```
Quit[];

SetDirectory[NotebookDirectory[]]
<< HurToolbox.m
```

```
D:\Dropbox\TAMU\Group\Project\Walker\five_link\AbsAngleJointTorque

HurToolbox for modeling and analysis of multibody systems 1.0.1.
HurToolbox mainly uses vector manipulation (vectors, dyadics).
Coordinates and matrix representation of the dyadics are also available.
Available methods: Newton-Euler
   Method, Euler-Lagrange Method, Hamiltonian Method, Kane's Method.
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Email questions, comments, or concerns to pilwonhur@tamu.edu.
```

```
HurInitialize[]

HurLoadData["data4.m"]

HurDefineRF[a, b, c, d, e]
HurDefineGeneralizedCoordinates[q1[t], q2[t], q3[t], q4[t], q5[t]]
HurDefineDCM[a, q1[t], {0, 0, 1}]
HurDefineDCM[b, q2[t], {0, 0, 1}]
HurDefineDCM[c, q3[t], {0, 0, 1}]
HurDefineDCM[d, q4[t], {0, 0, 1}]
HurDefineDCM[e, q5[t], {0, 0, 1}]

FootST = 0;
ShankSTCOM = lsa a2;
KneeST = (lsa + lsb) a2;
ThighSTCOM = KneeST + lta b2;
Hip = KneeST + (lta + ltb) b2;
TorsoCOM = Hip + lb / 2 c2;
ThighSWCOM = Hip - ltb d2;
KneeSW = Hip - (ltb + lta) d2;
ShankSWCOM = KneeSW - lsb e2;
FootSW = KneeSW - (lsb + lsa) e2;

HurDefineCOMPos[a, ShankSTCOM]
HurDefineCOMPos[b, ThighSTCOM]
HurDefineCOMPos[c, TorsoCOM]
HurDefineCOMPos[d, ThighSWCOM]
HurDefineCOMPos[e, ShankSWCOM]

HurKinematics[]
```

```
HurDefineMass[a, ms]
HurDefineMass[b, mt]
HurDefineMass[c, mb]
HurDefineMass[d, mt]
HurDefineMass[e, ms]
HurDefineInertia[a, {0, 0, 0, 0, 0, Is}]
HurDefineInertia[b, {0, 0, 0, 0, 0, It}]
HurDefineInertia[c, {0, 0, 0, 0, 0, Ib}]
HurDefineInertia[d, {0, 0, 0, 0, 0, It}]
HurDefineInertia[e, {0, 0, 0, 0, 0, Is}]
```

```
HurDefineVertical[n2]
```

```
Transpose[HurGetJacobian[FootST, a, n]].HurList2Column[{0, 0, 0, 0, 0, tau1}] +
 Transpose[HurGetJacobian[KneeST, a, n]].HurList2Column[{0, 0, 0, 0, 0, -tau2}] +
 Transpose[HurGetJacobian[KneeST, b, n]].HurList2Column[{0, 0, 0, 0, 0, tau2}] +
 Transpose[HurGetJacobian[Hip, b, n]].HurList2Column[{0, 0, 0, 0, 0, -tau3}] +
 Transpose[HurGetJacobian[Hip, c, n]].HurList2Column[{0, 0, 0, 0, 0, tau3}] +
 Transpose[HurGetJacobian[Hip, c, n]].HurList2Column[{0, 0, 0, 0, 0, -tau4}] +
 Transpose[HurGetJacobian[Hip, d, n]].HurList2Column[{0, 0, 0, 0, 0, tau4}] +
 Transpose[HurGetJacobian[KneeSW, d, n]].HurList2Column[{0, 0, 0, 0, 0, -tau5}] +
 Transpose[HurGetJacobian[KneeSW, e, n]].HurList2Column[{0, 0, 0, 0, 0, tau5}]
HurDefineNonConservativeForces[Flatten[%]]
```

```
{{tau1 - tau2}, {tau2 - tau3}, {tau3 - tau4}, {tau4 - tau5}, {tau5}}
```

```
{tau1 - tau2, tau2 - tau3, tau3 - tau4, tau4 - tau5, tau5}
```

```
HurELEquation[]
```

```
HurGlobalELEquation // MatrixForm
```

```
HurGlobalMMatrix // MatrixForm
```

```
HurGlobalCMatrix // MatrixForm
```

```
HurGlobalGVector // MatrixForm
```

```
dyn1 = HurGlobalELEquation[[1]]
HurToJulia[dyn1]
```

```
1/2*(-2*tau1+(2*tau2+(-2*g*lsa*mb*sin(q1)+(-2*g*lsb*mb*sin(q1)+(-4*g*lsa*ms*sin(q1)
  +(-2*g*lsb*ms*sin(q1)+(-4*g*lsa*mt*sin(q1)+(-4*g*lsb*mt*sin(q1)+(2*(lsa+lsb)*(ltb
  *(mb+(ms+mt))+lta*(mb+(ms+2*mt))))*sin((q1+-1*q2))*(q2d)^(2)+(lb*(lsa+lsb)*mb*sin
  ((q1+-1*q3))*(q3d)^(2)+(-2*lsa*lta*ms*sin((q1+-1*q4))*(q4d)^(2)+(-2*lsb*lta*ms*
  sin((q1+-1*q4))*(q4d)^(2)+(-2*lsa*ltb*ms*sin((q1+-1*q4))*(q4d)^(2)+(-2*lsb*ltb*ms
  *sin((q1+-1*q4))*(q4d)^(2)+(-2*lsa*ltb*mt*sin((q1+-1*q4))*(q4d)^(2)+(-2*lsb*ltb*
  mt*sin((q1+-1*q4))*(q4d)^(2)+(-2*lsa*lsb*ms*sin((q1+-1*q5))*(q5d)^(2)+(-2*(lsb)^
  (2)*ms*sin((q1+-1*q5))*(q5d)^(2)+(2*Is*q1dd+(2*(lsa)^(2)*mb*q1dd+(4*lsa*lsb*mb*
  q1dd+(2*(lsb)^(2)*mb*q1dd+(4*(lsa)^(2)*ms*q1dd+(4*lsa*lsb*ms*q1dd+(2*(lsb)^(2)*ms
  *q1dd+(4*(lsa)^(2)*mt*q1dd+(8*lsa*lsb*mt*q1dd+(4*(lsb)^(2)*mt*q1dd+(2*lsa*lta*mb*
  cos((q1+-1*q2))*q2dd+(2*lsb*lta*mb*cos((q1+-1*q2))*q2dd+(2*lsa*ltb*mb*cos((q1+-1*
  q2))*q2dd+(2*lsb*ltb*mb*cos((q1+-1*q2))*q2dd+(2*lsa*lta*ms*cos((q1+-1*q2))*q2dd+(
  2*lsb*lta*ms*cos((q1+-1*q2))*q2dd+(2*lsa*ltb*ms*cos((q1+-1*q2))*q2dd+(2*lsb*ltb*
  ms*cos((q1+-1*q2))*q2dd+(4*lsa*lta*mt*cos((q1+-1*q2))*q2dd+(4*lsb*lta*mt*cos((q1+
  -1*q2))*q2dd+(2*lsa*ltb*mt*cos((q1+-1*q2))*q2dd+(2*lsb*ltb*mt*cos((q1+-1*q2))*
  q2dd+(lb*lsa*mb*cos((q1+-1*q3))*q3dd+(lb*lsb*mb*cos((q1+-1*q3))*q3dd+(-2*lsa*lta*
  ms*cos((q1+-1*q4))*q4dd+(-2*lsb*lta*ms*cos((q1+-1*q4))*q4dd+(-2*lsa*ltb*ms*cos((
  q1+-1*q4))*q4dd+(-2*lsb*ltb*ms*cos((q1+-1*q4))*q4dd+(-2*lsa*ltb*mt*cos((q1+-1*q4)
  )*q4dd+(-2*lsb*ltb*mt*cos((q1+-1*q4))*q4dd+(-2*lsa*lsb*ms*cos((q1+-1*q5))*q5dd+-2
  *(lsb)^(2)*ms*cos((q1+-1*q5))*q5dd))))))))))))))))))))))))))))))))))))))))))))))
  )))))
```

```
dyn2 = HurGlobalELEquation[[2]]
HurToJulia[dyn2]
```

```
1/2*(-2*tau2+(2*tau3+(-2*g*lta*mb*sin(q2)+(-2*g*ltb*mb*sin(q2)+(-2*g*lta*ms*sin(q2)
  +(-2*g*ltb*ms*sin(q2)+(-4*g*lta*mt*sin(q2)+(-2*g*ltb*mt*sin(q2)+(-2*(lsa+lsb)*(
  ltb*(mb+(ms+mt))+lta*(mb+(ms+2*mt))))*sin((q1+-1*q2))*(q1d)^(2)+(lb*(lta+ltb)*mb*
  sin((q2+-1*q3))*(q3d)^(2)+(-2*(lta)^(2)*ms*sin((q2+-1*q4))*(q4d)^(2)+(-4*lta*ltb
  *ms*sin((q2+-1*q4))*(q4d)^(2)+(-2*(ltb)^(2)*ms*sin((q2+-1*q4))*(q4d)^(2)+(-2*lta
  *ltb*mt*sin((q2+-1*q4))*(q4d)^(2)+(-2*(ltb)^(2)*mt*sin((q2+-1*q4))*(q4d)^(2)+(-2
  *lsb*lta*ms*sin((q2+-1*q5))*(q5d)^(2)+(-2*lsb*ltb*ms*sin((q2+-1*q5))*(q5d)^(2)+(2
  *lsa*lta*mb*cos((q1+-1*q2))*q1dd+(2*lsb*lta*mb*cos((q1+-1*q2))*q1dd+(2*lsa*ltb*mb
  *cos((q1+-1*q2))*q1dd+(2*lsb*ltb*mb*cos((q1+-1*q2))*q1dd+(2*lsa*lta*ms*cos((q1+-1
  *q2))*q1dd+(2*lsb*lta*ms*cos((q1+-1*q2))*q1dd+(2*lsa*ltb*ms*cos((q1+-1*q2))*q1dd+
  (2*lsb*ltb*ms*cos((q1+-1*q2))*q1dd+(4*lsa*lta*mt*cos((q1+-1*q2))*q1dd+(4*lsb*lta*
  mt*cos((q1+-1*q2))*q1dd+(2*lsa*ltb*mt*cos((q1+-1*q2))*q1dd+(2*lsb*ltb*mt*cos((q1+
  -1*q2))*q1dd+(2*It*q2dd+(2*(lta)^(2)*mb*q2dd+(4*lta*ltb*mb*q2dd+(2*(ltb)^(2)*mb*
  q2dd+(2*(lta)^(2)*ms*q2dd+(4*lta*ltb*ms*q2dd+(2*(ltb)^(2)*ms*q2dd+(4*(lta)^(2)*mt
  *q2dd+(4*lta*ltb*mt*q2dd+(2*(ltb)^(2)*mt*q2dd+(lb*lta*mb*cos((q2+-1*q3))*q3dd+(lb
  *ltb*mb*cos((q2+-1*q3))*q3dd+(-2*(lta)^(2)*ms*cos((q2+-1*q4))*q4dd+(-4*lta*ltb*ms
  *cos((q2+-1*q4))*q4dd+(-2*(ltb)^(2)*ms*cos((q2+-1*q4))*q4dd+(-2*lta*ltb*mt*cos((
  q2+-1*q4))*q4dd+(-2*(ltb)^(2)*mt*cos((q2+-1*q4))*q4dd+(-2*lsb*lta*ms*cos((q2+-1*
  q5))*q5dd+-2*lsb*ltb*ms*cos((q2+-1*q5))*q5dd))))))))))))))))))))))))))))))))))))
  ))))))))))))
```

```
dyn3 = HurGlobalELEquation[[3]]
HurToJulia[dyn3]
```

```
1/4*(-4*tau3+(4*tau4+(-2*g*lb*mb*sin(q3)+(-2*lb*(lsa+lsb)*mb*sin((q1+-1*q3))*(q1d)^
  (2)+(-2*lb*(lta+ltb)*mb*sin((q2+-1*q3))*(q2d)^(2)+(2*lb*lsa*mb*cos((q1+-1*q3))*
  q1dd+(2*lb*lsb*mb*cos((q1+-1*q3))*q1dd+(2*lb*lta*mb*cos((q2+-1*q3))*q2dd+(2*lb*
  ltb*mb*cos((q2+-1*q3))*q2dd+(4*Ib*q3dd+(lb)^(2)*mb*q3dd))))))))))
```

```
dyn4 = HurGlobalELEquation[[4]]
HurToJulia[dyn4]
```

```
(-1*tau4+(tau5+(g*lta*ms*sin(q4)+(g*ltb*ms*sin(q4)+(g*ltb*mt*sin(q4)+((lsa+lsb)*(
  lta*ms+ltb*(ms+mt))*sin((q1+-1*q4))*(q1d)^(2)+((lta+ltb)*(lta*ms+ltb*(ms+mt))*sin
  ((q2+-1*q4))*(q2d)^(2)+(lsb*lta*ms*sin((q4+-1*q5))*(q5d)^(2)+(lsb*ltb*ms*sin((q4+
  -1*q5))*(q5d)^(2)+(-1*lsa*lta*ms*cos((q1+-1*q4))*q1dd+(-1*lsb*lta*ms*cos((q1+-1*
  q4))*q1dd+(-1*lsa*ltb*ms*cos((q1+-1*q4))*q1dd+(-1*lsb*ltb*ms*cos((q1+-1*q4))*q1dd
  +(-1*lsa*ltb*mt*cos((q1+-1*q4))*q1dd+(-1*lsb*ltb*mt*cos((q1+-1*q4))*q1dd+(-1*(lta
  )^(2)*ms*cos((q2+-1*q4))*q2dd+(-2*lta*ltb*ms*cos((q2+-1*q4))*q2dd+(-1*(ltb)^(2)*
  ms*cos((q2+-1*q4))*q2dd+(-1*lta*ltb*mt*cos((q2+-1*q4))*q2dd+(-1*(ltb)^(2)*mt*cos(
  (q2+-1*q4))*q2dd+(It*q4dd+((lta)^(2)*ms*q4dd+(2*lta*ltb*ms*q4dd+((ltb)^(2)*ms*
  q4dd+((ltb)^(2)*mt*q4dd+(lsb*lta*ms*cos((q4+-1*q5))*q5dd+lsb*ltb*ms*cos((q4+-1*q5
  ))*q5dd)))))))))))))))))))))))))))))
```

```
dyn5 = HurGlobalELEquation[[5]]
HurToJulia[dyn5]
```

