DT Sinusoidal Sequences

In the CT domain

$\sin \omega_0 t$ is periodic for every ω_0

signal repeats w/ $P = \frac{2\pi}{\omega_0}$ seconds

Its counterpart is much more complex.

consider DT sequence $x[n] = x(nT)$: sampled version

where n is integer $(-\infty, \infty)$

$T > 0$

then $x[n] = x[n+N] \Rightarrow$ periodic with N samples

$\Rightarrow N$ is fundamental <u>period</u>.

In CT, $\sin \omega_0 t$ is periodic for every ω_0

But in DT, $\boxed{\sin \omega_0 nT = \sin \omega_0 (n+N)T}$

$$= \sin \omega_0 nT \cos \omega_0 NT + \cos \omega_0 nT \sin \omega_0 NT$$

This holds iff $\cos \omega_0 NT = 1$ and $\sin \omega_0 NT = 0$

$\Rightarrow \omega_0 NT = k \cdot 2\pi$ or $N = \dfrac{2\pi k}{\omega_0 T}$

$\Rightarrow$ DT sinusoid $\sin \omega_0 nT$ is periodic iff

$\exists\ k > 0$ that makes $\dfrac{2k\pi}{\omega_0 T}$ integer

But this is not always possible!

For example) consider $\sin \omega_0 T n = \sin 2n$ $(\omega_0 T = 2)$

Because $N = \frac{2k\pi}{\omega_0 T} = \frac{2k\pi}{2} = k\pi$ and because π is irrational, there exists no k to make $N = k\pi$ an integer. $\therefore$ $\sin 2n$ is not periodic

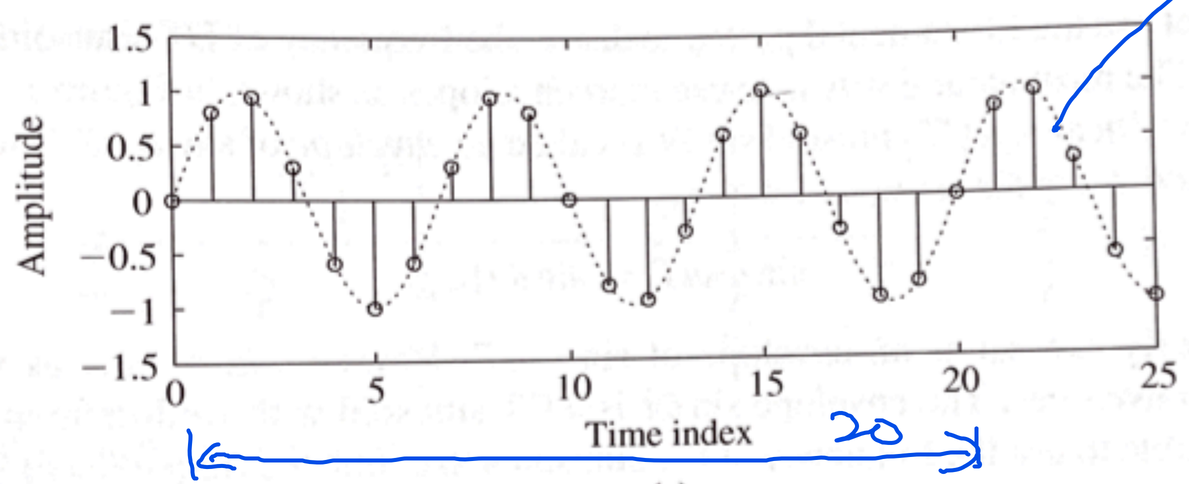
In order for such k to exist, $\omega_0 T$ must be rational # multiple of π.

