ESE/CSE 346
Probability Problems (Part 1)

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Electrical & Computer Engineering

13.

(a) $p = \frac{20}{60} = \frac{1}{3} \Rightarrow$ prob a circuit busy $\left(\frac{1}{3}\right)$

(b) P(at least one free circuit)

$= P(\text{find a free circuit})$

$= 1 - P(0 \text{ circuits free})$

$= 1 - \binom{3}{0} q^0 (1-q)^3 = 1 - (1-q)^3 = 1 - p^3$

$q = 1 - p$

$= P(\text{circuit free})$

(c)

$$P(\text{one exactly circuit free}) = \binom{3}{1} \underbrace{q}_{\text{circuit free}} (1-q)^2$$

(d)

$$\overline{\#\text{ busy circuits}} = 1 \cdot P(1\text{ busy}) + 2\,P(2\text{ busy}) + 3\,P(3\text{ busy})$$

$$= 1 \cdot \binom{3}{1} p(1-p)^2 + 2 \cdot \binom{3}{2} p^2(1-p) + 3 \underbrace{\binom{3}{3}}_{=1} p^3 \underbrace{(1-p)^0}_{=1}$$

$$= \boxed{3p(1-p)^2 + 6p^2(1-p) + 3p^3}$$

14)

P : P(computer is down)

P(at least one computer up in a city)

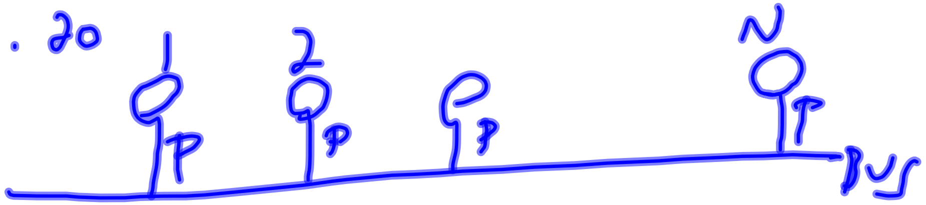
p: prob a computer down

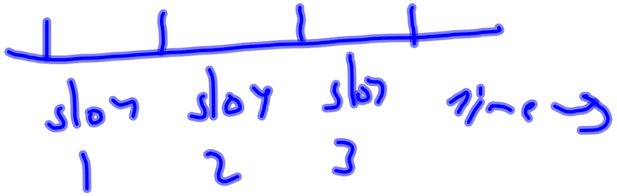
$$P\left(\begin{array}{l}\text{at least one}\\ \text{computer up in}\\ \text{a city}\end{array}\right) = 1 - P\left(\begin{array}{l}\text{both computers}\\ \text{down in}\\ \text{a city}\end{array}\right)$$

$$= 1 - p^2$$

$$P\left(\begin{array}{l}\text{at least one}\\ \text{computer up}\\ \text{in each city}\end{array}\right) = (1 - p^2)^3$$

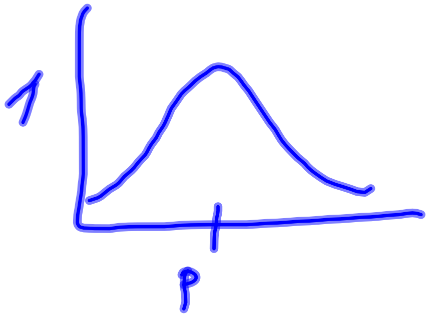
2.20





$$\text{Thrupt} = P(1 \text{ station xmits})$$

$$= \underline{Np(1-p)^{N-1}}$$

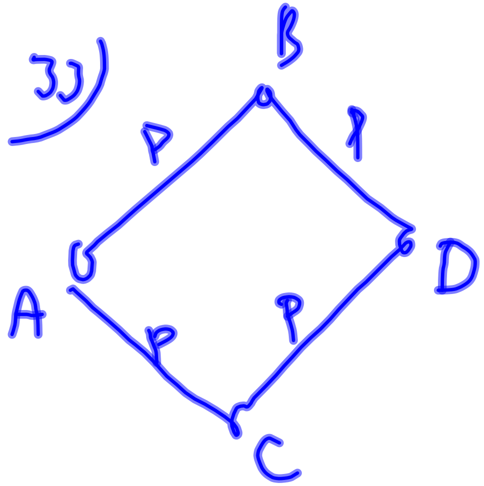


$$\frac{d\,\text{Thrupt}}{dp} = 0$$

$$\frac{1}{N} N = 1$$

$$P_{\text{optimum}} = \frac{1}{N}$$

33)



p: p(each link available)

