



Stony Brook University





Error Codes Part I: Hamming Codes

Instructor: Prof. Thomas Robertazzi

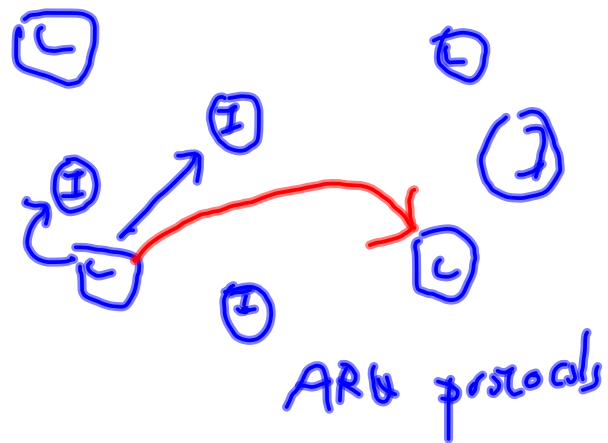
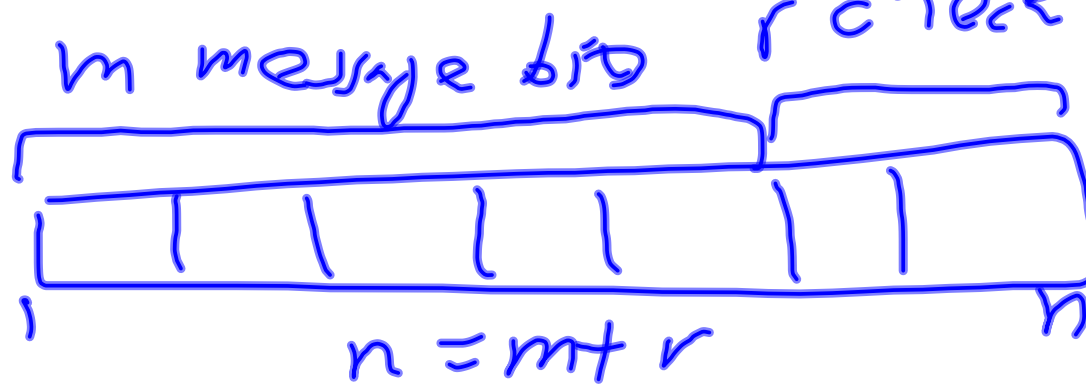
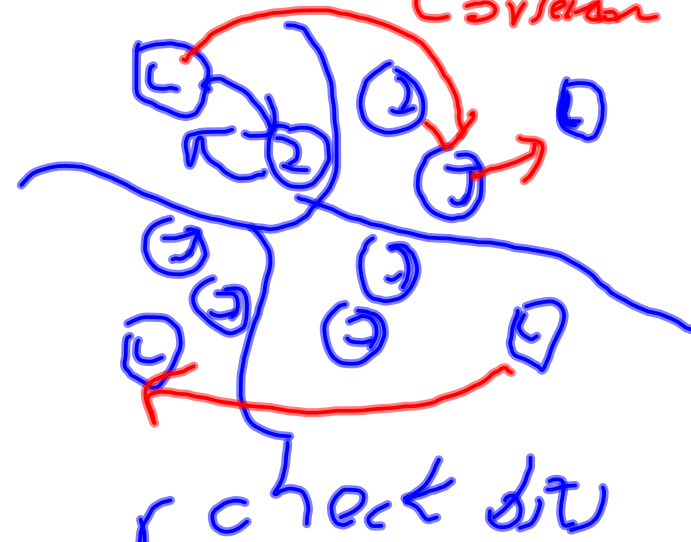
Electrical & Computer Engineering

Errors in Xmission :

- noise thermal noise
optical noise
- frequency distortion
- cosmic ray  
- crosstalk

