EE Formula Sheet

§1 - The Crystal Structure of Solids

Miller Index

1. Identify the intercepts: Locate the points where the plane intersects the crystallographic axes (usually denoted as a, b, and c).
2. Convert to fractional coordinates: Express these intercepts as fractions of the unit cell dimensions (a, b, c). To do this, divide each intercept by the respective unit cell dimension. If an intercept does not intersect the corresponding axis, use ∞ as the fraction.
3. Take reciprocals: Invert the fractional intercepts to obtain the reciprocal fractions.
4. Simplify the ratios: If any of the reciprocals are not integers, multiply all the indices by the smallest integer that makes them all whole numbers while maintaining the ratio. This ensures that the Miller indices are in the simplest form.
5. Enclose in parentheses: Write the indices as (hkl), where h, k, and l are the integers obtained after simplification. These are the Miller indices that represent the crystallographic plane.
6. Optional: If the plane is parallel to an axis (intercepts at infinity), write the corresponding Miller index as 0. For example, if a plane is parallel to the a-axis and intercepts the b and c-axes at infinity, its Miller indices would be $(0bc)$.

Principles of Quantum Mechanics

1. Superposition Principle: Quantum systems can exist in a linear combination of multiple states simultaneously, described by a wavefunction.
2. Wave-Particle Duality: Particles, such as electrons and photons, exhibit both wave-like and particle-like properties.
3. Uncertainty Principle: There is a fundamental limit to the precision with which certain pairs of properties, such as position and momentum, can be simultaneously known.
4. Quantum States and Operators: Quantum states are described by wavefunctions, and operators represent physical observables and transformations.
5. Measurement and Collapse: Measurement of a quantum system collapses its wavefunction to one of its possible states, with probabilities determined by the square of the amplitude of the wavefunction.
6. Quantum Entanglement: Entangled particles exhibit correlations that cannot be explained by classical physics, even when separated by large distances.
7. Quantum Tunneling: Particles can penetrate energy barriers that classical physics predicts they should not be able to cross.
8. Quantum Interference: Quantum systems can exhibit interference patterns when multiple pathways are available.
9. Quantum Information: Quantum mechanics plays a crucial role in the field of quantum computing and quantum cryptography, offering advantages in information processing and security.

Broglie Wavelength

Convert energy from eV to J:

$$E = 1.0\,\text{eV} = 1.60219 \times 10^{-19}\,\text{J}$$

Calculate momentum of the proton or electron:

$$p_{\text{proton}} = \sqrt{2 \cdot m_{\text{proton}} \cdot E}$$

Where: p_{proton} = momentum of the proton, m_{proton} = mass of the proton ($\approx 1.6726219 \times 10^{-27}$ kg), E = kinetic energy (in Joules)

$$p_{\text{electron}} = \sqrt{2 \cdot m_{\text{electron}} \cdot E}$$

Where: p_{electron} = momentum of the electron, m_{electron} = mass of the electron ($\approx 9.10938356 \times 10^{-31}$ kg), E = kinetic energy (in Joules)
Calculate the de Broglie wavelength:

$$\lambda = \frac{h}{p}$$

Where: λ = de Broglie wavelength, h = Planck's constant ($6.62607015 \times 10^{-34}$ m² kg/s)

Energy

The energy (E) of a photon is given by $E = h\nu$, where h is Planck's constant and ν is the frequency.
The frequency (ν) of a photon is inversely proportional to its wavelength (λ) and can be determined by the equation $\nu = \frac{c}{\lambda}$, where c is the speed of light.

Intrinsic Fermi Energy

$$n_i^2 = N_c N_v \cdot e^{\frac{-Eg}{kT}}$$

Where:

- $N_c = 2.8 \times 10^{19}\,\text{cm}^3$ for Si
- $N_v = 1.04 \times 10^{19}\,\text{cm}^3$ for Si
- E_g is the bandgap energy
- k is Boltzmann's constant $86 \times 10^{-6}\,\text{eV/K}$

