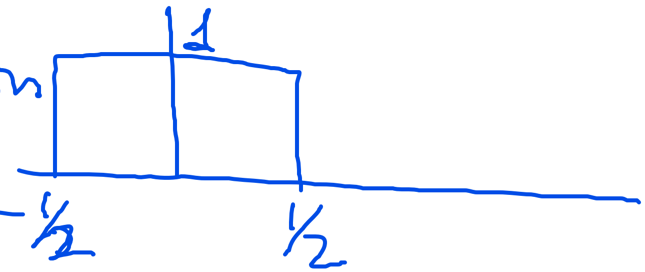
More Signal Shifting and Scaling

consider rectangular function



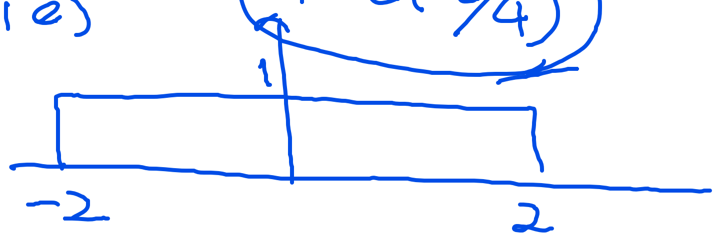
$$\text{rect}(t) = \begin{cases} 1 & |t| \le 1/2 \\ 0 & |t| > 1/2 \end{cases}$$

