

SAMPLE PROBLEM SET 1

[1] A uniform plane electromagnetic wave, propagating along the +z direction in a dielectric medium with a phase velocity $V_{ph} = 1.5 \times 10^8$ m/s, is linearly polarized along x. Given that the wave frequency is 1 GHz and that the wave electric field has a peak value of 10^{-3} V/m at $t=0$ and $z=0.1$ m, obtain

- i) the instantaneous expression for the wave electric field for any t and z ; and
- ii) the locations where the wave electric field is a positive maximum when $t=10^{-9}$ s.

[2] Obtain the wavelength of uniform plane EM wave in vacuum for the following frequencies:

- i) 60 Hz (power frequency), ii) 535-1605 kHz (AM radio) iii) 88-106 MHz (FM radio) iv) 4-8 GHz (C Band, used in satellite communication), v) 7.5×10^{14} - 1.75×10^{15} Hz (visible light), vi) 10^{18} Hz (typical frequency for x-rays).

[3] As an inverse to problem [2], find the frequencies of uniform plane electromagnetic waves in vacuum for the following wavelengths:

- i) 1 km; ii) 1 m; iii) 1 cm, iv) 1 mm, (v) 1 μ m; vi) 1 nm; v) 1 Angstrom ($=10^{-10}$ m or 0.1 nm).

[4] The electric field vector of a uniform plane EM wave propagating in vacuum is given by the phasor expression

$$\underline{E}(z) = -j2 e^{jkz} \underline{i}_x + 4e^{jkz} \underline{i}_y$$

- i) Write down the instantaneous field expression for the electric field
- ii) What is the locus of the tip of the instantaneous electric field vector $\underline{E}(z,t)$ in the xy plane, i.e., what is the wave polarization?

[5] The instantaneous expression for the electric field in a uniform plane electromagnetic wave propagating in a dielectric medium is

$$\underline{E}(z,t) = \underline{i}_x \cos[2\pi \cdot 10^9 t - (200\pi/3)z - 60^\circ], \text{ mV/m.}$$

Find:

- 1) the frequency of the wave in Hz
- 2) the wavelength of the wave in m
- 3) phase velocity V_{ph}
- 4) relative refractive index n_r of the medium; and
- 5) the polarization of the wave

SOLUTION TO SAMPLE PROBLEM SET 1

[1] $E_x = E_0 \cos(\omega t - kz + \phi)$

i) $f = 1 \text{ GHz} = 10^9 \text{ Hz} \rightarrow \omega = 2\pi f = 2\pi \cdot 10^9 \text{ Hz}$

$v_{ph} = \frac{\omega}{k}$, $\therefore k = \frac{\omega}{v_{ph}} = \frac{2\pi \cdot 10^9 \text{ rad/s}}{1.5 \times 10^8 \text{ m/s}} = \frac{20\pi}{1.5} \text{ m}^{-1} = \frac{40\pi}{3} \text{ m}^{-1}$

Also $E_0 = 10^{-3} \text{ V/m}$ (given)

The only unknown now is ϕ which is found using the initial condition

$E_x(\frac{1}{10} \text{ m}, 0) = E_0 \rightarrow 1 = \cos(-k \cdot \frac{1}{10} + \phi)$

Lowest soln is $-\frac{k}{10} + \phi = 0 \rightarrow \phi = \frac{k}{10} = \frac{40\pi}{3} \cdot \frac{1}{10} = \frac{4\pi}{3}$

\therefore Desired soln is $E_x(z, t) = 10^{-3} \cos[2\pi \cdot 10^9 t - \frac{40\pi z}{3} + \frac{4\pi}{3}] \text{ V/m}$ Answer

ii) $E_x(z, 10^{-9} \text{ s}) = 10^{-3} \cos[2\pi \cdot 10^9 \cdot 10^{-9} - \frac{40\pi z}{3} + \frac{4\pi}{3}] = 10^{-3} \cos[\frac{10\pi}{3} - \frac{40\pi z}{3}] \text{ V/m}$

The cosine has a maximum positive value when

$\frac{10\pi}{3} - \frac{40\pi z}{3} = 0, 2\pi, 4\pi, \dots$

or $\frac{40z}{3} = \frac{10}{3}, \frac{10}{3} + 2, \frac{10}{3} + 4, \dots$

yielding

$z = \frac{1}{4}, \frac{1}{10}, \frac{2}{5}, -\frac{1}{20}, \frac{11}{60}, \dots \text{ m}$ Answer

[2] $f\lambda = c \rightarrow f = 6 \text{ GHz}, \lambda = \frac{3 \times 10^8}{60} = 5 \times 10^6 \text{ m}$

$f = 535 - 1605 \text{ kHz} \rightarrow \lambda = 1.87 \times 10^2 \rightarrow 5.6 \times 10^2 \text{ m}$

$f = 88 - 106 \text{ MHz} \rightarrow \lambda = 2.83 \rightarrow 3.41 \text{ m}$

$f = 4 - 8 \text{ GHz} \rightarrow \lambda = 3.75 \times 10^{-2} \rightarrow 7.50 \times 10^{-2} \text{ m}$