§1.2 - The PN Junction

$V_{bi} = V_T \ln\left(\frac{N_a N_d}{n^2{}_i}\right)$, Built In barrier ($V_f$)
$C_j = C_{jo}\left(1 + \frac{V_R}{V_{bi}}\right)$, Junction Capacitance
$i_D = I_S\left(e^{\left(\frac{v_D}{nV_T}\right)} - 1\right)$, Diode Current, where $V_T = 26mV$ @ 300K

§1.3 - Diode Circuits: DC Analysis and Models

- Use KVL when $V_D \geq V_y$. Use open circuit when $V_D < V_y$
- Use PWL to plot the diode voltage where slope of diode cut in voltage is $m = 1/R_f$
- Use KVL formula of the circuit to plot the load line. (Arrange formula in slope-intercept form.)
- The *Q-point* is at the intersection of the PWL and load line plots.

§1.4 - Diode Circuits: AC Analysis and Models

First, analyze the DC portion, then the AC portion.

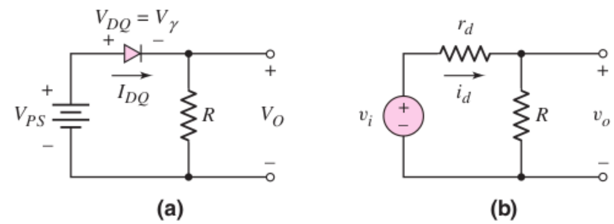

Figure 1.36 Equivalent circuits: (a) dc and (b) ac

$R_d = \frac{1}{g_d} = \frac{V_T}{I_{DQ}}$, Small Signal Diffusion Resistance
$i_d = \left(\frac{I_{DQ}}{V_T}\right) \cdot v_d = g_d \cdot v_d$, AC diode current
$v_d = \left(\frac{V_T}{I_{DQ}}\right) \cdot i_d = r_d \cdot i_d$, AC diode voltage
where g_d and r_d respectively, are the diode small-signal incremental conductance and resistance, also called the diffusion conductance and diffusion resistance.
$C_d = \left(\frac{d_Q}{dV_D}\right)$, Diffusion Capacitance

Small-Signal Equivalent Circuit

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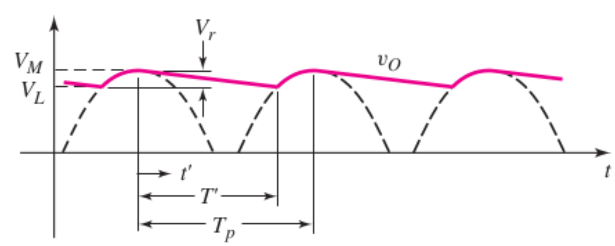
§2.1 - Rectifier Circuits

$\frac{v_i}{v_s} = \frac{N_1}{N_2}$, Transformer voltage to turn-ratio relationship.

Center tapped formulae

$v_s(max)) = v_o(max) + V_y$, Peak
$v_r(max) = 2v_S(max) - V_y$, Peak Inverse Voltage

Bridge rectifier formulae



$v_S(max) = v_O(max)$, Peak
$v_R(max) = v_S(max) - V_y$, Peak Inverse Voltage
$v_{o(t)} = V_M e^{t'/RC}$, Average Vout
$v_L = V_M e^{T'/RC}$, Minimum Vout
$v_r = V_M - V_L = \frac{V_M}{2fRC}$, Ripple on Vout

§2.4 - Clippers and Clampers

Clippers clip signals, and clampers shift the entire waveform.

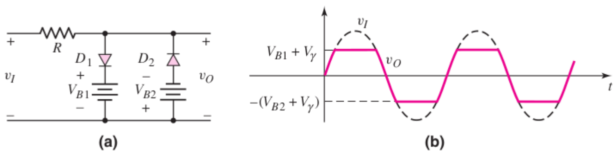


Figure 2.22 A parallel-based diode clipper circuit and its output response

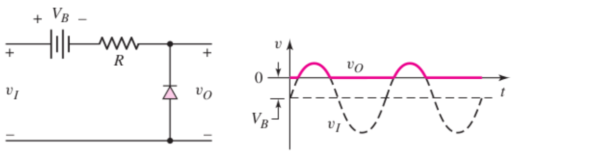


Figure 2.25 Series-based diode clipper circuit and resulting output response

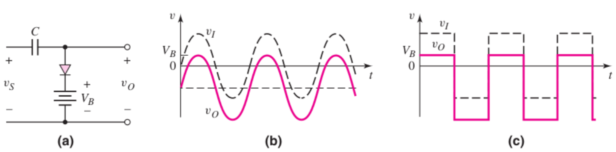


Figure 2.28 Action of a diode clamper circuit with a voltage source assuming an ideal diode ($V_r = 0$): (a) the circuit, (b) steady-state sinusoidal input and output signals, and (c) steady-state square-wave input and output signals