In general $\text{rect}\left(\frac{t - X}{T}\right)$

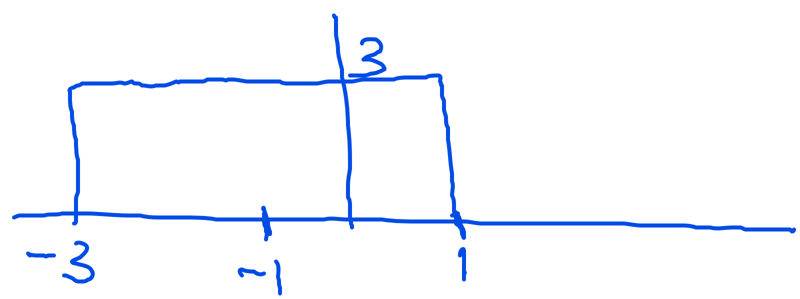
T: width

X: center

ie) $\text{rect}(t/4)$

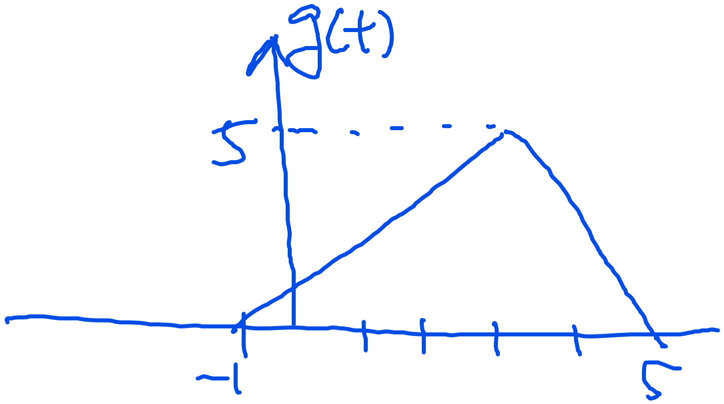


$3\,\text{rect}\left(\frac{t+1}{4}\right)$



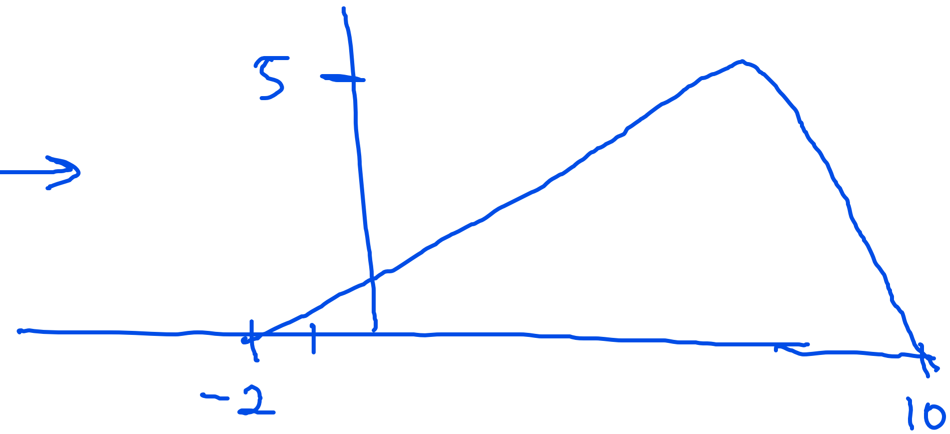
Time Scaling

$g(t) \longrightarrow g(t/a)$

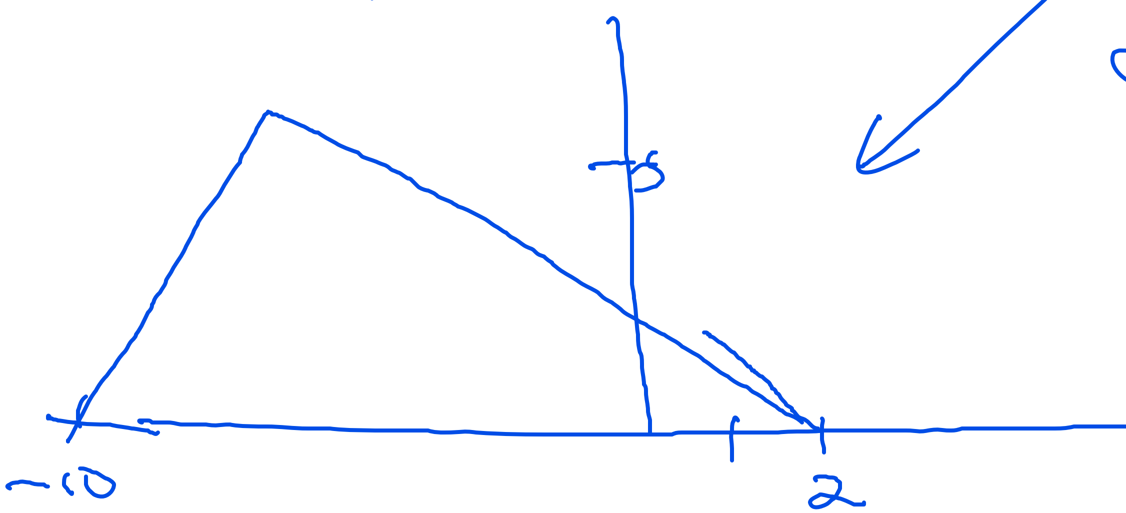


$\longrightarrow$

$g(t/2)$



$g(-t/2)$



flipped version

Shifting and Scaling Functions

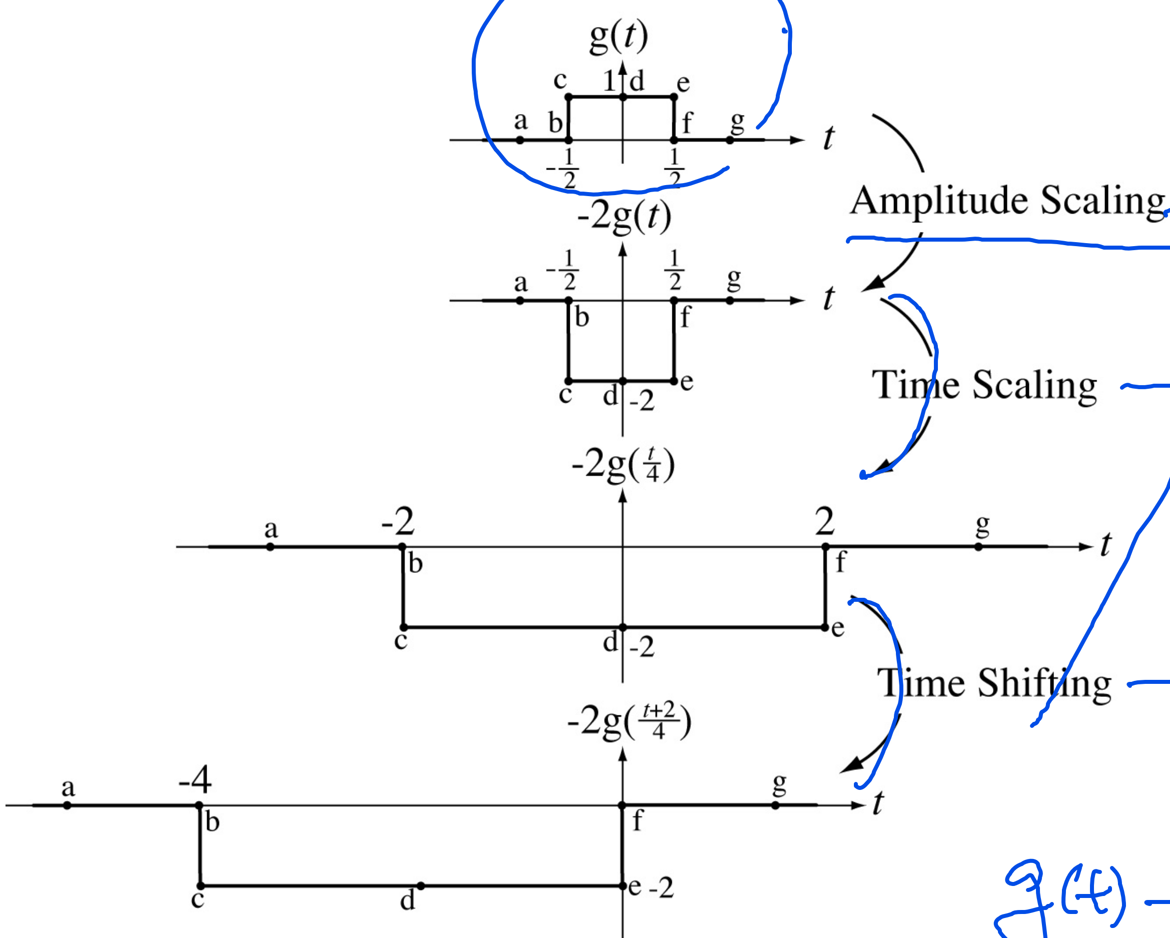
$$g(t) \longrightarrow A g\left(\frac{t - t_0}{a}\right)$$