For example, if $\omega_0 T = 0.3\pi$ then

$N = \frac{2k\pi}{0.3\pi} = \frac{20k}{3} \Rightarrow$ if $k=3$ then $N=20$

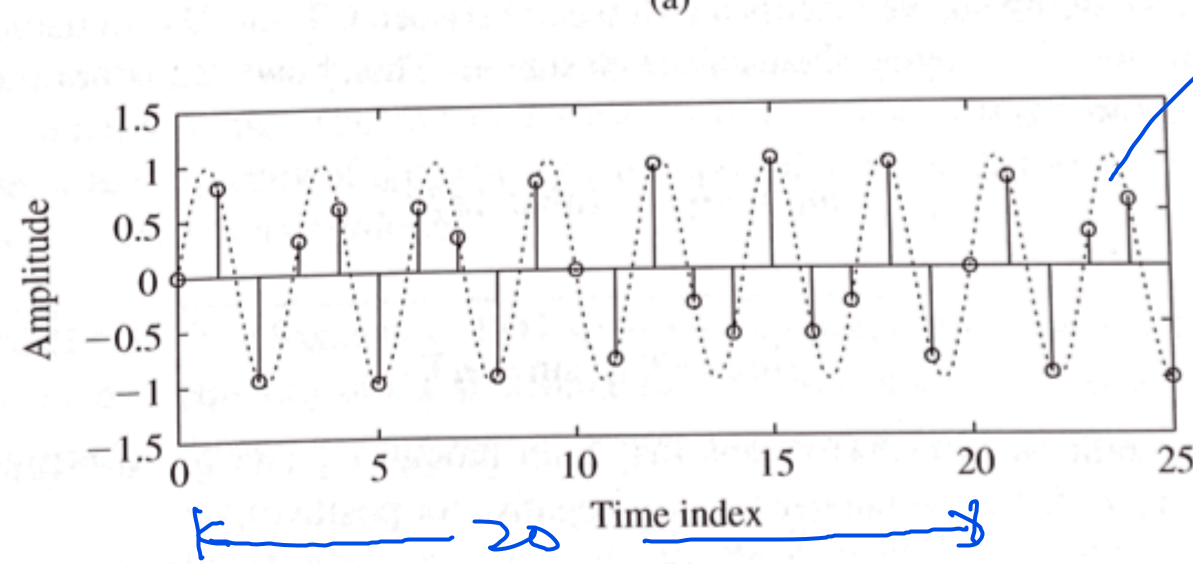
if $\omega_0 T = 0.7\pi$, then

$N = \frac{2k\pi}{0.7\pi} = \frac{20k}{7} \Rightarrow$ if $k=7$ then $N=20$



← $\sin 0.3\pi n$

(a)



← $\sin 0.7\pi n$

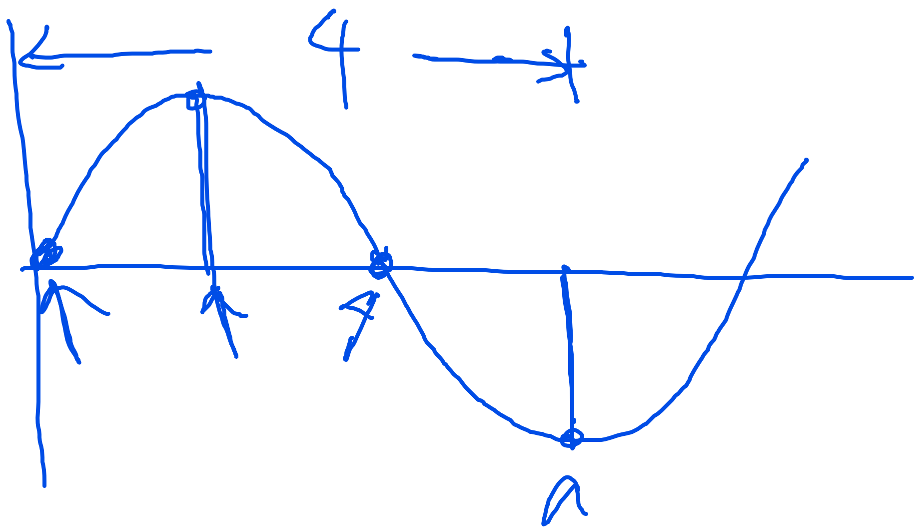
(b)

Simple Example

$\sin 0.5\pi n$ with $T = 1$

$$N = \frac{2k\pi}{0.5} = \frac{20k}{5} = 4k, \quad k = 1.$$

$N = 4$



Problem with DT Sinusoidal Sequence

$\sin 0.3\pi n$ and $\sin 0.7\pi n$ have the same fundamental period ($N=20$)