$f = 7.5 \times 10^{14} - 1.75 \times 10^{15} \text{ Hz} \rightarrow \lambda = 1.71 \times 10^{-7} \rightarrow 4.0 \times 10^{-7} \text{ m}$

$f = 10^{18} \rightarrow \lambda = 3 \times 10^{-10} \text{ m}$

[3] $f\lambda = c \rightarrow \lambda = 1 \text{ km}, f = \frac{3 \times 10^8 \text{ m/s}}{1 \times 10^3 \text{ m}} \rightarrow 3 \times 10^5 \text{ Hz}$

$\lambda = 1 \text{ m}, f = 3 \times 10^8 \text{ Hz}$

$\lambda = 1 \text{ cm}, f = 3 \times 10^{11} \text{ Hz}$

$\lambda = 1 \text{ mm}, f = 3 \times 10^{14} \text{ Hz}$

$\lambda = 10^{-6} \text{ m} \rightarrow f = 3 \times 10^{18} \text{ Hz}$

$\lambda = 10^{-9} \text{ m} \rightarrow f = 3 \times 10^{21} \text{ Hz}$

$\lambda = 10^{-10} \text{ m} \rightarrow f = 3 \times 10^{22} \text{ Hz}$

(2)

$$[4]. \underline{E}(z) = -j2 e^{jkz} \hat{z}_x + 4 e^{jkz} \hat{z}_y$$

$$i) \underline{E}(z,t) = \text{Re}[\underline{E}(z) e^{j\omega t}]$$

$$= 2 \sin(\omega t + kz) \hat{z}_x + 4 \cos(\omega t + kz) \hat{z}_y \quad \text{Answer}$$

$$ii) \text{ Take } \underline{E}(0,t) = 2 \sin \omega t \hat{z}_x + 4 \cos \omega t \hat{z}_y$$

$$\underline{E}(0,0) = \hat{z}_y 4$$

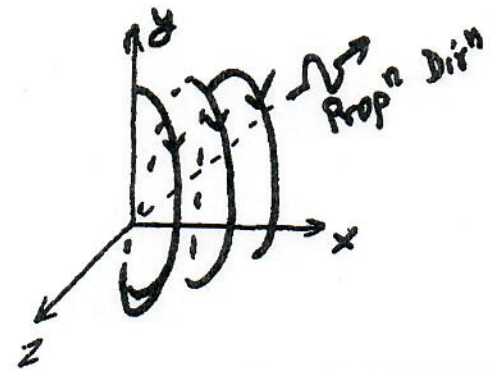
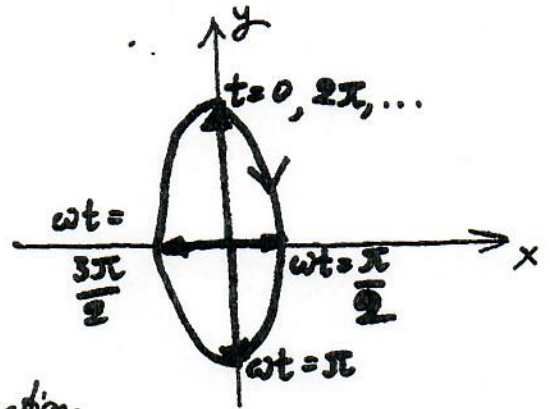
$$\underline{E}(0, \frac{T}{4}) = \hat{z}_x 2$$

$$\underline{E}(0, \frac{T}{2}) = -\hat{z}_y 4$$

$$\underline{E}(0, \frac{3T}{4}) = -\hat{z}_x 2$$

Signal is right elliptically polarized
about the $-z$ direction of propagation

Answer



$$[5] \underline{E}(z,t) = \hat{z}_x E_x(z,t) \text{ where}$$

$$E_x(z,t) = E_0 \cos(\omega t - kz - 60^\circ) \text{ mV/m}$$

$$\text{where } \omega = 2\pi \cdot 10^9 \text{ rad/s}, k = \frac{200\pi}{3} \text{ rad/m}, E_0 = 1 \frac{\text{mV}}{\text{m}}$$

Answer

$$i) f = \omega/2\pi = 10^9 \text{ Hz} = 1 \text{ GHz}$$

$$ii) k = 2\pi/\lambda \rightarrow \lambda = 2\pi/k = \frac{2\pi \cdot 3}{200\pi} \text{ m} = 0.03 \text{ m} \quad \text{Answer}$$

$$iii) v_{ph} = \frac{\omega}{k} = \frac{2\pi \cdot 10^9 \text{ rad/s}}{\frac{200\pi}{3} \text{ rad/m}} = \frac{3 \times 10^9}{100} \text{ m/s} = 3 \times 10^7 \text{ m/s} \quad \text{Answer}$$

iv) $n_r = (\text{vel of EMW in vacuum}) / (\text{vel of EMW in dielectric})$, i.e.,
 n_r is "a velocity reduction factor" relative to vel of EMW in vacuum
i.e., $\text{vel of EMW in dielectric} = \frac{\text{vel of EMW in vacuum}}{n_r}$

$$\therefore n_r = \frac{3 \times 10^8 \text{ m/s}}{3 \times 10^7 \text{ m/s}} = 10 \quad \text{Answer}$$

v) The UPEMW is linearly polarized along x Answer