① $g(t) \xrightarrow{\text{scaling } A} A g(t) \xrightarrow{t \to t/a} A g(t/a) \xrightarrow{t \to (t - t_0)} A g\left(\frac{t - t_0}{a}\right)$

② $g(t) \xrightarrow{\text{scaling } A} A g(t) \xrightarrow{t \to (t - t_0)} A g(t - t_0)$

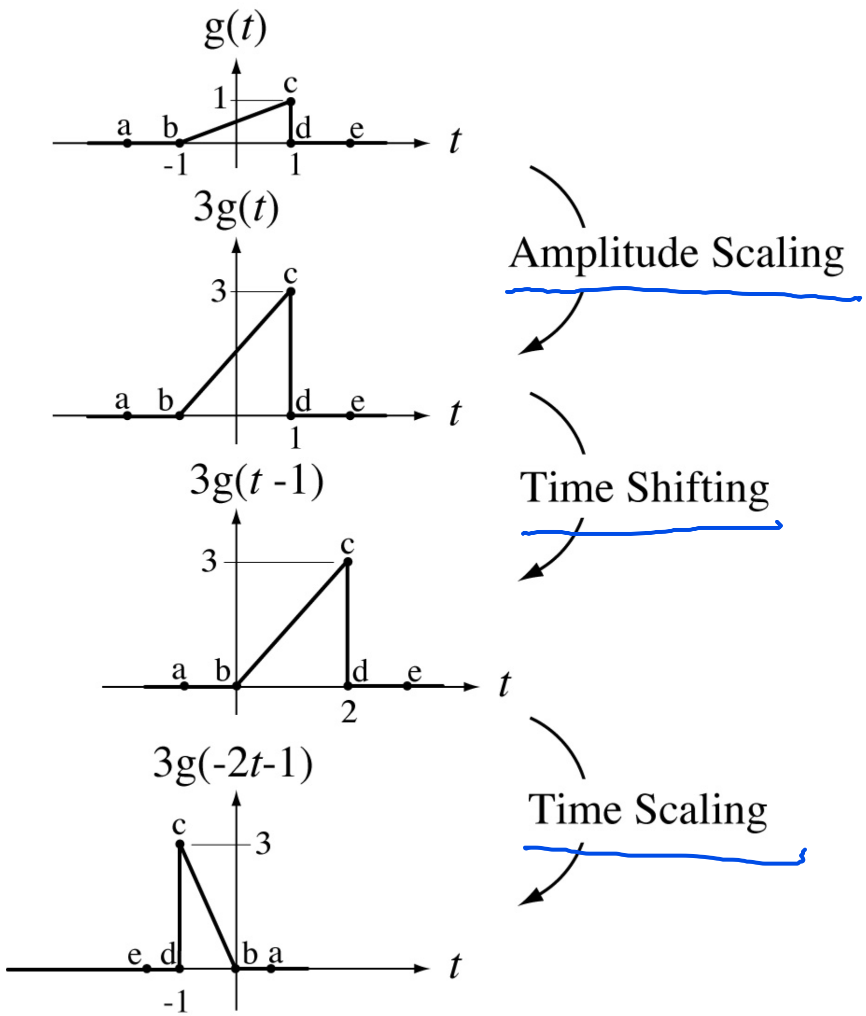
$$\xrightarrow{t \to t/a} A g\left(\frac{t}{a} - t_0\right) \neq A g\left(\frac{t - t_0}{a}\right)$$

Example 1



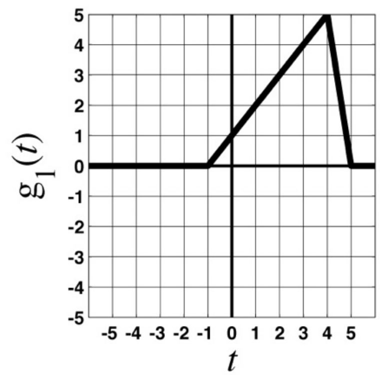
$$g(t) \longrightarrow A g\left(\frac{t - t_0}{a}\right)$$