Error Cols

Error Detection

Error ~~Correction~~
Correction

Parity Code

1 0 1 0 0 1 0 1
↑
parity
bit

parity
even^ codes

Hamming Code

$$\overbrace{1\ 0\ 1\ 0\ 1\ 0\ 1}^{n\ bits}$$

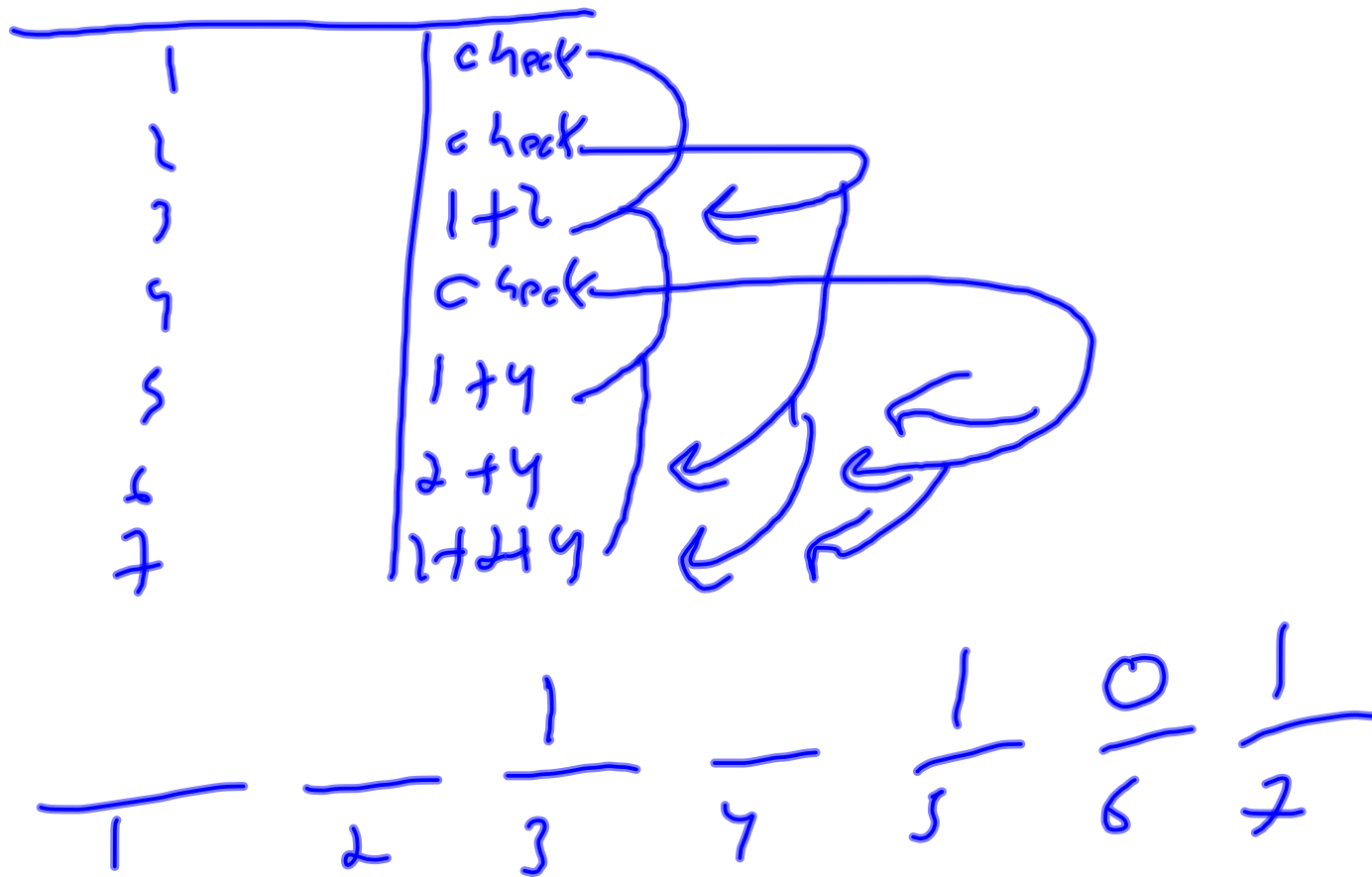
$$\underbrace{\hspace{1.5cm}}_m\ \underbrace{\hspace{1.5cm}}_{\text{message}}\ \underbrace{\hspace{1.5cm}}_r\ \underbrace{\hspace{1.5cm}}_{\text{check}}$$

2^m legal messages
 2^n any of n bits can have an error
 2^r code words
 $n = m + r$

