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Networks and Grids: Technology and Theory — Solutions Manual

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A Tour through Networking and Grids

1. There is no need to dig trenches for cables (though tower construction must be taken into account).
2. Fiber optics has the largest information carrying capacity (easily into the Terabits per second).
3. The electronic bottleneck refers to the fact that with current fiber speeds nodal electronics is often slower than fiber speeds so that the information throughput bottleneck is in the nodes.
4. An advantage of geosynchronous satellites is that they are in one spot in the sky all the time so a fixed directional antenna on earth can be used. An advantage of Low Earth Orbit Satellites (LEOS) is that because of their much lower orbits than those of geosynchronous satellites, they have much smaller channel propagation delay.
5. The use of infrared light is technologically possible and systems have been built, but the use of infrared is not that popular.
6. Ad hoc transmission is usually over much smaller distances than satellite transmission (as small as tens of feet for ad hoc networks compared to hundreds or thousands of kilometers for satellite transmission).
7. Wireless sensor networks may be used for such applications as environmental monitoring, interconnecting engine components and connecting computer components (i.e. Bluetooth technology).
8. As a packet moves down a layered protocol stack (is transmitted) each layer may append information to the packet so that it is at its largest at the bottom of the stack (physical layer). At the receiver, as the packet moves up the stack, information may be extracted at each layer so the packet is at its smallest at the top (application) layer.
9. Packets carried on a virtual circuit arrive in the same order they were originally transmitted as they follow a single sequential path.
10. A communication between peer network layer entities moves down the stack at the transmitter to the physical layer, across the network to the destination and up the destination stack to the appropriate network layer entity.

11. One function of the data link layer is to manage communication over a single link between a pair of nodes. Also, encryption is sometimes done at the data link layer.
12. The network layer manages communication over a multiplicity of links and nodes whereas the data link manages communication over a single link between a pair of nodes.
13. The transport layer is responsible for providing end to end communication over possibly unreliable sub-networks.
14. Thruput decreases in Ethernet under heavy loads because of the time wasted by collisions (in the CSMA versions of Ethernet).
15. With longer frames, as opposed to shorter frames, a bigger fraction of time is spent in useful transmission, as opposed to time spent in seizing the channel and collisions, so that utilization is higher.
16. A total of $3 \times 3 = 9$ symbols can be sent at once which is equivalent to 3 bits ($2^3 = 8$).
17. The use of unshielded twisted pair is a good thing because it is lightweight and relatively inexpensive.
18. If node A does not receive a Clear to Send message from node B it might try again to send a Request to Send or move to a different location and try again to send a RTS.
19. The use of a Clear to Send solves the hidden station problem in distributed wireless networks (some node over the radio horizon may try to transmit to node B at the same time as node A, causing a collision).
20. The amount of radio spectrum available for specific purposes is more limited than the equivalent bandwidth available on wired fiber.
21. If a network had to be constructed and taken down frequently, one would be better off using some combination of 802.11 WiFi and 802.15 Bluetooth technology as it is wireless, unlike standard Ethernet versions.
22. Using two 64 kps channels to carry 80 kps means that 128–80 or 48 kps of capacity is wasted.
23. In an ATM NNI connection 16 header bits are used for virtual channel identification (VCI) so that there are 2^{16} or approximately 64 thousand virtual channels (per virtual path).
24. ATM assumes that data is transported over low error rate fiber optics so only the header is protected by error coding. If errors in the data field cannot be tolerated, then a higher level protocol can provide error coding.
25. The VPI and VCI fields in the ATM header, like the rest of the header, are protected by error coding which makes misrouting extremely rare.
26. The use of ATM technology leads to service class independent switches. If one needed different switches for each class of traffic one would have an intractable traffic prediction problem. Since it is almost impossible to accurately predict demand by service class into the future one might install too few or too many of each class of switch, leading to problems in either providing inadequate capacity or over-investing in network facilities.

27. One application where it is critical to deliver data quickly is the transmission of stock prices. Also real time applications such as voice or video require packets be delivered within certain time limits (bounds).
28. A contract for an ATM session is difficult to define because there are so many quality of service parameters that could be used. Thus there are many possible contracts with many possible pricing options.
29. A T1 line has a data rate of approximately 1.5 Mbps. A SONET OC-3 channel has a data rate of approximately 155 Mbps.
30. Each byte entry in a SONET OC-1 frame has an equivalent data rate of 64 kbps. Thus to carry 1 Mbps one needs 1 Mbps/64 kbps or 16 entries of the $87 \times 9 = 783$ entries in the frame. Note that if the question is how many frames are needed to carry 1 Mbyte, not 1 Mbps, the answer is 1,000,000/783 or 1278 frames (neglecting path overhead).
31. A SONET add/drop multiplexer is a device that allows signals to be tapped off of and inserted onto a fiber.
32. In SONET protection fibers are backup fibers that can be brought into use if the service fibers that normally carry traffic do not function.
33. The line protocol layer in SONET is most similar to the data link layer in the OSI protocol stack.
34. More virtual paths are allowed on an ATM NNI link than a UNI link as the NNI link is like a trunk that is likely to carry much more traffic than a UNI network access link.
35. Transmitting in one byte of a SONET table has an effective data rate of 64 kbps. This is the standard data rate for an uncompressed digital telephone channel.
36. The approximate data rate of OC-3072 is 3072 times 51 Mbps (the OC-1 rate) or approximately 160 Gbps.
37. Each OC-192 channel has a data rate of approximately 10 Gbps so approximately 76 WDM OC-192 channels are needed to carry 760 Gbps.
38. The technology of WDM allows fiber already put in the ground (at great expense) to be upgraded through a simple replacement of transmitters and receivers to carry a much larger amount of traffic.
39. The basic idea behind grid computing is to allow a user to access a large distributed network of powerful computers and storage devices from anywhere on earth to carry out substantial computations of a scientific, economic or other nature. It is actually an old concept that predates the use of the word "grid".
40. To partake in grid computing a computer installation needs to be more open to use to outside users and entities, something not that well tolerated under old policies.
41. In the past when the (computer) resource owner was the key person, operations were optimized for high throughput. Making the user the key person necessitates a new set of requirements.

2

Fundamental Stochastic Models

Solution 2.1.

$$P_n(t+\Delta t) = P_n(t)P_{n,n}(\Delta t) + P_{n-1}(t)P_{n-1,n}(\Delta t)$$

$$n = 1, 2, \ldots + P_{n+1}(t)P_{n+1,n}(\Delta t)$$

$$P_0(t+\Delta t) = P_0(t)P_{0,0}(\Delta t) + P_1(t)P_{1,0}(\Delta t)$$

These equations relate the probability of n customers at time $t+\Delta t$ to the probabilities of $n-1, n$ and $n+1$ customers at time t. This is a first order model. Then:

$$\begin{aligned}
P_n(t+\Delta t) &= P_n(t)(1-\lambda(n)\Delta t)(1-\mu(n)\Delta t) \\
&\quad + P_{n-1}(t)\lambda(n-1)\Delta t \\
&\quad + P_{n+1}(t)\mu(n+1)\Delta t \\
P_0(t+\Delta t) &= P_0(t)(1-\lambda(0)\Delta t) + P_1(t)\mu(1)\Delta t
\end{aligned}$$

Making a difference equation:

$$\begin{aligned}
\frac{P_n(t+\Delta t) - P_n(t)}{\Delta t} &= -(\lambda(n)+\mu(n))P_n(t) \\
&\quad + \lambda(n-1)P_{n-1}(t) \\
&\quad + \mu(n+1)P_{n+1}(t) \quad n = 1, 2, \ldots \\
\frac{P_0(t+\Delta t) - P_0(t)}{\Delta t} &= -\lambda(0)P_0(t) + \mu(1)P_1(t)
\end{aligned}$$

If $\Delta t \to 0$:

$$\frac{dP_n(t)}{dt} = -(\lambda(n) + \mu(n))P_n(t) + \lambda(n-1)P_{n-1}(t) + \mu(n+1)P_{n+1}(t) \quad n = 1, 2, \ldots$$

$$\frac{dP_0(t)}{dt} = -\lambda(0)P_0(t) + \mu(1)P_1(t)$$

Solution 2.2. $\lambda = 400$ calls/sec

$$t = \frac{1}{400}\,\text{sec}, \quad \lambda t = 1$$

$$t = \frac{1}{1000}\,\text{sec}, \quad \lambda t = .4$$

$P_n(t) = \frac{(\lambda t)^n}{n!}e^{-\lambda t} \to$ Poisson distribution

n	$\lambda t = .4$	$\lambda t = 1.0$
0	.67	.368
1	.268	.368
2	.0536	.184

Solution 2.3. $\lambda = 400$ calls/sec

$$\overline{\#\ \text{calls}} \Rightarrow \lambda t = 400 \times \frac{1}{400} = 1\,\text{call}$$

$$\Rightarrow \lambda t = 400 \times \frac{1}{4} = 100\,\text{calls}$$

$$\Rightarrow \lambda t = 400 \times 1 = 400\,\text{calls}$$

Solution 2.4.

$$\lambda T = 400 \times .25 = 100\,\text{calls}$$

$$3\lambda T = 1200 \times .25 = 400\,\text{calls}$$

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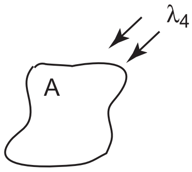
300 additional calls

Solution 2.5.



$$\lambda_A A = [\#\text{stations/area}][\text{area}] = \#\text{stations}$$

Solution 2.6. As i increases, it is less and less likely that the first arrival hasn't occurred yet.

Solution 2.7. Under a geometric distribution the expected time to the first breakdown is $\frac{1}{P}$ or 500 days. But P may increase with truck age.

Solution 2.8.

$$P[\text{Train of length } i] = P^{i-1}(1-P)$$

$$\begin{aligned}\overline{\text{Train length}} = \sum_{i=1}^{\infty} iP(i) &= \sum_{i=1}^{\infty} i(1-P)P^{i-1} \\ &= (1-P)\sum_{i=1}^{\infty} iP^{i-1} \\ &= \frac{(1-P)}{P}\sum_{i=1}^{\infty} iP^{i} \\ &= \frac{(1-P)}{P}\frac{P}{(1-P)^2} = \frac{1}{1-P}\end{aligned}$$

Solution 2.9.

$$\sigma^2 = P(1-P)$$

maximized at $P = \frac{1}{2}$ as the most uncertainty in the random variable at this value.

Solution 2.10.

$$\mathrm{E}[\#\text{arrivals for 1 slot}] = P$$

As slots are independent in Bernoulli process,

$$E[\#\text{arrivals for N slots}] = N_P$$

Solution 2.11. Use binomial distribution:

$$\begin{aligned} P[5 \text{ arrivals in 10 slots}] &= \binom{N}{n} P^n (1-P)^{N-n} \\ &= \binom{10}{5} P^5 (1-P)^5 \\ &= \binom{10}{5} (.2)^5 (.8)^5 \\ &= 252 \times .00032 \times .32768 \\ &= .264 \end{aligned}$$

Solution 2.12. Because

$$\binom{10}{7} = \binom{10}{3}$$

Solution 2.13.

(a)

$$P = \frac{20}{60} = \frac{1}{3} = .33 \quad q = 1 - P$$

(b)

$$\begin{aligned} &P(\text{at least one circuit free}) \\ &\quad = P(\text{finds a free circuit}) \\ &\quad = 1 - P(0 \ \text{ circuit free}) \\ &\quad = 1 - \binom{3}{0} q^{\circ} (1-q)^3 = 1 - P^3 \end{aligned}$$

(c)

$$P(\text{one circuit free}) = \binom{3}{1} q (1-q)^2$$

(d)

$$
\begin{aligned}
\overline{\#\text{ busy channels}} &= 1 \cdot P(1 \text{ busy}) + 2P(2 \text{ busy}) + 3P(3 \text{ busy}) \\
&= 1 \binom{3}{1} P(1-P)^2 + 2 \binom{3}{2} P^2(1-P) \\
&\quad +3 \binom{3}{3} P^3(1-P)^0 \\
&= 3P(1-P)^2 + 6P^2(1-P) + 3P^3
\end{aligned}
$$

Solution 2.14.

$$
\begin{aligned}
&P \triangleq \text{prob[a computer down]} \\
&P(\text{at least one computer up in } a \text{ city}) \\
&\quad = 1 - P(\text{both computers in } a \text{ city down}) \\
&\quad = 1 - P^2 \\
&P(\text{at least 1 computer up in } \textit{each} \text{ city}) \\
&\quad = (1 - P^2)^3
\end{aligned}
$$

Solution 2.15. No value appears to be feasible to reach .6 but in general one iterates on a calculator the formula:

$$
\begin{aligned}
&P(4 \text{ packets in } 10 \text{ slots}) \\
&\quad = \binom{10}{4} P^4(1-P)^6 \\
&\quad = 210P^4(1-P)^6
\end{aligned}
$$

Solution 2.16. For a binomial distribution the moment generating function is

$$P(Z) = (ZP + 1 = P)^N$$

A useful fact is

$$E[i^2] = \overset{\text{2nd derivative}}{E^{(2)}}[1] + E[i]$$

Then

$$E^{(2)}[1] = N(N-1)(ZP+1-P)^{N-2}P^2\big|_{Z=1}$$

$$E^{(2)}[1] = N(N-1)P^2$$

So

$$E[i^2] = N(N-1)P^2 + \overset{E[i]}{N_P}$$

But

$$\sigma^2 = E[i^2] - \overset{E[i]}{\mu}{}^2$$

$$\sigma^2 = N(N-1)P^2 + NP - (NP)^2$$

$$\sigma^2 = NP(1-P)$$

Solution 2.17. Use a Pascal distribution:

$$P_N(K) = \binom{N-1}{K-1} P^K (1-P)^{N-K}$$

$$= \binom{9}{4} P^5 (1-P)^5$$

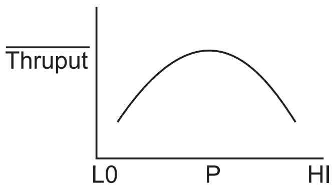
$$= \binom{9}{4} (.35)^5 (.65)^5$$

$$= 126 \times (.35)^5 (.65)^5 = .07678$$

Solution 2.18.

$$\mu = \frac{K}{P} = \frac{5}{.35} = 14.3$$

Solution 2.19.



mean thruput is maximized at a middle value, to the left load is light, to the right there are many collisions

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Solution 2.20.

$$\text{Thruput} = NP(1-P)^{N-1}$$

$$\frac{d\ \text{Thruput}}{dP} = \frac{d}{dP}(NP(1-P)^{N-1})$$

$$\cancel{N}P(N-1)(1-P)^{N-2}(-1) + \cancel{N}(1-P)^{N-1} = 0$$

$$-P(N-1) + (1-P) = 0$$

$$(1-P) = P(N-1)$$

$$1 - P = PN - P$$

$$\boxed{P = \frac{1}{N}}$$

Solution 2.21. Because there is less contention so a station can acquire the channel sooner.

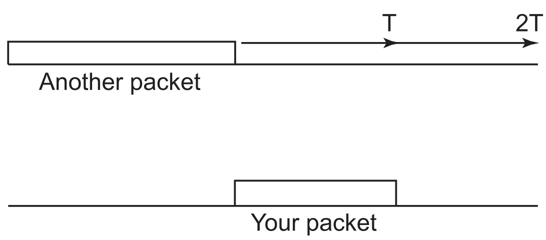
Solution 2.22.

$$U = \frac{1}{1+\frac{2BLe}{cF}} = \frac{1}{1+\frac{2\times 100\times 10^6\cdot 50\times 2.71}{3\times 10^8\times 512}}$$

$$= \frac{1}{1+.176} = .85 = 85\%$$

Solution 2.23. For Fast Ethernet a 1 Gbps Ethernet network maximum size is already about 50 m. Since the size can't be reduced further, it is better to either increase the minimum frame size, or use a switched hub rather than a shared media hub.

