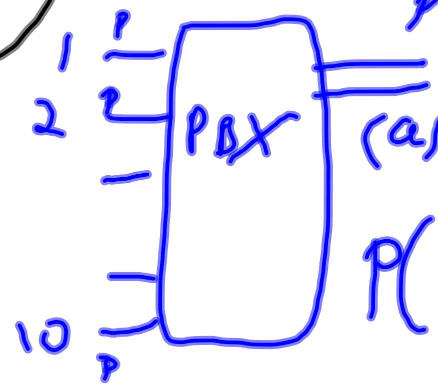
ESE/CSE 346
Probability Problems (Part II)

Instructor: Prof. Thomas Robertazzi

Electrical & Computer Engineering

2.36)

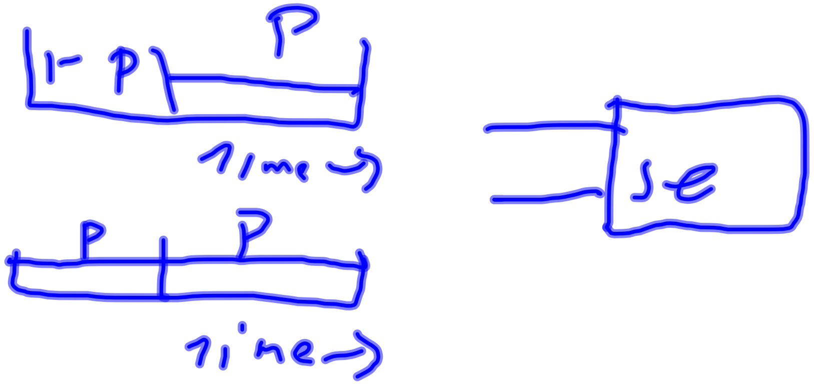


p: prob a phone seeks an outside line

(a) $$P\left(\begin{array}{l}n \text{ phones} \\ \text{seek an} \\ \text{outside line}\end{array}\right) = \underbrace{\binom{10}{n}} \underline{p^n (1-p)^{10-n}}$$

(b) $P(\text{blocking})$

$$= \sum_{n=3}^{10} \binom{10}{n} p^n (1-p)^{10-n}$$

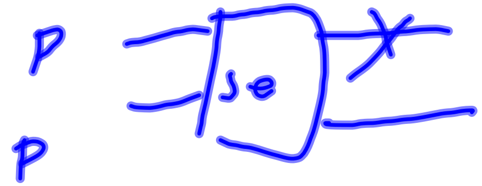


$$P\begin{pmatrix}\text{3 or more packets}\\ \text{in 2 time slots}\end{pmatrix} = P(\text{3 packets}) + P(\text{4 packets})$$

$$= \binom{4}{3} p^3 (1-p) + \overbrace{\binom{4}{4}}^{=1} p^4 \overbrace{(1-p)^0}^{=1}$$

$$= 4p^3(1-p) + p^4$$

38)

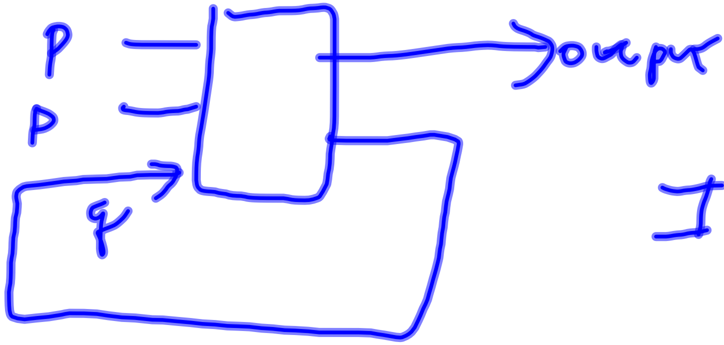


(a) $$P\left(\begin{matrix}\text{packet on}\\ \text{specific}\\ \text{output}\end{matrix}\right) = \overbrace{.5(2p(1-p))}^{\text{one arrival}} + \overbrace{1 \times p^2}^{\text{2 arrivals}}$$

(b) $$\overline{\#\text{ packets at output}} = \sum_i i\,p(i)$$

$$= 1 \times 2p(1-p) + 2p^2$$

39)



Implicit Equation

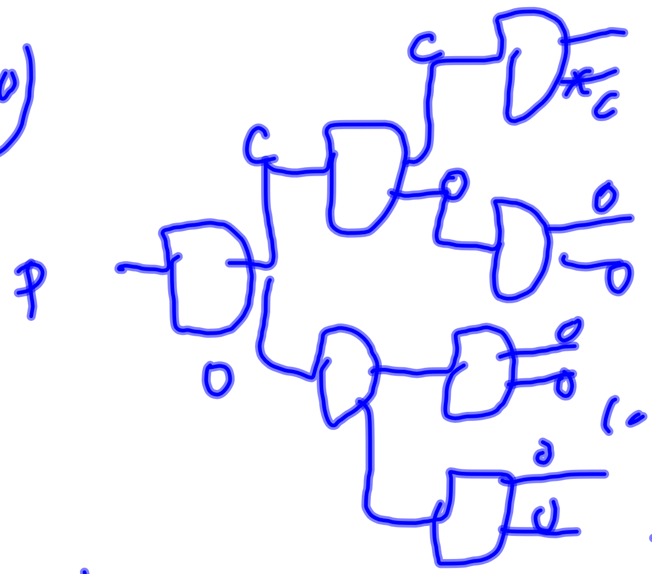
$$q = P\begin{pmatrix}\text{2 or more packets} \\ \text{at inputs}\end{pmatrix}$$

