LESSON 5

THE POYNTING VECTOR

I. WHAT YOU WILL LEARN IN THIS LESSON:

In this lesson, you will learn how to calculate the power carried by a wave which is given by the Poynting vector. A quick and simplified procedure for computing the instantaneous Poynting vector as well as the time-average Poynting vector is given here.

II. Instantaneous and phasor expressions for the electric and magnetic fields in a UPEMW:

Let the instantaneous expressions for the electric and magnetic field components of a UPEMW be

$$\underline{E}(\underline{r},t) = \hat{\underline{i}}_E \, E_0 \cos(\omega t - \underline{k}\cdot\underline{r} + \delta), \quad V/m$$

$$\underline{H}(\underline{r},t) = \hat{\underline{i}}_H \, H_0 \cos(\omega t - \underline{k}\cdot\underline{r} + \delta), \quad A/m$$

where $\underline{k} = k\,\hat{\underline{i}}_k$,

$$\hat{\underline{i}}_H = \hat{\underline{i}}_k \times \hat{\underline{i}}_E$$

and $\dfrac{E_0}{H_0} = \eta$

The corresponding phasor expressions $\bar{\underline{E}}(\underline{r})$ and $\bar{\underline{H}}(\underline{r})$ have expressions

$$\bar{\underline{E}}(\underline{r}) = \hat{\underline{i}}_E \, E_0 \, e^{-j(\underline{k}\cdot\underline{r} - \delta)}$$

and

$$\bar{\underline{H}}(\underline{r}) = \hat{\underline{i}}_H \, H_0 \, e^{-j(\underline{k}\cdot\underline{r} - \delta)}$$

III. Instantaneous Poynting vector $\underline{P}(\underline{r},t)$:

$$\underline{P}(\underline{r},t) = \underset{\uparrow V/m}{\underline{E}(\underline{r},t)} \times \underset{\uparrow A/m}{\underline{H}(\underline{r},t)} \quad [\text{Unit of } \underline{P} \text{ is } W/m^2]$$

$$\therefore \underline{P}(\underline{r},t) = \underline{i}_E E_o \sin(\omega t - \underline{k}\cdot\underline{r} + \delta) \times \underline{i}_H H_o \sin(\omega t - \underline{k}\cdot\underline{r} + \delta)$$

$$= \underbrace{\underline{i}_E \times \underline{i}_H}_{=\underline{i}_k} E_o H_o \sin^2(\omega t - \underline{k}\cdot\underline{r} + \delta)$$

Hence one may write

$$\underline{P}(\underline{r},t) = \underline{i}_P P(\underline{r},t) \text{ where}$$

$\underline{i}_P = \underline{i}_k$, and

$$\underline{P}(\underline{r},t) = \begin{cases} \underline{i}_k E_o H_o \sin^2(\omega t - \underline{k}\cdot\underline{r} + \delta) \\ \text{or} \\ \underline{i}_k \dfrac{E_o^2}{\eta} \sin^2(\omega t - \underline{k}\cdot\underline{r} + \delta) \\ \text{or} \\ \underline{i}_k \eta H_o^2 \sin^2(\omega t - \underline{k}\cdot\underline{r} + \delta) \end{cases}$$

Use any one of these three expressions for $P(r,t)$ in W/m^2

IV. Time-average Poynting vector $\underline{P}_{av}(\underline{r})$:

$$\underline{P}_{av}(\underline{r}) = \text{Re}\left[\underline{P}_{complex}(\underline{r})\right]$$

where

$$\underline{P}_{complex}(\underline{r}) = \begin{cases} \frac{1}{2}\, \underline{\bar{E}}(\underline{r}) \times \underline{\bar{H}}(\underline{r})^* & \text{if amplitude values are used} \\ \text{or} \\ \underline{\bar{E}}(\underline{r}) \times \underline{\bar{H}}(\underline{r})^* & \text{if RMS values are used} \end{cases}$$