Solution 2.24.



Another packet can be anywhere in a "window" of 2T and overlap the packet of interest, causing a collision.

Solution 2.25. Slotted Aloha:

$$S = Ge^{-G}$$
$$\frac{dS}{dG} = e^{-G} - Ge^{-G} = 0$$
$$1 - G = 0$$
$$G = 1$$
$$S = 1 \times e^{-1} = .368$$

Pure Aloha:

$$S = Ge^{-2G}$$
$$\frac{dS}{dG} = e^{-2G} - 2Ge^{-2G} = 0$$
$$1 - 2G = 0$$
$$G = \frac{1}{2}$$
$$S = \frac{1}{2}e^{-2\frac{1}{2}} = .184$$

Solution 2.26. Pure Aloha has a larger delay for large G because of its e^{2G} term, versus slotted Aloha's e^{G} term.

Solution 2.27.

$$\begin{aligned} F_0(x) &= \int_0^x 2\gamma(1-\chi)d\chi \\ &= 2\gamma\chi - \frac{2\gamma\chi^2}{2}\bigg|_0^x \\ &= 2\gamma x - \gamma x^2 \\ &= \gamma x(2-x) \end{aligned}$$

Solution 2.28.

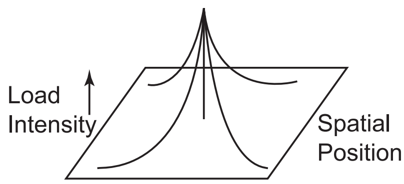
$$
\begin{aligned}
F_{O,DR}(x) &= \int_0^x (f_T(\chi) - f_R(\chi))d\chi \\
&= \int_0^x (2\gamma(1-\chi) - 2\gamma\chi)d\chi \\
&= \int_0^x (2\gamma - 4\gamma\chi)d\chi \\
&= 2\gamma\chi - 2\gamma\chi^2\Big|_0^x \\
&= 2\gamma x - 2\gamma x^2 = 2\gamma x(1-x)
\end{aligned}
$$

Solution 2.29. $\text{Load}_x = 2^{2\ell-1} - 2^{\ell-1}$

ℓ	$2\ell-1$	$\ell-1$	Load_x
2	3	1	6
5	9	4	496
10	19	9	523, 776

Solution 2.30. This is left as an exercise to the reader. Note that there are two components: one is traffic From/To nodes below the link and the other component is traffic between nodes below the link going to nodes on the other side of the link. Be careful on the indexing.

Solution 2.31.



Solution 2.32.

$$P\begin{pmatrix}\text{station xmts on}\\ \text{a given bus}\end{pmatrix} = \frac{P}{M}$$

$$\text{Thruput one bus} = \binom{N}{1}\left(\frac{P}{M}\right)^1\left(1-\frac{P}{M}\right)^{N-1}$$

$$\text{Thruput system} = MN\frac{P}{M}\left(1-\frac{P}{M}\right)^{N-1} \qquad N \text{ station}$$

$$M \text{ buses}$$

$$= NP\left(1-\frac{P}{M}\right)^{N-1} \qquad R = M$$

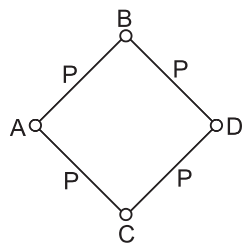
where R is # of connections to different buses per station.

Solution 2.33. P: Probability a link available

(a)

$$q = P^2$$

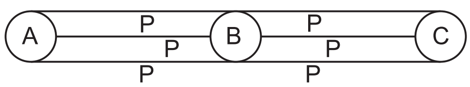
(b)



$$P\begin{bmatrix}\text{at least}\\ \text{one A to D path}\\ \text{available}\end{bmatrix} = 1 - P\begin{bmatrix}\text{no AD}\\ \text{path available}\end{bmatrix}$$

$$= 1-(1-P^2)^2$$

Solution 2.34.



P: probability a link is in use

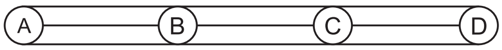
(a)

$$\overline{\#\text{ Busy Links}} = \sum_{n=1}^{N} n\binom{N}{n}P^n(1-P)^{N-n} = NP$$

(b)

$$P\begin{bmatrix}\text{at least one}\\ \text{idle path from}\\ \text{A to C}\end{bmatrix} = \left(P\begin{pmatrix}\text{at least 1 idle}\\ \text{link from A to B}\end{pmatrix}\right)^2$$
$$= \left(1 - P\begin{pmatrix}\text{no idle links}\\ \text{from A to B}\end{pmatrix}\right)^2$$
$$= (1-P^3)^2$$

Solution 2.35.



(a) P: Probability a link is busy

$$P\begin{pmatrix}\text{A to D}\\ \text{call blocked}\end{pmatrix} = P\begin{pmatrix}\text{at least one adjacent pair}\\ \text{is completely blocked}\end{pmatrix}$$
$$= 1 - P\begin{pmatrix}\text{at least one idle link}\\ \text{between each adjacent pair}\end{pmatrix}$$
$$= 1 - \left(P\begin{pmatrix}\text{at least one idle link between}\\ \text{an adjacent pair of nodes}\end{pmatrix}\right)^3$$
$$= 1-(1-P^3)^3$$

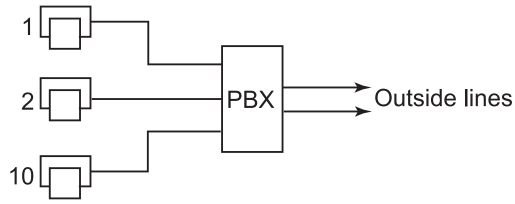
(b)

$$P\begin{pmatrix}\text{at least}\\ \text{two paths (available)}\\ \text{from A to D}\end{pmatrix} = P\begin{pmatrix}\text{at least 2}\\ \text{available links between}\\ \text{every adjacent pair of nodes}\end{pmatrix}$$
$$= (\underbrace{3(1-P)^2P}_{\text{2 links available}} + \underbrace{(1-P)^3}_{\text{3 links available}})^3$$

Solution 2.36. P: Probability a phone seeks on outside line.

(a)

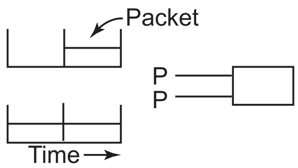
$$P\begin{pmatrix}\text{n phones}\\ \text{seek an}\\ \text{outside line}\end{pmatrix} = \underbrace{\binom{10}{n}}_{\text{binomial distribution}} P^n(1-P)^{10-n}$$



(b)

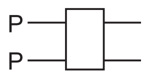
$$\begin{aligned} P(\text{blocking}) &= P(> 2 \text{ phones seek an outside line}) \\ &= \sum_{n=3}^{10} \binom{10}{n} P^n (1-P)^{10-n} \end{aligned}$$

Solution 2.37.



$$\begin{aligned} P\begin{pmatrix} \text{3 or more packets} \\ \text{in 2 consecutive time slots} \end{pmatrix} &= \binom{4}{3} P^3(1-P) + \binom{4}{4} P^4 (1-P)^0 \\ &= \binom{4}{3} P^3(1-P) + P^4 \end{aligned}$$

Solution 2.38.



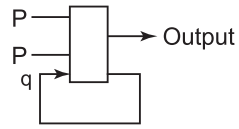
(a)

$$P\begin{pmatrix} \text{Packet on} \\ \text{specific} \\ \text{output} \end{pmatrix} = \underbrace{.5}_{P\begin{pmatrix} \text{Single packet goes to} \\ \text{specific output} \end{pmatrix}} \overbrace{(2P(1-P))}^{\text{one arrival}} + \overbrace{P^2}^{\text{two arrivals}}$$

(b)

$$\overline{\#\text{ packets at outputs}} = 1 - 2 \cdot P(1-P) + 2 \cdot P^2$$

Solution 2.39.



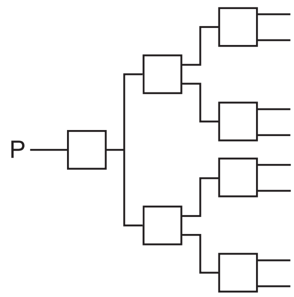
(a)

$$\begin{aligned} q &= P\begin{pmatrix}\text{2 or more packets}\\ \text{at inputs}\end{pmatrix} \\ q &= 2P(1-P)q + P^2(1-q) + P^2 q \\ q &= 2P(1-P)q + P^2 \end{aligned}$$

(b)

$$q = \frac{P^2}{1 - 2P(1-P)}$$

Solution 2.40.



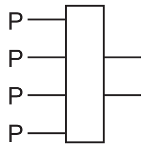
C: Probability a packet appears at switching element output if there is an input packet.

(a) For a given output, the probability a copy appears is $C^3 P$.

(b)

$$\overline{\#\ \text{Copies}} \cong \sum_{n=1}^{8} n \binom{8}{n} (C^3 P)^n (1 - C^3 P)^{8-n}$$

Solution 2.41.

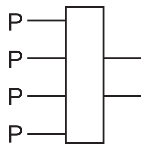


(a) $P(n \text{ arrivals}) = \binom{4}{n} P^n (1-P)^{4-n}$

(b)

$$\begin{aligned} P(\text{at least 1 arrival}) &= 1 - P(0 \text{ arrivals}) \\ &= 1 - \binom{4}{0} P^0 (1-P)^{4-0} \\ &= 1 - (1-P)^4 \end{aligned}$$

Solution 2.42.



(a)

$$\begin{aligned} P\begin{pmatrix}\text{packet dropped} \\ \text{(bird eye's view)}\end{pmatrix} &= P(3 \text{ or more arrivals}) \\ &= \binom{4}{3} P^3 (1-P) + \binom{4}{4} P^4 (1-P)^0 \end{aligned}$$

(b)

$$P\left(\frac{\text{Tagged arriving}}{\text{packet dropped}}\right) = \frac{1}{3}P(2\ \underline{\text{other}}\text{ packets arrive}) + \frac{1}{2}P\left(\begin{array}{c}3\ \underline{\text{other}}\\ \text{packets}\\ \text{arrive}\end{array}\right)$$

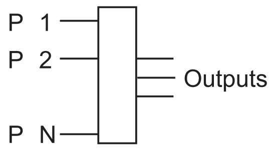
$$= \frac{1}{3}\binom{3}{2}P^2(1-P)\binom{3}{3}P^3(1-P)^0$$

(c)

$$\overline{\text{Thruput}} = 1 \cdot P(1\text{ arrival}) + 2P(2\text{ or more arrivals})$$

$$= 1 \cdot \binom{4}{1}P(1-P)^3 + 2\sum_{n=2}^{4}\binom{4}{n}P^n(1-P)^{4-n}$$

Solution 2.43.



(a)

$$\overline{\text{Thruput}} = 1 \cdot P(1\text{ arrival}) + 2P(2\text{ arrivals}) + 3P(3\text{ or more arrivals})$$

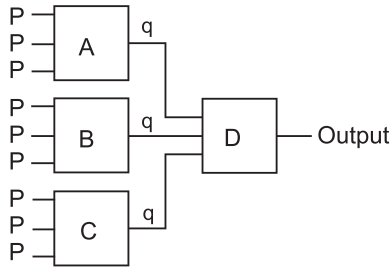
$$= 1 \cdot \binom{N}{1}P(1-P)^{N-1} + 2\binom{N}{2}P^2(1-P)^{N-2}$$

$$+3\sum_{n=3}^{N}\binom{N}{n}P^n(1-P)^{N-n}$$

(b)

$$\overline{\#\text{ Dropped packets}} = \sum_{n=4}^{N}(n-3)\binom{N}{n}P^n(1-P)^{N-n}$$

Solution 2.44.



(a) For either A, B or C

$$\begin{aligned} q &= P(1 \text{ or more arrivals to element}) \\ &= 1-(1-P)^3 \end{aligned}$$

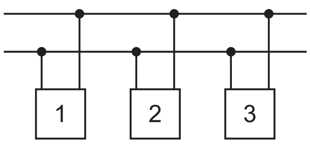
(b)

$$\begin{aligned} \overline{\text{Thruput}} &= P(1 \text{ or more arrivals to D}) \\ &= 1-(1-q)^3 \end{aligned}$$

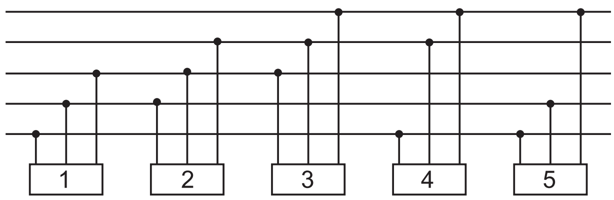
Solution 2.45. $\frac{NR}{m}$ should be integer:

$$\begin{aligned} N &\rightarrow \# \text{ stations} \\ M &\rightarrow \# \text{ buses} \\ R &\rightarrow \# \text{ connections/stations} \end{aligned}$$

(a) $N = 3 \quad R = 2 \quad M = 2$



(b) $N = 5 \quad R = 3 \quad M = 5$



3

Queueing Models

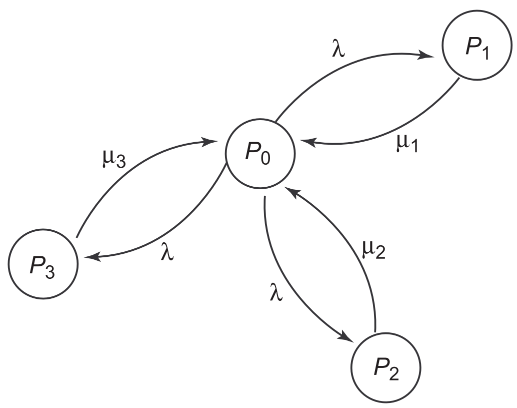
1. In an M/M/1 queue the arrival process is Poisson and the service time distribution is (independent) negative exponential. In a Geom/Geom/1 queue the arrival process is Bernoulli and a customer in the server completes service in each slot with independent probability p (i.e. service times follow a geometric distribution).
2. To say the M/M/1 queue is memoryless means that the queue's state is completely summarized by the current number of customers in the queue. There is no need to include the time since the last arrival or the time the current customer in the server has been in service. A knowledge of this past history has no increased predictive power. This is because the negative exponential distribution which underlies the arrival (Poisson) and service processes is "memoryless". It is the only continuous time memoryless distribution. The geometric distribution is the only discrete time memoryless distribution.
3. Electric current flows in electric circuits. Probability flux flows in Markov chains. The probability flux flowing on a link is the average number of times a second the system state transits the transition. Current can flow in either direction in a simple resistive electric circuit. Current flow direction is preset by transition direction in a Markov chain. Voltage/current sources guarantee non-zero flows of current in electric circuits. The normalization equation guarantees non-zero flows of probability flux in Markov chains.
4. Using Little's Law for a packet buffer, either mean thruput, mean delay or mean number of packets in the buffer can be found simply using the formula from the other two quantities.
5. Global balance holds that in equilibrium the total flow of probability flux into a Markov chain state equals the total flow of probability flux out of the same Markov chain state. The "total" flow is the flow summed over all incoming/outgoing transitions to/from a state. Global balance applies to any Markov chain in equilibrium. Local balance only applies to certain Markov chains. Under local balance the flow in and out of certain subsets

of transitions to/from a state balances. Local balance is a characteristic of Markov chains associated with product from queueing or Petri nets.