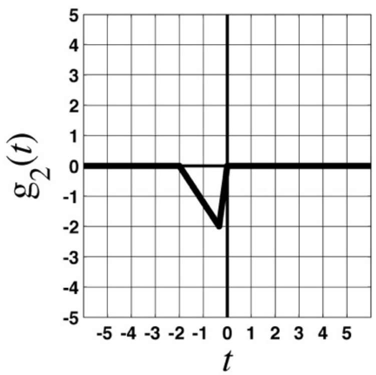
Example 2



$g(t) \longrightarrow$

$A g(bt - t_0)$



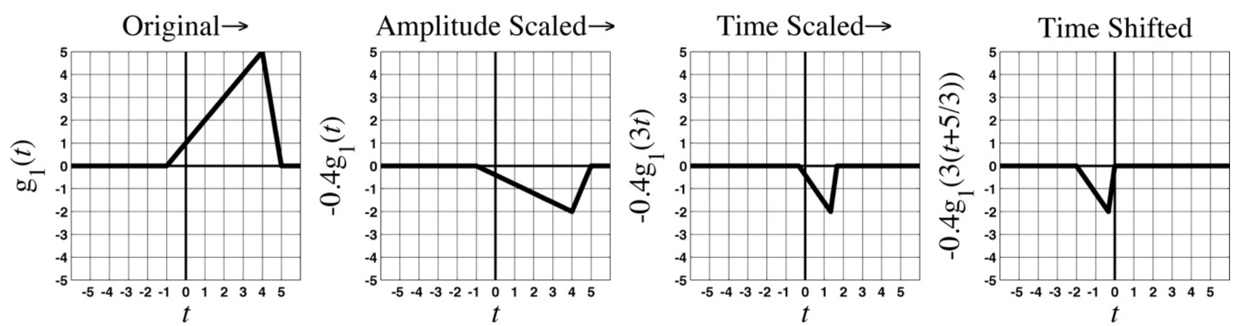


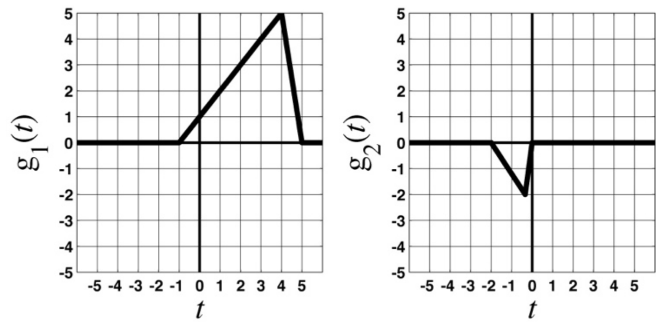
If $g_2(t) = Ag_1((t - t_0)/w)$, what are A, t_0 and w?

Amplitude: $5 \rightarrow -2 \Rightarrow A = -0.4 \Rightarrow g_1(t) \rightarrow -0.4g_1(t)$

Width: $6 \rightarrow 2 \Rightarrow w = 1/3 \Rightarrow -0.4g_1(t) \rightarrow -0.4g_1(3t)$

Shift: $5/3 \Rightarrow t_0 = -5/3 \Rightarrow -0.4g_1(3t) \rightarrow -0.4g_1(3(t + 5/3))$



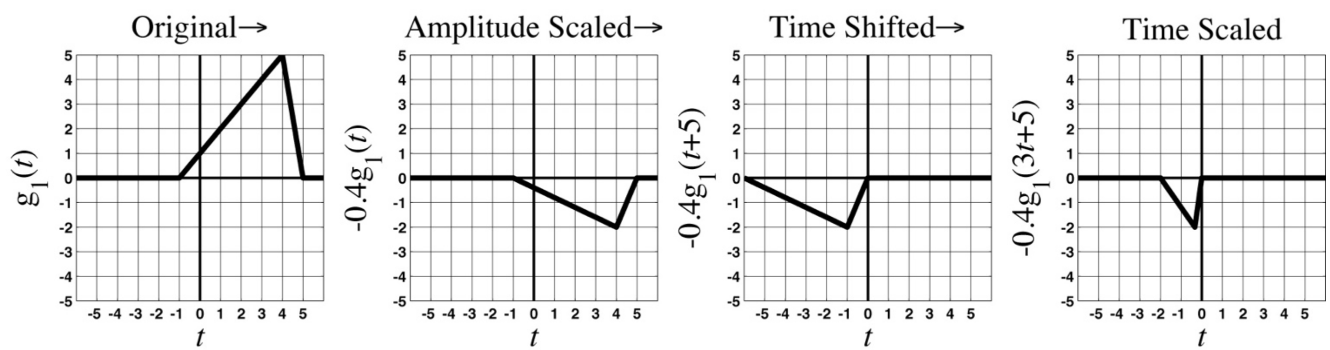


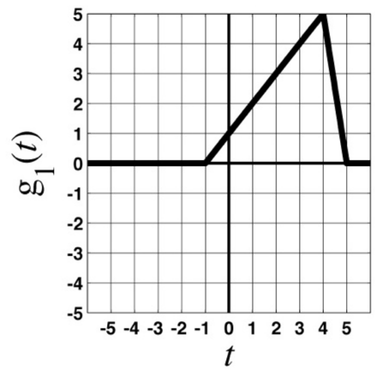
If $g_2 = Ag_1(wt - t_0)$, what are A, t_0 and w?

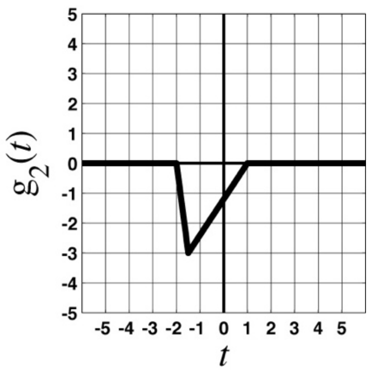
Amplitude: $5 \rightarrow -2 \Rightarrow A = -0.4 \Rightarrow g_1(t) \rightarrow -0.4g_1(t)$

Shift: $5 \Rightarrow t_0 = -5 \Rightarrow -0.4g_1(t) \rightarrow -0.4g_1(t + 5)$

Width: $6 \rightarrow 2 \Rightarrow w = 3 \Rightarrow -0.4g_1(t + 5) \rightarrow -0.4g_1(3t + 5)$





