ESE/EEO 306
Random Signals and Systems
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Assignment 4A
Joint Random Variables - Marginals & Independence

1 Assignment

1. Random variables X_1 and X_2 have the joint PMF $p_{X_1,X_2}(x_1, x_2)$ given by the following table:

$p_{X_1,X_2}(x_1,x_2)$	$x_2=-1$	$x_2=0$	$x_2=1$
$x_1=-1$	3/16	1/16	0
$x_1=0$	1/6	1/6	1/6
$x_1=1$	0	1/8	1/8

   (a) Compute the marginal PMF $p_{X_1}(x_1)$.
   (b) Compute the marginal PMF $p_{X_2}(x_2)$.
   (c) Compute the probability $P(X_1 = X_2)$.
   (d) Are X_1 and X_2 independent?

2. Random variables X_1 and X_2 have the joint probability mass function (PMF)

$$p_{X_1,X_2}(x_1,x_2) = \begin{cases} cx_1x_2, & x_1 = 1,2,4; \quad x_2 = 1,3 \\ 0, & \text{otherwise.} \end{cases}$$

   (a) What is the value of the constant c.
   (b) What is $P(X_1 < X_2)$?
   (c) What is $P(X_1 > X_2)$?
   (d) What is $P(X_1 = X_2)$?
   (e) What is $P(X_2 = 3)$?

3. Random variables X_1 and X_2 have the joint probability mass function (PMF)

$$p_{X_1,X_2}(x_1,x_2) = \begin{cases} c|x_1+x_2|, & x_1 = -2,0,2; \quad x_2 = -1,0,1 \\ 0, & \text{otherwise.} \end{cases}$$

   (a) What is the value of the constant c.
   (b) What is $P(X_1 < X_2)$?
   (c) What is $P(X_1 > X_2)$?
   (d) What is $P(X_1 = X_2)$?
   (e) What is $P(X_1 < 1)$?

4. Random variables N and K have the joint probability mass function (PMF)

$$p_{N,K}(n,k) = \begin{cases} (1-p)^{n-1}\, p/n, & k = 1,2\ldots n; \quad n = 1,2,\ldots \\ 0, & \text{otherwise.} \end{cases}$$

Find the marginal PMFs $p_N(n)$ and $p_K(k)$.

5. Random variables X_1 and X_2 have the joint PDF

$$f_{X_1,X_2}(x_1,x_2) = \begin{cases} 6e^{-2x_1}e^{-3x_2}, & x_1 \geq 0, \quad x_2 \geq 0, \\ 0, & \text{otherwise.} \end{cases}$$

(a) What is the marginal PDF $f_{X_1}(x_1)$?

(b) What is the marginal PDF $f_{X_2}(x_2)$?

(c) What is $P(X_1 \leq X_2)$?

(d) Are X_1 and X_2 independent?

6. Random variables X_1 and X_2 have the joint PDF

$$f_{X_1,X_2}(x_1,x_2) = \begin{cases} c, & x_1 + x_2 \leq 1, \quad x_1 \geq 0, \quad x_2 \geq 0, \\ 0, & \text{otherwise.} \end{cases}$$

(a) What is the value of the constant c?

(b) What is probability $P(X_1 \leq X_2)$?