6. Global balance equations are usually solved using linear equations with one equation per state. Unfortunately the number of states often goes up exponentially with the size of a system. Moreover the computational complexity of general linear equation solution is proportional to the cube of the number of equations. Thus the computational burden can be overwhelming.
7. From Burke's theorem, if an M/M/1 queue input follows a Poisson process, the output process is also Poisson.
8. Yes, the arrival rate can be greater for an M/M/1 finite buffer queue as all this means is that the buffer is often full and many customers are turned away. By way of contrast, for an infinite buffer M/M/1 queue if the arrival rate is greater than the service rate then the queue is unstable (the waiting line continually increases in size).
9. The Erlang B formula and Erlang C formula both involve a Poisson input of calls and negative exponential service time distributions for each parallel server. However the Erlang B formula is appropriate when there is no waiting line so that if an incoming call doesn't immediately find an idle server, it is dropped. The Erlang C formula applies to systems with a queue so an incoming call is queued if it doesn't immediately find an idle server until it is at the head of the line of the queue and an idle server becomes available.
10. The queueing based memory model is appropriate for computer systems where incoming jobs have memory requirements and memory is a shared resource.
11. A Markov chain imbedded at the departure instants is the system Markov chain that exists only at departure instants.
12. For queueing network product form results we assume Poisson inputs (for an open network), negative exponential servers and (independent) random routing.
13. Traffic equations model the mean throughput (flow) of customers through a queueing network.
14. The mean value analysis algorithm is based on Little's Law. It is also based on the insight that an arriving customer in a closed Markovian queueing network "sees" a number of customers that follows the equilibrium distribution with one less customer in the network.
15. When a negative customer enters a queue with a least one positive customer, the negative customer and one of the positive customers are instantly removed from the network. When a negative customer enters an empty queue, the negative customer is instantly removed from the queue and dropped from the system.
16. Non-product networks can in certain cases be solved with simple recursions. In general numerical linear equations are necessary.

17. Petri networks model concurrency, serializability, synchronization and resource sharing.
18. A specific marking (placement of tokens to places) is a state.
19. Analytical results, though sometimes difficult to obtain, are usually simple to solve and give insight into performance tradeoffs.
20. Simulation results will not always match experimental results because there may be real world factors that influence an experiment that are not programmed into a particular simulation. A simulation will not indicate anything beyond its programming.
21. Markovian queueing networks, Markovian negative customer queueing networks and certain stochastic Petri nets have product form solutions.

Solution 3.22.



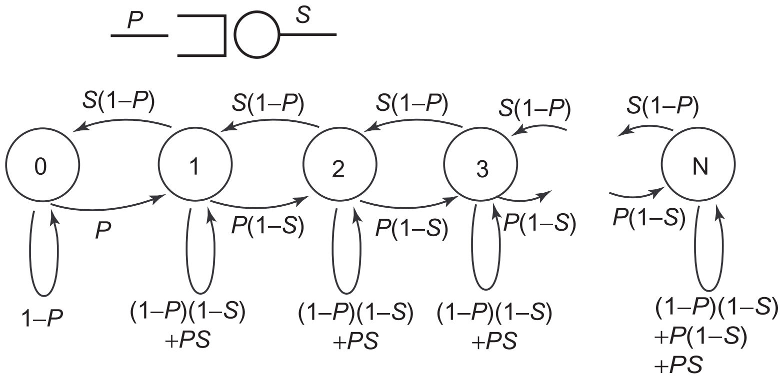
(a) Using a boundary across each arm:

$$
\begin{array}{lclcl}
\mu_1 P_1 = \lambda P_0 & & P_1 = \dfrac{\lambda}{\mu_1} P_0 & & P_0 + P_1 + P_2 + P_3 = 1 \\
\mu_2 P_2 = \lambda P_0 & \Rightarrow & P_2 = \dfrac{\lambda}{\mu_2} P_0 & \Rightarrow & P_0 + \dfrac{\lambda}{\mu_1} P_0 + \dfrac{\lambda}{\mu_2} P_0 + \dfrac{\lambda}{\mu_3} P_0 = 1 \\
\mu_3 P_3 = \lambda P_0 & & P_3 = \dfrac{\lambda}{\mu_3} P_0 & & P_0 = \dfrac{1}{1 + \frac{\lambda}{\mu_1} + \frac{\lambda}{\mu_2} + \frac{\lambda}{\mu_3}}
\end{array}
$$

(b) $P_B = 1 - P_0 =$ Utilization

Solution 3.23.

(a)



(b)

$$n = 1: \qquad P_n = \frac{P}{S(1-P)} P_0$$

$$n = 2, 3 \ldots N: \quad P_n = \frac{P}{S(1-P)} \left(\frac{P(1-S)}{S(1-P)} \right)^{n-1} P_0$$

$$n = 1, 2 \ldots N: \quad P_n = \frac{P^n (1-S)^{n-1}}{S^n (1-P)^n} P_0$$

(c)

$$P_0 = \frac{1}{\sum_{i=1}^{\infty} \frac{P^i (1-S)^{i-1}}{S^i (1-P)^i} + 1}$$

from normalization equation

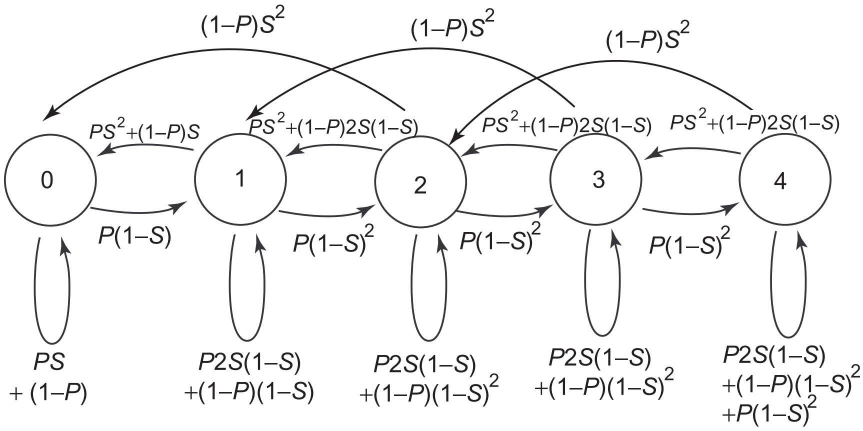
$$P_0 = \frac{(1-S)}{\sum_{i=1}^{\infty} \frac{P^i (1-S)^i}{S^i (1-P)^i} + 1 - S}$$

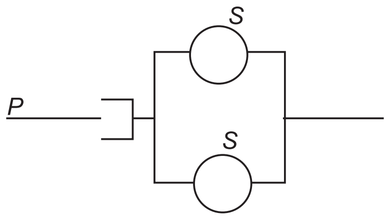
$$P_0 = \frac{(1-S)}{\sum_{i=0}^{\infty} \frac{P^i (1-S)^i}{S^i (1-P)^i} - S}$$

$$P_0 = \frac{1-S}{\frac{1}{1-\frac{P(1-S)}{S(1-P)}} - S}$$

WITH ALGEBRA: $P_0 = 1 - \dfrac{P}{S}$

Solution 3.24.





Solution 3.25.

n	$P(n)$
0	.15
1	.20
2	.35
3	.30

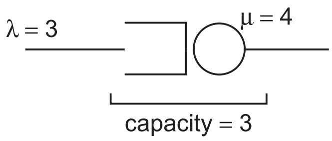
$$\overline{n} = 1 \times .2 + 2 \times .35 + 3 \times .3$$

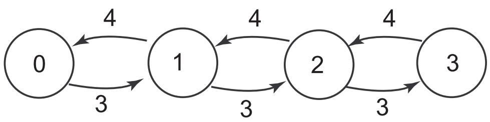
$$= .2 + .7 + .9 = 1.8$$

$$\overline{\text{Thruput}} = \mu(.2 + .35 + .3) = .85\,\mu$$

$$\overline{\text{Delay}} = \frac{\overline{n}}{\overline{\text{Thruput}}} = \frac{1.8}{.85\,\mu} = \frac{2.11}{\mu}$$

Solution 3.26.





(a)

$$P_1 = \frac{3}{4}P_0 \qquad P_0 + P_1 + P_2 + P_3 = 1$$

$$P_2 = \frac{3}{4}P_1 = \frac{9}{16}P_0 \qquad P_0 = \frac{1}{1 + \frac{3}{4} + \frac{9}{16} + \frac{27}{64}}$$

$$P_3 = \frac{3}{4}P_2 = \frac{27}{64}P_0 \qquad = \frac{1}{2.7343} = .36571$$

$$P_1 = .27428 \qquad P_2 = .20571$$

$$P_3 = .15429$$

(b)

$$\overline{\text{Delay}} = \frac{\overline{n}}{\overline{\overline{\text{Thruput}}}}$$

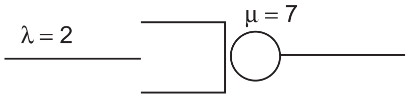
$$= \frac{1P_1 + 2P_2 + 3P_3}{4(P_1 + P_2 + P_3)} = .45270\,\text{sec}$$

(c)

$$\overline{\text{Delay}_{\text{buffer}}} = \overline{\text{Delay}_{\text{Total}}} - \overline{\text{Delay}_{\text{Server}}}$$

$$= .45270 - \frac{1}{4} = .2027\,\text{sec}$$

Solution 3.27.



(a) Infinite Buffer

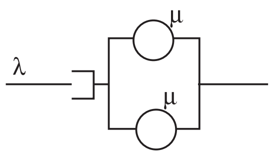
$$P_0 = 1 - \rho = 1 - \frac{2}{7} = .714$$
$$U = 1 - P_0 = \rho = 2/7 = .28571$$

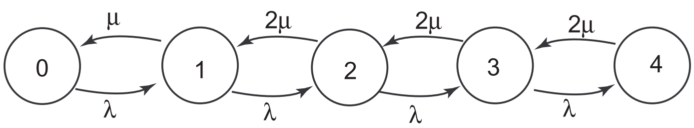
(b) Finite Buffer ($N = 4$)

$$U = 1 - P_0$$
$$P_0 = \frac{1 - \frac{\lambda}{\mu}}{1 - \left(\frac{\lambda}{\mu}\right)^{N+1}}$$
$$= \frac{1 - \frac{2}{7}}{1 - \left(\frac{2}{7}\right)^5} = \frac{5/7}{.998} = .71564$$
$$U = 1 - P_0 = 1 - .71564 = .28436$$

(c) The infinite buffer queue has a larger utilization as no customer is turned away.

Solution 3.28.





$$P_1 = \frac{\lambda}{\mu} P_0$$

$$P_2 = \frac{\lambda}{2\mu} P_1 = \frac{\lambda^2}{2\mu^2} P_0$$

$$P_3 = \frac{\lambda}{2\mu} \frac{\lambda^2}{2\mu^2} P_0 = \frac{\lambda^3}{4\mu^3} P_0$$

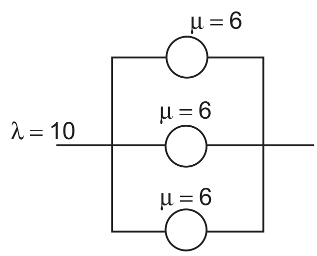
$$P_4 = \frac{\lambda}{2\mu} \frac{\lambda^3}{4\mu^3} P_0 = \frac{\lambda^4}{8\mu^4} P_0$$

$$P_0 + P_1 + P_2 + P_3 + P_4 = 1$$

$$P_0 = \frac{1}{1 + \frac{\lambda}{\mu} + \frac{\lambda^2}{2\mu^2} + \frac{\lambda^3}{4\mu^3} + \frac{\lambda^4}{8\mu^4}}$$

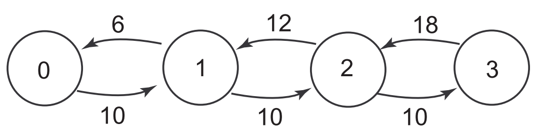
$$P_B = P_4 = \frac{\frac{1}{8}\left(\frac{\lambda}{4}\right)^4}{1 + \frac{\lambda}{\mu} + \frac{\lambda^2}{2\mu^2} + \frac{\lambda^3}{4\mu^3} + \frac{\lambda^4}{8\mu^4}}$$

Solution 3.29.



(a) Avg # empty servers = $3P_0 + 2P_1 + P_2$ where P_i is the equilibrium probability of i customers in the queueing system.

(b)



$$P_1 = \frac{10}{6} P_0$$

$$P_2 = \frac{10}{12}\ P_1 = \frac{100}{72} P_0$$

$$P_3 = \frac{10}{18}\ P_2 = \frac{1000}{1296} P_0$$

$$P_0 + P_1 + P_2 + P_3 = 1$$

$$P_0 = \frac{1}{1 + \frac{10}{6} + \frac{100}{72} + \frac{1000}{1296}}$$

$$P_0 = .207161125$$

Avg # empty servers

$$= 3P_0 + 2P_1 + P_2$$

$$= 1.5997$$

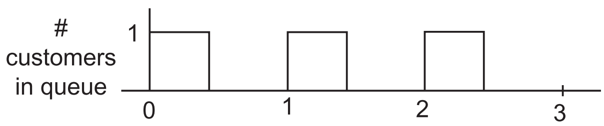
Solution 3.30.

$$\mu = \frac{60}{12} = 5\,\text{calls/hour}$$

$$\lambda = \frac{60}{9} = 6.66\,\text{calls/hour}$$

(a) M/M/m, m parallel servers with a queue, blocked calls are queued.
(b) Erlang C

Solution 3.31.



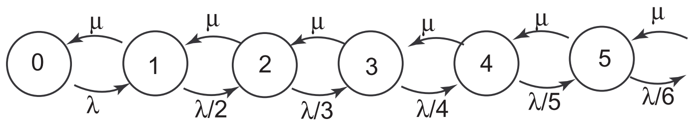
$$\begin{aligned}\text{Avg \# Customers in queue} &= \frac{1}{2}\times 1+\frac{1}{2}\times 0\\ &= \frac{1}{2}\end{aligned}$$

Solution 3.32. Since no customers are lost,

$$\text{Thruput} = \lambda,$$

Solution 3.33.

(a)



(b)

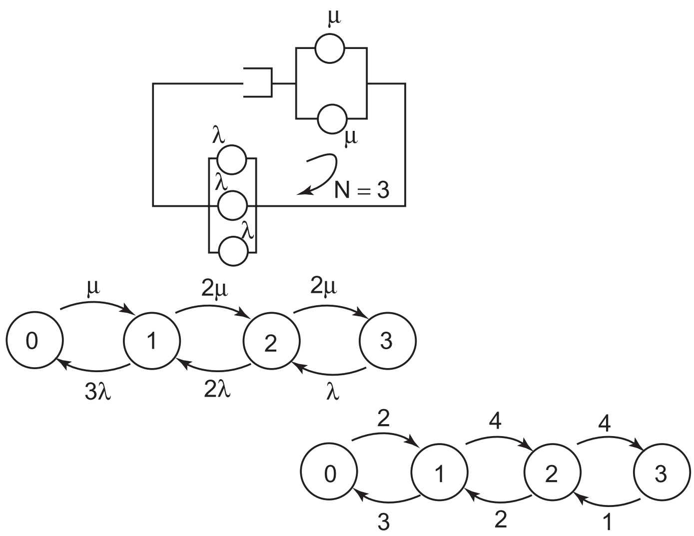
$$\begin{aligned} P_n &= \frac{\lambda^n}{n!\mu^n}P_0 \quad n = 1,2,3\ldots\\ \text{as}\quad P_1 &= \frac{\lambda}{\mu}P_0\\ P_2 &= \frac{\lambda}{2\mu}P_1 = \frac{\lambda^2}{2\mu^2}P_0\\ P_3 &= \frac{\lambda}{3\mu}P_2 = \frac{\lambda^3}{6\mu^3}P_0\\ &\vdots \end{aligned}$$

(c)

$$\begin{aligned} &P_0+P_1+P_2+P_3+\ldots = 1\\ &P_0\left(\sum_{n=0}^{\infty}\frac{\lambda^n}{n!\mu^n}\right) = 1\\ &P_0 = \frac{1}{\sum_{n=0}^{\infty}\frac{\lambda^n}{n!\mu^n}} = \frac{1}{e^{\lambda/\mu}} = e^{-\lambda/\mu} \end{aligned}$$

from well known summation formula.