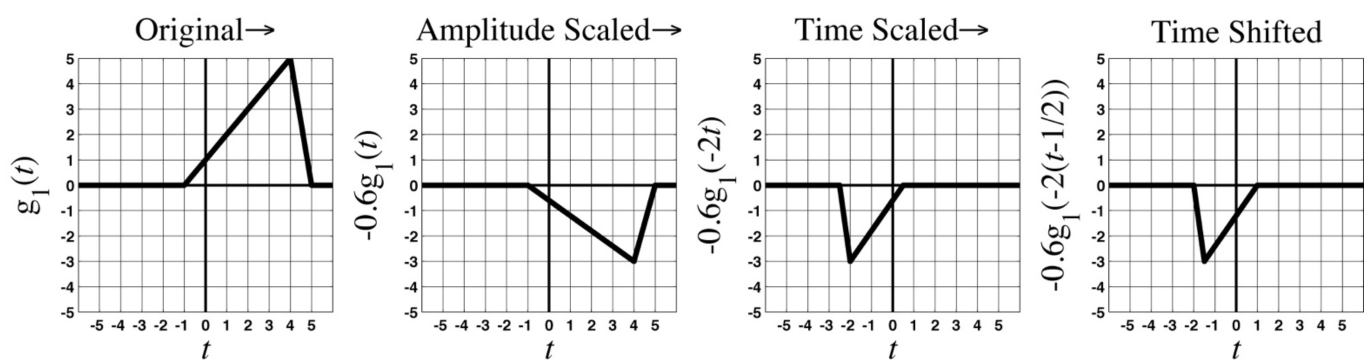


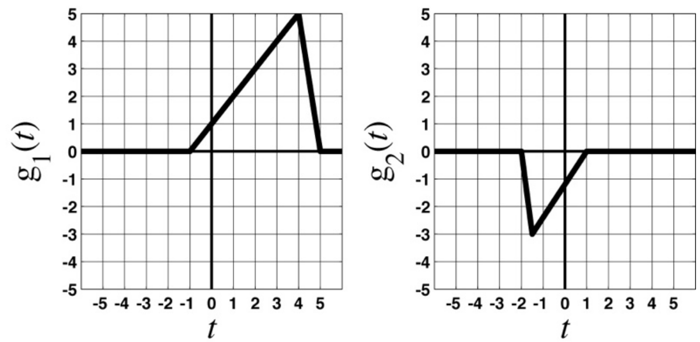
If $g_2 = Ag_1(w(t - t_0))$, what are A, t_0 and w?

Amplitude: $5 \rightarrow -3 \Rightarrow A = -0.6 \Rightarrow g_1(t) \rightarrow -0.6g_1(t)$

Width: $6 \rightarrow -3 \Rightarrow w = -2 \Rightarrow -0.6g_1(t) \rightarrow -0.6g_1(-2t)$

Shift: $1/2 \Rightarrow t_0 = 1/2 \Rightarrow -0.6g_1(-2t) \rightarrow -0.6g_1(-2(t - 1/2))$



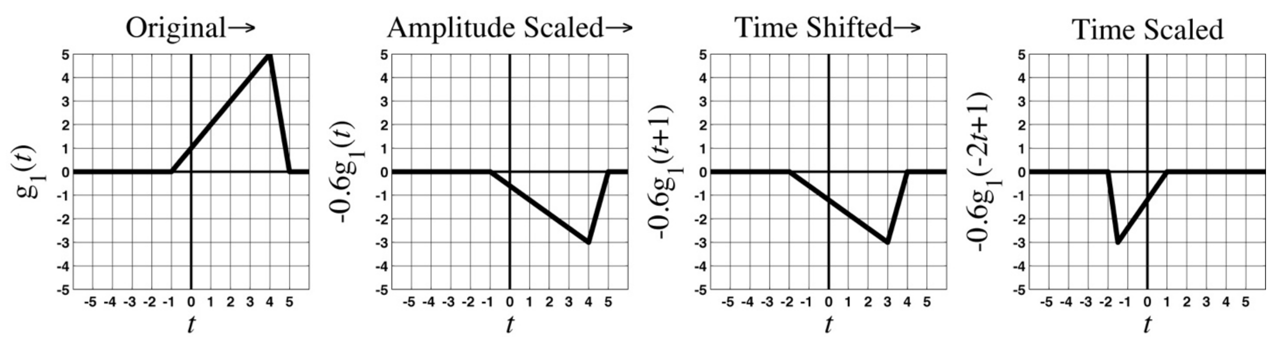


If $g_2 = Ag_1(t/w - t_0)$, what are A, t_0 and w?

Amplitude: $5 \rightarrow -3 \Rightarrow A = -0.6 \Rightarrow g_1(t) \rightarrow -0.6g_1(t)$