$$q = \binom{2}{1} p(1-p)q + p^2(1-q) + p^2 q$$

$$q = 2p(1-p)q + p^2 \underbrace{[(1-q+q)]}_{=1}$$

$$\boxed{q = \frac{p^2}{1-2p(1-p)}}$$

4a)



c: prob a packet appears at output if packet at input

(a) For a given output

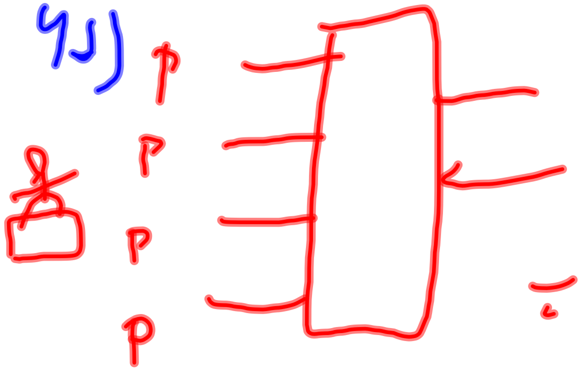
$P(\text{copy}) = P\,c^3$

(b)

$$\overline{\#\text{ Copies}} \approx \sum_{n=1}^{8} n\binom{8}{n}(c^3 p)(1-c^3 p)^{8-n}$$

$= \sum n p_n$

4a)



$P(\text{a packet dropped (birds eye view)})$

$= P(\text{3 or more packets arrive})$

$= \binom{4}{3}p^3(1-p) + \underbrace{\binom{4}{4}}_{=1}p^4\underbrace{(1-p)^0}_{=1}$

$= 4p^3(1-p) + p^4$

(b) $P(\text{tagged arriving packet is dropped})$

$= \frac{1}{3}P\left(\begin{array}{c}\text{2 other}\\ \text{packets}\\ \text{arrive}\end{array}\right) + \frac{2}{4}P\left(\begin{array}{c}\text{3 other}\\ \text{packets}\\ \text{arrive}\end{array}\right)$

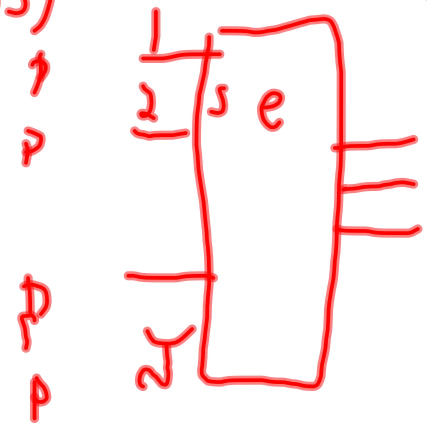
$= \frac{1}{3}\binom{3}{2}p^2(1-p) + \frac{1}{2}\binom{3}{3}p^3(1-p)^0$

42 continued

$$\text{Thruput} = 1 \cdot P(1 \text{ arrival}) + 2\, P(\geq 2 \text{ arrive})$$

$$\sum n\, P(n) = 1 \cdot \binom{4}{1} p (1-p)^3 + 2 \sum_{n=2}^{4} \binom{4}{n} p^n (1-p)^{4-n}$$

43)



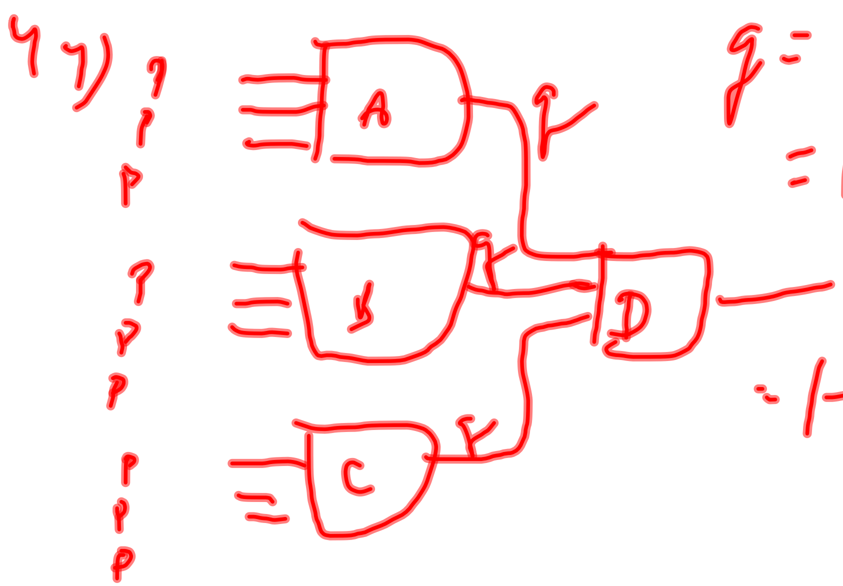
(a) Throughput

$$= 1P(1 \text{ arrival}) + 2P(2 \text{ arrivals}) + 3P(\geq 3 \text{ arrivals})$$

$$= 1\binom{N}{1} p(1-p)^{N-1} + 2\binom{N}{2} p^2(1-p)^{N-1} + 3\sum_{n=3}^{N}\binom{N}{n} p^n (1-p)^{N-n}$$

b) # DROPPED PACKETS

$$= \sum_{n=4}^{N} (n-3)\binom{N}{n} p^n (1-p)^{N-n}$$



$q = P(1 \text{ or more arrival})$

$= 1 - P(0 \text{ arrivals at input})$

$= 1 - (1-p)^3$

Thruput $= P(1 \text{ or more arrival at } D)$

$= 1 - (1-q)^3$