Solution 3.34. (a)



(b)

$$P_1 = \frac{3}{2} P_0$$

$$P_2 = \frac{1}{2} P_1 = \frac{3}{4} P_0$$

$$P_3 = \frac{1}{4} P_2 = \frac{3}{16} P_0$$

$$P_0 + P_1 + P_2 + P_3 = 1$$

$$P_0 + \frac{3}{2} P_0 + \frac{3}{4} P_0 + \frac{3}{16} P_0 = 1$$

$$P_0 = \frac{1}{1 + \frac{3}{2} + \frac{3}{4} + \frac{3}{16}} = \frac{16}{55}$$

(c)

$$\text{Thruput upper queue} = 1 \times P_1 + 2P_2 + 2P_3 = \frac{54}{55}$$

Solution 3.35.

$$P(\underline{n}) = \left(\prod_{i=1}^{k} \frac{a_i^{n_i}}{n_i!}\right) G^{-1}(\Omega)$$

$$P(\underline{n}) = \left(\frac{(1/2)^{n_1}}{n_1!}\right)\left(\frac{(1/3)^{n_2}}{n_2!}\right) \Big/ \sum_{\underline{n}\in\Omega} \left(\frac{(1/2)^{n_1}}{n_1!}\right)\left(\frac{(1/3)^{n_2}}{n_2!}\right)$$

$$\sum n_1 + 3n_2 \le 1000, \quad n_1, n_2 = \text{X} \;\; \text{M bytes}$$

Solution 3.36.

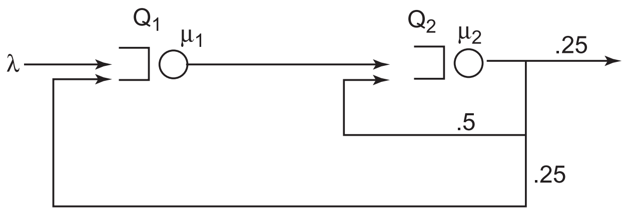
$$\text{M/D/1} \quad E[n] = \rho + \frac{\rho^2}{2(1-\rho)}$$

$$\text{M/M/1} \quad E[n] = \frac{\rho}{1-\rho}$$

ρ	M/M/1	M/D/1
.1	.111	.106
.2	.250	.225
.4	.667	.533
.6	1.50	1.05
.8	4.00	2.40
.9	9.00	4.95
.99	99.0	50.0

- As $\rho \to 1$
  $\underset{\text{M/M/1}}{E[n]} = 2\times \underset{M/D/1}{E[n]}$
- M/M/1 has larger average queue size because of longer (more variable) distribution tail.

Solution 3.37. (a)



$$\theta_1 = \lambda + .25\theta_2$$
$$\theta_2 = \theta_1 + .5\theta_2$$
Traffic equations

$$\theta_2 = 2\theta_1$$
$$\downarrow$$
$$\theta_1 = \lambda + .5\theta_1$$

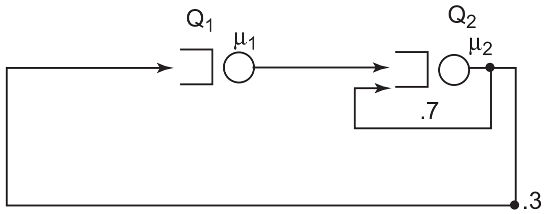
$$\boxed{\begin{aligned}\theta_1 &= 2\lambda \\ \theta_2 &= 4\lambda\end{aligned}}$$

(b)

$$P(\underline{n}) = \left(\frac{2\lambda}{\mu_1}\right)^{n_1} \left(\frac{4\lambda}{\mu_2}\right)^{n_2} P(\underline{0})$$

$$P(\underline{0}) = \frac{1}{\sum_{\underline{n}} \left(\left(\frac{2\lambda}{\mu_1}\right)^{n_1} \left(\frac{4\lambda}{\mu_2}\right)^{n_2}\right)}$$

Solution 3.38.



$$\left.\begin{aligned}\theta_1 &= .3\theta_2 \\ \theta_2 &= \theta_1 + .7\theta_2\end{aligned}\right\}\text{Traffic equations}$$

$$\theta_2 = \frac{10}{3}\theta_1$$

so $\quad \theta_1 = 1, \theta_2 = 10/3$

Solution 3.39. Global Balance Equation:

$$\sum_{i=1}^{M} \mu_i P(\underline{n}) = \sum_{i=1}^{M}\sum_{j=1}^{M} \mu_j r_{ji} P(\underline{n} + 1_j - 1_i)^*$$

Traffic Equations:

---

* Here $\underline{1}_i = [0, 0, 0 \ldots \underset{\substack{\uparrow \\ i\text{th position}}}{1}, \ldots 0, 0]$

$$\theta_j = \sum_{j=1}^{M} r_{ji}\theta_j \quad i = 1, 2, \ldots M$$

Manipulate the Traffic Equations:

$$1 = \sum_{j=1}^{M} r_{ji}\frac{\theta_j}{\theta_i} \quad i = 1, 2, \ldots M$$

Multiply the left hand side of the global balance equation by this expression for one:

$$\sum_{i=1}^{M}\sum_{j=1}^{M} \mu_i r_{ji}\frac{\theta_j}{\theta_i} P(\underline{n})$$
$$= \sum_{i=1}^{M}\sum_{j=1}^{M} \mu_j r_{ji} P(\underline{n} + \underline{1}_j - \underline{1}_i)$$

Rearrange as

$$\sum_{i=1}^{M}\sum_{j=1}^{M} r_{ji}\left(\frac{\theta_j}{\theta_i}\mu_i P(\underline{n}) - \mu_j P(\underline{n} + \underline{1}_j - \underline{1}_i)\right) = 0$$

If the parenthesis term is 0, this equation is satisfied when

$$\frac{\theta_j}{\theta_i}\mu_i P(\underline{n}) = \mu_j P(\underline{n} + \underline{1}_j - \underline{1}_i) \quad i, j = 1, 2, \ldots M$$

Manipulating:

$$P(\underline{n}) = \left(\frac{\theta_i}{\mu_i}\right)\left(\frac{\theta_j}{\mu_j}\right)^{-1} P(\underline{n} + \underline{1}_j - \underline{1}_i) \quad i, j = 1, 2, \ldots M$$

which yields

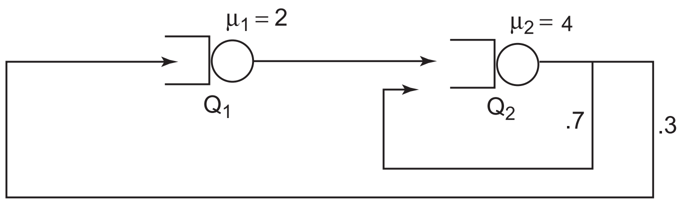
$$P(\underline{n}) = \left(\frac{\theta_i}{\mu_i}\right) P(\underline{n} - \underline{1}_i)$$

or

$$\mu_i P(\underline{n}) = \theta_i P(\underline{n} - \underline{1}_i),$$

which is the desired result.

Solution 3.40. Refering to problem 3-38:



From 3-38, $\theta_2 = \frac{10}{3}\theta_1$ or $\theta_1 = 1$ $\theta_2 = 10/3$

$\underline{N = 1}$

$$\overline{\tau_1}(1) = .5$$

$$\overline{\tau_2}(1) = .25$$

$$\overline{T}(1) = \frac{1}{1 + .5 + \frac{10}{3} \times .25} = .75$$

$$\overline{n}_1(1) = .75 \times 1 \times .5 = .375$$

$$\overline{n}_2(1) = .75 \times \frac{10}{3} \times .25 = .625$$

$\underline{N = 2}$

$$\overline{\tau}_1(2) = .5 + .5 \times .375 = .6875$$

$$\overline{\tau}_2(2) = .25 + .25 \times .625 = .40625$$

$$\overline{T}(2) = \frac{2}{1 \times .6875 + \frac{10}{3} \times .40625} = .9796$$

$$\overline{n}_1(2) = .9796 \times 1 \times .6875 = .6735$$

$$\overline{n}_2(2) = .9796 \times \frac{10}{3} \times .40625 = 1.327$$

$\underline{N = 3}$

$$\overline{\tau}_1(3) = .5 + .5 \times .6735 = .83675$$

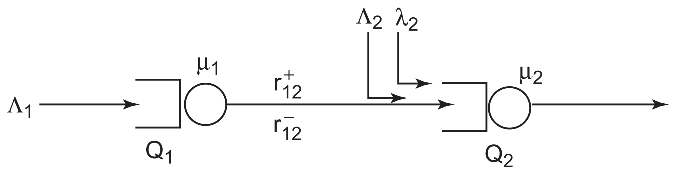
$$\overline{\tau}_2(3) = .25 + .25 \times 1.327 = .58175$$

$$\overline{T}(3) = \frac{3}{1 \times .83675 + \frac{10}{3} \times .58175} = 1.081$$

$$\overline{n}_1(3) = 1.081 \times 1.0 \times .83675 = .905$$

$$\overline{n}_2(3) = 1.081 \times \frac{10}{3} \times .58175 = 2.096$$

Solution 3.41.



$$q_1 = \frac{\wedge_1}{\mu_1}$$

$$q_2 = \frac{\wedge_2 + q_1\mu_1 r_{12}^{+}}{\mu_2 + q_1\mu_1 r_{12}^{-} + \lambda_2}$$

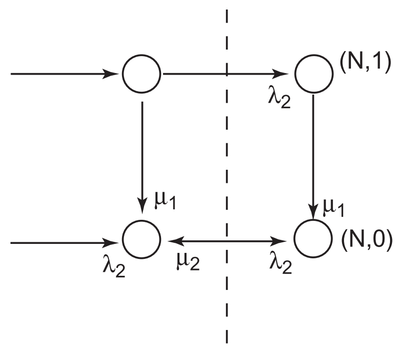
Or:

$$q_2 = \frac{\wedge_2 + \wedge_1 r_{12}^{+}}{\mu_2 + \wedge_1 r_{12}^{-} + \lambda_2}$$

And:

$$P(\underline{n}) = \prod_{i=1}^{2}(1 - q_i){q_i}^{n_i}$$

Solution 3.42. For eg. (3.250):



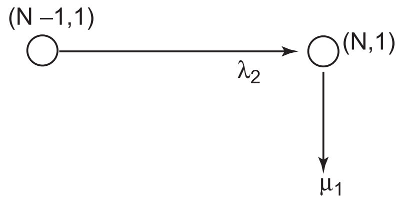
Balancing flow across the boundary,

$$\lambda_2(P(N-1,0) + P(N-1,1)) = \mu_2 P(N,0)$$

Leading to

$$P(N,0) = \frac{\lambda_2}{\mu_2}[P(N-1,0) + P(N-1,1)]$$

For eg. (3.251)

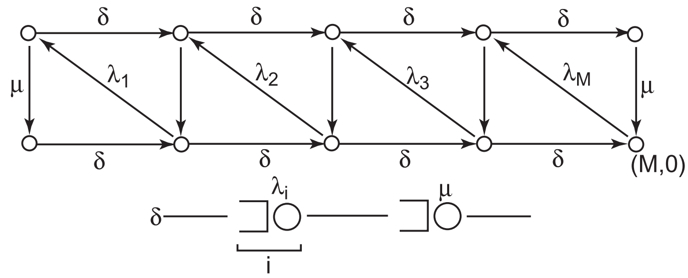


A global balance eg. at $(N, 1)$ yields

$$\lambda_2 P(N-1,1) = \mu_1 P(N,1)$$

$$P(N,1) = \frac{\lambda_2}{\mu_1}(P(N-1,1))$$

Solution 3.43. (a)



(b) Initially:

$$p(0,0) = 1.0 \quad p(0,1) = \left(\frac{\delta}{\mu}\right) p(0,0)$$

Recursively $i = 1, 2, \ldots M-1$

$$p(i,0) = \left(\frac{\delta}{\lambda_i}\right) [p(i-1,0) + p(i-1,1)]$$

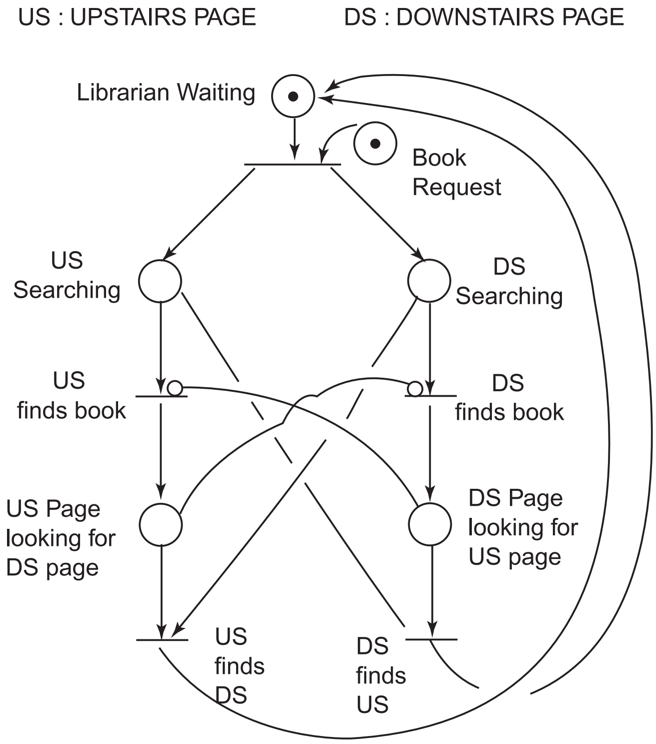
$$p(i,1) = \left(\frac{\delta}{\mu}\right) [p(i-1,1) + p(i,0)]$$

At the (right) boundary:

