Lesson 3: Uniform Plane Electromagnetic Waves (UPEMWs)

1 Key Learning Objectives

- Understand UPEMWs propagating in a lossless infinite medium (vacuum or dielectric).
- Recognize that a UPEMW consists of two UPWs: one for the electric field and one for the magnetic field.
- Learn the fundamental properties of UPEMWs.

2 Wave Function of UPEMWs

UPEMWs are vector wave functions, meaning that both electric and magnetic fields have direction in space in addition to their spatial and temporal dependence. A simple case of a UPEMW propagating in the z-direction is expressed as:

$$E(z,t) = i_E E_0 \cos(\omega t - kz) \quad \text{(1a)} \tag{1}$$

$$H(z,t) = i_H H_0 \cos(\omega t - kz) \quad \text{(1b)} \tag{2}$$

where:

- i_E and i_H are unit vectors representing the directions of the electric and magnetic fields, respectively.
- E_0 and H_0 are amplitude constants.
- k is the wavenumber.
- ω is the angular frequency.

A more general form of the wave function is:

$$E(z,t) = i_E E_0 \cos(\omega t - \mathbf{k} \cdot \mathbf{r}) \quad \text{(2a)} \tag{3}$$

$$H(z,t) = i_H H_0 \cos(\omega t - \mathbf{k} \cdot \mathbf{r}) \quad \text{(2b)} \tag{4}$$

where $\mathbf{k}$ is the wave vector.

3 Properties of UPEMWs

3.1 Property 1: Synchronization of Fields

The space-time factor $\cos(\omega t - \mathbf{k} \cdot \mathbf{r})$ is the same for both E and H, meaning:

- Electric and magnetic fields are in phase.
- Their maxima and minima occur at the same points in space and time.

3.2 Property 2: Right-Handed Triad of Unit Vectors

The unit vectors i_E, i_H, and i_K form a right-handed coordinate system:

$$i_E \times i_H = i_K \quad \text{(3a)} \tag{5}$$

$$i_H \times i_K = i_E \quad \text{(3b)} \tag{6}$$

$$i_K \times i_E = i_H \quad \text{(3c)} \tag{7}$$

These relationships provide a mnemonic for determining an unknown unit vector given two known vectors. The right-hand rule can also be used to determine the direction of the unknown unit vector.

3.3 Property 3: Relationship Between E_0 and H_0

The amplitudes of the electric and magnetic fields are related by the characteristic impedance η of the medium:

$$\frac{E_0}{H_0} = \eta \quad (4) \tag{8}$$

where η is given by:

$$\eta = \sqrt{\frac{\mu_0 \mu_r}{\epsilon_0 \epsilon_r}} \quad (5) \tag{9}$$

with:

- $\mu_0 = 4\pi \times 10^{-7}$ H/m (permeability of free space).
- μ_r = relative permeability of the medium ($\mu_r = 1$ for a nonmagnetic medium).
- $\epsilon_0 = 8.854 \times 10^{-12}$ F/m (permittivity of free space).
- ϵ_r = relative permittivity (dielectric constant of the medium).

For free space:

$$\eta_0 = 377\Omega \quad (6) \tag{10}$$

4 Phase Velocity of UPEMWs

The phase velocity of UPEMWs is given by:

$$v_p = \frac{c}{n_r} \quad (7) \tag{11}$$

where n_r is the relative refractive index, defined as the velocity reduction factor relative to the speed of light in vacuum ($c = 3 \times 10^8$ m/s). The refractive index is:

$$n_r = \sqrt{\mu_r \epsilon_r} \quad (8) \tag{12}$$

5 Conclusion

- A UPEMW consists of an electric and magnetic field propagating together.
- Both fields are in phase and perpendicular to each other and to the direction of propagation.
- The amplitudes of the fields are related through the characteristic impedance of the medium.
- The phase velocity depends on the refractive index of the medium.