Shift: $1 \Rightarrow t_0 = -1 \Rightarrow -0.6g_1(t) \rightarrow -0.6g_1(t+1)$

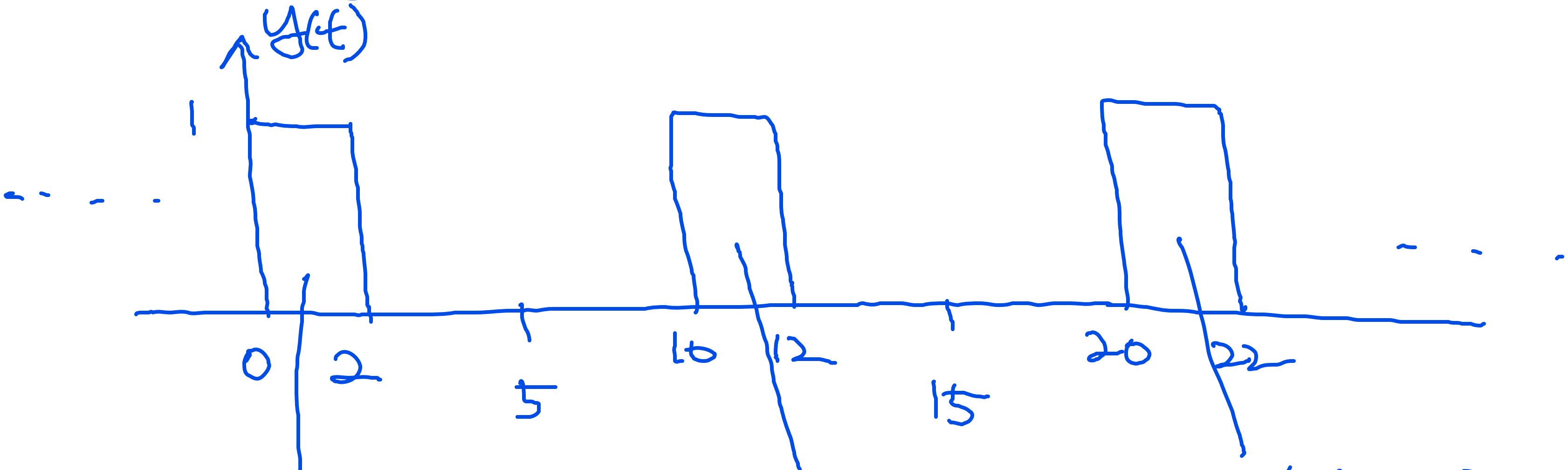
Width: $6 \rightarrow -3 \Rightarrow w = -1/2 \Rightarrow -0.6g_1(t+1) \rightarrow -0.6g_1(-2t+1)$



Scaling Property of Impulse

$$\boxed{\delta(a(t-t_0)) = \frac{1}{|a|}\delta(t-t_0)}$$

Periodic Pulses



$x(t) = g(t) - g(t-2)$ $\qquad x(t-10)$ $\qquad x(t-20)$

$$y(t) = \cdots \; x(t) + x(t-10) + x(t-20) + \cdots$$

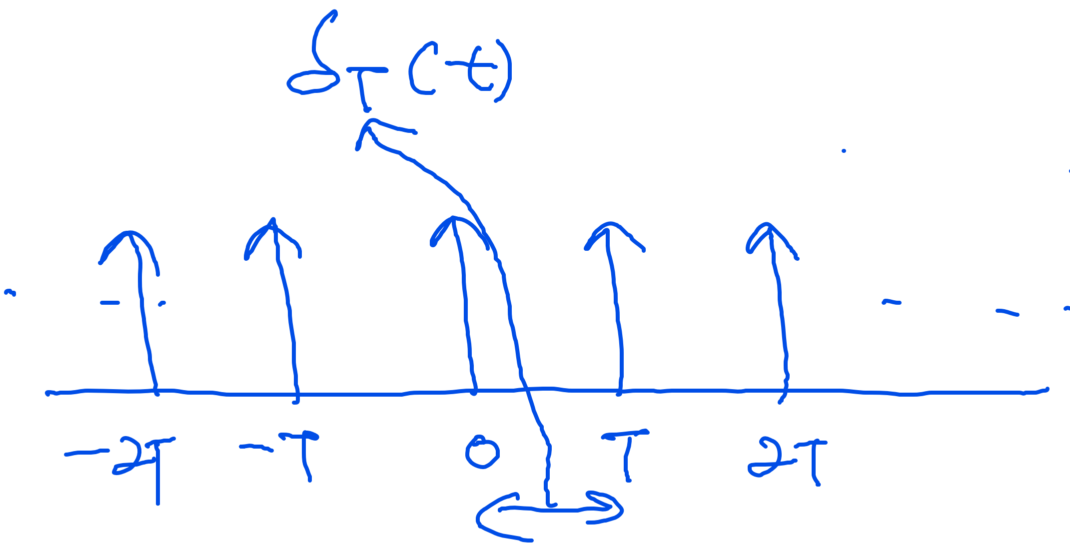
$$= \sum_{n=0}^{\infty} x(t - n \cdot 10)$$

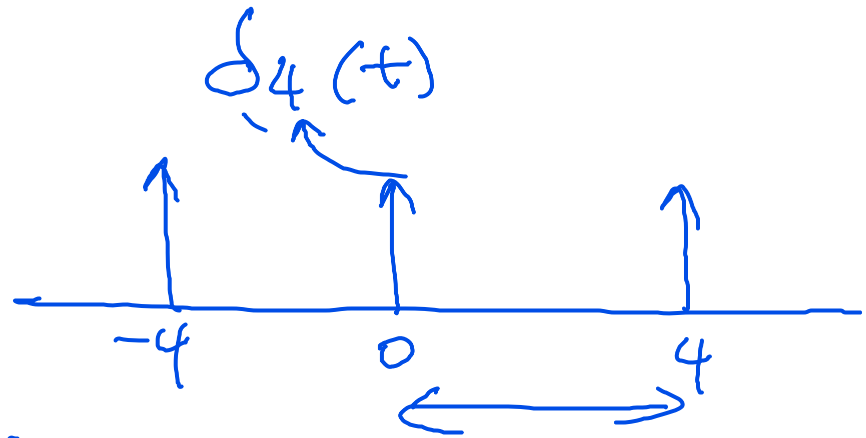
$$\boxed{= \sum_{n=0}^{\infty} g(t - n \cdot 10) - g(t - 2 - n \cdot 10)}$$