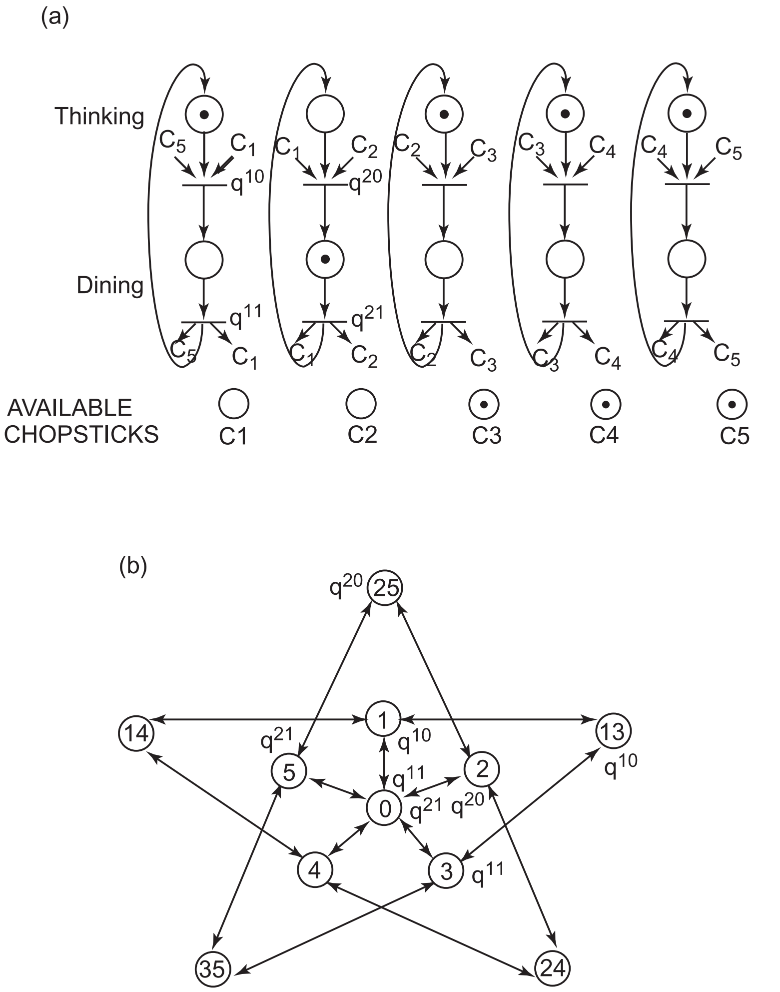
$$p(M,0) = \left(\frac{\delta}{\lambda_M}\right)[P(M-1,0) + P(M-1,1)]$$

$$P(M,1) = \left(\frac{\delta}{\mu}\right)P(M-1,1)$$

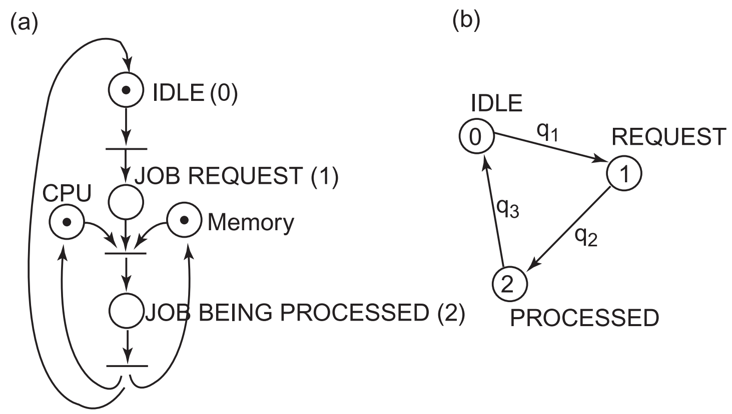
Solution 3.44.



Solution 3.45.



Solution 3.46.



$$q_1 \; P_{\text{IDLE}} = q_3 \; P_{\text{PROCESSED}}$$

$$\boxed{P_{\text{PROCESSED}} = \tfrac{q_1}{q_3} \; P_{\text{IDLE}}}$$

$$\boxed{P_{\text{JOB REQUEST}} = \tfrac{q_1}{q_2} P_{\text{IDLE}}}$$

LIKEWISE:

$$\left(1 + \frac{q_1}{q_3} + \frac{q_1}{q_2}\right) P_{\text{IDLE}} = 1$$

$$P_{\text{IDLE}} = \frac{1}{1 + \frac{q_1}{q_3} + \frac{q_1}{q_2}} = \frac{q_2 q_3}{q_2 q_3 + q_1 q_2 + q_1 q_3}$$

$$P_{\text{PROCESSED}} = \longrightarrow = \frac{q_1 q_2}{q_1 q_3 + q_1 q_2 + q_1 q_3}$$

$$P_{\text{JOB REQUEST}} = \longrightarrow = \frac{q_1 q_3}{q_2 q_3 + q_1 q_2 + q_1 q_3}$$

4

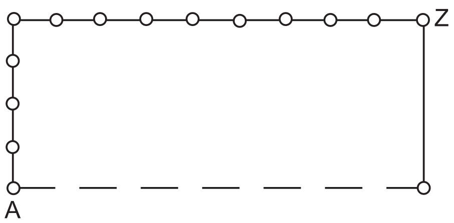
Fundamental Deterministic Algorithms

1. See below.
2. The Dijkstra and Ford-Fulkerson algorithms each find all the shortest paths from a root node to all of the other nodes in the network in one run.
3. To find the shortest paths between every pair of nodes in a network, one runs either the Dijkstra or Ford-Fulkerson algorithm N times, each time using a different node as the root node.
4. To find the k shortest link disjoint paths between a pair of nodes one first runs a shortest path algorithm to find the shortest path between the pair of nodes. Links along this path are then removed from the network. Again, one runs a shortest path algorithm, finds the shortest path between the two nodes and removes the links from this path from the network. One does this k times or until the remaining network is partitioned between the two nodes.
5. The “algorithm table” is used in the calculation of shortest paths and contains intermediate as well as final results. A “routing table” is generated from the algorithm table and is placed in each node. It allows the output port to the next neighbor along a path be looked up for an outgoing packet or circuit.
6. See below.
7. A typical way to find routes under source routing is to flood discovery packets that maintain a record of the routes they take. When a discovery packet(s) arrives at the destination, the destination can extract the path and send a packet back to the source with the route. If several discovery packets arrive at a destination, the destination can select the “best” route according to some metric.
8. Under pure flooding packets are sent out from a node in all directions. A node receiving a flooded packet forwards it on all of its outgoing links except the link it arrived on. The number of packets generated under flooding can be reduced if there is some sense of direction so packets are

only forwarded in directions of interest or directions away from their point of generation.

9. Paths between nodes in a hierarchical network are sometimes longer than direct connections because they are not necessarily shortest paths.
10. The concept of switching elements is useful in VLSI design because it is easy with VLSI to replicate simple switching element building blocks many times on a chip.
11. A disadvantage of putting multiple addresses into packet for multicasting is the packet length increase (though there are less total packets) and the need for software modifications to the nodes. A disadvantage of the use of spanning trees for multicasting is the need for software modifications to the nodes.
12. Because it takes more energy in a nonlinear way to go further distances (usually according to some power law), a simple calculation shows the total energy consumed in a series of short hops is significantly smaller than the energy consumed in one large hop with equivalent total distance.
13. Reactive routing in ad hoc networks is more efficient than proactive routing when only a small fraction of potential paths are in use as under reactive routing information is only maintained for paths in use.
14. The complexity of protocol verification for large systems increases combinatorially. This necessitates the use of efficient protocol verification algorithms.
15. Under a "deadlock" a system freezes and can not move into a further state. Under "livelock" messages are continually transmitted and received with no useful work accomplished.
16. Under an unspecified reception a message in the channel may be received but not as initially specified in the design.
17. The bits in four positions differ between 00001111 and 11001100 so the Hamming distance between the two code words is four.
18. In Figure 4.10, a legitimate codeword can be conceivably corrupted in such a way that it becomes a legitimate codeword in a different region and so the error is not detected. This occurs either with low probability or with types of errors that are outside the design of the code.
19. Yes, try a few examples to see this.
20. If the codeword "250" is received, the error is not detected as the codeword is divisible by the generator number 25. See problem 4.18.

Solution 4.1.



Any shortest path from A to Z has 4 Ups (U) and 9 Rights (R)

$$4 \text{ shortest paths} = \binom{9+4}{4} = \binom{9+4}{9} = \binom{13}{9} = 715 \,\text{paths}.$$

Solution 4.6. NODE C Routing Table

Destination	Nearest Neighbor
A	F
B	B
D	E
E	E
F	F

Solution 4.21.

Dijkstra Algorithm

N	B	C	D	E	F	G
$\{A\}$	2	∞	10	∞	10	∞
$\{A, B\}$	2	3	10	∞	10	∞
$\{A - C\}$	2	3	10	4	10	4
$\{A - C, E\}$	2	3	5	4	10	4
$\{A - C, E, G\}$	2	3	5	4	5	4
$\{A - E, G\}$	2	3	5	4	5	4
$\{A - G\}$	2	3	5	4	5	4

Ford-Fulkerson Algorithm

	B	C	D	E	F	G
Initialize	$(\cdot, \infty)$	$(\cdot, \infty)$	$(\cdot, \infty)$	$(\cdot, \infty)$	$(\cdot, \infty)$	$(\cdot, \infty)$
1	$(A, 2)$	$(B, 3)$	$(A, 10)$	$(C, 4)$	$(A, 10)$	$(C, 4)$
2	$(A, 2)$	$(B, 3)$	$(E, 5)$	$(C, 4)$	$(G, 5)$	$(C, 4)$
3	$(A, 2)$	$(B, 3)$	$(E, 5)$	$(C, 4)$	$(G, 5)$	$(C, 4)$

Solution 4.22.

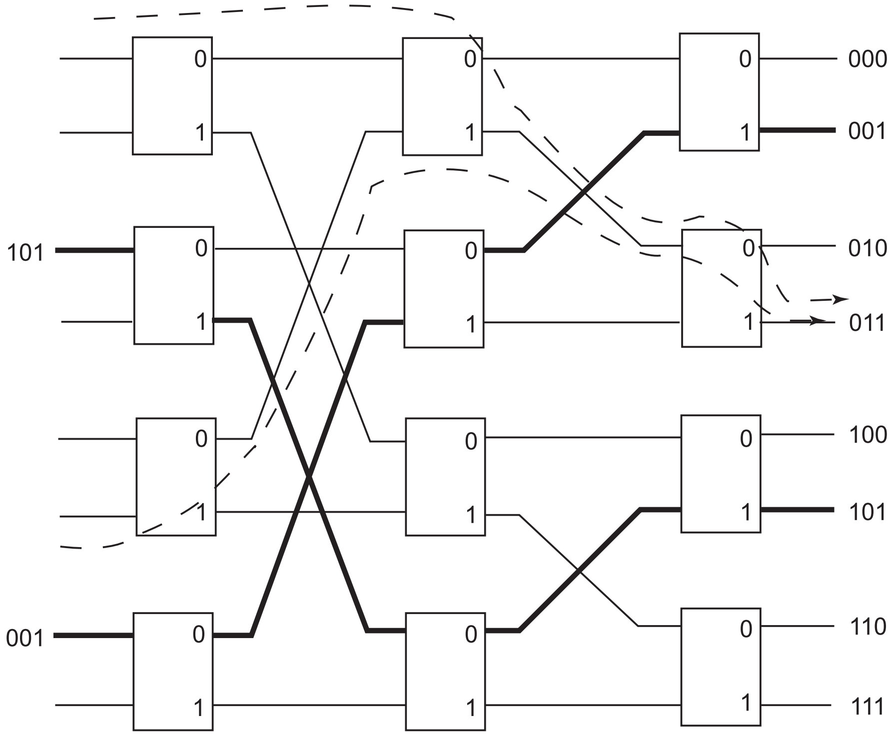


Fig. 4.4. An 8×8 delta network with two paths from specific inputs to outputs indicated.

Solution 4.23.

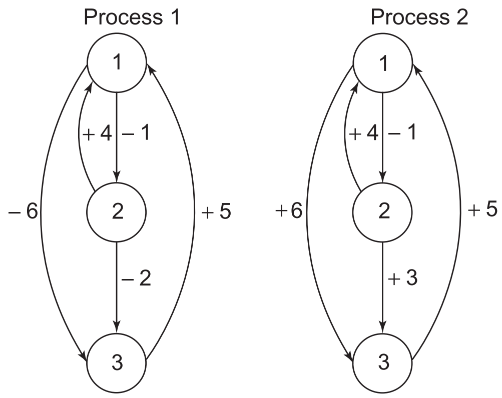
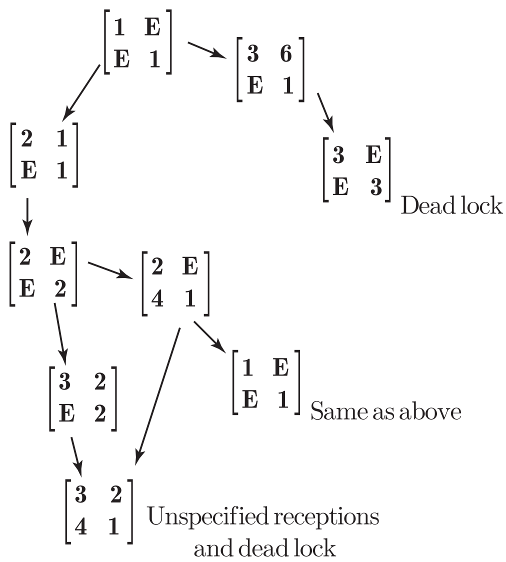
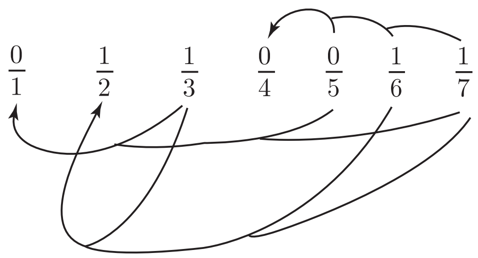


Fig. 4.17. A reachability tree problem

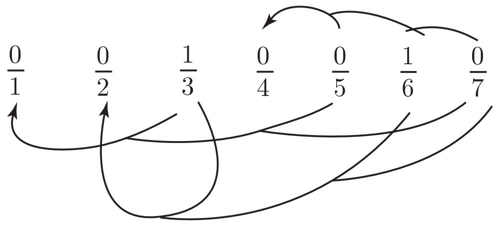


Solution 4.24. 1011000 → odd parity so there is an error.

Solution 4.25.

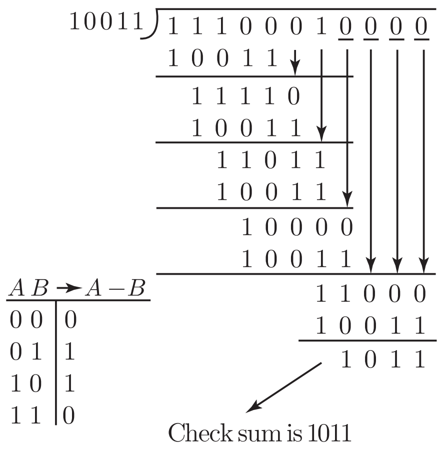


Solution 4.26.



1: odd ↘
2: even $\quad 1 + 4 = 5$, 5th bit in error
4: odd ↗

Solution 4.27.



Solution 4.28.

$$\begin{array}{r|ccccccccc}
10011 & 1&0&0&0&1&\underline{0}&\underline{1}&\underline{1}&\underline{1}\\
 & 1&0&0&1&1&&&&\\ \hline
 & 0&0&0&1&0&0&1&1&\\
 & &&&1&0&0&1&1&\\ \hline
 & &&&&&&&0&1
\end{array}$$

remainder!
There is an error.