$$-\text{tau5} + \text{g lsb ms Sin}[\text{q5}[\text{t}]] + \text{lsb}\left(\text{lsa} + \text{lsb}\right)\text{ms Sin}[\text{q1}[\text{t}] - \text{q5}[\text{t}]]\,\text{q1}'[\text{t}]^2 + \text{lsb}\left(\text{lta} + \text{ltb}\right)\text{ms Sin}[\text{q2}[\text{t}] - \text{q5}[\text{t}]]\,\text{q2}'[\text{t}]^2 - \text{lsb lta ms Sin}[\text{q4}[\text{t}] - \text{q5}[\text{t}]]\,\text{q4}'[\text{t}]^2 - \text{lsb ltb ms Sin}[\text{q4}[\text{t}] - \text{q5}[\text{t}]]\,\text{q4}'[\text{t}]^2 - \text{lsa lsb ms Cos}[\text{q1}[\text{t}] - \text{q5}[\text{t}]]\,\text{q1}''[\text{t}] - \text{lsb}^2\,\text{ms Cos}[\text{q1}[\text{t}] - \text{q5}[\text{t}]]\,\text{q1}''[\text{t}] - \text{lsb lta ms Cos}[\text{q2}[\text{t}] - \text{q5}[\text{t}]]\,\text{q2}''[\text{t}] - \text{lsb ltb ms Cos}[\text{q2}[\text{t}] - \text{q5}[\text{t}]]\,\text{q2}''[\text{t}] + \text{lsb lta ms Cos}[\text{q4}[\text{t}] - \text{q5}[\text{t}]]\,\text{q4}''[\text{t}] + \text{lsb ltb ms Cos}[\text{q4}[\text{t}] - \text{q5}[\text{t}]]\,\text{q4}''[\text{t}] + \text{Is q5}''[\text{t}] + \text{lsb}^2\,\text{ms q5}''[\text{t}]$$

```
(-1*tau5+(g*lsb*ms*sin(q5)+(lsb*(lsa+lsb)*ms*sin((q1+-1*q5))*(q1d)^(2)+(lsb*(lta+
  ltb)*ms*sin((q2+-1*q5))*(q2d)^(2)+(-1*lsb*lta*ms*sin((q4+-1*q5))*(q4d)^(2)+(-1*
  lsb*ltb*ms*sin((q4+-1*q5))*(q4d)^(2)+(-1*lsa*lsb*ms*cos((q1+-1*q5))*q1dd+(-1*(lsb
  )^(2)*ms*cos((q1+-1*q5))*q1dd+(-1*lsb*lta*ms*cos((q2+-1*q5))*q2dd+(-1*lsb*ltb*ms*
  cos((q2+-1*q5))*q2dd+(lsb*lta*ms*cos((q4+-1*q5))*q4dd+(lsb*ltb*ms*cos((q4+-1*q5))
  *q4dd+(Is*q5dd+(lsb)^(2)*ms*q5dd))))))))))))
```

```
HurUnifyTriadsCoord[FootSW, n] // MatrixForm
```