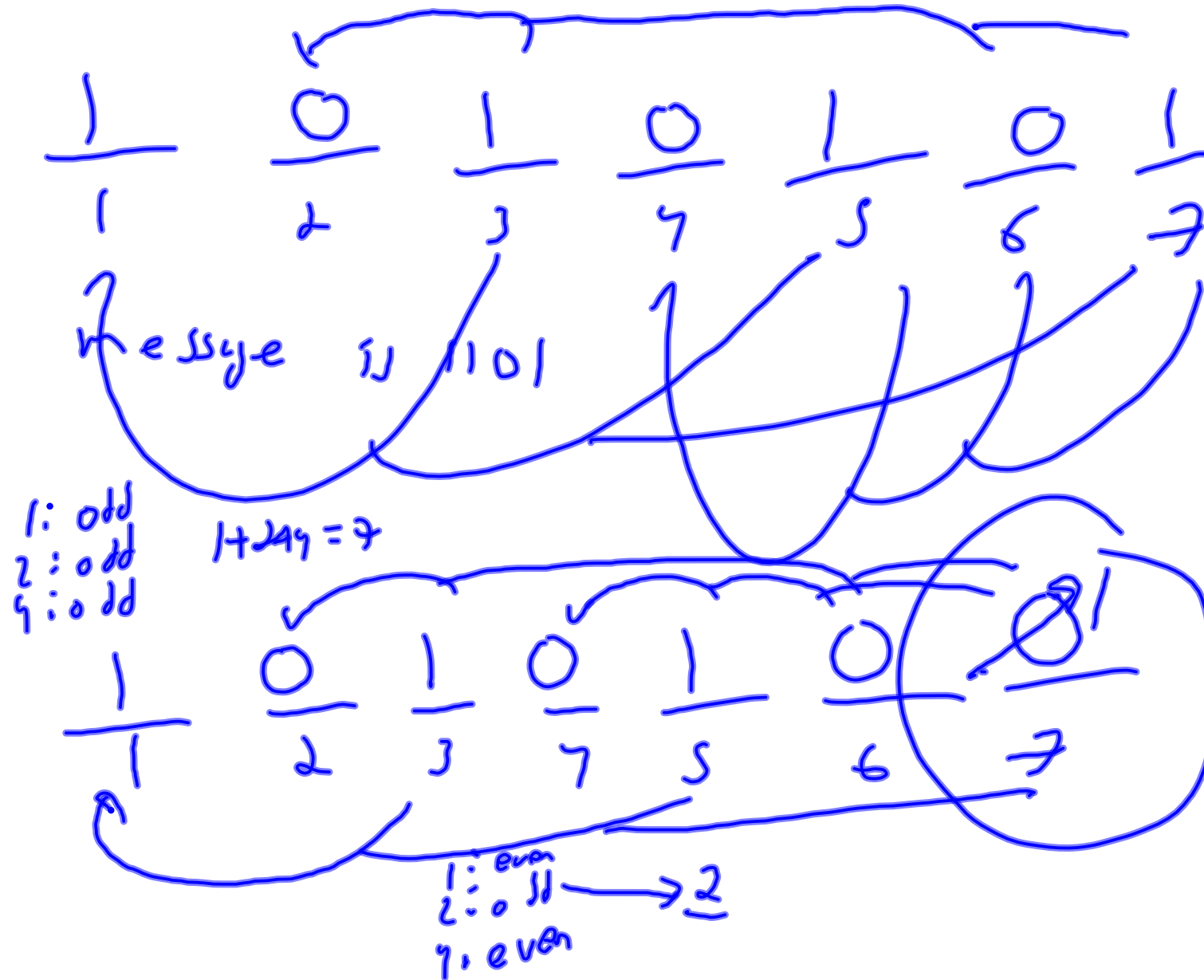
$2^m (n+1) \leq 2^n$
 $2^m (m+r+1) \leq 2^{m+r}$

$m = 6$

r	$2^m (m+r+1)$	2^{m+r}	Result
1	$2^6 (6+1+1) = 12$	$2^7 = 128$	no
2	$2^6 (6+2+1) = 18$	$2^8 = 256$	no
3	$2^6 (6+3+1) = 24$	$2^9 = 512$	no
4	$2^6 (6+4+1) = 32$	$2^{10} = 1024$	yes



message: 110



1	Check	
2	check	1: even
3	1+2	2: even
4	check	4: even
5	1+4	
6	2+4	
7	1+2+4	