EEO399

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ELECTROMAGNETIC WAVES & TRANSMISSION LINES

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Copy of Original Assignment

Homework 3

[1] The following information is known about a uniform plane electromagnetic wave propagating in the +x direction in a lossless non-magnetic dielectric medium of infinite extent:

i) The wave is a monochromatic or single-frequency wave with a frequency of f = 6 GHz.

ii) The relative permittivity of the dielectric medium of propagation is $\varepsilon_r = 9$.

iii) The wave is linearly polarized in the y direction.

iv) The amplitude of the electric field component of the wave is 10 V/m.

v) The value of the electric field at t = 0 and x = 0 is E(0,0) = 1 V/m.

Obtain the instantaneous and phasor expressions of the wave electric and magnetic fields

[2] A uniform plane electromagnetic wave propagates in a lossless dielectric medium of infinite extent. The electric field in the wave has the *instantaneous* expression

$\mathbf{E}(\mathbf{r},t) = (\mathbf{i_y} + \mathbf{i_z}\sqrt{3})\ \cos(24\pi.\ 10^9 t + 160\sqrt{3}\pi y - 160\pi z + 20^0)$, V/m.

i) What is the wavelength of the wave in m?

ii) What is the direction of propagation of the wave?

iii) What is the relative refractive index of the dielectric medium?

iv) What is the instantaneous expression of the wave magnetic field?

v) What is the phasor expression for the wave electric field?

[3] The electric field in a UPEMW of frequency 3 GHz propagating in vacuum is comprised of two components which are in spatial and temporal quadrature. The phasor expression of the wave electric field is given to be

$\mathbf{E}(y) = e^{-jky}\,\mathbf{i_x} - j\,10\,e^{-jky}\,\mathbf{i_z}$, V/m.

Find:

i) the instantaneous expression for the wave electric field $\mathbf{E}(y,t)$;

ii) the instantaneous expression for the wave magnetic field H(y,t);

iii) the polarization of the wave

iv) the phasor expression for the magnetic electric field.

1 Problem 1: Uniform Plane Electromagnetic Wave

Given the parameters:

- Frequency: $f = 6$ GHz
- Relative Permittivity: $\varepsilon_r = 9$
- Linear Polarization: y direction
- Amplitude of Electric Field: 10 V/m
- Initial Electric Field: $E(0,0) = 1$ V/m

The wave number is given by:

$$k = \frac{\omega}{v_p} = \frac{2\pi f}{c/\sqrt{\varepsilon_r}} = \frac{2\pi(6\times10^9)}{3\times10^8/3} = 4\pi\times10^2 \text{ rad/m} \tag{1}$$

The phase velocity is:

$$v_p = \frac{c}{\sqrt{\varepsilon_r}} = \frac{3\times10^8}{3} = 10^8 \text{ m/s} \tag{2}$$

1.1 Instantaneous Electric Field Expression

$$E(x,t) = 10\cos(\omega t - kx + \phi)\hat{y} \tag{3}$$

1.2 Phasor Electric Field Expression

$$\tilde{E}(x) = 10e^{-jkx}\hat{y} \tag{4}$$

1.3 Instantaneous Magnetic Field Expression

The intrinsic impedance of the medium is:

$$\eta = \frac{\eta_0}{\sqrt{\varepsilon_r}} = \frac{377}{3} \approx 125.67\ \Omega \tag{5}$$

$$H(x,t) = \frac{E_0}{\eta}\cos(\omega t - kx + \phi)\hat{z} \tag{6}$$

1.4 Phasor Magnetic Field Expression

$$\tilde{H}(x) = \frac{10}{125.67}e^{-jkx}\hat{z} \tag{7}$$

2 Problem 2: Plane Wave in a Lossless Dielectric Medium

Given the electric field:

$$E(r,t) = (\hat{y} + \hat{z}\sqrt{3})\cos(24\pi \times 10^9 t + 160\sqrt{3}\pi y - 160\pi z + 200) \tag{8}$$

2.1 Wavelength Calculation

The wave number magnitude is:

$$k = \sqrt{(160\sqrt{3}\pi)^2 + (160\pi)^2} \tag{9}$$

$$k = 320\pi \text{ rad/m} \tag{10}$$

The wavelength is:

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{320\pi} = \frac{1}{160} \text{ m} \approx 6.25 \text{ mm} \tag{11}$$

2.2 Propagation Direction

$$\hat{k} = \frac{(160\sqrt{3}\pi\hat{y} - 160\pi\hat{z})}{320\pi} = \frac{1}{2}\hat{y} - \frac{1}{2}\hat{z} \tag{12}$$

Thus, the wave propagates in the direction $\frac{1}{2}\hat{y} - \frac{1}{2}\hat{z}$.

2.3 Relative Refractive Index

$$n_r = \frac{c}{v_p} = \frac{3 \times 10^8}{v_p} \tag{13}$$

2.4 Instantaneous Magnetic Field Expression

$$H(r,t) = \frac{1}{\eta}\left(\hat{k} \times E(r,t)\right) \tag{14}$$

2.5 Phasor Expression for the Electric Field

$$\tilde{E}(r) = (\hat{y} + \hat{z}\sqrt{3})e^{j(160\sqrt{3}\pi y - 160\pi z + 200)} \tag{15}$$

3 Problem 3: Electromagnetic Wave in Vacuum

Given the phasor expression:

$$E(y) = e^{-jky}\hat{x} - j10e^{-jky}\hat{z} \tag{16}$$

3.1 Instantaneous Electric Field Expression

$$E(y,t) = \text{Re}\{(e^{-jky}\hat{x} - j10e^{-jky}\hat{z})e^{j\omega t}\} \tag{17}$$

3.2 Instantaneous Magnetic Field Expression

$$H(y,t) = \frac{1}{\eta}\left(\hat{k} \times E(y,t)\right) \tag{18}$$

3.3 Polarization of the Wave

Elliptical polarization.

3.4 Phasor Expression for the Magnetic Field

$$\tilde{H}(y) = \frac{1}{\eta}\left(\hat{k} \times \tilde{E}(y)\right) \tag{19}$$