Here the case where amplitude values are used is illustrated. Thus,

$$\underline{P}_{complex}(\underline{r}) = \frac{1}{2}\,\overline{E}(\underline{r}) \times \overline{H}(r)^{*}$$

Now $\overline{E}(\underline{r}) = \underline{i}_E\, E_o\, e^{-j(\underline{k}\cdot\underline{r} - \delta)}$

and $\overline{H}(\underline{r}) = \underline{i}_H\, H_o\, e^{-j(\underline{k}\cdot\underline{r} - \delta)}$

$$\therefore\ \underline{P}_{complex}(\underline{r}) = \frac{1}{2}\,\underbrace{\underline{i}_E \times \underline{i}_H}_{=\underline{i}_k}\, E_o H_o\, \underbrace{e^{-j(\underline{k}\cdot\underline{r} - \delta)}\,\underbrace{\left[e^{-j(\underline{k}\cdot\underline{r} - \delta)}\right]^{*}}_{=e^{+j(\underline{k}\cdot\underline{r} - \delta)}}}_{=1}$$

$$\therefore\ \underline{P}_{complex}(\underline{r}) = \begin{cases} \frac{1}{2}\,\underline{i}_k\, E_o H_o \\ \text{or} \\ \frac{1}{2}\,\underline{i}_k\, \frac{E_o^2}{\eta} \\ \text{or} \\ \frac{1}{2}\,\underline{i}_k\, \eta H_o^2 \end{cases}$$

$\Leftarrow$ use any one of these three formulas

$$\therefore\ \underline{P}_{av}(\underline{r}) = \left\{ \begin{array}{c} \frac{1}{2}\,\underline{i}_k\, E_o H_o \\ \text{or} \\ \frac{1}{2}\,\underline{i}_k\, E_o^2/\eta \\ \text{or} \\ \frac{1}{2}\,\underline{i}_k\, \eta H_o^2 \end{array} \right\}\ \mathrm{W/m^2}$$

SAMPLE PROBLEM

[1] A UPEMW propagates in an infinite, lossless dielectric medium. The instantaneous expression for the electric field component of the wave is given to be

$$\underline{E}(\underline{r},t) = \underline{i}_x\, 6 \sin(120\pi \times 10^8 t - 55.23\pi y + 23.44\pi z + 50^\circ),\ V.$$

Find :

i) the wave frequency f

ii) wavenumber λ of the wave

iii) direction of propagation of the wave

iv) relative permittivity ϵ_r of the wave

v) phasor expression $\underline{\bar{E}}(\underline{r})$ of the electric field

vi) instantaneous expression $\underline{H}(\underline{r},t)$ of the wave magnetic field

vii) instantaneous Poynting vector $\underline{P}(\underline{r},t)$ W/m^2

viii) time-average Poynting vector $\underline{P}_{av}(\underline{r})$ W/m^2

ix) time-average power in W intercepted by a planar loop antenna oriented perpendicular to the y-axis

Solution:

Rewrite $\underline{E}(\underline{r},t)$ as

$$\underline{E}(\underline{r},t) = \underline{i}_E\, E_o \sin(\omega t - k_y y - k_z z + 50^\circ)$$