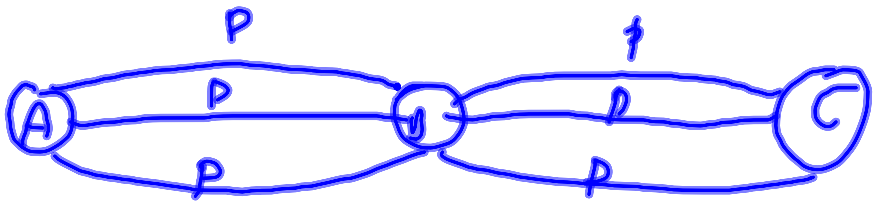
(a) $\xi = p^2 = P(AD \text{ path available})$

(a)

(b) $P\left[\begin{array}{c}\text{at least one} \\ A \text{ to } D \text{ path} \\ \text{available}\end{array}\right] = 1 - P\left[\begin{array}{c}\text{no AD} \\ \text{path available}\end{array}\right]$

$= 1 - (1 - p^2)^2$

3a)



p : link in use

(a)

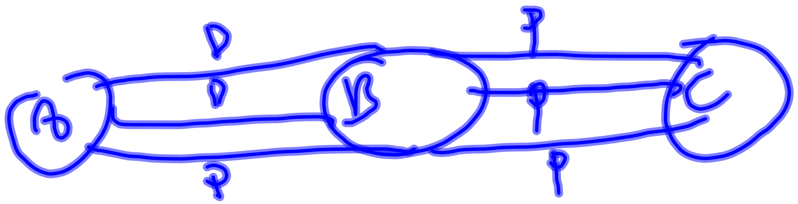
$$E[\#\ \text{busy links}] = \sum n\, p(n) = \sum_{n=1}^{6} n\, p^{n} (1-p)^{6-n} \binom{6}{n}$$

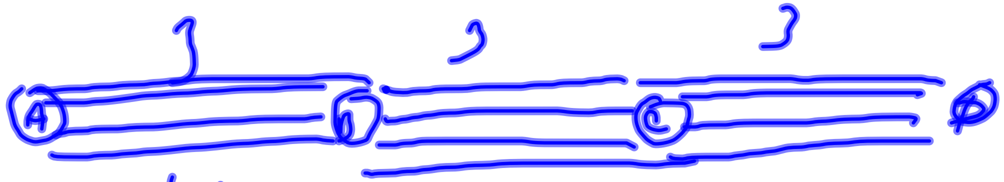
(b)

$P\big[$ at least

$$P\left[\begin{array}{c}\text{at least one}\\ \text{idle path from}\\ A \text{ to } C\end{array}\right] = \left(P\left(\begin{array}{c}\text{at least 1 idle}\\ \text{link from } A \text{ to } B\end{array}\right)\right)^2$$

$$= \left(1 - P\left(\begin{array}{c}\text{no idle links}\\ \text{from } A \text{ to } B\end{array}\right)\right)^2 = \left(1 - p^3\right)^2$$





p: prob link busy

(a)

$$P\begin{pmatrix}\text{A to D}\\ \text{all blocked}\end{pmatrix} = \boxed{P\begin{pmatrix}\text{at least one adjacent pair}\\ \text{is completely blocked}\end{pmatrix}}$$

$$= 1 - P\begin{pmatrix}\text{at least one idle}\\ \text{link between each}\\ \text{adjacent pair}\end{pmatrix}$$

$$= 1 - \left(P\begin{pmatrix}\text{at least one idle link}\\ \text{between an adjacent}\\ \text{pair of nodes}\end{pmatrix}\right)^3$$

$$= 1 - \left(\underline{1} - \underline{p^3}\right)^3$$