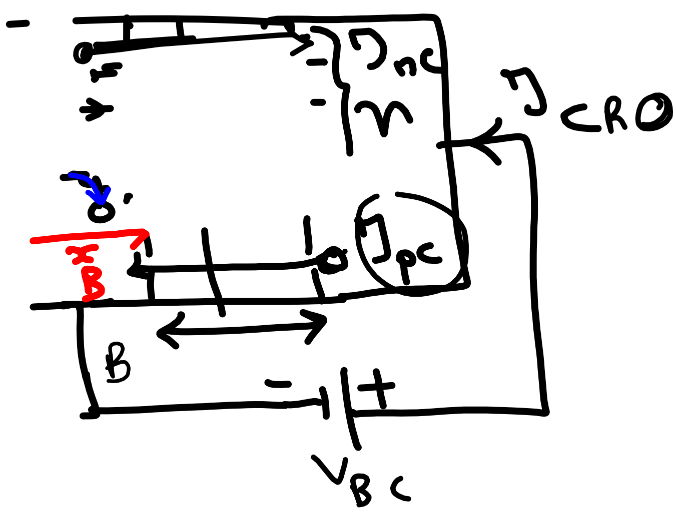
If $I_E = 0$, $I_C = I_{CBO}$ : saturation current

DC Common Emitter current gain

$$\beta \equiv \frac{I_C}{I_B}$$

$$\beta = \frac{I_C}{I_B} = \frac{I_C}{I_E - I_C} = \frac{(I_C/I_E)}{1 - \left(\frac{I_C}{I_E}\right)} \simeq \frac{\alpha}{1-\alpha}$$

$$\boxed{\beta = \frac{\alpha}{1-\alpha}}$$



$J_{nC}$
$n$
$J_{CBO}$
$J_{pC}$
$x_B$
B
$V_{BC}$

$$\gamma = \frac{1}{1 + \frac{N_B}{N_E}\frac{D_E}{D_B}\frac{x_B}{x_E}} = \frac{J_{nE}}{J_{nE} + J_{pE}}$$

We want $\gamma \to 1$, we need

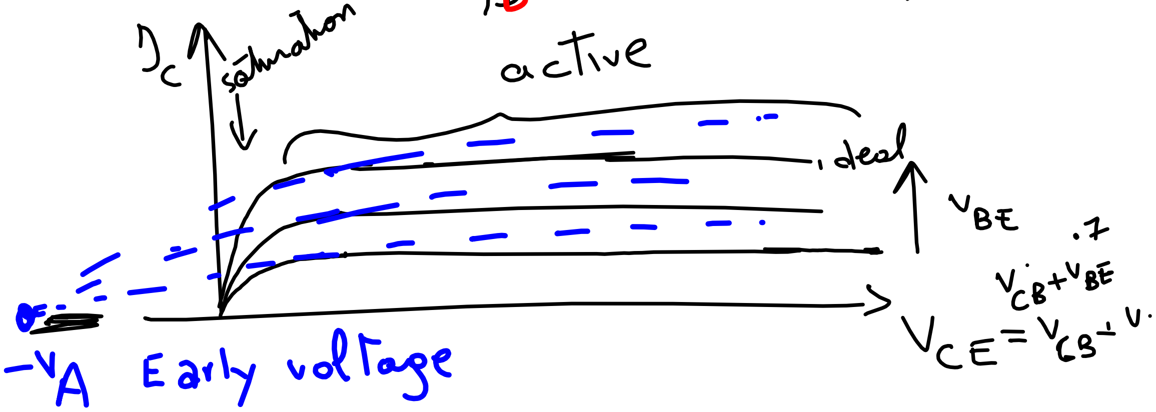
$\frac{N_B}{N_E} << 1 \Rightarrow N_B << N_E$