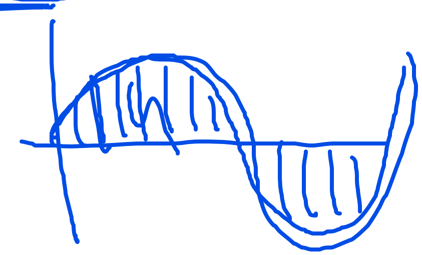
$\Rightarrow$ we cannot define the frequency of $\sin w_0 T n$ as in the CT case.

① $\sin \omega_0 T n$ may not be periodic

② Even if $\sin \omega_0 T n$ is periodic, the frequency should not be defined as in CT case

Let $\sin \bar{\omega} t$ is an envelop of $\sin \omega_0 n T$

$\Rightarrow \sin \omega_0 n T = \sin \bar{\omega} t \Big|_{t=nT}$



$\Rightarrow \sin \omega_0 t$ is an envelop of $\sin \omega_0 n T$

but there are other envelops (???)

In CT, if $\underline{\omega_1 \neq \omega_2} \Rightarrow \underline{\sin \omega_1 t \neq \sin \omega_2 t}$

But in DT,

★ if $\omega_1 = \omega_2 \pmod{2\pi/T} \Rightarrow \underline{\sin \omega_1 nT = \sin \omega_2 nT}$

if $\underline{\omega_1 - \omega_2 = \frac{k2\pi}{T}}$ (Note. ω_1 and ω_2 are separated by multiple of $\frac{2\pi}{T}$)

$\sin \omega_1 nT = \sin\left[\left(\omega_2 + \frac{2\pi k}{T}\right) nT\right] = \sin(\omega_2 nT + 2kn\pi)$

$= \sin \omega_2 nT \cos 2\pi nk + \cos \omega_2 nT \sin 2kn\pi$

1 0

For example, consider $\sin 0.4 nT$, if $T = 5$

then $\frac{2\pi}{T} = 2 \times \frac{3.14}{5} \approx 1.256$

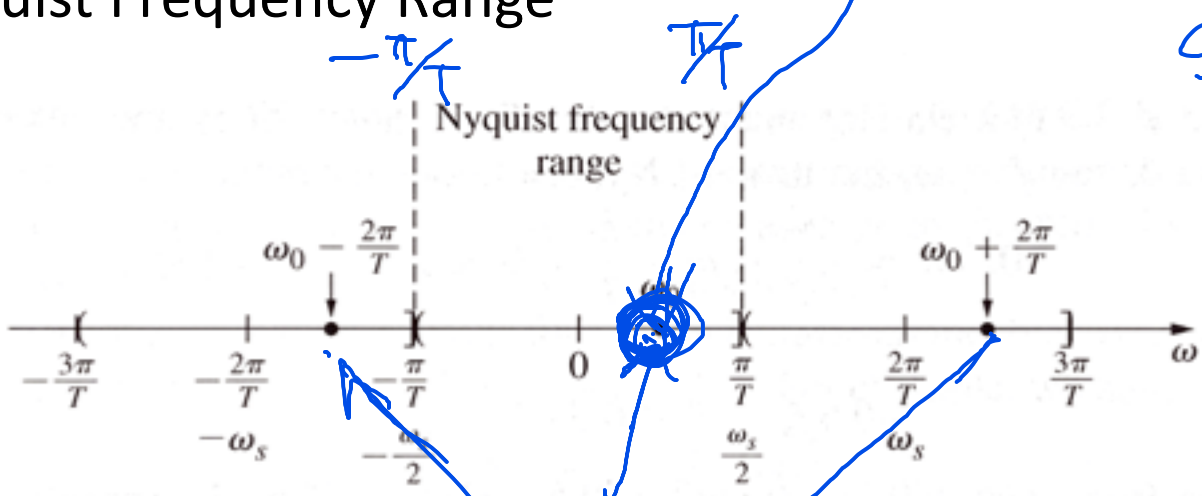
0.4, $(0.4 + 1.256)$, $(0.4 - 1.256)$, $0.4 + (2)(1.256)$