Periodic Impulse Function

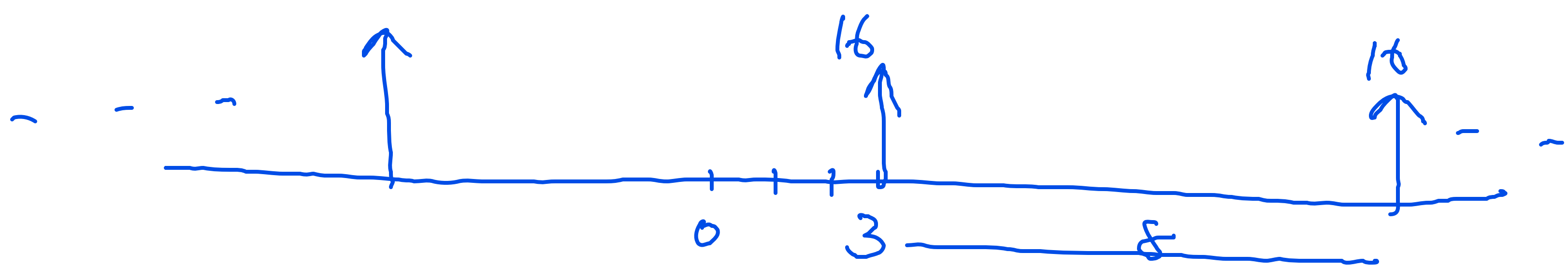
$$\delta_T(t) = \sum_{-\infty}^{\infty} \delta(t-nT)$$

period





$$16\,\delta_8(t-3)$$



Signal Energy and Power

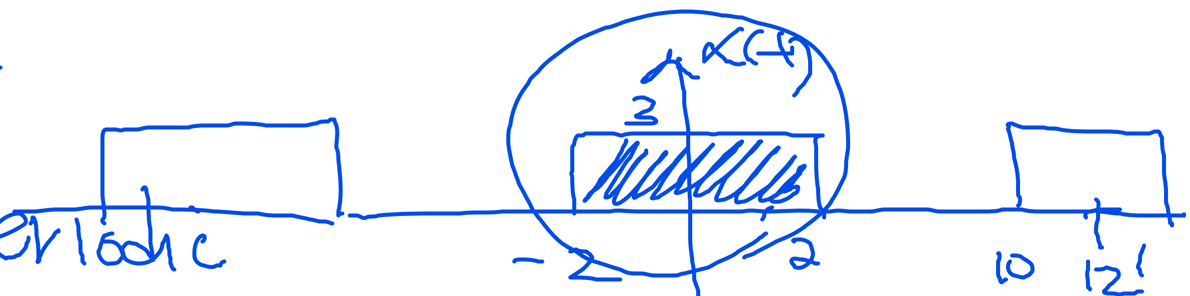
given a signal $x(t)$

If $x(t)$ is periodic with period T.

$$P[x(t)] = \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2 \, dt$$

(periodic signal)

If $x(t)$ is not periodic



$$\boxed{P = \frac{1}{12}\int_{-2}^{2} 3^2 \, dt}$$

$$E[x(t)] = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

(non periodic)