$$\begin{pmatrix} -\left(\text{lsa} + \text{lsb}\right)\text{Sin}[\text{q1}[\text{t}]] - \left(\text{lta} + \text{ltb}\right)\text{Sin}[\text{q2}[\text{t}]] + \left(\text{lta} + \text{ltb}\right)\text{Sin}[\text{q4}[\text{t}]] + \left(\text{lsa} + \text{lsb}\right)\text{Sin}[\text{q5} \\ \left(\text{lsa} + \text{lsb}\right)\text{Cos}[\text{q1}[\text{t}]] + \left(\text{lta} + \text{ltb}\right)\text{Cos}[\text{q2}[\text{t}]] - \left(\text{lta} + \text{ltb}\right)\text{Cos}[\text{q4}[\text{t}]] - \left(\text{lsa} + \text{lsb}\right)\text{Cos}[\text{q5}[\\ 0 \\ \text{n} \end{pmatrix}$$

```
stepLength = HurUnifyTriadsCoord[FootSW, n][[1]]
HurToJulia[stepLength]
```

$$-\left(\text{lsa} + \text{lsb}\right)\text{Sin}[\text{q1}[\text{t}]] - \left(\text{lta} + \text{ltb}\right)\text{Sin}[\text{q2}[\text{t}]] + \left(\text{lta} + \text{ltb}\right)\text{Sin}[\text{q4}[\text{t}]] + \left(\text{lsa} + \text{lsb}\right)\text{Sin}[\text{q5}[\text{t}]]$$

```
(-1*(lsa+lsb)*sin(q1)+(-1*(lta+ltb)*sin(q2)+((lta+ltb)*sin(q4)+(lsa+lsb)*sin(q5))))
```

```
stepHeight = HurUnifyTriadsCoord[FootSW, n][[2]]
HurToJulia[stepHeight]
```