$\Rightarrow \underline{\sin 0.4 nT = \sin(1.656 nT) = \sin(-0.856) nT}$

$\Rightarrow$ these sinusoids have the same sequences

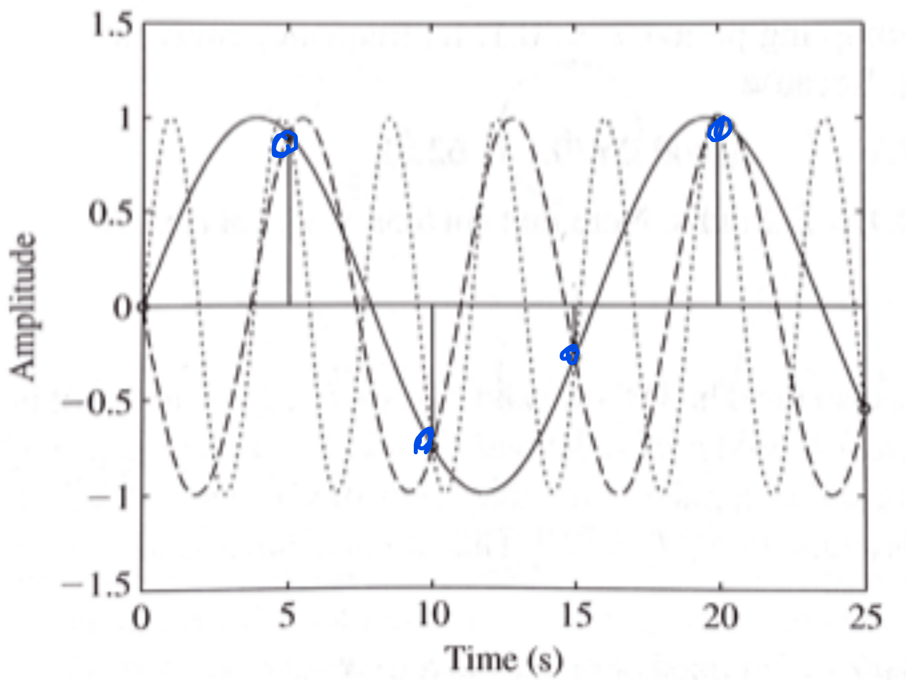
Nyquist Frequency Range

Nyquist freq ⇒ corresponds to CT counterpart



Same sinusoidal sequences CDT

Def: The frequency of DT sinusoidal sequence $\sin \omega_0 nT$, periodic or not, is defined as the frequency of CT sinusoidal function $\sin \bar{\omega} t$ with smallest $|\bar{\omega}|$ such that the sample of $\sin \bar{\omega} t$, with sampling period T, equals $\sin \omega_0 nT$



every DT sinusoid.

sin won't has infinitely

many CT sinusoid

envelops

Sampling and Frequency Aliasing

CT signals $x(t)$ $\xrightarrow[T]{}$ DT sequence $x(nT) = x[n]$

Q: what should be the value of T?

Consider $x(t) = \sin 3t \Rightarrow \omega_0 = 3$ rad/sec.

$x[n] = \sin 3nT$

NFR $[-\pi/T, \pi/T]$

T	NFR $[-\pi/T, \pi/T]$	freq of $\sin 3nT$
0.1	$[-31.4, 31.4]$	3 within NFR
0.5	$[-15.7, 15.7]$	3 within NFR
1	$(-3.14, 3.14)$	3 within NFR
1.8	$[-1.74, 1.74]$	3 is not within NFR
2.6	$(-1.2, 1.2]$	