§3.1 - MOS Field-Effect Transistor

N-Channel

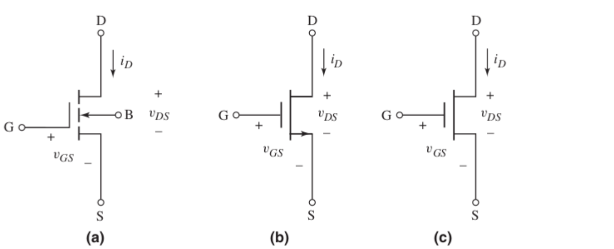


Figure 3.12 The n-channel enhancement-mode MOSFET: (a) conventional circuit symbol, (b) circuit symbol that will be used in this text, and (c) a simplified circuit symbol used in more advanced texts

$v_{DS}(sat) = v_{GS} - V_{TN}$, Saturation Voltage, where V_{TN} is the threshold voltage.
$i_D = K_n\left[2(v_{GS} - V_{TN})v_{DS} - v_{DS}^2\right]$, I-V Characteristic in non-saturation.
$i_D = K_n(v_{GS} - V_{TN})^2$, I-V Characteristic in saturation.
$C_{ox} = \epsilon_{ox}/t_{ox}$, Oxide capacitance per unit area.
$\epsilon_{ox} = (3.9)(8.85 \times 10^{-14}\,\mathrm{F/cm})$, Oxide permittivity for Si devices.
$K_n = \frac{W\mu_n C_{ox}}{2L}$, Conduction Parameter
$K_n = \frac{k'_n}{2} \cdot \frac{W}{L}$, Conduction Parameter
$k'_n = \mu_n C_{ox}$, Process conduction parameter.
μ_n, Electron mobility in the inversion layer.

P-Channel

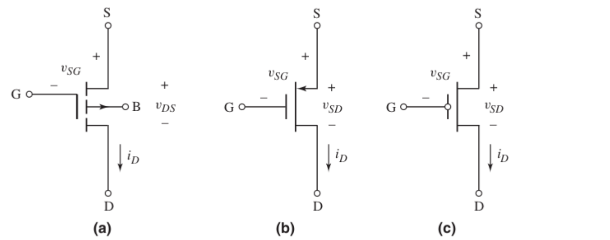


Figure 3.13 The p-channel enhancement-mode MOSFET: (a) conventional circuit symbol, (b) circuit symbol that will be used in this text, and (c) a simplified circuit symbol used in more advanced texts

$i_D = K_p\left[2(v_{SG} - V_{TP})v_{SD} - v_{SD}^2\right]$, I-V Characteristic in non-saturation.
$i_D = K_p(v_{SG} - V_{TP})^2$, I-V Characteristic in saturation.
$K_p = \frac{W\mu_p C_{ox}}{2L}$, Conduction Parameter
$K_p = \frac{k'_p}{2} \cdot \frac{W}{L}$, Conduction Parameter
$k'_p = \mu_p C_{ox}$

Table 3.1 Summary of the MOSFET current–voltage relationships

NMOS	PMOS
Nonsaturation region ($v_{DS} < v_{DS}$(sat))	Nonsaturation region ($v_{SD} < v_{SD}$(sat))
$i_D = K_n[2(v_{GS} - V_{TN})v_{DS} - v_{DS}^2]$	$i_D = K_p[2(v_{SG} + V_{TP})v_{SD} - v_{SD}^2]$
Saturation region ($v_{DS} > v_{DS}$(sat))	Saturation region ($v_{SD} > v_{SD}$(sat))
$i_D = K_n(v_{GS} - V_{TN})^2$	$i_D = K_p(v_{SG} + V_{TP})^2$
Transition point	Transition point
$v_{DS}(\mathrm{sat}) = v_{GS} - V_{TN}$	$v_{SD}(\mathrm{sat}) = v_{SG} + V_{TP}$
Enhancement mode	Enhancement mode
$V_{TN} > 0$	$V_{TP} < 0$
Depletion mode	Depletion mode
$V_{TN} < 0$	$V_{TP} > 0$

