EEO 306
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Name:_____

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Sample Final
Total Score: 36 points
All problems carry equal weight of 6 points each
Please show your work and justify your answers

1. In a game of Blackjack, the tens and face cards (10's, Jacks, Queens and Kings) count as 10 points, and Aces count as either 1 or 11 points. A blackjack occurs if the sum of the two cards is 21 (counting Ace as 11 points). A player is dealt two cards. What is the probability that the player has Blackjack?

2. Let X be a Poisson random variable with parameter λ.

   (a) What is the probability that X is even?

   (b) Simplify the above expression (X is even), by utilizing the following expansions for e^{λ} and $e^{-\lambda}$:

$$e^{\lambda} = 1 + \lambda + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} + \frac{\lambda^4}{4!} \dots$$

   and

$$e^{-\lambda} = 1 - \lambda + \frac{\lambda^2}{2!} - \frac{\lambda^3}{3!} + \frac{\lambda^4}{4!} \dots$$

3. A plane is missing, and it is assumed that it is equally likely to have gone down in any of three possible regions. Let $1-\alpha_i$ denote the probability that the plane will be found upon a search of the ith region when the plane is, in fact, in that region, $i = 1, 2, 3$. (The constants α_i are called overlook probabilities because they represent the probabilities of overlooking the plane.) What is the conditional probability that the plane is in the ith region, given that a search of region 1 is unsuccesful?

4. Random variables X_1 and X_2 have the joint PMF $p_{X_1,X_2}(x_1, x_2)$ given by the following table:

$p_{X_1,X_2}(x_1, x_2)$	$x_2 = -1$	$x_2 = 0$	$x_2 = 1$
$x_1 = -1$	0	0	1/3
$x_1 = 0$	0	1/3	0
$x_1 = 1$	1/3	0	0

(a) Compute the marginal PMF $p_{X_1}(x_1)$.

(b) Compute the marginal PMF $p_{X_2}(x_2)$.

(c) Compute the probability $P(X_1 < X_2)$.

(d) Are X_1 and X_2 independent?

5. (a) An Urn contains N white balls and M black balls. Draw a ball **with replacement** until a black ball is selected. What is the probability that **exactly** k draws are needed?

   (b) What is the expected number of draws needed to observe a black ball?

6. Sketch the ensemble, that is, realizations of the random process

$$X(t) \;=\; A cos(2\pi \boldsymbol{f} t),$$

where $\boldsymbol{f}$ is a uniform random variable $\mathcal{U}(10, 20)$. That is, $\boldsymbol{f}$ is uniformly distributed in the range $[10, 20]$ Hz and $A = 5$ is a constant.

Blank Sheet