Lesson 4: Phasor Wavefunctions

1 Key Learning Objectives

- Understand phasors as a tool for simplifying sinusoidal steady-state analysis.
- Learn how phasors simplify mathematical operations on sinusoidal signals.
- Apply phasor techniques to analyze electrical circuits.

2 Phasors in Steady-State Sinusoidal Analysis

Phasors allow us to represent sinusoidal signals using complex numbers, greatly simplifying calculations such as addition, subtraction, multiplication, and division.

Consider the series LR circuit in Fig. 1 driven by a sinusoidal voltage source. The circuit's governing equation, derived from Kirchhoff's Voltage Law (KVL), is:

$$L\frac{di(t)}{dt} + Ri(t) = V_{\mathrm{m}}\cos(\omega t) \tag{1}$$

where:

- R is resistance,
- L is inductance,
- V_{m} is the voltage amplitude,
- ω is the angular frequency.

3 Trigonometric Solution

The steady-state solution for $i(t)$ must have the same frequency as the forcing function:

$$i(t) = I_m\cos(\omega t - \phi) \tag{2}$$

Substituting this into the differential equation and solving for I_m and ϕ gives:

$$I_m = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \quad \text{(Magnitude)} \tag{3}$$

$$\tan\phi = \frac{\omega L}{R} \quad \text{(Phase angle)} \tag{4}$$

Thus, the final expression for $i(t)$ is:

$$i(t) = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}}\cos(\omega t - \phi) \tag{5}$$

4 Phasor Solution

Rewriting the governing equation in complex form:

$$L\frac{di_x(t)}{dt} + Ri_x(t) = V_m e^{j\omega t} \tag{6}$$

Solving this algebraically gives:

$$I = \frac{V_m}{R + j\omega L} \tag{7}$$

Expressing in polar form:

$$I = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} e^{-j\phi} \tag{8}$$

Using Euler's formula:

$$i(t) = \Re\{Ie^{j\omega t}\} = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos(\omega t - \phi) \tag{9}$$

which matches the trigonometric result.

5 Phasors and the Frequency Domain

A sinusoidal signal can be represented either in the time domain or frequency domain:

- Time domain: $i(t) = I_m \cos(\omega t - \phi)$
- Frequency domain: $I = \frac{V_m}{R+j\omega L}$

Moving to the frequency domain simplifies many calculations, making it a useful technique for circuit analysis.

6 Conclusion

- Phasors convert differential equations into algebraic equations, simplifying steady-state sinusoidal analysis.
- The magnitude and phase of the solution are easily obtained in phasor form.
- Moving between the time and frequency domains enhances problem-solving efficiency.