Stony Brook University

EEO319

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Electromagnetic Waves & Transmission Lines

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Copy of Original Assignment

Homework 2

[1] A uniform plane wavefunction has the instantaneous expression

$\Phi(x,t) = 5 \sin(200\pi t + 0.4\pi x + 30^0)$.

Identify or calculate the following:

- i) the radian frequency ω of the wave in radns/s
- ii) the frequency f in Hz of the wave
- iii) the direction of propagation of the wave
- iv) the wavelength λ in m of the wave
- v) the amplitude A of the wave, and
- vi) the phase velocity v_{ph} in m/s of the wave.

[2] A uniform plane scalar wave represented by the function $\Phi(x,t)$ is specified as having the following properties:

The wave propagates in the -x direction

Wave amplitude = 10

Wave frequency = 500 Hz

Wave's phase velocity = 100 m/s

The wavefunction $\Phi(0,0)$ at x = 0 and t = 0 has the value $\Phi(0,0) = 5$.

Find the expression $\Phi(x,t)$ of the wave function.

[3] Consider two scalar UPW's of the same amplitude which have incrementally different frequencies and thus incrementally different wavelengths. Both waves are given to be propagating in the +z direction. Show that the effect of interference between the two UPW's is to generate an amplitude-modulated carrier wave (cw) signal moving in the +z direction such that the cw component of the signal propagates with phase velocity $v_p = \omega/k$ while the envelope signal propagates with group velocity $v_g = d\omega/dk$.

1 Wave Analysis

1.1 Problem 1

A uniform plane wavefunction has the instantaneous expression:

$$\Phi(x,t) = 5\sin(200\pi t + 0.4\pi x + 300) \tag{1}$$

Identify or calculate the following:

1. The radian frequency ω of the wave in rad/s:
$$\omega = 200\pi \text{ rad/s} \tag{2}$$
2. The frequency f in Hz of the wave:
$$f = \frac{\omega}{2\pi} = \frac{200\pi}{2\pi} = 100 \text{ Hz} \tag{3}$$
3. The direction of propagation of the wave:
The wave propagates in the negative x-direction since the wave phase term is $(\omega t + kx + \phi)$.
4. The wavelength λ in meters:
$$k = 0.4\pi, \quad \lambda = \frac{2\pi}{k} = \frac{2\pi}{0.4\pi} = 5 \text{ m} \tag{4}$$
5. The amplitude A of the wave:
$$A = 5 \tag{5}$$
6. The phase velocity v_{ph} in m/s of the wave:
$$v_{ph} = \frac{\omega}{k} = \frac{200\pi}{0.4\pi} = 500 \text{ m/s} \tag{6}$$

1.2 Problem 2

A uniform plane scalar wave represented by the function $\Phi(x,t)$ is given with the following properties:

- The wave propagates in the $-x$ direction.
- Wave amplitude = 10.
- Wave frequency = 500 Hz.
- Wave's phase velocity = 100 m/s.
- The wavefunction $\Phi(0,0)$ at $x = 0$ and $t = 0$ has the value $\Phi(0,0) = 5$.

Find the expression $\Phi(x,t)$ of the wave function.

$$\omega = 2\pi f = 2\pi \times 500 = 1000\pi \text{ rad/s} \tag{7}$$

$$k = \frac{\omega}{v_{ph}} = \frac{1000\pi}{100} = 10\pi \text{ rad/m} \tag{8}$$

Since the wave propagates in the $-x$ direction, the wavefunction takes the form:

$$\Phi(x,t) = 10\sin(\omega t + kx + \phi) \tag{9}$$

To satisfy $\Phi(0,0) = 5$:

$$10\sin(\phi) = 5 \Rightarrow \sin(\phi) = 0.5 \Rightarrow \phi = \frac{\pi}{6} \tag{10}$$

Thus, the wave function is:

$$\Phi(x,t) = 10\sin(1000\pi t + 10\pi x + \frac{\pi}{6}) \tag{11}$$

1.3 Problem 3

Consider two scalar UPW's of the same amplitude which have incrementally different frequencies and thus incrementally different wavelengths. Both waves are given to be propagating in the $+z$ direction.

Let the two waves be:

$$\Phi_1 = A\cos(\omega_1 t - k_1 z) \tag{12}$$

$$\Phi_2 = A\cos(\omega_2 t - k_2 z) \tag{13}$$

Using trigonometric identities, their sum can be rewritten as an amplitude-modulated wave:

$$\Phi = \Phi_1 + \Phi_2 = 2A\cos\left(\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}z\right)\cos\left(\omega_c t - k_c z\right) \tag{14}$$

where:

$$\omega_c = \frac{\omega_1 + \omega_2}{2}, \quad k_c = \frac{k_1 + k_2}{2}, \tag{15}$$

$$\Delta\omega = \omega_2 - \omega_1, \quad \Delta k = k_2 - k_1 \tag{16}$$

The carrier wave propagates with phase velocity:

$$v_p = \frac{\omega_c}{k_c} \tag{17}$$

The envelope wave propagates with group velocity:

$$v_g = \frac{d\omega}{dk} \tag{18}$$

This shows that the interference of two UPWs results in an amplitude-modulated signal with the carrier wave moving at the phase velocity and the envelope moving at the group velocity.