$$\left(\text{lsa} + \text{lsb}\right)\text{Cos}[\text{q1}[\text{t}]] + \left(\text{lta} + \text{ltb}\right)\text{Cos}[\text{q2}[\text{t}]] - \left(\text{lta} + \text{ltb}\right)\text{Cos}[\text{q4}[\text{t}]] - \left(\text{lsa} + \text{lsb}\right)\text{Cos}[\text{q5}[\text{t}]]$$

```
((lsa+lsb)*cos(q1)+((lta+ltb)*cos(q2)+(-1*(lta+ltb)*cos(q4)+-1*(lsa+lsb)*cos(q5))))
```

```
HurGlobalMMatrix
HurToJulia[%]
```

```
[(Is+(2*lsa*lsb*(mb+(ms+2*mt))+((lsb)^(2)*(mb+(ms+2*mt))+(lsa)^(2)*(mb+2*(ms+mt))))
  ))  (lsa+lsb)*(ltb*(mb+(ms+mt))+lta*(mb+(ms+2*mt)))*cos((q1+-1*q2))
  1/2*lb*(lsa+lsb)*mb*cos((q1+-1*q3))
  -1*(lsa+lsb)*(lta*ms+ltb*(ms+mt))*cos((q1+-1*q4))
  -1*lsb*(lsa+lsb)*ms*cos((q1+-1*q5));(lsa+lsb)*(ltb*(mb+(ms+mt))+lta*(mb+(ms+2*mt))
  ))*cos((q1+-1*q2))
  (It+(2*lta*ltb*(mb+(ms+mt))+((ltb)^(2)*(mb+(ms+mt))+(lta)^(2)*(mb+(ms+2*mt)))))
  1/2*lb*(lta+ltb)*mb*cos((q2+-1*q3))
  -1*(lta+ltb)*(lta*ms+ltb*(ms+mt))*cos((q2+-1*q4))
  -1*lsb*(lta+ltb)*ms*cos((q2+-1*q5));1/2*lb*(lsa+lsb)*mb*cos((q1+-1*q3))
  1/2*lb*(lta+ltb)*mb*cos((q2+-1*q3))  (Ib+1/4*(lb)^(2)*mb)
  0 0;-1*(lsa+lsb)*(lta*ms+ltb*(ms+mt))*cos((q1+-1*q4))
  -1*(lta+ltb)*(lta*ms+ltb*(ms+mt))*cos((q2+-1*q4))  0
  (It+((lta)^(2)*ms+(2*lta*ltb*ms+(ltb)^(2)*(ms+mt))))
  lsb*(lta+ltb)*ms*cos((q4+-1*q5));-1*lsb*(lsa+lsb)*ms*cos((q1+-1*q5))
  -1*lsb*(lta+ltb)*ms*cos((q2+-1*q5))  0
  lsb*(lta+ltb)*ms*cos((q4+-1*q5))  (Is+(lsb)^(2)*ms)]
```

```
HurGlobalCMatrix
HurToJulia[%]
```

```
[0  (lsa+lsb)*(ltb*(mb+(ms+mt))+lta*(mb+(ms+2*mt)))*sin((q1+-1*q2))*q2d
  1/2*lb*(lsa+lsb)*mb*sin((q1+-1*q3))*q3d
  -1*(lsa+lsb)*(lta*ms+ltb*(ms+mt))*sin((q1+-1*q4))*q4d
  -1*lsb*(lsa+lsb)*ms*sin((q1+-1*q5))*q5d;-1*(lsa+lsb)*(ltb*(mb+(ms+mt))+lta*(mb+(
  ms+2*mt)))*sin((q1+-1*q2))*q1d 0 1/2*lb*(lta+ltb)*mb*sin((q2+-1*q3))*q3d
  -1*(lta+ltb)*(lta*ms+ltb*(ms+mt))*sin((q2+-1*q4))*q4d
  -1*lsb*(lta+ltb)*ms*sin((q2+-1*q5))*q5d;-1/2*lb*(lsa+lsb)*mb*sin((q1+-1*q3))*q1d
  -1/2*lb*(lta+ltb)*mb*sin((q2+-1*q3))*q2d 0 0
  0;(lsa+lsb)*(lta*ms+ltb*(ms+mt))*sin((q1+-1*q4))*q1d
  (lta+ltb)*(lta*ms+ltb*(ms+mt))*sin((q2+-1*q4))*q2d 0 0
  lsb*(lta+ltb)*ms*sin((q4+-1*q5))*q5d;lsb*(lsa+lsb)*ms*sin((q1+-1*q5))*q1d
  lsb*(lta+ltb)*ms*sin((q2+-1*q5))*q2d 0 -1*lsb*(lta+ltb)*ms*sin((q4+-1*q5))*q4d 0]
```

```
HurGlobalGVector
HurToJulia[HurList2Column[%]]
```

```
[-1*g*(lsb*(mb+(ms+2*mt))+lsa*(mb+2*(ms+mt)))*sin(q1);-1*g*(ltb*(mb+(ms+mt))+lta*(
  mb+(ms+2*mt)))*sin(q2);-1/2*g*lb*mb*sin(q3);g*(lta*ms+ltb*(ms+mt))*sin(q4);g*lsb*
  ms*sin(q5)]
```

```
JacFootSW = HurGetJacobian[FootSW, e, n];
JacFootSW.HurList2Column[{q1'[t], q2'[t], q3'[t], q4'[t], q5'[t]}]
verticalVel = %[[2, 1]]
HurToJulia[verticalVel]
```

```
(-1*(lsa+lsb)*sin(q1)*q1d+(-1*(lta+ltb)*sin(q2)*q2d+((lta+ltb)*sin(q4)*q4d+(lsa+lsb
  )*sin(q5)*q5d)))
```

```
JacFootSW.HurList2Column[{q1'[t], q2'[t], q3'[t], q4'[t], q5'[t]}]
horizontalVel = %[[1, 1]]
HurToJulia[horizontalVel]
```

```
(-1*(lsa+lsb)*cos(q1)*q1d+(-1*(lta+ltb)*cos(q2)*q2d+((lta+ltb)*cos(q4)*q4d+(lsa+lsb
  )*cos(q5)*q5d)))
```

```
HurSaveData["data1.m", "FootST", "ShankSTCOM", "KneeST", "ThighSTCOM", "Hip", "TorsoCOM",
 "ThighSWCOM", "KneeSW", "ShankSWCOM", "FootSW", "dyn1", "dyn2", "dyn3", "dyn4", "dyn5",
 "stepLength", "stepHeight", "verticalVel", "horizontalVel", "JacFootSW", "invans"]
```

```
HurUnifyTriadsCoord[FootSW, n];
HurToJulia[%[[1]]]
```

```
(-1*(lsa+lsb)*sin(q1)+(-1*(lta+ltb)*sin(q2)+((lta+ltb)*sin(q4)+(lsa+lsb)*sin(q5))))
```

```
HurUnifyTriadsCoord[FootSW, n];
HurToJulia[%[[2]]]
```

```
((lsa+lsb)*cos(q1)+((lta+ltb)*cos(q2)+(-1*(lta+ltb)*cos(q4)+-1*(lsa+lsb)*cos(q5))))
```

```
HurUnifyTriadsCoord[KneeSW, n];
HurToJulia[%[[1]]]
```

```
(-1*(lsa+lsb)*sin(q1)+-1*(lta+ltb)*(sin(q2)+-1*sin(q4)))
```

```
HurUnifyTriadsCoord[KneeSW, n];
HurToJulia[%[[2]]]
```

```
((lsa+lsb)*cos(q1)+(lta+ltb)*(cos(q2)+-1*cos(q4)))
```

```
HurUnifyTriadsCoord[Hip, n];
HurToJulia[%[[1]]]
```

```
(-1*(lsa+lsb)*sin(q1)+-1*(lta+ltb)*sin(q2))
```

```
HurUnifyTriadsCoord[Hip, n];
HurToJulia[%[[2]]]
```

```
((lsa+lsb)*cos(q1)+(lta+ltb)*cos(q2))
```

```
HurUnifyTriadsCoord[TorsoCOM, n];
HurToJulia[%[[1]]]
```

```
(-1*(lsa+lsb)*sin(q1)+(-1*(lta+ltb)*sin(q2)+-1/2*lb*sin(q3)))
```

```
HurUnifyTriadsCoord[TorsoCOM, n];
HurToJulia[%[[2]]]
```

```
((lsa+lsb)*cos(q1)+((lta+ltb)*cos(q2)+1/2*lb*cos(q3)))
```

```
HurUnifyTriadsCoord[KneeST, n];
HurToJulia[%[[1]]]
```

```
-1*(lsa+lsb)*sin(q1)
```

```
HurUnifyTriadsCoord[KneeST, n];
HurToJulia[%[[2]]]
```

```
(lsa+lsb)*cos(q1)
```

```
Transpose[HurGetJacobian[Hip, c, n]].
  HurList2Column[{forcex, forcey, 0, 0, 0, 0}] // MatrixForm
```

```
HurToJulia[%[[2, 1]] ]
```

```
HurGetJacobian[Hip, c, n] // MatrixForm
```

```
HurTurnOffSimplify[]
```

```
False
```

```
invans = HurELInverse[]
```

```
{{q1''[t] →
    -(((-((Is + lsb^2 ms) (… 1 …) - (… 1 …) (… 1 … + … 1 …))) (… 1 …) +
          … 1 …) (… 1 …) - … 1 …) /
      (-(-((Ib + lb^2 mb/4) (Is + lsb^2 ms) (… 1 …) - … 1 …) (… 1 …) +
          (… 1 …) … 1 …) (… 1 …) + … 1 …)), … 3 …, q5''[t] → … 1 …}}
```

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```
q5''[t] /. invans[[1]];
HurToJulia[%]
```

Whole body COM

```
COMWhole = Total[Table[HurGlobalCOMPos[[i]] * HurGlobalMass[[i]],
    {i, 2, Length[HurGlobalRF]}]] / Total[HurGlobalMass]
```

$$\frac{1}{mb + 2\,ms + 2\,mt}\left(\left(\frac{c2\,lb}{2} + a2\,(lsa + lsb) + b2\,(lta + ltb)\right) mb + a2\,lsa\,ms + (-e2\,lsb + a2\,(lsa + lsb) + b2\,(lta + ltb) - d2\,(lta + ltb))\,ms + (a2\,(lsa + lsb) + b2\,lta)\,mt + (a2\,(lsa + lsb) - d2\,ltb + b2\,(lta + ltb))\,mt\right)$$

```
HurToJulia[HurUnifyTriadsCoord[COMWhole, n][[2]]]
```

```
((mb+(2*ms+2*mt)))^(-1)*(lsa*ms*cos(q1)+(mt*((lsa+lsb)*cos(q1)+lta*cos(q2)))+(mb*((
  lsa+lsb)*cos(q1)+((lta+ltb)*cos(q2)+1/2*lb*cos(q3))))+(mt*((lsa+lsb)*cos(q1)+((lta
  +ltb)*cos(q2)+-1*ltb*cos(q4)))+ms*((lsa+lsb)*cos(q1)+((lta+ltb)*cos(q2)+(-1*(lta+
  ltb)*cos(q4)+-1*lsb*cos(q5))))))))
```

Whole body COM linear momentum