$A\,B$	$A-B$
0 0	0
0 1	1
1 0	1
1 1	0

5

Divisible Load Modeling for Grids

1. A divisible load is a communication/computing load that can be arbitrarily partitioned among processors and links. There is thus a very fine granularity to the load (there have been some papers on coarser granularity). Continuous variable are used to represent the fraction of load assigned to a given link or processor. There are also no precedence relations among the data.
2. The basic divisible load model is a linear model which leads to the usual tractable mathematics of linear models such a those for queueing models or electric circuits.
3. Consider a single node distributing load to its children nodes in a single level tree (star) type network. Under sequential distribution load is distributed to one child at a time. Under simultaneous distribution load is distributed to all children concurrently. A hybrid load distribution strategy is multi-installment (multi-round) distribution where load is sent sequentially to children from the root in repeating installments (rounds).
4. As the number of children increases when using sequential load distribution, a saturation in performance (solution time and speedup) is observed. This is due to the fact that no matter how many children there are, at any point in time only one child is receiving load. Simultaneous scheduling avoids this problem by feeding load concurrently to all children present.
5. Under staggered start, each child processor in a single level tree network first receives load from the root and only after all of this load is received starts computing. Under simultaneous start a child node starts computing as soon as load begins to be received and reception of load and computing may go on concurrently for some period of time.
6. A front end processor performs communication duties for a main processor, thus the main processor can concentrate on computation. The use of front ends leads to a faster solution as communication and computation can proceed concurrently.
7. Speedup is defined as the ratio of the time to solve a computational problem on one processor divided by the time for N homogeneous processors

to solve the problem. Since the N processor solution should be faster than the one processor solution the ratio is greater than 1.0. However speedup is usually less than N because of overhead, communication delays and serialization.

8. In the Gantt type chart there is one graph per processor. Time is on the horizontal axis. Communication is shown above the horizontal axis and computation is shown below it. Usually the chart starts at time $t = 0$.
9. Speedup increases nonlinearly for nonlinear models because of the nonlinear nature of the computing time. It is more efficient to process load in fragments on a number of processors than on one processor. However the results of individual processors processing fragments of load must often be combined in a "post-processing" phase of computation that adds overhead. The amount of improvement in overall speedup for nonlinear problems is thus problem specific.
10. An equivalent processor is a processor that has identical operating characteristics (solution time and speedup) as the network of processors and links that it replaces. The concept of equivalent elements is a basic property of linear models. To find the overall performance of a network of elements one can aggregate elements until a single processor results with known processing speed that is identical to the processing speed of the overall network.
11. As the number of nodes is increased in a linear daisy chain under store and forward load distribution, load for the further processors has to be relayed from the load originating processor over many links to where it is actually processed. This repetitive transmission of the same load is a form of overhead that results in a saturated network performance. The same effect occurs for store and forward load distribution in tree networks as depth is increased. The use of virtual cut through switching, where load is relayed to the next node before being completely received, can mitigate this saturation.
12. It is not worth distributing load from one node to the next in a linear daisy chain if the link speed is too slow relative to the computing speed of the first node. In this case one obtains a faster solution if the first node processes the remaining load rather than communicating it to the next node.
13. Many (often unpredictable) events occur in a computer that take effort on the part of the processor, thus leaving less available processor effort for a divisible job(s) of interest. Such events include background job(s) start and termination.
14. Figure 5.16 corresponds to a single level tree (star) network where the root has inverse processing speed w and each child has inverse processing speed w_{eq}. The root performs sequential distribution. The root has a front end so it can compute and communicate simultaneously. The children implement staggered start and so each child node waits until it has completely received its load before starting its computation.

15. The equivalence between divisible load scheduling policies and Markov chains is surprising because the former is a deterministic model and the latter is a stochastic model. It is not completely surprising because both correspond to linear models.
16. Some divisible load scheduling policies do not have a Markov chain analog.

Solution 5.17.

$$\text{speedup} = 1 + k_1 \left[1 + \sum_{i=2}^{m} \left(\prod_{l=2}^{i} q_l\right)\right]$$

$$q_l = 1 - \sigma \quad \text{for homogeneous new where} \quad \sigma = zT_{cm}/wT_{cp}$$

$$\text{speedup} = 1 + k_1 \left[1 + \sum_{i=2}^{m} (1-\sigma)^{i-1}\right] \quad \text{Let} \quad i \to i+1$$

$$\text{speedup} = 1 + k_1 \left[1 + \sum_{i=1}^{m-1} (1-\sigma)^{i}\right]$$

$$= 1 + k_1 \left[\sum_{i=0}^{m-1} (1-\sigma)^{i}\right]$$

$$= 1 + k_1 \left[\frac{1-(1-\sigma)^m}{\sigma}\right] \quad \text{as} \quad \sum_{n=0}^{N} X^n = \frac{1-X^{N+1}}{1-X} \quad \text{and}$$

$$k_1 = w_0/w_1$$

so

$$\text{speedup} = 1 + \tfrac{w_0}{w_1}\left[\tfrac{1-(1-\sigma)^m}{\sigma}\right] \tag{5.21}$$

Solution 5.18.

$$\alpha_n \left(\sum_{i=1}^{n} r^{n-i} + 1 - r^{n-1}\right) = 1 \tag{5.112}$$

From the next equation in the text:

$$\alpha_n \left(\frac{r^n - 1}{r-1} + 1 - r^{n-1}\right) = 1$$

$$\alpha_n(\cancel{r}^n - 1 + r - 1 - \cancel{r}^n + r^{n-1}) = r - 1$$

$$\alpha_n(r^{n-1} + r - 2) = r - 1$$

$$\alpha_n = \frac{r-1}{r^{n-1}+r-2} \tag{5.113}$$

Solution 5.19. From the equation after (5.128)

$$\alpha_n \left(c^i + \sum_{j=1}^{n} r^{n-j} - r^{n-1} \right) = 1$$

From the equation before (5.129):

$$\begin{aligned}
&\alpha_n \left(c^i + \frac{r^n - 1}{r - 1} - r^{n-1} \right) = 1 \\
&\alpha_n (rc^i - c^i + \cancel{r}^{\,n} - 1 - \cancel{r}^{\,n} + r^{n-1}) = r - 1 \\
&\alpha_n (c^i(r - 1) + r^{n-1} - 1) = r - 1 \\
&\alpha_n = \frac{r - 1}{c^i(r - 1) + r^{n-1} - 1}
\end{aligned}$$

Let "i" represent the ith level subtree. Then for the ith level subtree:

$$\alpha_n = \frac{r_i - 1}{c^i(r_i - 1) + r_i^{n-1} - 1} \tag{5.129}$$

where

$$r_i = \frac{w_{eq}^i \; T_{cp} + z \; T_{cm}}{w_{eq}^i \; T_{cp}} \qquad c^i = \frac{w_{eq}^i}{w}$$

(see (5.123–127))

Solution 5.20. From the equations after (5.166),

$$\alpha_n \left(r^{n-2} c^i + \sum_{j=1}^{n} r^{n-j} - r^{n-1} \right) = 1$$

From the equation before (5.167):

$$\begin{aligned}
&\alpha_n \left(r^{n-2} c^i + \frac{r^n - 1}{r - 1} - r^{n-1} \right) = 1 \\
&\alpha_n (r^{n-1} c^i - r^{n-2} c^i + \cancel{r}^{\,n} - 1 - \cancel{r}^{\,n} + r^{n-1}) = r - 1 \\
&\alpha_n ((c^i + 1) r^{n-1} - c^i r_i^{n-2} - 1) = r - 1 \\
&\alpha_n = \frac{r_i - 1}{(c^i + 1) r_i^{n-1} - c^i r_i^{n-2} - 1}
\end{aligned} \tag{5.167}$$

where

$$\left.\begin{aligned} r_i &= \frac{w_{eq}^i\ T_{cp} + z\ T_{cm}}{w_{eq}^i\ T_{cp}} \\ c^i &= \frac{w_{eq}^i\ T_{cp} + z\ T_{cm}}{w\ T_{cp}} \end{aligned}\right\} \qquad \text{after} \quad (5.165)$$

Solution 5.21. From (5.174) and (5.175),

$$T_{N-1} \geq T_N$$

$$\hat{\alpha}_{N-1} w_{N-1} T_{cp} \geq (1-\hat{\alpha}_{N-1}) z_{N-1} T_{cm} + (1-\hat{\alpha}_{N-1}) w_N T_{cp}$$

with $(\alpha_{N-1} + \alpha_N)$ canceling above

$$\hat{\alpha}_{N-1}(w_{N-1} T_{cp} + z_{N-1} T_{cm} + w_N\ T_{cp}) \geq z_{N-1}\ T_{cm} + w_N T_{cp}$$

$$\hat{\alpha}_{N-1} \geq \frac{z_{N-1}\ T_{cm} + w_N\ T_{cp}}{w_{N-1}\ T_{cp} + z_{N-1}\ T_{cm} + w_N\ T_{cp}} \tag{5.176}$$

Solution 5.22. (a)

$$w_{eq}^{fc} = \frac{z\rho + w_{eq}^{fc}}{w + z\rho + w_{eq}^{fe}}\ w \tag{5.189}$$

$$(w_{eq}^{fc})^2 + (\cancel{w} + z\ \rho - \cancel{w}) w_{eq}^{fe} = z\rho w$$

$$(w_{eq}^{fc})^2 + z\ \rho w_{eq}^{fc} - z\rho w = 0$$

Solving the quadratic equation:

$$w_{eq}^{fe} = (-z\rho + \sqrt{(z\rho)^2 + 4wz\rho})/2 \tag{5.190}$$

Solution 5.22. (b)

$$w_{eq}^{nfe} = (1-\hat{\alpha}_{i-1}) z_{i-1}\ T_{cm} + \hat{\alpha}_{i-1} w_{i-1}\ T_{cp} \tag{5.186}$$

But:

$$\begin{aligned} \rho \quad &= T_{cm}/T_{cp} \qquad & w_{i-1} &= w \\ z_{i-1} &= z & w_i \quad &= w_{eq}^{nfe} \end{aligned}$$

So:

$$\hat{\alpha}_{N-1} = \frac{w_N}{w_{N-1} + w_N} \ \rightarrow\ \frac{w_{eq}^{nfe}}{w + w_{eq}^{nfe}}$$

And:

$$w_{eq}^{nfe} = \frac{wz\rho}{w + w_{eq}^{nfe}} + \frac{w_{eq}^{nfe}}{w + w_{eq}^{nfe}}\ w$$

Cross-multiplying:

$$w\ \cancel{w_{eq}^{nfe}} + (w_{eq}^{nfe})^2 = wz\rho + \cancel{w_{eq}^{nfe}}\ w$$

And:

$$w_{eq}^{nfe} = \sqrt{wz\rho} \tag{5.191}$$

Solution 5.23. $\alpha_o + \alpha_l + \alpha_r = 1$

$$\frac{w_{eq}^{\infty}}{w} + \frac{w_{eq}^{\infty}\ T_{cp}}{z\ T_{cm} + w_{eq}^{\infty}\ T_{cp}} + \left(\frac{w_{eq}^{\infty}\ T_{cp}}{z\ T_{cm} + w_{eq}^{\infty}\ T_{cp}}\right) = 1$$

$$w_{eq}^{\infty} = w\left(1 - \frac{w_{eq}^{\infty}\ T_{cp}}{z\ T_{cm} + w_{eq}^{\infty}\ T_{cp}} - \left(\frac{w_{eq}^{\infty}\ T_{cp}}{z\ T_{cm} + w_{eq}^{\infty}\ T_{cp}}\right)^2\right)$$

This expression can be iterated to find w_{eq}^{∞}. With algebra, it is equivalent to (5.196).

Solution 5.24.

T_{cm}: Time to transmit entire load over channel.
T_{cp}: Time to process entire load on a processor.
(These definitions are somewhat different from those in the rest of the chapter).

(a) Improperly formulated — my apologies.
(b) (Cheng 88) Since T_{cp}/N is overlapped with T_{cm}/N except for the last processor,

$$\begin{aligned} T_T &= \frac{T_{cm}}{N}(N-1+N-2+\cdots+1) + \frac{T_{cp}}{N} \\ &= \frac{N-1}{2}T_{cm} + \frac{T_{cp}}{N} \end{aligned}$$

$$\frac{dT_T}{dN} = 0 \Rightarrow\ N_{\text{optimal}} = \sqrt{\frac{2T_{cp}}{T_{cm}}} = \sqrt{2/\rho}$$

(c)

$$T_T = \frac{T_{cm}}{N}(N-1+N-2+\cdots+1) + \frac{T_{cp}}{N}$$
$$+T_s(1+2+3+\cdots+N-1)$$
$$T_T = \frac{N-1}{2}T_{cm} + \frac{T_{cp}}{N} + T_s\frac{(N-1)N}{2}$$
$$\frac{dT_T}{dN} = 0 \Rightarrow \; N^3T_s + N^2T_{cm} - 2T_{cp} = 0$$

and solve for N_{optimal}.

Solution 5.25. (a) *Sequential*:

$$q_i = 1-\sigma = 1 - \frac{z\;T_{cm}}{w\;T_{cp}} = 1-\frac{1}{3} = \frac{2}{3} \tag{5.20}$$

$$w_{eq} = \frac{1}{1+k_1\left[1+\sum_{i=2}^{m}\left(\prod_{l=z}^{1} q_l\right)\right]}w_0 \tag{5.16}$$

$$\text{but} \quad k_1 = \frac{w_0}{w_1} = 1$$

$$\text{so} \quad w_{eq} = \frac{6}{1+1\left[1+\frac{1}{3}+\left(\frac{1}{3}\right)^2\right]} = 2.45$$

$$\text{speedup} = 1 + \frac{w_0}{w}\left[\frac{1-(1-\sigma)^m}{\sigma}\right] \tag{5.21}$$
$$= 1 + \left[\frac{1-(2/3)^3}{1/3}\right] = 3.11$$