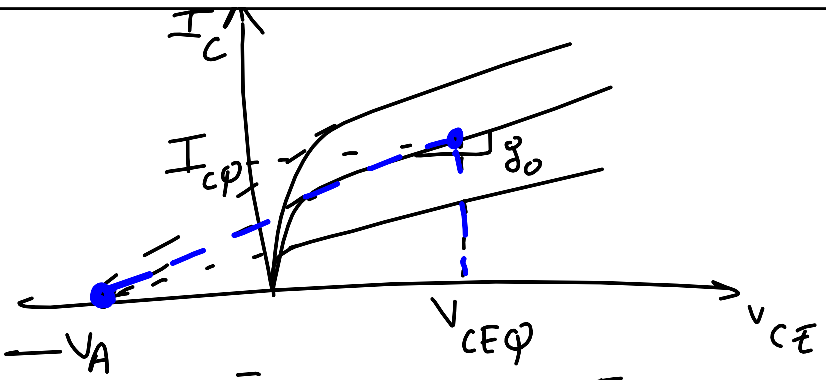
$\frac{x_B}{x_E} << 1 \Rightarrow x_B << x_E \Rightarrow$ base to be narrow

# Nonidealities

## 1) Base Width Modulation Effect

$$J_C \simeq \ominus e \frac{D_B n_{BO}}{x_B}\left(e^{V_{BE}/V_t} - 1\right)$$





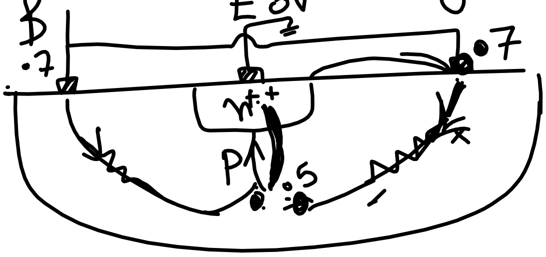

$$g_o = \frac{\Delta I_C}{\Delta V_{CE}} = \frac{I_{CQ} - 0}{V_{CEQ} - (-V_A)} = \frac{I_{CQ}}{V_{CEQ} + V_A}$$

$$g_o \simeq \frac{I_{CQ}}{V_A}$$

$$r_o = \frac{1}{g_o} = \frac{V_A}{I_{CQ}}$$ output resistance

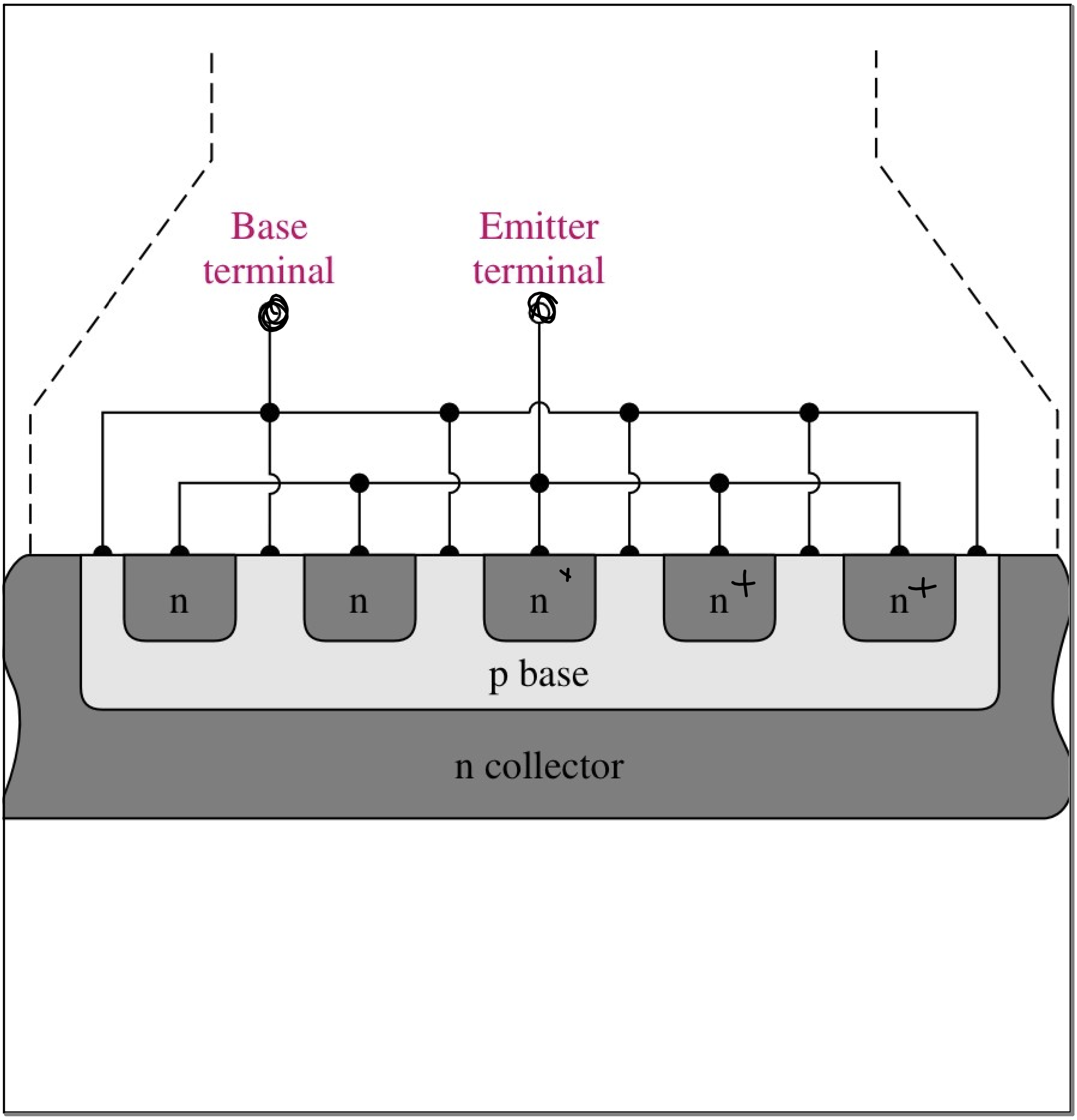
As $V_{CE} \uparrow \Rightarrow V_{CB} \uparrow \Rightarrow x_B \downarrow \Rightarrow I_C \uparrow$