```
LinearMomentumWholeBody =
  D[HurUnifyTriadsCoord[COMWhole, n], t] * Total[HurGlobalMass] // Simplify;
LinearMomentumWholeBody // MatrixForm
```

$$\begin{pmatrix} -(lsb\,(mb + ms + 2\,mt) + lsa\,(mb + 2\,(ms + mt)))\,Cos[q1[t]]\,q1'[t] - (ltb\,(mb + ms + mt) + lta\,(mb + \\ -(lsb\,(mb + ms + 2\,mt) + lsa\,(mb + 2\,(ms + mt)))\,Sin[q1[t]]\,q1'[t] - (ltb\,(mb + ms + mt) + lta\,(mb + \end{pmatrix}$$

Whole body COM linear momentum rate change

```
LinearMomentumRateWholeBody =
  D[LinearMomentumWholeBody, t] * Total[HurGlobalMass] // Simplify;
LinearMomentumRateWholeBody // MatrixForm
```

$$\begin{pmatrix} (mb + 2\,(ms + mt))\,((lsb\,(mb + ms + 2\,mt) + lsa\,(mb + 2\,(ms + mt)))\,Sin[q1[t]]\,q1'[t]^2 + (ltb\,(mb + \\ (mb + 2\,(ms + mt))\,(-(lsb\,(mb + ms + 2\,mt) + lsa\,(mb + 2\,(ms + mt)))\,Cos[q1[t]]\,q1'[t]^2 - (ltb\,(mb \end{pmatrix}$$

Whole body Angular Momentum