§3.2 - Mosfet DC Analysis

Establishes the DC operating point, Q. This is I_D and V_{DS}
A resistor on the source leg provides stability via negative feedback at the expense of reducing gain. Alternatively a CC bias may be used to increase stability without limiting gain.

Common Source Amplifiers

N-Type

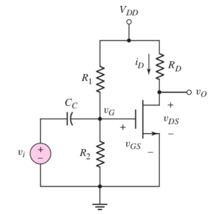


Figure 3.24 An NMOS common-source circuit

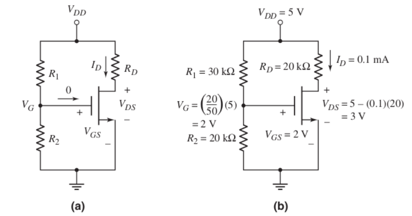


Figure 3.25 (a) The dc equivalent circuit of the NMOS common-source circuit and (b) the NMOS circuit for Example 3.3, showing current and voltage values

$v_G = v_{GS} = \left(\frac{R2}{R1+R2}\right)V_{DD}$
$I_D = K_n(V_{GS} - V_{TN})^2$
$V_{DS} = V_{DD} - I_D R_D$
$P_T = I_D V_{DS}$, Power
If $V_{DS} > V_{DS}(sat)$, where $V_{DS}(sat) = V_{GS} - V_{TN}$, then the transistor is biased in the saturation region

P-Type

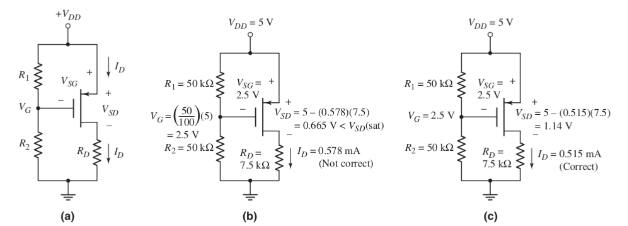


Figure 3.26 (a) A PMOS common-source circuit, (b) the PMOS common-source circuit for Example 3.4 showing current and voltage values when the saturation-region bias assumption is incorrect, and (c) the circuit for Example 3.4 showing current and voltage values when the nonsaturation-region bias assumption is correct

$v_G = \left(\frac{R2}{R1+R2}\right)V_{DD}$
$v_{SG} = V_{DD} - V_G$
$I_D = K_p(V_{SG} + V_{TP})^2$
$V_{SD} = V_{DD} - I_D R_D$
$P_T = I_D V_{DS}$, Power
If $V_{SD} > V_{SD}(sat)$, where $V_{SD}(sat) = V_{SG} + V_{TP}$, then the transistor is biased in the saturation region

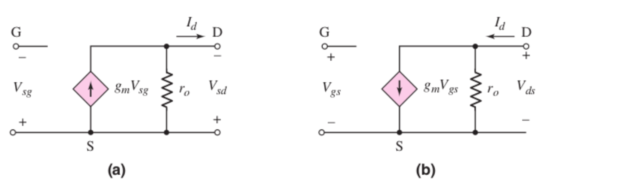


Figure 4.11 Small signal equivalent circuit of a p-channel MOSFET showing (a) the conventional voltage polarities and current directions and (b) the case when the voltage polarities and current directions are reversed.

§4.1 - Mosfet amplifier

$g_m = 2\sqrt{K_n I_{DQ}}$, Trans-conductance
$r_o = \frac{1}{\lambda I_D}$

$r_{eq} = \frac{1}{g_m}$, Small Sig curr source equivalent resistance.

