ESE/EEO 306
Random Signals and Systems
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**Assignment 2B**
**Discrete Random Variables:**
**Binomial, Poisson and Geometric**

# 1 Assignment

1. Two fair dice are rolled and the absolute value of the difference of the outcomes is denoted by the random variable $X$. What are the possible values $X$ takes, and the associated probabilities?

2. The random variable $X$ has the probabilities listed in the table below. What is the cumulative distribution function (CDF) of $X$?

| **x** | **P(X=x)** |
|---|---|
| 1 | 0.5 |
| 2 | 0.2 |
| 3 | 0.1 |
| 4 | 0.2 |

3. You are given a binomial random variable $X$, with parameters $n = 8$ and $p = 0.1$. Determine the CDF and PMF of $X$ and plot these.

4. Five fair coins are flipped. If the outcomes are assumed independent, find the probability mass function (PMF) of the number of heads obtained.

5. A communication system consists of $n$ components each of which will function independently with probability $p$. The total system will operate effectively if **at least half** of its components function.
   (a) What is the probability that the total system will operate effectively if $n = 3$?
   (b) What is the probability that the total system will operate effectively if $n = 5$?
   (c) For what values of $p$ will a 5-component system be more likely to operate effectively than a 3-component system?

6. From a deck of 52 cards, we draw cards at random **with replacement**. The drawing is successive until an ace is drawn.
   (a) Determine the probability that **exactly** 10 draws are needed. That is, we observe an ace for the first time in the tenth draw.
   (b) Determine the probability that **at least** 10 draws are needed.

7. Using plotting software, plot the probability mass function (PMF) of a Poisson random variable for the parameter value: (a) $\lambda = 3$ (b) $\lambda = 5$

8. Messages that arrive at a computer in a period of one hour are modeled by the Poisson PMF

$$p_X(x) \; = \; e^{-\lambda}\frac{\lambda^x}{x!}, \quad x = 0, 1, 2, 3, \ldots .$$

   We are given that the parameter $\lambda = 15$.

   (a) Determine the probability that **exactly** 3 messages arrive in one hour.

   (b) Determine the probability that **no more than** 9 messages arrive in one hour.

9. The random variable $X$ has a Poisson distribution, such that $P(X = 1) = P(X = 2)$. Find $P(X = 4)$.