```
Table[HurCross[HurGlobalCOMPos[[ i ]] - COMWhole, HurGlobalLinearMomentum[[i]], n ] +
  HurGlobalAngularMomentum[[i]], {i, 2, Length[HurGlobalRF]}]
HurUnifyTriadsCoord[Total[%], n] // Simplify
```

$$\Big\{ a3\, Is\, q1'[t] + n3 \Big(\cdots 96 \cdots + \frac{\cdots 1 \cdots}{\cdots 1 \cdots^2} + \big(2\, lsa\, ltb\, ms^3 \sin[q1[t]] \sin[q4[t]]\, q1'[t]\big) \Big/ \big(mb + 2\, ms + 2\, mt\big)^2 + \big(lsa\, ltb\, mb\, ms\, mt \sin[q1[t]] \sin[q4[t]]\, q1'[t]\big) \Big/ \big(mb + 2\, ms + 2\, mt\big)^2 + \big(2\, lsa\, lta\, ms^2\, mt \sin[q1[t]] \sin[q4[t]]\, q1'[t]\big) \Big/ \big(mb + 2\, ms + 2\, mt\big)^2 + \big(4\, lsa\, ltb\, ms^2\, mt \sin[q1[t]] \sin[q4[t]]\, q1'[t]\big) \Big/ \big(mb + 2\, ms + 2\, mt\big)^2 + \big(2\, lsa\, ltb\, ms\, mt^2 \sin[q1[t]] \sin[q4[t]]\, q1'[t]\big) \Big/ \big(mb + 2\, ms + 2\, mt\big)^2 + \big(lsa\, lsb\, mb\, ms^2 \sin[q1[t]] \sin[q5[t]]\, q1'[t]\big) \Big/ \big(mb + 2\, ms + 2\, mt\big)^2 + \big(2\, lsa\, lsb\, ms^3 \sin[q1[t]] \sin[q5[t]]\, q1'[t]\big) \Big/ \big(mb + 2\, ms + 2\, mt\big)^2 + \big(2\, lsa\, lsb\, ms^2\, mt \sin[q1[t]] \sin[q5[t]]\, q1'[t]\big) \Big/ \big(mb + 2\, ms + 2\, mt\big)^2 \Big),$$

$$b3\, It\, q2'[t] + n3 \left(\frac{lsa \cdots 7 \cdots \cdots 1 \cdots}{\cdots 1 \cdots^2} + \cdots 310 \cdots + \frac{\cdots 1 \cdots}{\cdots 1 \cdots} \right),$$

$$\cdots 1 \cdots,$$

$$\cdots 1 \cdots,$$

$$e3\, Is\, q5'[t] + n3 \Big(\big(lsa\, lsb\, mb\, ms^2 \cos[q1[t]] \cos[q1[t] - q5[t]] \cos[q5[t]]\, q1'[t]\big) \Big/ \big(mb + 2\, ms + 2\, mt\big)^2 + \big(lsb^2\, mb\, ms^2 \cos[q1[t]] \cos[q1[t] - q5[t]] \cos[q5[t]]\, q1'[t]\big) \Big/ \big(mb + 2\, ms + 2\, mt\big)^2 + \big(2\, lsa\, lsb\, ms^3 \cos[q1[t]] \cos[q1[t] - q5[t]] \cos[q5[t]]\, q1'[t]\big) \Big/ \big(mb + 2\, ms + 2\, mt\big)^2 + \cdots 1036 \cdots + \frac{4\, lsb^2\, mb\, ms\, mt \sin[q5[t]]^2\, q5'[t]}{(mb+2\, ms+2\, mt)^2} + \frac{6\, lsb^2\, ms^2\, mt \sin[q5[t]]^2\, q5'[t]}{(mb+2\, ms+2\, mt)^2} + \frac{4\, lsb^2\, ms\, mt^2 \sin[q5[t]]^2\, q5'[t]}{(mb+2\, ms+2\, mt)^2} \Big) \Big\}$$

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$$\left\{0,\ 0,\ \frac{1}{2\left(\text{mb}+2\left(\text{ms}+\text{mt}\right)\right)}\right.$$
$$\Big(\left(2\,\text{lsb}\,\text{ms}\left(\text{ltb}\left(\text{mb}+\text{ms}+\text{mt}\right)+\text{lta}\left(\text{mb}+\text{ms}+2\,\text{mt}\right)\right)\text{Cos}[\text{q1}[\text{t}]-\text{q2}[\text{t}]]+\text{lb}\,\text{lsb}\,\text{mb}\,\text{ms}\right.$$
$$\text{Cos}[\text{q1}[\text{t}]-\text{q3}[\text{t}]]+2\left(\text{Is}\,\text{mb}+2\,\text{Is}\,\text{ms}+\text{lsb}^2\,\text{mb}\,\text{ms}+\text{lsb}^2\,\text{ms}^2+2\,\text{Is}\,\text{mt}+2\,\text{lsb}^2\,\text{ms}\,\text{mt}-\right.$$
$$\left.\left.\text{lsb}\,\text{ms}\left(\text{lta}\,\text{ms}+\text{ltb}\left(\text{ms}+\text{mt}\right)\right)\text{Cos}[\text{q1}[\text{t}]-\text{q4}[\text{t}]]-\text{lsb}^2\,\text{ms}^2\,\text{Cos}[\text{q1}[\text{t}]-\text{q5}[\text{t}]]\right)\right)$$
$$\text{q1}'[\text{t}]+\Big(2\,\text{lsb}\,\text{ms}\left(\text{ltb}\left(\text{mb}+\text{ms}+\text{mt}\right)+\text{lta}\left(\text{mb}+\text{ms}+2\,\text{mt}\right)\right)\text{Cos}[\text{q1}[\text{t}]-\text{q2}[\text{t}]]+$$
$$\text{lb}\,\text{mb}\left(\text{lta}\,\text{ms}+\text{ltb}\left(\text{ms}+\text{mt}\right)\right)\text{Cos}[\text{q2}[\text{t}]-\text{q3}[\text{t}]]+$$
$$2\left(\text{It}\,\text{mb}+2\,\text{It}\,\text{ms}+\text{lta}^2\,\text{mb}\,\text{ms}+2\,\text{lta}\,\text{ltb}\,\text{mb}\,\text{ms}+\text{ltb}^2\,\text{mb}\,\text{ms}+\text{lta}^2\,\text{ms}^2+\right.$$
$$2\,\text{lta}\,\text{ltb}\,\text{ms}^2+\text{ltb}^2\,\text{ms}^2+2\,\text{It}\,\text{mt}+\text{ltb}^2\,\text{mb}\,\text{mt}+2\,\text{lta}^2\,\text{ms}\,\text{mt}+2\,\text{lta}\,\text{ltb}\,\text{ms}\,\text{mt}+$$
$$2\,\text{ltb}^2\,\text{ms}\,\text{mt}+\text{ltb}^2\,\text{mt}^2-\left(\text{lta}\,\text{ms}+\text{ltb}\left(\text{ms}+\text{mt}\right)\right)^2\text{Cos}[\text{q2}[\text{t}]-\text{q4}[\text{t}]]-$$
$$\left.\text{lsb}\,\text{ms}\left(\text{lta}\,\text{ms}+\text{ltb}\left(\text{ms}+\text{mt}\right)\right)\text{Cos}[\text{q2}[\text{t}]-\text{q5}[\text{t}]]\right)\Big)\,\text{q2}'[\text{t}]+$$
$$2\,\text{Ib}\,\text{mb}\,\text{q3}'[\text{t}]+4\,\text{Ib}\,\text{ms}\,\text{q3}'[\text{t}]+\text{lb}^2\,\text{mb}\,\text{ms}\,\text{q3}'[\text{t}]+4\,\text{Ib}\,\text{mt}\,\text{q3}'[\text{t}]+$$
$$\text{lb}^2\,\text{mb}\,\text{mt}\,\text{q3}'[\text{t}]+$$
$$\text{lb}\,\text{lsb}\,\text{mb}\,\text{ms}\,\text{Cos}[\text{q1}[\text{t}]-\text{q3}[\text{t}]]\,\text{q3}'[\text{t}]+$$
$$\text{lb}\,\text{lta}\,\text{mb}\,\text{ms}\,\text{Cos}[\text{q2}[\text{t}]-\text{q3}[\text{t}]]\,\text{q3}'[\text{t}]+$$
$$\text{lb}\,\text{ltb}\,\text{mb}\,\text{ms}\,\text{Cos}[\text{q2}[\text{t}]-\text{q3}[\text{t}]]\,\text{q3}'[\text{t}]+$$
$$\text{lb}\,\text{ltb}\,\text{mb}\,\text{mt}\,\text{Cos}[\text{q2}[\text{t}]-\text{q3}[\text{t}]]\,\text{q3}'[\text{t}]+$$
$$\text{lb}\,\text{lta}\,\text{mb}\,\text{ms}\,\text{Cos}[\text{q3}[\text{t}]-\text{q4}[\text{t}]]\,\text{q3}'[\text{t}]+$$
$$\text{lb}\,\text{ltb}\,\text{mb}\,\text{ms}\,\text{Cos}[\text{q3}[\text{t}]-\text{q4}[\text{t}]]\,\text{q3}'[\text{t}]+$$
$$\text{lb}\,\text{ltb}\,\text{mb}\,\text{mt}\,\text{Cos}[\text{q3}[\text{t}]-\text{q4}[\text{t}]]\,\text{q3}'[\text{t}]+$$
$$\text{lb}\,\text{lsb}\,\text{mb}\,\text{ms}\,\text{Cos}[\text{q3}[\text{t}]-\text{q5}[\text{t}]]\,\text{q3}'[\text{t}]+$$
$$2\,\text{It}\,\text{mb}\,\text{q4}'[\text{t}]+4\,\text{It}\,\text{ms}\,\text{q4}'[\text{t}]+2\,\text{lta}^2\,\text{mb}\,\text{ms}\,\text{q4}'[\text{t}]+$$
$$4\,\text{lta}\,\text{ltb}\,\text{mb}\,\text{ms}\,\text{q4}'[\text{t}]+2\,\text{ltb}^2\,\text{mb}\,\text{ms}\,\text{q4}'[\text{t}]+$$
$$2\,\text{lta}^2\,\text{ms}^2\,\text{q4}'[\text{t}]+4\,\text{lta}\,\text{ltb}\,\text{ms}^2\,\text{q4}'[\text{t}]+2\,\text{ltb}^2\,\text{ms}^2\,\text{q4}'[\text{t}]+$$
$$4\,\text{It}\,\text{mt}\,\text{q4}'[\text{t}]+2\,\text{ltb}^2\,\text{mb}\,\text{mt}\,\text{q4}'[\text{t}]+4\,\text{lta}^2\,\text{ms}\,\text{mt}\,\text{q4}'[\text{t}]+$$
$$4\,\text{lta}\,\text{ltb}\,\text{ms}\,\text{mt}\,\text{q4}'[\text{t}]+4\,\text{ltb}^2\,\text{ms}\,\text{mt}\,\text{q4}'[\text{t}]+$$
$$2\,\text{ltb}^2\,\text{mt}^2\,\text{q4}'[\text{t}]-2\,\text{lsb}\,\text{lta}\,\text{ms}^2\,\text{Cos}[\text{q1}[\text{t}]-\text{q4}[\text{t}]]\,\text{q4}'[\text{t}]-$$
$$2\,\text{lsb}\,\text{ltb}\,\text{ms}^2\,\text{Cos}[\text{q1}[\text{t}]-\text{q4}[\text{t}]]\,\text{q4}'[\text{t}]-$$
$$2\,\text{lsb}\,\text{ltb}\,\text{ms}\,\text{mt}\,\text{Cos}[\text{q1}[\text{t}]-\text{q4}[\text{t}]]\,\text{q4}'[\text{t}]-$$
$$2\,\text{lta}^2\,\text{ms}^2\,\text{Cos}[\text{q2}[\text{t}]-\text{q4}[\text{t}]]\,\text{q4}'[\text{t}]-4\,\text{lta}\,\text{ltb}\,\text{ms}^2\,\text{Cos}[\text{q2}[\text{t}]-\text{q4}[\text{t}]]\,\text{q4}'[\text{t}]-$$
$$2\,\text{ltb}^2\,\text{ms}^2\,\text{Cos}[\text{q2}[\text{t}]-\text{q4}[\text{t}]]\,\text{q4}'[\text{t}]-4\,\text{lta}\,\text{ltb}\,\text{ms}\,\text{mt}\,\text{Cos}[\text{q2}[\text{t}]-\text{q4}[\text{t}]]\,\text{q4}'[\text{t}]-$$
$$4\,\text{ltb}^2\,\text{ms}\,\text{mt}\,\text{Cos}[\text{q2}[\text{t}]-\text{q4}[\text{t}]]\,\text{q4}'[\text{t}]-2\,\text{ltb}^2\,\text{mt}^2\,\text{Cos}[\text{q2}[\text{t}]-\text{q4}[\text{t}]]\,\text{q4}'[\text{t}]+$$
$$\text{lb}\,\text{lta}\,\text{mb}\,\text{ms}\,\text{Cos}[\text{q3}[\text{t}]-\text{q4}[\text{t}]]\,\text{q4}'[\text{t}]+\text{lb}\,\text{ltb}\,\text{mb}\,\text{ms}\,\text{Cos}[\text{q3}[\text{t}]-\text{q4}[\text{t}]]\,\text{q4}'[\text{t}]+$$
$$\text{lb}\,\text{ltb}\,\text{mb}\,\text{mt}\,\text{Cos}[\text{q3}[\text{t}]-\text{q4}[\text{t}]]\,\text{q4}'[\text{t}]+2\,\text{lsb}\,\text{lta}\,\text{mb}\,\text{ms}\,\text{Cos}[\text{q4}[\text{t}]-\text{q5}[\text{t}]]\,\text{q4}'[\text{t}]+$$
$$2\,\text{lsb}\,\text{ltb}\,\text{mb}\,\text{ms}\,\text{Cos}[\text{q4}[\text{t}]-\text{q5}[\text{t}]]\,\text{q4}'[\text{t}]+2\,\text{lsb}\,\text{lta}\,\text{ms}^2\,\text{Cos}[\text{q4}[\text{t}]-\text{q5}[\text{t}]]\,\text{q4}'[\text{t}]+$$
$$2\,\text{lsb}\,\text{ltb}\,\text{ms}^2\,\text{Cos}[\text{q4}[\text{t}]-\text{q5}[\text{t}]]\,\text{q4}'[\text{t}]+4\,\text{lsb}\,\text{lta}\,\text{ms}\,\text{mt}\,\text{Cos}[\text{q4}[\text{t}]-\text{q5}[\text{t}]]\,\text{q4}'[\text{t}]+$$
$$2\,\text{lsb}\,\text{ltb}\,\text{ms}\,\text{mt}\,\text{Cos}[\text{q4}[\text{t}]-\text{q5}[\text{t}]]\,\text{q4}'[\text{t}]+2\,\text{Is}\,\text{mb}\,\text{q5}'[\text{t}]+$$
$$4\,\text{Is}\,\text{ms}\,\text{q5}'[\text{t}]+2\,\text{lsb}^2\,\text{mb}\,\text{ms}\,\text{q5}'[\text{t}]+2\,\text{lsb}^2\,\text{ms}^2\,\text{q5}'[\text{t}]+4\,\text{Is}\,\text{mt}\,\text{q5}'[\text{t}]+$$
$$4\,\text{lsb}^2\,\text{ms}\,\text{mt}\,\text{q5}'[\text{t}]-2\,\text{lsb}^2\,\text{ms}^2\,\text{Cos}[\text{q1}[\text{t}]-\text{q5}[\text{t}]]\,\text{q5}'[\text{t}]-$$
$$2\,\text{lsb}\,\text{lta}\,\text{ms}^2\,\text{Cos}[\text{q2}[\text{t}]-\text{q5}[\text{t}]]\,\text{q5}'[\text{t}]-2\,\text{lsb}\,\text{ltb}\,\text{ms}^2\,\text{Cos}[\text{q2}[\text{t}]-\text{q5}[\text{t}]]\,\text{q5}'[\text{t}]-$$
$$2\,\text{lsb}\,\text{ltb}\,\text{ms}\,\text{mt}\,\text{Cos}[\text{q2}[\text{t}]-\text{q5}[\text{t}]]\,\text{q5}'[\text{t}]+\text{lb}\,\text{lsb}\,\text{mb}\,\text{ms}\,\text{Cos}[\text{q3}[\text{t}]-\text{q5}[\text{t}]]\,\text{q5}'[\text{t}]+$$
$$2\,\text{lsb}\,\text{lta}\,\text{mb}\,\text{ms}\,\text{Cos}[\text{q4}[\text{t}]-\text{q5}[\text{t}]]\,\text{q5}'[\text{t}]+2\,\text{lsb}\,\text{ltb}\,\text{mb}\,\text{ms}\,\text{Cos}[\text{q4}[\text{t}]-\text{q5}[\text{t}]]\,\text{q5}'[\text{t}]+$$
$$2\,\text{lsb}\,\text{lta}\,\text{ms}^2\,\text{Cos}[\text{q4}[\text{t}]-\text{q5}[\text{t}]]\,\text{q5}'[\text{t}]+2\,\text{lsb}\,\text{ltb}\,\text{ms}^2\,\text{Cos}[\text{q4}[\text{t}]-\text{q5}[\text{t}]]\,\text{q5}'[\text{t}]+$$
$$\left.4\,\text{lsb}\,\text{lta}\,\text{ms}\,\text{mt}\,\text{Cos}[\text{q4}[\text{t}]-\text{q5}[\text{t}]]\,\text{q5}'[\text{t}]+2\,\text{lsb}\,\text{ltb}\,\text{ms}\,\text{mt}\,\text{Cos}[\text{q4}[\text{t}]-\text{q5}[\text{t}]]\,\text{q5}'[\text{t}]\right),\ \text{n}\Big\}$$