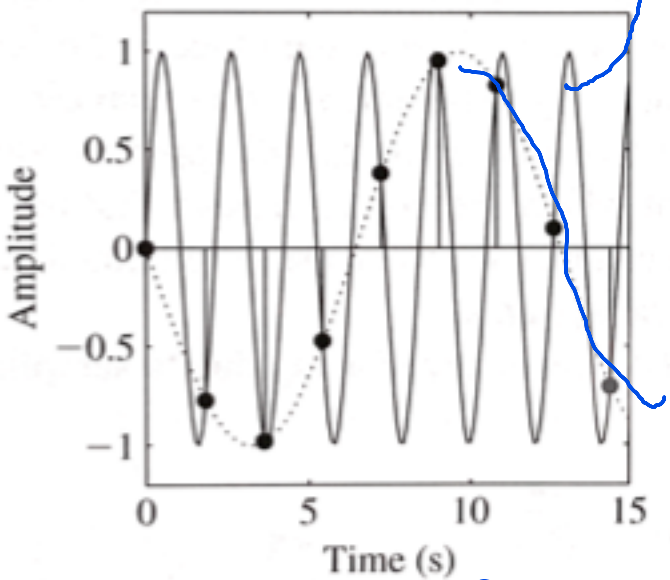
$\frac{2\pi}{1.8}$

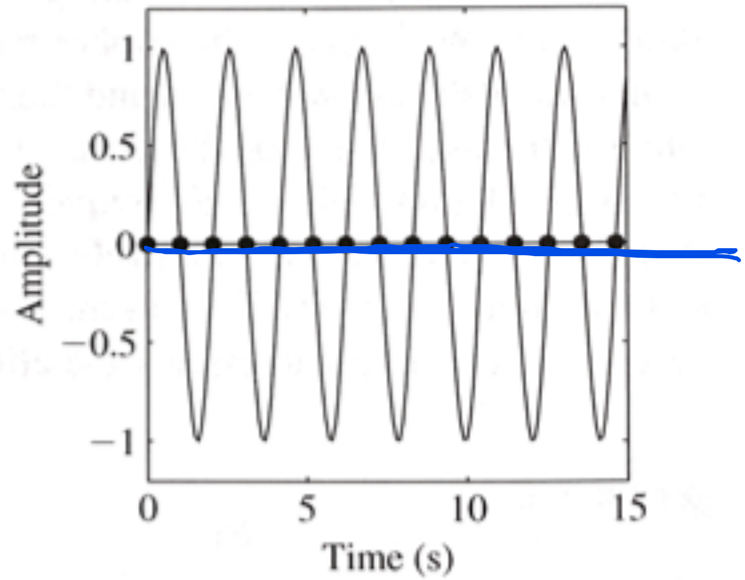
ω_1 $\quad 3 - \underset{\omega_2}{3.48} = 0.48$

$3 - 2.4 = 0.6$. $\quad \frac{2\pi}{2.4}$

$$x(t) = \sin 3t$$
$$x(n) = \sin 3nT$$

Example:





$T = 1.8$ $\qquad$ $T = \pi/3$

dashed: $\sin 0.48nT$

Ex) Consider the CT signal

$$\boxed{x(t) = \cos 50t + 2\cos 70t}$$

2 freq : 50, 70 rad/sec

$$x(nT) = \cos 50nT + 2\cos 70nT$$

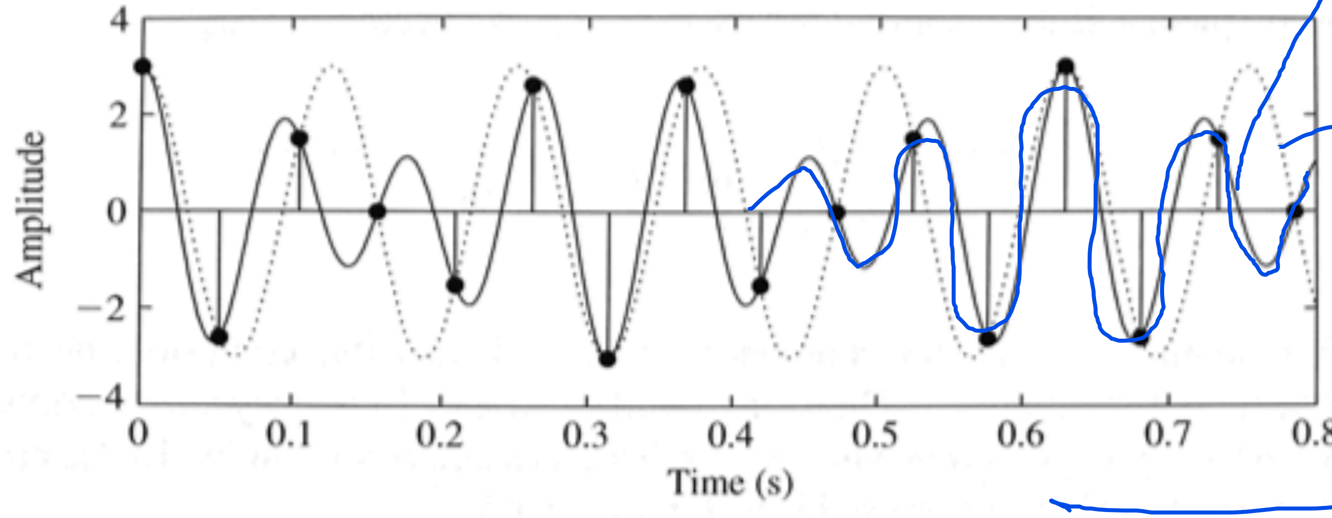
let $T = \pi/60 \Rightarrow \omega_s = 2\pi f_s = \frac{2\pi}{\pi/60} = 120$ rad/sec

NFR $[-\pi/T, \pi/T] \Rightarrow (-60, 60)$

Since 70 is out of the range, $70 \equiv -50 \pmod{\frac{2\pi}{1} = 120}$

$$x(nT) = \cos 50nT + 2\cos(-50)nT$$

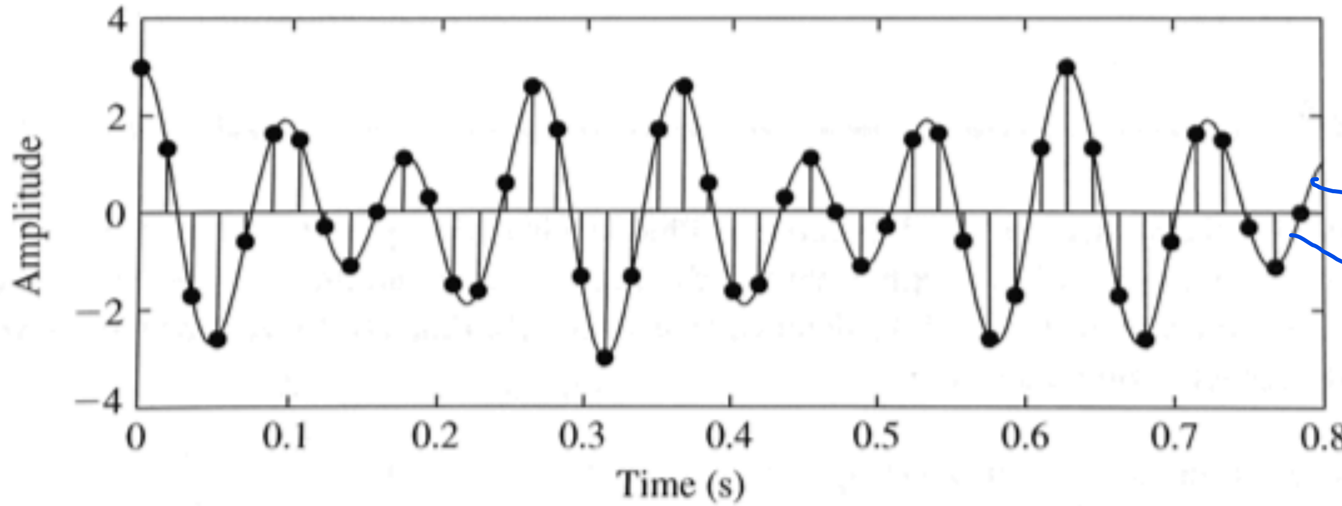
$$= \boxed{3\cos 50nT}$$



(a)

— x(t)

- - - aliased version 3 cos 50πt



(b)

$T = \pi/180$

x(t)

x(nT)