2) Current Crowding




$V_{BE}(\text{edges}) > V_{BE}(\text{center}) \Rightarrow$

$I_E(\text{edges}) > I_E(\text{center})$


Base
terminal
Emitter
terminal
n
n
n
n+
n+
p base
n collector

Breakdown

1) Punch through

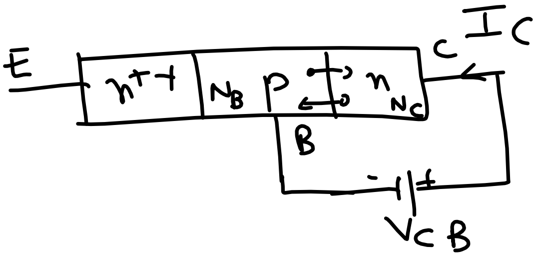
$$x_p = \left[ \frac{2\epsilon_s}{e} \frac{(V_{bi} - V_a)}{N_B} \frac{N_C}{(N_B + N_C)} \right]^{1/2}$$

At punchthrough, $x_p = W_B$ ; $-V_a \equiv V_{PT}$

$$V_{PT} \simeq \frac{e W_B^2}{2\epsilon_s} \frac{N_B (N_C + N_B)}{N_C}$$

2) Avalanche

a) Common Base

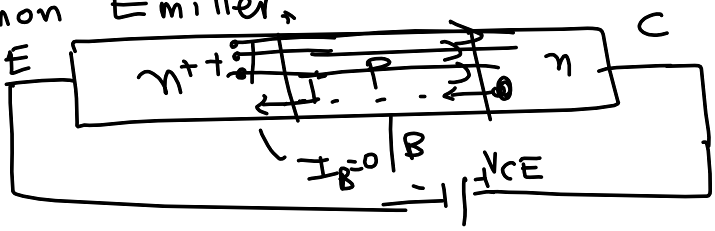



$$I_C = \alpha I_E + I_{CBO}$$

$$BV_{CBO} = \frac{\epsilon_s \mathcal{E}_{cr}^2}{2eN_C}$$

Assume $N_B \gg N_C$

b) Common Emitter



$$BV_{CEO} = \frac{BV_{CBO}}{\sqrt[n]{\beta}} \quad ; \; 3 < n \leq 6$$