```
JacWhole = HurGetJacobian[COMWhole, n, n][[1 ;; 2, ;;]] // Simplify;
JacWhole // MatrixForm
```

$$\begin{pmatrix} -\frac{(\text{lsb}\,(\text{mb}+\text{ms}+2\,\text{mt})+\text{lsa}\,(\text{mb}+2\,(\text{ms}+\text{mt})))\,\text{Cos}[\text{q1}[\text{t}]]}{\text{mb}+2\,(\text{ms}+\text{mt})} & -\frac{(\text{ltb}\,(\text{mb}+\text{ms}+\text{mt})+\text{lta}\,(\text{mb}+\text{ms}+2\,\text{mt}))\,\text{Cos}[\text{q2}[\text{t}]]}{\text{mb}+2\,(\text{ms}+\text{mt})} & -\frac{\text{lb}\,\text{mb}\,\text{Cos}[\text{q3}[\text{t}]]}{2\,(\text{mb}+2\,(\text{ms}+\text{mt}))} & \left(\right. \\ -\frac{(\text{lsb}\,(\text{mb}+\text{ms}+2\,\text{mt})+\text{lsa}\,(\text{mb}+2\,(\text{ms}+\text{mt})))\,\text{Sin}[\text{q1}[\text{t}]]}{\text{mb}+2\,(\text{ms}+\text{mt})} & -\frac{(\text{ltb}\,(\text{mb}+\text{ms}+\text{mt})+\text{lta}\,(\text{mb}+\text{ms}+2\,\text{mt}))\,\text{Sin}[\text{q2}[\text{t}]]}{\text{mb}+2\,(\text{ms}+\text{mt})} & -\frac{\text{lb}\,\text{mb}\,\text{Sin}[\text{q3}[\text{t}]]}{2\,(\text{mb}+2\,(\text{ms}+\text{mt}))} & \left(\right. \end{pmatrix}$$

HurToJulia[JacWhole]

```
[-1*((mb+2*(ms+mt)))^(-1)*(lsb*(mb+(ms+2*mt))+lsa*(mb+2*(ms+mt)))*cos(q1)
  -1*((mb+2*(ms+mt)))^(-1)*(ltb*(mb+(ms+mt))+lta*(mb+(ms+2*mt)))*cos(q2)
  -1/2*lb*mb*((mb+2*(ms+mt)))^(-1)*cos(q3)
  ((mb+2*(ms+mt)))^(-1)*(lta*ms+ltb*(ms+mt))*cos(q4)
  lsb*ms*((mb+2*(ms+mt)))^(-1)*cos(q5);-1*((mb+2*(ms+mt)))^(-1)*(lsb*(mb+(ms+2*mt))
  )+lsa*(mb+2*(ms+mt)))*sin(q1)
  -1*((mb+2*(ms+mt)))^(-1)*(ltb*(mb+(ms+mt))+lta*(mb+(ms+2*mt)))*sin(q2)
  -1/2*lb*mb*((mb+2*(ms+mt)))^(-1)*sin(q3)
  ((mb+2*(ms+mt)))^(-1)*(lta*ms+ltb*(ms+mt))*sin(q4)
  lsb*ms*((mb+2*(ms+mt)))^(-1)*sin(q5)]
```

NJacWhole = NullSpace[JacWhole] // Simplify;
NJacWhole // MatrixForm (*I should expect 3 5D column vectors. It's weird that Mathematica gives 3x5 matrix, not 5x3 matrix. Anyway, a vector with 5 element is the basis vector for the null space.*)

$$\begin{pmatrix} -\frac{lsb\,ms\,Csc[q1[t]-q2[t]]\,Sin[q2[t]-q5[t]]}{lsb\,(mb+ms+2\,mt)+lsa\,(mb+2\,(ms+mt))} & \frac{lsb\,ms\,Csc[q1[t]-q2[t]]\,Sin[q1[t]-q5[t]]}{ltb\,(mb+ms+mt)+lta\,(mb+ms+2\,mt)} & 0 & 0 \\ -\frac{(lta\,ms+ltb\,(ms+mt))\,Csc[q1[t]-q2[t]]\,Sin[q2[t]-q4[t]]}{lsb\,(mb+ms+2\,mt)+lsa\,(mb+2\,(ms+mt))} & \frac{(lta\,ms+ltb\,(ms+mt))\,Csc[q1[t]-q2[t]]\,Sin[q1[t]-q4[t]]}{ltb\,(mb+ms+mt)+lta\,(mb+ms+2\,mt)} & 0 & 1 \\ \frac{lb\,mb\,Csc[q1[t]-q2[t]]\,Sin[q2[t]-q3[t]]}{2\,(lsb\,(mb+ms+2\,mt)+lsa\,(mb+2\,(ms+mt)))} & -\frac{lb\,mb\,Csc[q1[t]-q2[t]]\,Sin[q1[t]-q3[t]]}{2\,(ltb\,(mb+ms+mt)+lta\,(mb+ms+2\,mt))} & 1 & 0 \end{pmatrix}$$

JacWhole.HurList2Column[NJacWhole[[1, ;;]]] // Simplify