§6 - BJT Amplifier

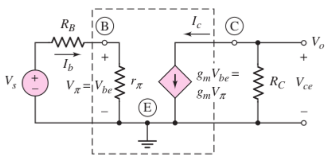


Figure 6.11 The small-signal equivalent circuit of the common-emitter circuit shown in Figure 6.3. The small-signal hybrid-π model of the npn bipolar transistor is shown within the dotted lines.

$g_m = 2\sqrt{K_n I_{DQ}}$,
$g_m = \frac{I_D}{V_{GS}}$,
$g_m = 2K_n(V_{GS} - V_{TH})$,
$g_m = \frac{I_C}{V_T H}$,
$r_o = \frac{1}{\lambda I_{DQ}}$,
$r_o = \frac{V_A}{I_C}$,
$r_\pi = \frac{V_T}{I_B}$,
$r_\pi = \frac{\beta}{g_m}$,
$A_v = -g_m \cdot R_C || R_L$, Voltage Gain Formula

Transistor DC Equivalent

$V_{th} = \frac{V_{cc}}{R_1 + R_2} \cdot R_2$,
$R_{th} = R_1 || R_2$,
$V_{ce}(sat) \approxeq 0.2(typ)$,
$I_E \approxeq I_C$, In active region
$-\frac{1}{R_E - R_C}$, load line slope, where R_C & R_E are from the AC or DC equivalent circuit. A load line plot is I_C vs V_{CE}
$I_{RE} = I_B(\beta + 1)R_E$,

Terminology

Common Source: Input connected to gate, output connected to drain.
Common Drain (Source Follower): Input connected to gate, output connected to source.
Common Gate: Input connected to source, output connected to drain.

Transistor formulas

$I_C = \beta \cdot I_B$, Conduction Parameter
$I_B = \frac{I_E}{\beta + 1}$,
$\alpha = \frac{I_C}{I_E}$, Current Ratio
$I_C = I_E - I_B$, Kirchhoff's Current Law
$V_{CE} = V_{BE} + V_{CB}$, Voltage Relationships
$I_C = I_{C0}\left(e^{\frac{V_{BE}}{V_T}} - 1\right)$, BJT Current Equation
$I = I_0 \cdot \left(e^{\frac{V}{n \cdot V_T}} - 1\right)$, Schottky Diode Equation
$I_D = \frac{1}{2}\mu_n C_{ox} \frac{W}{L}(V_{GS} - V_{TH})^2$, MOSFET Drain Current Equation

$I_D = \mu_n C_{ox} \frac{W}{L}\left[(V_{GS} - V_{TH})V_{DS} - \frac{V_{DS}^2}{2}\right]$, MOSFET Drain Current Equation (Triode Region)
$g_m = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D}$, Transconductance Parameter
$A_v = -g_m \cdot R_D$, Voltage Gain Formula

EE General Formulae

$rms = \frac{1}{\sqrt{2}}$,
$V = I \cdot R$, Ohm's law.
$P = V \cdot I$, DC Power.
$P = V \cdot I \cdot \cos(\theta)$, AC power.
$E = P \cdot t$, Energy.
$C = \frac{Q}{V}$, Capacitance.
$V = L \cdot \frac{di}{dt}$, Inductance.
$\tau = R \cdot C$, Time constant to reach 63.2% of capacitors final voltage.
$\tau = \frac{L}{R}$, Time constant to reach 63.2% of inductors final value.
$\frac{N_1}{N_2} = \frac{V_1}{V_2}$, Transformer turns ratio.
$V_{\text{peak}} = \sqrt{2} \cdot V_{\text{rms}}$, Peak AC Voltage.
$V_{\text{rms}} = \frac{V_{\text{peak}}}{\sqrt{2}}$, RMS AC Voltage.
$V_{\text{avg}} = \frac{1}{T}\int_0^T V(t)\,dt$, RMS AC Voltage.
$V_{\text{out}} = V_{\text{in}} \cdot \frac{R_2}{R_1 + R_2}$, voltage divider.
$R_{\text{eq}} = R_1 + R_2 + \ldots + R_n$, series resistors.
$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}$, Parallel resistors.
$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots + \frac{1}{C_n}$, Series capacitors.
$C_{\text{eq}} = C_1 + C_2 + \ldots + C_n$, parallel capacitors.