where $\underline{i}_E = \underline{i}_x$

$E_o = 6$ V/m

$\omega = 120\pi \times 10^8$ rad/s

$k_y = 173.42$ rad/m

$k_z = -73.61$ rad/m

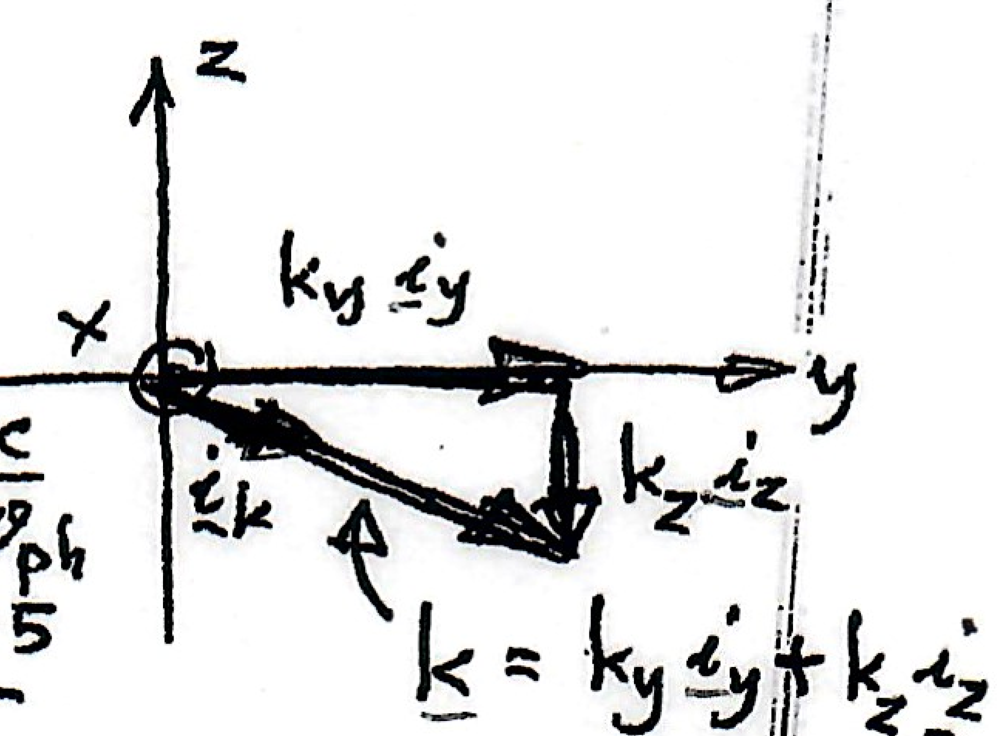
Finding the expression for instantaneous Poynting vector $\underline{P}(\underline{r},t)$:

First find $k = \sqrt{k_y^2 + k_z^2} = \sqrt{173.42^2 + 73.61^2} = 188.4 \ \frac{rad}{m}$

$$\underline{i}_k = \underline{i}_y \frac{k_y}{k} + \underline{i}_z \frac{k_z}{k}$$

$$= \underline{i}_y \, 0.921 - \underline{i}_z \, 0.391$$

$$v_{ph} = \omega/k = \frac{12\pi \times 10^9}{188.4} = 2\times 10^8 \, m/s \rightarrow n_r = \frac{c}{v_{ph}} = 1.5$$



$$\therefore \ \underline{P}(\underline{r},t) = \underline{i}_k \frac{E_0^2}{\eta} \cos^2(\omega t - \underline{k}\cdot\underline{r} + \delta)$$

$$= \underline{i}_k \frac{36}{377/n_r} \cos^2(120\pi \times 10^8 t - 173.42y + 73.61z + 50^\circ) \ W/m^2$$

$$= (\underline{i}_y 0.921 - \underline{i}_z 0.391) \frac{36\times 1.5}{377} \cos^2(120\pi \times 10^8 t - 173.42y + 73.61z + 50^\circ) \ W/m^2$$

Answer

Now find the expression for the time-av Poynting vector

$$\underline{P}_{av}(\underline{r}) = \frac{1}{2}\underline{i}_k \frac{E_0^2}{\eta} \qquad \eta = \frac{\eta_0}{n_r} = \frac{377}{1.5}$$

$$= \frac{1}{2}(\underline{i}_y 0.921 - \underline{i}_z 0.391) \underbrace{\frac{36\times 1.5}{377}}_{=0.143}$$

$$= 0.071 (\underline{i}_y 0.921 - \underline{i}_z 0.391) \ W/m^2$$

Answer