For the α's:

$$\alpha_1 = \frac{1}{\frac{1}{k_1}+1+\sum_{i=2}^{m}\left(\prod_{l=2}^{1} q_l\right)} = \frac{1}{1+1+\frac{2}{3}+\left(\frac{2}{3}\right)^2} = .3214$$

$$\alpha_0 = \frac{1}{k_1}\;\alpha_1 = \alpha_1 = .3214$$

$$\alpha_2 = q_i\;\alpha_1 = \frac{2}{3}\;.3214 = .2143 \qquad \sum \alpha_i \approx 1$$

$$\alpha_3 = q_i\;\alpha_2 = \frac{2}{3}\;.2143 = 1.428$$

(b) *Simultaneous Distribution with Staggered Start*

$$\alpha_1 = \frac{1}{\left[\frac{1}{k_1} + 1 + \sum_{i=2}^{m}\left(\prod_{l=2}^{i} q_l\right)\right]} \tag{5.31}$$

$$\text{where} \quad k_1 = \frac{w_0\ T_{cp}}{w_1\ T_{cp} + z_1 T_{cm}} = \frac{\sigma}{\sigma + 2} = 3/4$$

$$q_l = 1$$

$$\alpha_1 = \frac{1}{\left[\frac{4}{3} + 1 + 1 + 1^2\right]} = .2308$$

$$w_{eq} = \frac{1}{k_1}\ \alpha_1\ w_0 = \frac{4}{3} \times .2308 \times 6 = 1.84 \quad \text{where} \quad T_{cp} = 1$$

$$\text{speedup} = 1 + k_1 m = 1 + \frac{3}{4} \times 3 = 3.25$$

From above, for $\alpha' s$, $\alpha_1 = .2308$

$$\alpha_0 = \frac{1}{k_1}\ \alpha_1 = \frac{4}{3} \times .2308 = .3077$$

$$\alpha_2 = q\ \alpha_1 = 1 \times .2308 = .2308$$

$$\alpha_3 = q\ \alpha_2 = 1 \times .2308 = .2308 \qquad \sum \alpha_i \cong 1$$

(c) *Simultaneous Distribution with Simultaneous Start*

$$\alpha_1 = \frac{1}{\left[\frac{1}{k_1} + 1 + \sum_{i=2}^{m}\left(\prod_{l=2}^{i} q_l\right)\right]} \tag{5.51}$$

$$k_1 = \frac{w_0}{w_1} = 1 \qquad q_l = 1$$

$$\alpha_1 = \frac{1}{[1 + 1 + 1 + 1^2]} = .25$$

$$w_{eq} = \frac{1}{k_1}\ \alpha_1\ w_0 = 1 \times .25 \times 6 = 1.5 \quad \text{where} \quad T_{cp} = 1$$

$$\text{speedup} = 1 + k_1 m = 4.0$$

For the α's:

$$\alpha_0 = \frac{1}{k_1}\,\alpha_1 = \alpha_1 = .25$$

$$\alpha_2 = q\ \alpha_1 = \alpha_1 = .25$$

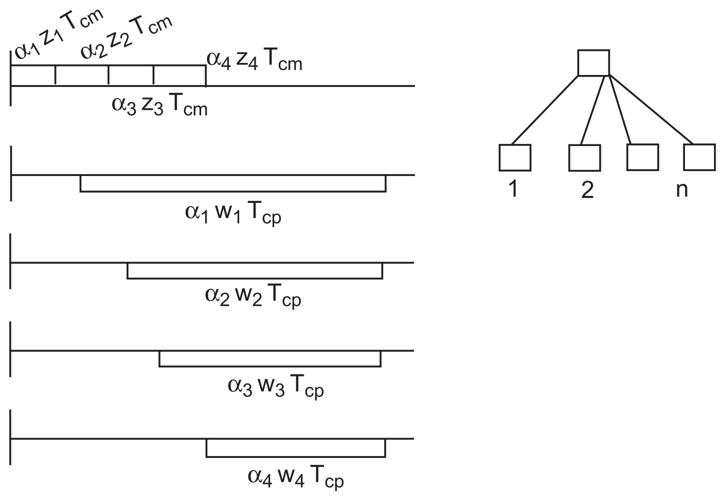
$$\alpha_3 = q\ \alpha_2 = \alpha_2 = .25 \qquad \sum \alpha_i\text{'s} = 1$$

(d) The w_{eq} and speedup are decreasing (increasing), as they should be, since sequential is the worst strategy, and simultaneous distribution with simultaneous start is the best strategy.

	Sequential	Simul. Dist., Staggered Start	Simul. Dist., Simul. Start
w_{eq}	2.45	1.84	1.5
Speedup	3.11	3.25	4.0

Note that all speedups ≤ 4 (there are four processors).

Solution 5.26.



(a) Timing Equations:

$$N-1eqs \quad \begin{cases} \alpha_1\ w_1\ T_{cp} = \alpha_2 z_2\ T_{cm} + \alpha_2\ w_2\ T_{cp} \\ \alpha_2\ w_2\ T_{cp} = \alpha_3 z_3\ T_{cm} + \alpha_3\ w_3\ T_{cp} \\ \vdots \\ \alpha_i\ w_i\ T_{cp} = \alpha_{i+1} z_{i+1}\ T_{cm} + \alpha_{i+1}\ w_{i+1}\ T_{cp} \\ \vdots \\ \alpha_{N-1}\ w_{N-1}\ T_{cp} = \alpha_N z_N\ T_{cm} + \alpha_N\ w_N\ T_{cp} \end{cases}$$

$$+\alpha_1\ \alpha_2 + \cdots + \alpha_N = 1$$

(b)

$$\alpha_i = \frac{w_{i-1}\ T_{cp}}{z_i\ T_{cm} + w_i\ T_{cp}} \quad \alpha_{i-1}$$

$$= q_i\ \alpha_{i-1} = \left(\prod_{l=2}^{i} q_l\right) \times \alpha_1$$

$$\alpha_1 + \alpha_2 + \cdots + \alpha_N = 1$$

$$\left[1 + \sum_{i=2}^{N}\left(\prod_{l=2}^{i} q_l\right)\right] \ \alpha_i = 1$$

$$\alpha_1 = \frac{1}{1 + \sum_{i=2}^{N}\left(\prod_{l=2}^{i}\ q_l\right)}$$

(c)

$$T_f = \alpha_1(z_1\ T_{cm} + w_1 T_{cp}) = \frac{z_1\ T_{cm} + w_1\ T_{cp}}{1 + \sum_{i=2}^{N}\left(\prod_{l=2}^{i}\ q_l\right)}$$

$$\text{speedup} = \frac{T_0}{T_f} = \frac{w\ T_{cp}\ \left(1 + \sum_{i=2}^{N}\left(\prod_{l=2}^{i}\ q_l\right)\right)}{z\ T_{cm} + w\ T_{cp}}$$

$$= \frac{w\ T_{cp}}{z\ T_{cm} + w\ T_{cp}}\left(1 + \sum_{i=2}^{N}\left(\frac{w\ T_{cp}}{z\ T_{cm} + w\ T_{cp}}\right)^{i-1}\right)$$

can be further simplified using summation formula.

Solution 5.27. (Cheng 88, Appendix)

Case I: The N^{th} processor stops first. Assume $w = z = 1$ without loss of generality.

$$T_T = \alpha T_{cp}$$

$$\alpha\ T_{cp} \geq (1-\alpha)(T_{cm} + T_{cp})$$

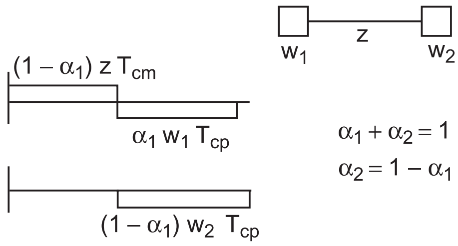
$$\alpha \geq \frac{T_{cm} + T_{cp}}{T_{cm} + 2\ T_{cp}} \qquad (*)$$

$$\min\ (T_T) = \min\ (\alpha)\ T_{cp}$$

where $\min(\alpha)$ occurs when $(*)$ has equality or both processors stop at the same time.

Case II: Similar to Case I.

Solution 5.28.



$$\alpha_1\ w_1\ T_{cp} = (1-\alpha_1) w_2\ T_{cp}$$

$$\alpha_1\ (w_1\ T_{cp} + w_2\ T_{cp}) = w_2\ T_{cp}$$

$$\alpha_1 = \frac{w_2\ T_{cp}}{w_1\ T_{cp} + w_2\ T_{cp}} = \frac{w_2}{w_1 + w_2}$$

$$T_f = (1-\alpha_1)\ z\ T_{cm} + \alpha_1\ w_1\ T_{cp}$$
$$= z\ T_{cm} + \alpha_1\ (w_1\ T_{cp} - z\ T_{cm})$$

$w_1\ T_{cp}$ should be $> z\ T_{cm}$, otherwise seek to max α_1 and all load processed on P_1 as communication is too slow.

Solution 5.29.

(a) $$\text{speedup}|_{\text{staggered start}} = 1 + k_1\ m = 1 + \frac{w_0\ T_{cp}}{w\ T_{cp} + z\ T_{cm}}\ m$$

If $z\ T_{cm} \to 0$

$$\text{speedup}|_{\text{staggered start}} = 1 + \frac{w_0}{w}\ m = \text{speedup}|_{\text{simultaneous start}}$$

(b)

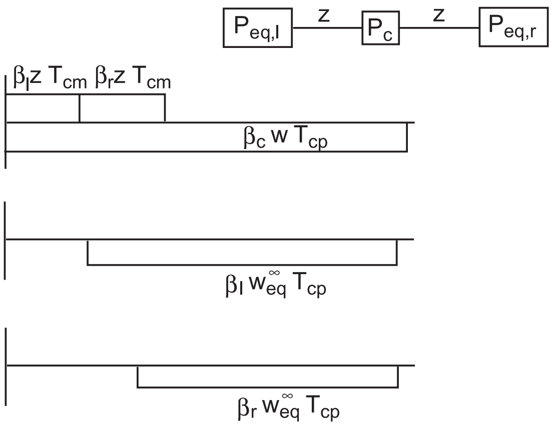
$$1 + \frac{w_0}{w}\ m \overset{?}{>} 1 + \frac{w\ T_{cp}}{w\ T_{cp} + z\ T_{cm}}\ m$$

$$\frac{\cancel{w_0}}{w} > \frac{\cancel{w_0}\ T_{cp}}{w\ T_{cp} + z\ T_{cm}}$$

Cross-multiply:

$$w\ T_{cp} + z\ T_{cm} > w\ T_{cp} \quad \text{confirmed!}$$

Solution 5.30.



(a)

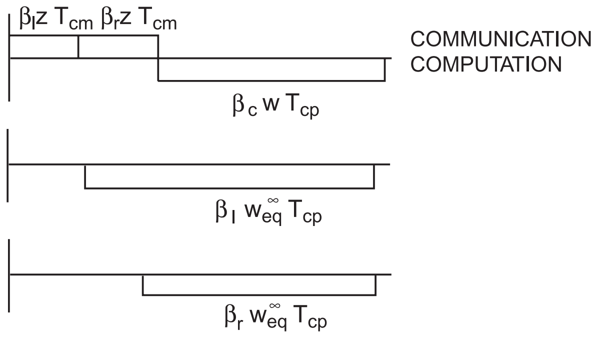
$$\begin{aligned} T_{fe}^{\infty} &= \beta_c\ w\ T_{cp} \\ &= \beta_l\ z\ T_{cm} + \beta_l\ w_{eq}^{\infty}\ T_{cp} \\ &= (\beta_l + \beta_r)\ z\ T_{cm} + \beta_r\ w_{eq}^{\infty}\ T_{cp} \\ &= \underset{\substack{\uparrow \\ \text{system}}}{w_{eq,s}},T_{cp} \qquad \beta_c + \beta_r + \beta_l = 1 \end{aligned}$$

(b) Using algebra:

$$w_{eq,s} = \frac{w(z\ T_{cm} + w_{eq}^{\infty}\ T_{cp})}{z\ T_{cm} + w_{eq}^{\infty}\ T_{cp} + w\ T_{cp} + \frac{w\ w_{eq}^{\infty}\ T_{cp}^2}{z\ T_{cm} + w_{eq}^{\infty}\ T_{cp}}}$$

(c) Calculate w_{eq}^{∞} using boundary results and then substituting in (b)

Solution 5.31. (Bataineh 97)

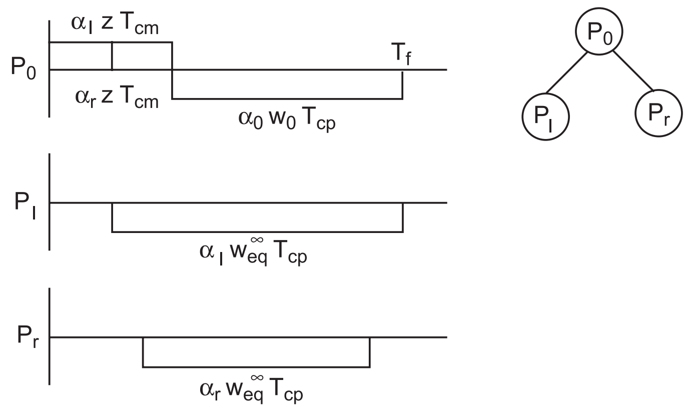


(a)

$$\begin{aligned} T^{\infty}_{nfe} &= (\beta_l + \beta_r)\ z\ T_{cm} + \beta_c\ w\ T_{cp} \\ &= \beta_l\ z\ T_{cm} + \beta_l\ w^{\infty}_{eq}\ T_{cp} \\ &= (\beta_l + \beta_r)\ z\ T_{cm} + \beta_r\ w^{\infty}_{eq}\ T_{cp} \\ &= w^{\infty}_{eq,s}\ T_{cp} \qquad \beta_c + \beta_l + \beta_r = 1 \end{aligned}$$

(b) $w^{\infty}_{eq,s} = \dfrac{w\ (z\ T_{cm} + w^{\infty}_{eq}\ T_{cp})^2}{(w^{\infty}_{eq}\ T_{cp})^2 + w^{\infty}_{eq}\ T^2_{cp}\ w + w\ T_{cp}(z\ T_{cm} + w^{\infty}_{eq}\ T_{cp})}$

Solution 5.32.



(a) Timing equations:

$$\begin{aligned} T_f &= (\alpha_l + \alpha_r)\, z\ T_{cm} + \alpha_0\ w\ T_{cp} \\ &= \alpha_l\ z\ T_{cm} + \alpha_l\ w_{eq}^{\infty}\ T_{cp} \\ &= (\alpha_l + \alpha_r)\, z\ T_{cm} + \alpha_r\ w_{eq}^{\infty}\ T_{cp} = w_{eq,s}^{\infty}\ T_{cp} \end{aligned}$$

(b) With algebra:

$$w_{eq,s}^{\infty} = \frac{w(z\ T_{cm} + w_{eq}^{\infty}\ T_{cp})^2}{(w_{eq}^{\infty}\ T_{cp})^2 + 2w\ w_{eq}^{\infty}\ T_{cp}^2 + w\ z\ T_{cp}\ T_{cm}}$$

(c) With more algebra:

$$(w_{eq}^{\infty})^3 + w(w_{eq}^{\infty})^2 - [w\ z\ \rho]\ w_{eq}^{\infty} - w\ z^2\ \rho^2 = 0$$

Solution 5.33.

(a) (Sohn):

$$\begin{aligned} T_f - T_n &= \alpha_n\ w_n\ T_{cp} \quad n = 1, 2, \ldots, N \\ T_n - T_{n-1} &= \alpha_n\ \overline{z}_{n-1}^{n}\ (T)\ T_{cm} \quad n = 1, 2, \ldots, N \\ \overline{z}_{n-1}^{n}(T) &= \left(E \left\{ \frac{1}{z_{n-1}^{n}(T)} \right\} \right)^{-1} \\ &= \frac{T_n - T_{n-1}}{\int_{T_{n-1}}^{T_n} \frac{1}{z(T)} dT} \end{aligned}$$

$z_{n-1}^{n}\ (T)$ is inverse of the time avg. of the applied channel speed in interval (T_{n-1}, T_n)

$$\frac{d}{dT} z(T) = \sum_{k=0}^{\infty}\ S_k \delta(t - t_k)\, z$$

$$S_k = \begin{cases} +1 & \text{for arrival} \\ -1 & \text{for departure} \end{cases}$$

$$z(T) = \sum_{k=0}^{\infty} S_k\ \mu\,(t - t_k)\, z$$

$$\frac{1}{z(T)} = \sum_{k=0}^{\infty} \left(\sum_{j=0}^{k} S_j \right)^{-1} [\mu\,(t - t_k) - \mu\,(t - t_{k+1})] \frac{1}{z}$$

$$\int_{T_{n-1}}^{T_n} \frac{1}{z(T)} = \frac{T_n}{z(T_n)} - \frac{T_{n-1}}{z(T_{n-1})}$$

$$
\begin{aligned}
T_n - T_{n-1} &= \alpha_n \, \overline{z}^n_{n-1}(T) \, T_{cm} \\
&= \alpha_n \, T_{cm} \, \frac{T_n - T_{n-1}}{\int_{T_{n-1}}^{T_n} \frac{1}{z(t)} dt} \\
\alpha_n &= \frac{1}{T_{cm}} \int_{T_{n-1}}^{T_n} \frac{1}{z(T)} dT \\
&= \frac{1}{T_{cm}} \left[\frac{T_n}{z(T_n)} - \frac{T_{n-1}}{z(T_{n-1})} - \sum_{k=x_{n-1}+1}^{x_n} \left(\frac{1}{z(t_k)} - \frac{1}{z(t_{k-1})} \right) t_k \right]
\end{aligned}
$$

(b) One can solve for $\alpha' s$ versus finish time and select those $\alpha' s$ that sum to one as being correct (Sohn 98).