Convert Polar to Rectangular

$$x = r\cos\theta$$
$$y = r\sin\theta$$

Exact Slope of a Tangent Line

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

Basic integration Rules

$\int kf(u)du = k\int f(u)du + C$,
$\int [f(u) \pm g(u)]du = \int f(u)du \pm \int g(u)du$, $\int du = u + C$,
$\int u^n du = \frac{u^{n+1}}{n+1} + C, n \neq -1$, $\int \frac{du}{u} = \ln|u| + C$,
$\int \frac{u}{du} = \frac{u^2}{2} + C$, $\int e^u du = e^u + C$,$\int e^{4u} = \frac{e^{4u}}{4} + C$,
$\int a^u du = \left(\frac{1}{\ln a}\right)a^u + C$,

Some Integrals

$\int \sin u du = -\cos u + C$, $\int \cos u du = \sin u + C$,
$\int \tan u du = -\ln|\cos u| + C$, $\int \cot u du = \ln|\sin u| + C$,
$\int \sec u du = \ln|\sec u + \tan u| + C$,
$\int \csc u du = -\ln|\csc u + \cot u| + C$, $\int \sec^2 u du = \tan u + C$,
$\int \csc^2 u du = -\cot u + C$, $\int \sec u \tan u du = \sec u + C$,
$\int \csc u \cot u du = -\csc u + C$, $\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin\frac{u}{a} + C$,
$\int \frac{du}{a^2 + u^2} = \frac{1}{a}\arctan\frac{u}{a} + C$, $\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a}\text{arcsec}\frac{|u|}{a} + C$,
$\int \sin 3x = -\frac{1}{3}\cos 3x$, $\int e^{-4x} = \frac{e^{-4x}}{-4}$

$\int kdx = kx + C$, $\int xdx = \frac{1}{2}x^2 + C$, $\int x^2 dx = \frac{1}{3}x^3 + C$,
$\int \frac{1}{x}dx = \ln|x| + C$, $\int e^x dx = e^x + C$,$\int k^u du = \frac{k^u}{\ln u} + C$,
$\int \ln x dx = x\ln x - x + C$, $\int \cos x dx = \sin x + C$,
$\int \sin x dx = -\cos x + C$, $\int \sec^2 x dx = \tan x + C$,
$\int x^n dx = \frac{x^{n+1}}{n+1} + C$, $\int \tan x = -\ln(\cos x) + C$,

Integration by Parts

$$\int udv = uv - \int vdu$$

Some Identities

$\sin 2x = 2\sin x\cos x$

Pythagorean:

$\sin^2 x + \cos^2 x = 1$, $1 + \tan^2 x = \sec^2 x$, $1 + cot^2 x = \csc^2 x$

Reciprocal:

$\sin x = \frac{1}{\csc x}$, $\cos x = \frac{1}{\sec x}$, $\tan x = \frac{\sin x}{\cos x} = \frac{1}{\cot x}$
$\csc x = \frac{1}{\sin x}$, $\sec x = \frac{1}{\cos x}$, $\cot x = \frac{\cos x}{\sin x} = \frac{1}{\tan x}$

Half Angle:

$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$, $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$

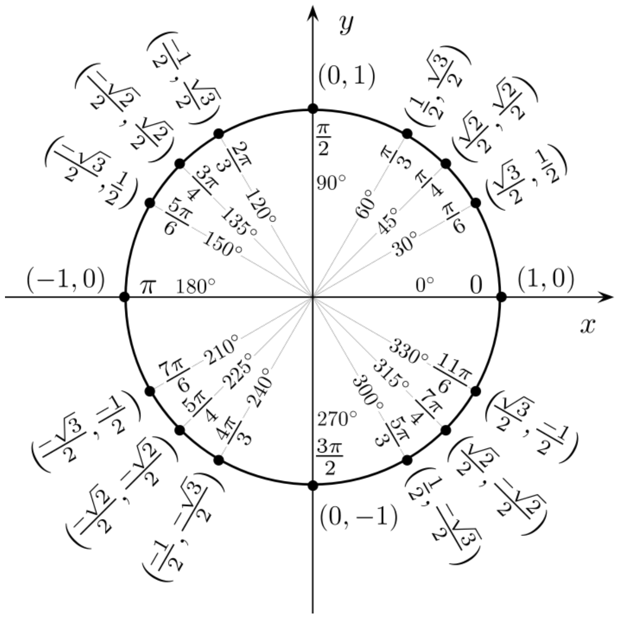
Additional Notes:

