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Error Codes Part III: CRC Code Performance

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Receiver $\dfrac{R(x)}{G(x)} = \dfrac{T(x) + E(x)}{G(x)} \Rightarrow \cancel{\dfrac{T(x)}{G(x)}}$?

$\dfrac{E(x)}{G(x)}$?

$T(x) = 1\,0\,0\,1\,1\,0\,1\,0$

$E(x) = 0\,1\,0\,0\,0\,0\,0\,1$

$R(x) = 1\,1\,0\,1\,1\,0\,1\,1$

Single Bit Errors

$E(x) = 0\,0\,0\,1\,0\,0\,0\,0$

$E(x) = x^i$ power of 2

If $G(x) \geq 2$ terms (not power of 2)
remainder will be produced

$\frac{32}{\underbrace{1+2+4}_{7}} \rightarrow$ remainder

G(x) has 2
or more terms

Double Bit Error $E(x) = 0010001000$

$E(x) = x^i + x^j \quad i > j$

$= x^j (x^{i-j} + 1)$

$= \dfrac{x^j (x^k + 1)}{G(x)} \qquad k = i - j$

Suf Cond → $G(x)$ does not divide $x^k + 1$

example: $x^{15} + x^{14} + 1$ will not divide $x^k + 1$ evenly for k up to 32,768

$$\frac{E(x) = 0100100001100}{G(x)}$$

there's no polynomial with an odd # terms divisible evenly by $(x+1)$ → So pick generator polynomials that $(x+1)$ is a factor → catch all odd # errors

Proof: Assume $E(x)$ has odd # terms and is divisible by $x+1$ — NO

$E(x) = (x+1)Q(x)$

Let $x=1$

$E(1) = (1+1)Q(1)$

$= 0 \cdot Q(1) = 0$

A	B	A⊕B
0	0	0
0	1	1
1	0	1
1	1	0

But if $E(x)$ has odd # terms, $E(1) = 1$

$x^5 + x^2 + 1 \Big|_{x=1} = \underline{1+1}+1 = 0+1 = 1$

Burst Errors

$E(x) = 00111111000$

$$E(x) = \underline{x^i(x^{k-i} + \cdots + 1)}$$

~~G(x)~~ $G(x)$

If $G(x) \Rightarrow x^0 = 1$

then x^i is not a factor

so remainder is not 0

Burst Errors

If burst $< r+1$ (where r is # check bits) including a term of $+1$ ($1x^0$) in $G(x)$ will catch all bursts $\leq r$

If bursts $> r+1$ / several short bursts

$$P\begin{pmatrix}\text{bad}\\ \text{message}\\ \text{undetected}\end{pmatrix} = \left(\frac{1}{2}\right)^{r-1}$$

International standards

$$x^{12}+x^{11}+x^{3}+x^{2}+x^{1}+1$$

$$x^{16}+x^{15}+x^{2}+1$$

$$x^{16}+x^{12}+x^{5}+1$$

ARQ protocols

✓ All have +1

✓ All have $(x+1)$ as a factor

✓ all have 3rd term