ESE/EEO 306
Random Signals and Systems
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Assignment 1A
Probability Basics and Conditional Probability

1 Assignment

1. A coin is tossed three times, and the outcomes, heads or tails, are noted. Find
   (a) the sample space.
   (b) the set A corresponding to the event "the total number of heads is 2".
   (c) the set B corresponding to the event "the outcome of the first toss is heads ".
   (d) the set $A \cap B$ and describe the event that corresponds to this set.

2. Four balls numbered $1, 2, 3$ and 4 are in an urn. Two balls are drawn randomly from the urn. The order of drawing is important.
   (a) Find the sample space of the experiment.
   (b) Let the event A be described by "one of the drawn balls is 2 ". Find the set that defines A.
   (c) Let the event B be described by "the absolute value of the difference of the drawn balls is one ". Find the set that defines B.
   (d) Find the set that corresponds to the event defined by the statement "A occurs but B does not ".

3. We roll a fair dice, and define the events $A = \{1, 2, 3\}$, $B = \{2, 4, 6\}$ and $C = \{2, 3, 4, 5\}$. Calculate the probabilities of the following events:
   (a) A, B and C.
   (b) $A \cap B$, $A \cap C$, $B \cap C$ and $A \cap B \cap C$.
   (c) $A \cup B$, $A \cup C$, $B \cup C$ and $A \cup B \cup C$.
   (d) $(A \cap B)^C$ and $(A \cup B)^C$, directly and using De Morgan's laws.

4. Show that if events A and B are independent, then so are
   (a) A and B^C
   (b) A^C and B
   (c) A^C and B^C

5. A coin is tossed three times, and the outcomes, heads or tails, are noted. All the elementary outcomes in the sample space have equal probabilities. Find the probabilities that

   (a) the first two outcomes are heads.

   (b) there are no heads.

   (c) there are more heads than tails.

6. Two fair dice are rolled. Let their scores be represented by X_1 and X_2. Find the probability $P(X_1 = 4|X_1 + X_2 = 10)$.

7. We roll two dice. Assume all 36 possibilities are equally likely. Let X_1 and X_2 be the result of the first and the second dice, respectively. Let S be the sum of the scores, that is $S = X_1 + X_2$. Calculate the following:

   (a) $P(S = k)$, for $k = 2, 3, \ldots 12$.

   (b) $P(X_1 = 2|S = k)$ for $k = 2, 3, \ldots 12$.

   (c) $P(X_1 = 6|S = k)$ for $k = 2, 3, \ldots 12$.

8. We roll a fair dice, and define the events $A = \{1, 2, 3\}$, $B = \{2, 3, 4, 5\}$ and $C = \{2, 4, 5, 6\}$. Show that the probabilities satisfy the chain rule

$$P(A, B, C) = P(A|B, C)P(B|C)P(C)$$

9. A random number N of dice is thrown. Let A_i be the event that $N = i$, and

$$P(A_i) = \frac{1}{2^i}, \quad i = 1, 2, 3, \ldots$$

   (a) Compute the probabilities $P(A_i)$, $i = 1, 2, 3, 4$.

   (b) Let us denote the sum of the scores by S. What is the probability that S is 4?

10. We have two dice. The first dice is fair, that is, all outcomes are equally likely. The second dice shows a 2 with probability $1/2$. We choose a dice at random and observe the face 2. What is the probability that we chose the second dice?

11. An insurance company divides people into two categories

   - accident-prone, call this event A.
   - not accident-prone, event A^C.

   They have statistics that an accident-prone person will have an accident in a 1-year period with probability 0.4. Further, the probability of accident for a not accident-prone person in a 1-year period is 0.05. It is also given that $P(A) = 0.3$.

   (a) What is the probability that a new policy holder will have an accident during the first year of the policy?

   (b) Suppose a new policy holder has an accident in the first year of the policy. What is the probability that he (or she) is accident prone?