$\ln(x * y) = \ln(x) + \ln(y)$, $\ln(x/y) = \ln(x) - \ln(y)$
$\ln x^a = a\ln x$, $\tan\theta = \frac{\sin\theta}{\cos\theta}$
$ax^2 + bx + c = 0$, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$\ln a = c \equiv e^c = a$
$\sqrt[n]{a} = a^{\frac{1}{n}}$, $a^{-n} = \frac{1}{a^n}$, $\sqrt[n]{a^m} = a^{\frac{m}{n}}$, $a^0 = 1$, $(a^m)^n = a^{mn}$,
$a^m * a^n = a^{m+n}$, $\frac{a^m}{a^n} = a^{m-n}$, Rewrite $\sqrt{5x}$ as $\sqrt{5}\sqrt{x}$,

Some Derivatives:

$\frac{d}{du}\sin u = (\cos u)u'$, $\frac{d}{du}\cos u = -(\sin u)u'$,
$\frac{d}{du}\tan u = (\sec^2 u)u'$, $\frac{d}{du}\cot u = -(\csc^2 u)u'$,
$\frac{d}{du}\sec u = (\sec u\tan u)u'$, $\frac{d}{du}\csc u = -(\csc u\cot u)u'$,
$\frac{d}{du}\arcsin u = \frac{u'}{\sqrt{1 - u^2}}$, $\frac{d}{du}\arccos u = \frac{-u'}{\sqrt{1 - u^2}}$,
$\frac{d}{du}\arctan u = \frac{u'}{1 + u^2}$, $\frac{d}{du}\text{arccot}\ u = \frac{-u'}{1 + u^2}$,
$\frac{d}{du}\text{arcsec}\ u = \frac{u'}{|u|\sqrt{u^2 - 1}}$, $\frac{d}{du}\text{arccsc}\ u = \frac{-u'}{|u|\sqrt{u^2 - 1}}$

$\frac{d}{du}[\ln u] = \frac{1}{u}u'$, $\frac{d}{dx}[e^{-x}] = -e^{-x}$, $e^{\ln a} = a$
$\frac{d}{du}[\sqrt{u}] = \frac{u'}{2\sqrt{u}}$, $e^{3x} = 3e^{3x}$, $\frac{d}{dx}[x] = 1$, $\frac{d}{dx}[c] = 0$,
$\frac{d}{du}[\frac{1}{u}] = \frac{1}{u^2}$, $\frac{du}{u} = \ln|u|$,



y
x
$(0,1)$
$(1,0)$
$(-1,0)$
$(0,-1)$
$\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right)$
$\left(\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)$
$\left(\frac{\sqrt{3}}{2},\frac{1}{2}\right)$
$\left(-\frac{1}{2},\frac{\sqrt{3}}{2}\right)$
$\left(-\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)$
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$\left(\frac{1}{2},-\frac{\sqrt{3}}{2}\right)$
$\left(\frac{\sqrt{2}}{2},-\frac{\sqrt{2}}{2}\right)$
$\left(\frac{\sqrt{3}}{2},-\frac{1}{2}\right)$
$\frac{\pi}{2}$ 90°
$\frac{\pi}{3}$ 60°
$\frac{\pi}{4}$ 45°
$\frac{\pi}{6}$ 30°
0 0°
$\frac{2\pi}{3}$ 120°
$\frac{3\pi}{4}$ 135°
$\frac{5\pi}{6}$ 150°
π 180°
$\frac{7\pi}{6}$ 210°
$\frac{5\pi}{4}$ 225°
$\frac{4\pi}{3}$ 240°
$\frac{3\pi}{2}$ 270°
$\frac{5\pi}{3}$ 300°
$\frac{7\pi}{4}$ 315°
$\frac{11\pi}{6}$ 330°