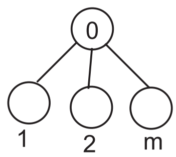
Solution 5.34.

$$
\alpha_n = \frac{1}{T_{cp}} \int_{T_n}^{T_f} \frac{1}{w_n(T)} dt
$$

$$
\sum_{i=1}^{n} \alpha_i = \frac{1}{T_{cm}} \int_0^{T_n} \frac{1}{z(T)} dt
$$

see (Sohn 98) for full explanation.

Solution 5.35.



$$
\text{Start with} \left\{
\begin{array}{c}
\alpha_1 \; w_1 \; T_{cp} \le T_f \\
\alpha_1 \; z_1 \; T_{cm} + \alpha_2 \; w_2 \; T_{cp} \le T_f \\
\alpha_1 \; z_1 \; T_{cm} + \alpha_2 \; z_2 \; T_{cm} + \alpha_3 \; w_3 \; T_{cp} \le T_f \\
\left(\sum_{i=1}^{m} \alpha_i \; z_i \; T_{cm} + \alpha_m \; w_m \; T_{cp} \right) \le T_f \\
T_f = \alpha_0 \; w_0 \; T_{cp}
\end{array}
\right.
$$

m.p.: min $T_f = \min \alpha_0 \; w_0 \; T_{cp}$

$$\alpha_1\ w_1\ T_{cp} - \alpha_0\ w_0\ T_{cp} \leq 0$$

$$\alpha_1\ z_1\ T_{cm} + \alpha_2\ w_2\ T_{cp} - \alpha_0\ w_0\ T_{cp} \leq 0$$

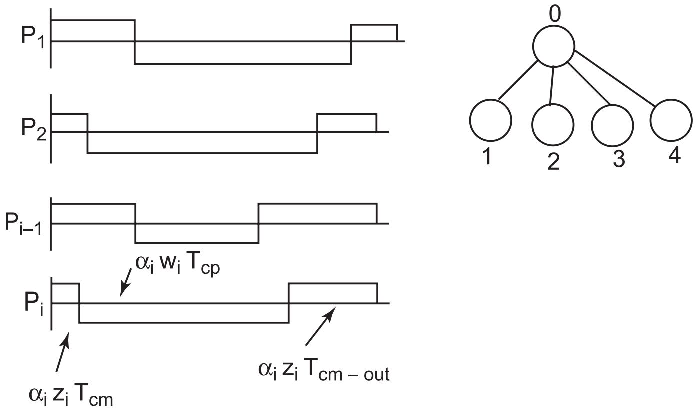
$$\alpha_1\ z_1\ T_{cm} + \alpha_2\ z_2\ T_{cm} + \alpha_3\ w_3\ T_{cp} - \alpha_0\ w_0\ T_{cp} \leq 0$$

$$\sum_{i=1}^{m} \alpha_i\ z_i\ T_{cm} + \alpha_n\ w_m\ T_{cp} - \alpha_0\ w_0\ T_{cp} \leq 0$$

$$\alpha_0 + \alpha_1 + \alpha_2 + \cdots + \alpha_m - 1 = 0$$

$$\alpha_1, \alpha_2, \ldots, \alpha_m \geq 0$$

Solution 5.36.



$$\alpha_1(z_1\ T_{cm} + w_1\ T_{cp} + z_1\ T_{cm-out}) = \alpha_2(z_2\ T_{cm} + w_2\ T_{cp} + z_2\ T_{cm-out})$$

Let

$$q_i = \frac{z_{i-1}\ T_{cm} + w_{i-1}\ T_{cp} + z_{i-1}\ T_{cm-out}}{z_i\ T_{cm} + w_i\ T_{cp} + z_i\ T_{cm-out}}$$

$$\text{Then} \quad \alpha_i = q_i\ \alpha_{i-1} \quad i = 2, 3, \ldots, m$$

Then we can use the equations of Section 5.2.2 with the new q_i to solve for the $\alpha's$ and the speedup.

Solution 5.37. *(From Hung PhD Thesis, 2003)*:

Each child begins to receive load once the root has received the load for itself and all its children. The root without data storage scheduling in Fig. 5 is of interest as it represents a typical store and forward switching behavior where

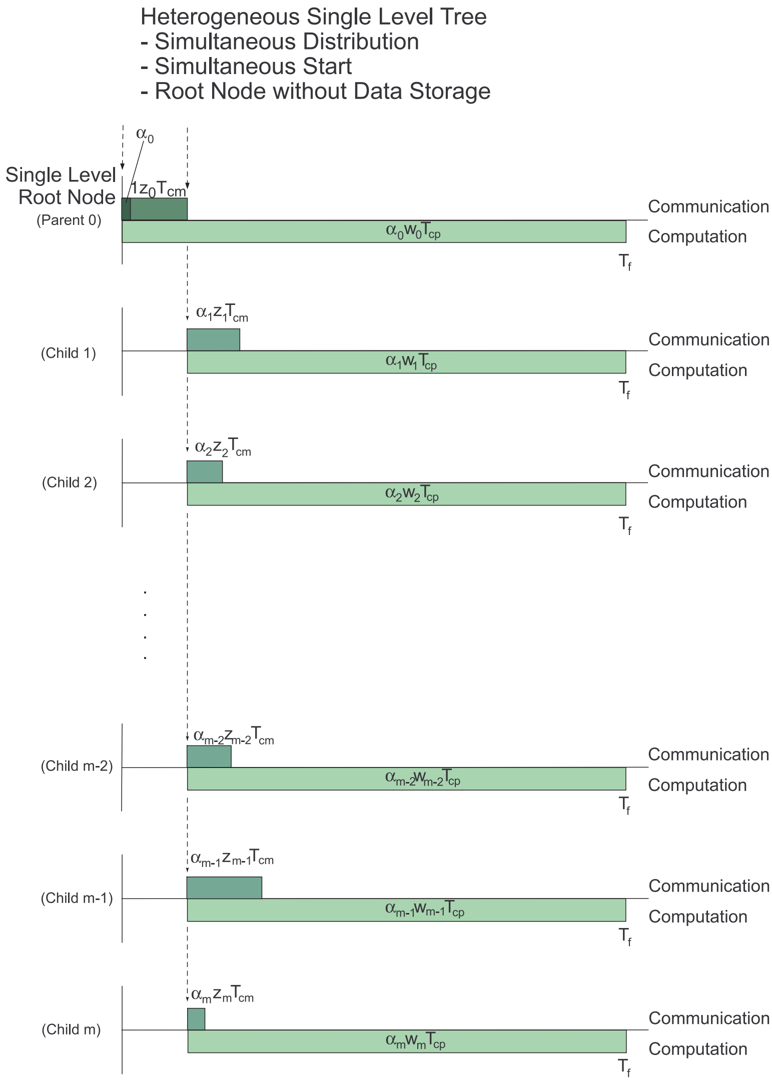


Fig. 5. Timing diagram of single level tree using simultaneous distribution and simultaneous start for a root node without data storage

the entire load for a subtree is received by the parent (root) before the children commence receiving and processing load. Each processor begins computing the received data as soon as the data starts to arrive. Other scheduling variations are certainly possible.

From the timing diagram of Fig. 5, the fundamental recursive equations of the system can be formulated as follows:

$$\alpha_0 \ w_0 \ T_{cp} = 1 \cdot z_0 \ T_{cm} + \alpha_1 \ w_1 \ T_{cp} \tag{5.24}$$

$$\alpha_{i-1} \ w_{i-1} \ T_{cp} = \alpha_i \ w_i \ T_{cp} \quad i = 2, 3, \ldots, m \tag{5.25}$$

The normalization equation for this single level tree is

$$\alpha_0 + \alpha_1 + \alpha_2 + \cdots + \alpha_m = 1 \tag{5.26}$$

This gives $m + 1$ linear equations with $m + 1$ unknowns.

From (24), we obtain

$$\alpha_0 = \frac{z_0 \ T_{cm}}{w_0 \ T_{cp}} + \frac{w_1 \ T_{cp}}{w_0 \ T_{cp}} \alpha_1 = \sigma_0 + \frac{1}{k_1} \alpha_1 \tag{5.27}$$

Here σ_0 is defined as $(z_0 \ T_{cm})/(w_0 \ T_{cp})$ and k_1 as w_0/w_1. From (25), the solution of α_i is

$$\alpha_i = \frac{w_{i-1} \ T_{cp}}{w_i \ T_{cp}} \alpha_{i-1} = q_i \ \alpha_{i-1} \quad i = 2, 3, \ldots, m \tag{5.28}$$

where $q_i = w_{i-1}/w_i$.

Equation (28) can be represented as

$$\alpha_i = q_i \ \alpha_{i-1} = \left(\prod_{l=2}^{i} q_l \right) \alpha_1 \quad i = 2, 3, \ldots, m \tag{5.29}$$

Employing (27) and (29), the normalization equation (26) becomes

$$\sigma_0 + \frac{1}{k_1} \alpha_1 + \alpha_1 + \sum_{2}^{m} \alpha_i = 1 \tag{5.30}$$

$$\left[\frac{1}{k_1} + 1 + \sum_{i=2}^{m} \left(\prod_{l=2}^{i} q_l \right) \right] \alpha_1 = 1 - \sigma_0 \tag{5.31}$$

Consequently,

$$\alpha_1 = \frac{1 - \sigma_0}{\left[\frac{1}{k_1} + 1 + \sum_{i=2}^{m} \left(\prod_{l=2}^{i} q_l \right) \right]} \tag{5.32}$$

From Fig. 5, the finish time is achieved as:

$$T_{f,m} = \alpha_0 \ w_0 \ T_{cp} = \left[\sigma_0 + \frac{1}{k_1} \alpha_1 \right] w_0 \ T_{cp} \tag{5.33}$$

The term, $T_{f,m}$, is the finish time of a single divisible job solved on the entire tree, consisting of one root node as well as m child nodes. Now, collapsing a single level tree into a single node, one can obtain the finish time of the single level tree as follows.

$$T_{f,m} = w_{eq}\ T_{cp} = \alpha_0\ w_0\ T_{cp} = \left[\sigma_0 + \frac{1}{k_1}\alpha_1\right] w_0\ T_{cp} \tag{5.34}$$

On the other hand, $T_{f,0}$ is defined as the solution time for the entire divisible load solved on the root processor.

$$T_{f,0} = \alpha_0\ w_0\ T_{cp} = 1 \times w_0\ T_{cp} = w_0\ T_{cp} \tag{5.35}$$

According to Definition 1 in Section 2, $\gamma_{eq} = w_{eq}/w_0 = T_{f,m}/T_{f,0}$, one obtains the value of γ_{eq} by (35) dividing (34).

$$\begin{aligned}\gamma_{eq} &= \sigma_0 + \frac{1}{k_1}\alpha_1 \\ &= \sigma_0 + \frac{1}{k_1} \times \frac{1-\sigma_0}{\left[\frac{1}{k_1} + 1 + \sum_{i=2}^{m}\left(\prod_{l=2}^{i} q_l\right)\right]} \\ &= \frac{1+\sigma_0\ k_1\left[1+\sum_{i=2}^{m}\left(\prod_{l=2}^{i} q_l\right)\right]}{1+k_1\left[1+\sum_{i=2}^{m}\left(\prod_{l=2}^{i} q_l\right)\right]}\end{aligned} \tag{5.36}$$

Speedup is intuitively obtained by $T_{f,0}/T_{f,m}$, or $1/\gamma_{eq}$; therefore,

$$\mathit{Speedup} = \frac{1}{\gamma_{eq}} = \frac{1+k_1\left[1+\sum_{i=2}^{m}\left(\prod_{l=2}^{i} q_l\right)\right]}{1+\sigma_0\ k_1\left[1+\sum_{i=2}^{m}\left(\prod_{l=2}^{i} q_l\right)\right]} \tag{5.37}$$

Two cases are discussed:

1) *General Case:* Since $\prod_{l=2}^{i} ql$ can be simplified as w_1/w_i, γ_{eq} can be derived from (36) as

$$\gamma_{eq} = \frac{1+\sigma_0\frac{w_0}{w_1}\left[1+\sum_{i=2}^{m}\frac{w_1}{w_i}\right]}{1+\frac{w_0}{w_1}\left[1+\sum_{i=2}^{m}\frac{w_1}{w_i}\right]} = \frac{1+\sigma_0\ w_0\sum_{i=1}^{m}\frac{1}{w_i}}{1+w_0\sum_{i=1}^{m}\frac{1}{w_i}} \tag{5.38}$$

Thus, the value of speedup becomes

$$\mathit{Speedup} = \frac{1+w_0\sum_{i=1}^{m}\frac{1}{w_i}}{1+\sigma_0\ w_0\sum_{i=1}^{m}\frac{1}{w_i}} \tag{5.39}$$

2) *Homogeneous Case:* As a special case, consider the situation of a homogeneous network where all children processors have the same inverse computing speed and all links have the same inverse transmission speed. In

other words, $w_i = w$ and $z_i = z$ for $i = 1, 2, \ldots, m$. Note that the root inverse computing speed, w_0 can be different from those $w_i, i = 1, 2, \ldots, m$. Consequently,

$$k_1 = \frac{w_0}{w_1} = \frac{w_0}{w} \tag{5.40}$$

$$q_i = \frac{w_{i-1}}{w_i} = \frac{w}{w} = 1 \quad i = 2, 3, \ldots, m$$

$$\gamma_{eq} = \frac{1 + \sigma_0 \ k_1 \Big[1 + \sum_{i=2}^{m} \Big(\prod_{l=2}^{i} q_l\Big)\Big]}{1 + k_1 \Big[1 + \sum_{i=2}^{m} \Big(\prod_{l=2}^{i} q_l\Big)\Big]}$$

$$= \frac{1 + \sigma_0 \ m\frac{w_0}{w}}{1 + m\frac{w_0}{w}} = \frac{w + \sigma_0 \ m \ w_0}{w + m \ w_0} \tag{5.41}$$

$$\text{Speedup} = \frac{1 + m\frac{w_0}{w}}{1 + \sigma_0 m\frac{w_0}{w}} = \frac{w + m w_0}{w + \sigma_0 m w_0} \tag{5.42}$$

Solution 5.38.

$$\sum_{i=1}^{N} \alpha_{i,j} \ w_i \ T_{cp} = \sum_{i=1}^{N} \alpha_{i,j+1} \ w_{j+1} \ T_{cp} \quad j = 1, 2, \ldots, M-1$$

$$\sum_{i=1}^{N} \alpha_j \ L_i \ w_j = \sum_{i=1}^{N} \alpha_{j+1} \ L_i \ w_{j+1} \quad j = 1, 2, \ldots, M-1$$

Using this and $\sum\limits_{i=1}^{M} \alpha_i = 1$,

$$\alpha_j = \frac{1}{w_j\Big(\sum\limits_{x=1}^{M} \frac{1}{w_x}\Big)} \quad j = 1, 2, \ldots, M$$

$$\alpha_{i,j} = \frac{1}{w_j\Big(\sum\limits_{x=1}^{M} \frac{1}{w_x}\Big)} L_i$$

$$T(M) = \sum_{i=1}^{N} \alpha_{i,M} \ w_M \ T_{cp}$$