Elementary DT Signals and Their Manipulation

- Notation

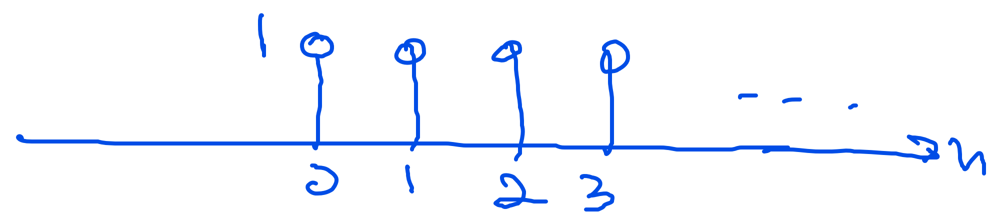
$x[n] = x(nT)$ $\quad x(t)\big|_T$

T: sampling period

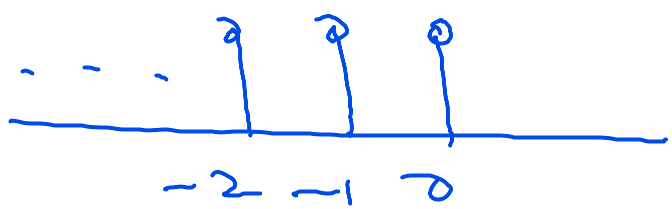
n: time index

- Step Sequence

$$g[n] = \begin{cases} 1 & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$



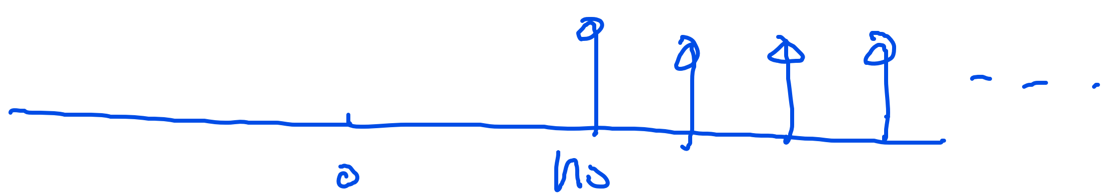
$g[-n]$



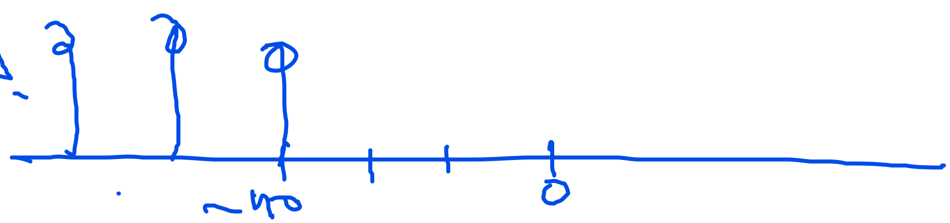
Shifted by n_0

n_0

$g[n-n_0]$

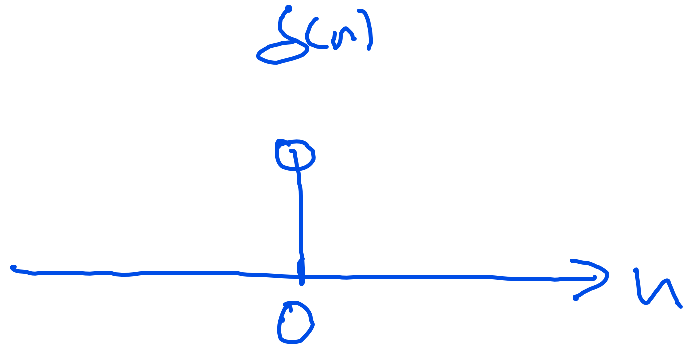


$g[-n-n_0]$

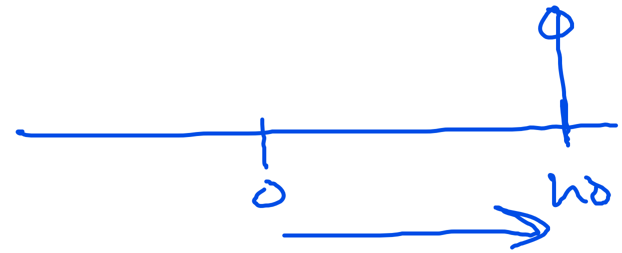


- Impulse Sequence

$$\delta[n] = \begin{cases} 1 & \text{for } n = 0 \\ 0 & \text{for } n \neq 0 \end{cases}$$



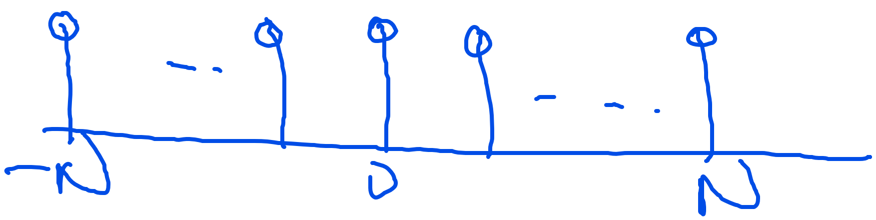
$$\delta[n - n_0] = \begin{cases} 1 & \text{for } n = n_0 \\ 0 & \text{for } n \neq n_0 \end{cases}$$



- Window Sequence

let N be a positive integer

$$w_N[n] = \begin{cases} 1 & \text{for } -N \le n \le N \\ 0 & \text{for } n < -N, \; n > N \end{cases}$$



window of length $2N+1$

$$w_N[n] = \delta[n+N] - \delta[n-N-1]$$

$$= \delta[n+N]\,\delta[-n+N]$$

- Counterpart of Impulse Integration

$$\boxed{x_a(t) = \int_0^{\infty} x(\tau)\,\delta(t-\tau)\,d\tau = x(t)}$$

$$x[n] = x[0]\,\delta[n-0] + x[1]\,\delta[n-1] + \cdots$$

$$\boxed{= \sum_{k=0}^{\infty} x[k]\,\delta[n-k]}$$

Ex) Consider a sequence

$x[0] = 1$

$x[1] = 0$

$x[2] = -2$

$x[3] = 1$

$x[4] = -3$

$$\Rightarrow x[n] = \delta[n] - 2\delta[n-2] + \delta[n-3] - 3\delta[n-4]$$

- Real Exponent Sequence

Sampling the CT signal $x(t) = e^{-at}$ with T

$\Rightarrow \underline{x(nT)} = e^{-a\underline{nT}} = b^n$ where $b = e^{-aT}$

Thus $x[n] = b^n$ for some real b.

Similarly $x[n] \to 0$ $|b| < 1$

time constant $t_c = \frac{-1}{\ln|b|}$

hence $x[n] \to 0$, $\underline{5t_c} = \underline{\text{round}}\left(\frac{-5}{\ln|b|}\right)$