Stony Brook University

<div align="center">

Assignment 4B
Expectations - One Random Variable

</div>

1 Assignment

1. Find the mean and variance of the uniform discrete random variable X that takes values from the set $\{1, 2, \ldots n\}$.

   Hint: Use the identities

   $$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

   and

   $$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}.$$

2. Determine the mean and variance of the binomial random variable X, with PMF given by

   $$f_X(x) = \binom{n}{x} p^x (1-p)^{n-x}, \qquad x = 0, 1, \ldots n.$$

3. Determine the mean and variance of the geometric random variable X, with PMF given by

   $$f_X(x) = (1-p)^{x-1} p, \qquad x = 1, 2, 3, \ldots.$$

4. Let X be a random variable, with PDF

   $$f_X(x) = \frac{1}{2} e^{-|x|}, \qquad -\infty < x < \infty.$$

   Find the mean and variance of X.

5. The random variable X has a Poisson distribution, with probability mass function (PMF) given by:
   $$f_X(x) = e^{-\lambda} \frac{\lambda^x}{x!}, \qquad \lambda > 0, \ x = 0, 1, 2, 3, \ldots$$
   Compute the expected value $E[X(X-1)]$.

6. We roll a fair dice twice. Let X denote the sum of the two rolls. Find the expected value of X.

7. The probability of a bit error in a communication channel is 10^{-3}. Find the expected value of the number of errors in a frame of 1000 transmitted bits.