ESE/EEO 306
Random Signals and Systems
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Assignment 3A
Continuous Random Variables

1 Assignment

1. A random variable X has the probability density function(PDF)

$$f_X(x) = \begin{cases} cx(1-x), & 0 \le x \le 1, \\ 0, & \text{otherwise.} \end{cases}$$

   (a) Determine
      i. the constant c,
      ii. The probability $P(1/2 \le X \le 3/4)$, and
      iii. The cumulative distribution function (CDF) $F_X(x)$.

   (b) Are the following properties of the CDF satisfied?
      i. $F_X(-\infty) = 0$.
      ii. $F_X(\infty) = 1$.
      iii. For $x_1 < x_2$, $F_X(x_1) \le F_X(x_2)$. Try with some numeric values.

2. The random variable X is uniformly distributed over $(0, 10)$. Calculate the following probabilities:

   (a) $P(X < 3)$.
   (b) $P(X > 6)$.
   (c) $P(3 < X < 8)$.

3. The CDF of a random variable X is given by

$$F_X(x) = \begin{cases} 1 - e^{-\lambda x}, & x \ge 0, \\ 0, & \text{otherwise.} \end{cases}$$

   Calculate the following probabilities:

   (a) $P(X > 5)$.
   (b) $P(2 \le X \le 3)$.

4. The lifetime of a light bulb is modeled by an exponential random variable, with parameter $\lambda = \frac{1}{400\ hours}$. What is the probability that the light bulb operates for more than 500 hours?

5. A machine has four engines, and the machine can work properly only when there are at least 3 engines in good condition. The lifetime of each engine can be modeled as an independent exponential random variable, with parameter λ. Let Y be the lifetime of the machine. Find the CDF of Y.

6. X is a normal random variable with parameters $\mu = 3$ and $\sigma^2 = 4$. Calculate the following probabilities:

   (a) $P(2 < X < 3)$.

   (b) $P(X > 3)$.

   (c) $P(|X - 3| < 4)$.

   Note that we also write $X \sim \mathcal{N}(3, 4)$. You will be making use of the CDF table for the standard normal distribution, given in the posted lecture.

7. Given CDF of the standard normal distribution $\mathcal{N}(0, 1)$,

$$\Phi(z) \;=\; \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-u^2/2} du, \qquad -\infty < z < \infty,$$

   show that

$$\Phi(z) + \Phi(-z) \;=\; 1.$$