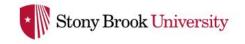
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Assignment 1A Probability Basics and Conditional Probability

1 Assignment

- 1. A coin is tossed three times, and the outcomes, heads or tails, are noted. Find
 - (a) the sample space.
 - (b) the set A corresponding to the event "the total number of heads is 2".
 - (c) the set B corresponding to the event "the outcome of the first toss is heads".
 - (d) the set $A \cap B$ and describe the event that corresponds to this set.
- 2. Four balls numbered 1,2,3 and 4 are in an urn. Two balls are drawn randomly from the urn. The order of drawing is important.
 - (a) Find the sample space of the experiment.
 - (b) Let the event A be described by "one of the drawn balls is 2". Find the set that defines A.
 - (c) Let the event B be described by "the absolute value of the difference of the drawn balls is one". Find the set that defines B.
 - (d) Find the set that corresponds to the event defined by the statement "A occurs but B does not".
- 3. We roll a fair dice, and define the events $A = \{1, 2, 3\}$, $B = \{2, 4, 6\}$ and $C = \{2, 3, 4, 5\}$. Calculate the probabilities of the following events:
 - (a) A, B and C.
 - (b) $A \cap B$, $A \cap C$, $B \cap C$ and $A \cap B \cap C$.
 - (c) $A \cup B$, $A \cup C$, $B \cup C$ and $A \cup B \cup C$.
 - (d) $(A \cap B)^C$ and $(A \cup B)^C$, directly and using De Morgan's laws.
- 4. Show that if events A and B are independent, then so are
 - (a) A and B^C
 - (b) A^C and B
 - (c) A^C and B^C

- 5. A coin is tossed three times, and the outcomes, heads or tails, are noted. All the elementary outcomes in the sample space have equal probabilities. Find the probabilities that
 - (a) the first two outcomes are heads.
 - (b) there are no heads.
 - (c) there are more heads than tails.
- 6. Two fair dice are rolled. Let their scores be represented by X_1 and X_2 . Find the probability $P(X_1 = 4|X_1 + X_2 = 10)$.
- 7. We roll two dice. Assume all 36 possibilities are equally likely. Let X_1 and X_2 be the result of the first and the second dice, respectively. Let S be the sum of the scores, that is $S = X_1 + X_2$. Calculate the following:
 - (a) P(S = k), for $k = 2, 3, \dots 12$.
 - (b) $P(X_1 = 2|S = k)$ for $k = 2, 3, \dots 12$.
 - (c) $P(X_1 = 6|S = k)$ for $k = 2, 3, \dots 12$.
- 8. We roll a fair dice, and define the events $A = \{1, 2, 3\}$, $B = \{2, 3, 4, 5\}$ and $C = \{2, 4, 5, 6\}$. Show that the probabilities satisfy the chain rule

$$P(A, B, C) = P(A|B, C)P(B|C)P(C)$$

9. A random number N of dice is thrown. Let A_i be the event that N=i, and

$$P(A_i) = \frac{1}{2^i}, \quad i = 1, 2, 3, \dots$$

- (a) Compute the probabilities $P(A_i)$, i = 1, 2, 3, 4.
- (b) Let us denote the sum of the scores by S. What is the probability that S is 4?
- 10. We have two dice. The first dice is fair, that is, all outcomes are equally likely. The second dice shows a 2 with probability 1/2. We choose a dice at random and observe the face 2. What is the probability that we chose the second dice?

11. An insurance company divides people into two categories

- accident-prone, call this event A.
- not accident-prone, event A^C .

They have statistics that an accident-prone person will have an accident in a 1-year period with probability 0.4. Further, the probability of accident for a not accident-prone person in a 1-year period is 0.05. It is also given that P(A) = 0.3.

- (a) What is the probability that a new policy holder will have an accident during the first year of the policy?
- (b) Suppose a new policy holder has an accident in the first year of the policy. What is the probability that he (or she) is accident prone?