[Problem 1.] Score: 6 points

Urn I contains 2 red (R) and 8 blue (B) balls. Urn II contains 9 red and 6 blue balls. We select at random one of the two urns and choose a ball at random from it. Find the probability that the selected ball is red.

Solution 1

$$P(R) = \frac{2}{10} \times \frac{1}{2} + \frac{9}{15} \times \frac{1}{2}$$

$$= \frac{2}{5}$$

Instruction Comments

The above presented solution will earn you 2 points out of 6 points, for lack of justification.

Solution 2

We utilize the total probability theorem:

$$P(R) = P(R|U_I) \cdot P(U_I) + P(R|U_{II}) \cdot P(U_{II}),$$

where U_I and U_{II} denote urns I and II, respectively;

$P(R|U_I)$ – probability of selecting a red ball from U_I.

$P(R|U_{II})$ – probability of selecting a red ball from U_{II}.

$P(U_I)$ – probability of selecting U_I

$P(U_{II})$ – probability of selecting U_{II}

We also have the following probabilities

$P(R|U_I) = \frac{2}{10}$ 2R out of 10 total

$P(R|U_{II}) = \frac{9}{15}$ 9R out of 15 total

$P(U_I) = P(U_{II}) = 1/2$, given

Therefore, the required probability is

$$P(R) = \frac{2}{10} \times \frac{1}{2} + \frac{9}{15} \times \frac{1}{2}$$

$$= \frac{2}{5}.$$

Instructor Comments

This is the correct solution with justification and will earn you 6 points.

[Problem 2] Score 6 points

Let X be a normal random variable with parameters $\mu = 3$ and $\sigma^2 = 4$. Calculate the probability $P(3 < X < 4)$.

Solution 1

$$P(3 < X < 4) = \Phi(0.5) - \Phi(0)$$

From the CDF $\Phi(\cdot)$ tables,

$$\Phi(0) = 0.5$$

$$\Phi(0.5) = 0.6915$$

Therefore,

$$P(3 < X < 4) = 0.6915 - 0.5$$
$$= 0.1915$$

Instructor Comments:

This solution lacks justification (several missing steps) and will earn you 2.5 points out of 6 points.

$\boxed{\text{Solution 2}}$

To determine the required probability, we utilize the transformation of variables

$$Z = \frac{X - \mu}{\sigma}.$$

Therefore,

$$P(3 < X < 4)$$

$$= P\left(\frac{3-\mu}{\sigma} < Z < \frac{4-\mu}{\sigma}\right)$$

$$= \Phi\left(\frac{4-\mu}{\sigma}\right) - \Phi\left(\frac{3-\mu}{\sigma}\right),$$

where $\Phi(\cdot)$ is the CDF of standard normal distribution.

Given $\mu = 3$, $\sigma = 2$, this gives

$$P(3 < X < 4) = \Phi\left(\frac{4-3}{2}\right) - \Phi\left(\frac{3-3}{2}\right)$$

$$= \Phi(0.5) - \Phi(0)$$

From the tables of $\Phi(\cdot)$,

$$\Phi(0) = 0.5$$

$$\Phi(0.5) = 0.6915$$

Therefore,

$$P(3 < X < 4) = 0.6915 - 0.5$$

$$= 0.1915$$

Instructor Comments

This is the correct solution with justification and will earn you 6 points.