Lesson 1: Uniform Plane Harmonic Traveling Waves

1 Key Learning Objectives

- Understand the concept of a uniform plane sinusoidal (harmonic) wave.
- Define and quantify wave parameters: **amplitude (A), frequency (f), wavelength (λ)**.
- Learn about **phase velocity ($\mathbf{v}_p$)** and **group velocity ($\mathbf{v}_g$)**.
- Differentiate between **non-dispersive** and **dispersive** wave propagation.

2 Uniform Plane Wave (UPW) Function

A traveling wave is described by the equation:

$$\Phi(z,t) = A\cos(\omega t - kz) \tag{1}$$

2.1 Wave Parameters

$$\omega = 2\pi f \quad \text{(Angular frequency in radians/s)} \tag{2}$$

$$k = \frac{2\pi}{\lambda} \quad \text{(Wavenumber in radians/m)} \tag{3}$$

$$T = \frac{1}{f} \quad \text{(Time period in seconds)} \tag{4}$$

$$\lambda = \frac{2\pi}{k} \quad \text{(Wavelength in meters)} \tag{5}$$

3 Wave Propagation

3.1 +z Propagation

If the wave propagates in the +z direction, it follows:

$$\Phi(z,t) = A\cos(\omega t - kz) \tag{6}$$

3.2 -z Propagation

If the wave propagates in the -z direction, the function is:

$$\Phi(z,t) = A\cos(\omega t + kz) \tag{7}$$

4 Phase Velocity

The phase velocity, v_p, is the speed at which a constant phase point moves:

$$v_p = \frac{\omega}{k} \tag{8}$$

5 Group Velocity

For a wave packet consisting of multiple waves, the group velocity v_g is given by:

$$v_g = \frac{d\omega}{dk} \tag{9}$$

5.1 Key Insights

- **Phase velocity** (v_p) describes the motion of the wave profile.
- **Group velocity** (v_g) describes the speed at which energy propagates.
- In **nondispersive media**, $v_p = v_g$.
- In **dispersive media**, $v_p \neq v_g$.

For electromagnetic waves in vacuum:

$$v_p \cdot v_g = c^2 \tag{10}$$

where c is the speed of light in vacuum.

6 Wavefronts and Uniform Plane Waves

A **wavefront** is a surface of constant phase. Since $\Phi(z,t)$ does not depend on x or y, it remains uniform on any $z =$ constant plane, defining a **uniform plane wave**.

7 Conclusion

- A wave is a periodic disturbance that moves with a well-defined velocity.
- A **uniform plane wave (UPW)** is characterized by:
  - Amplitude A
  - Frequency f (or angular frequency ω)
  - Wavelength λ
- The **phase velocity** v_p determines wave profile motion, while the **group velocity** v_g dictates energy transport.
- If $v_p = v_g$, the wave propagates **nondispersively**; otherwise, it is **dispersive**.