```
{{0}, {0}}
```

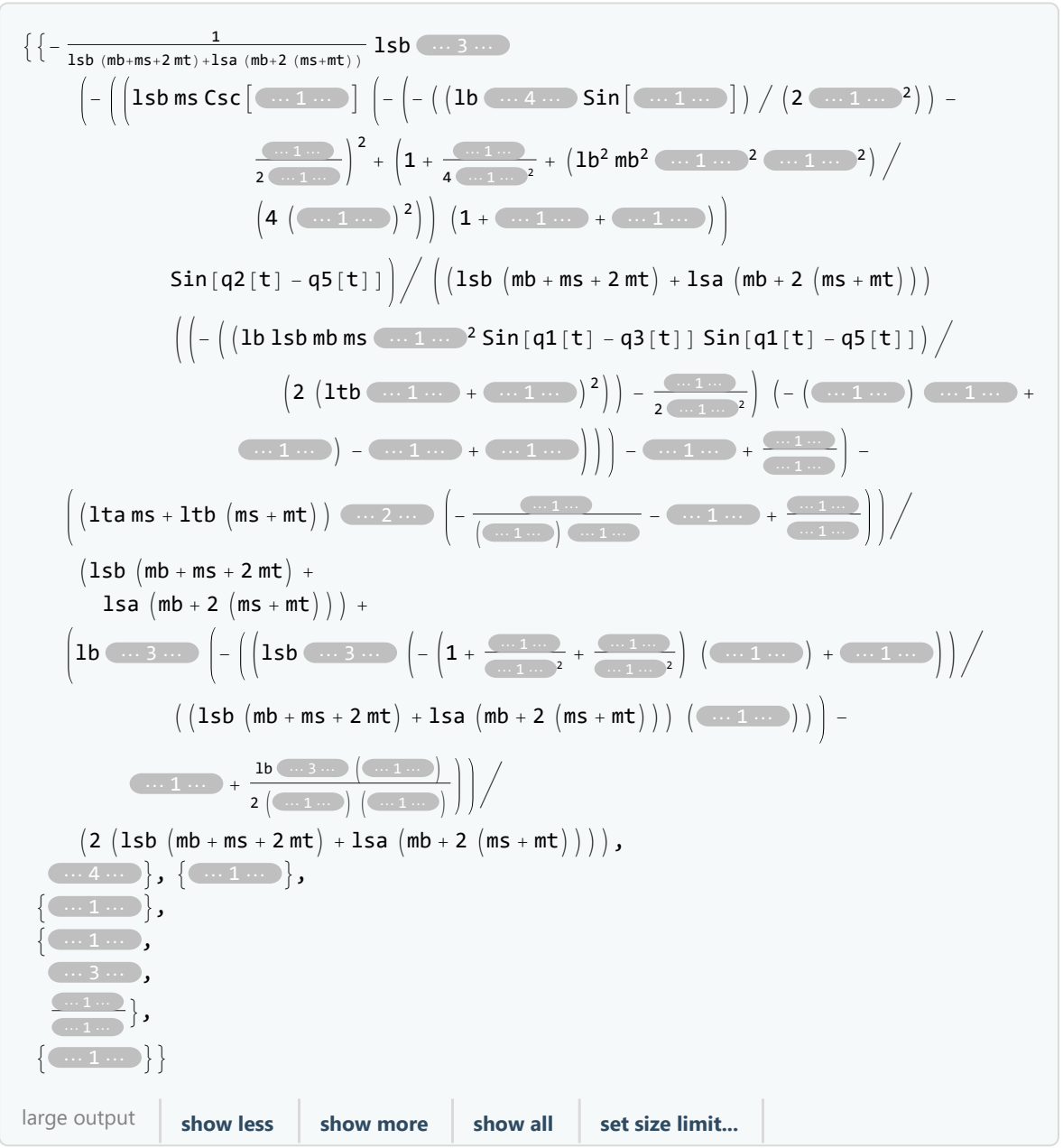
Nmat = Transpose[NJacWhole];
Nmat // MatrixForm

$$\begin{pmatrix} -\frac{lsb\,ms\,Csc[q1[t]-q2[t]]\,Sin[q2[t]-q5[t]]}{lsb\,(mb+ms+2\,mt)+lsa\,(mb+2\,(ms+mt))} & -\frac{(lta\,ms+ltb\,(ms+mt))\,Csc[q1[t]-q2[t]]\,Sin[q2[t]-q4[t]]}{lsb\,(mb+ms+2\,mt)+lsa\,(mb+2\,(ms+mt))} & \frac{lb\,mb\,Csc[q1[t]-q2} \\ \frac{lsb\,ms\,Csc[q1[t]-q2[t]]\,Sin[q1[t]-q5[t]]}{ltb\,(mb+ms+mt)+lta\,(mb+ms+2\,mt)} & \frac{(lta\,ms+ltb\,(ms+mt))\,Csc[q1[t]-q2[t]]\,Sin[q1[t]-q4[t]]}{ltb\,(mb+ms+mt)+lta\,(mb+ms+2\,mt)} & -\frac{lb\,mb\,Csc[q1[t]-q2} \\ 0 & 0 & \\ 0 & 1 & \\ 1 & 0 & \end{pmatrix}$$

HurToJulia[Nmat]

```
[-1*lsb*ms*((lsb*(mb+(ms+2*mt))+lsa*(mb+2*(ms+mt))))^(-1)*csc((q1+-1*q2))*sin((q2+
  -1*q5))
  -1*(lta*ms+ltb*(ms+mt))*((lsb*(mb+(ms+2*mt))+lsa*(mb+2*(ms+mt))))^(-1)*csc((q1+-
  1*q2))*sin((q2+-1*q4))
  1/2*lb*mb*((lsb*(mb+(ms+2*mt))+lsa*(mb+2*(ms+mt))))^(-1)*csc((q1+-1*q2))*sin((q2
  +-1*q3));lsb*ms*((ltb*(mb+(ms+mt))+lta*(mb+(ms+2*mt))))^(-1)*csc((q1+-1*q2))*sin
  ((q1+-1*q5))
  (lta*ms+ltb*(ms+mt))*((ltb*(mb+(ms+mt))+lta*(mb+(ms+2*mt))))^(-1)*csc((q1+-1*q2)
  )*sin((q1+-1*q4))
  -1/2*lb*mb*((ltb*(mb+(ms+mt))+lta*(mb+(ms+2*mt))))^(-1)*csc((q1+-1*q2))*sin((q1+
  -1*q3));0 0 1;0 1 0;1 0 0]
```

```
Projector = Nmat.Inverse[Transpose[Nmat].Nmat].Transpose[Nmat]
```



large output | show less | show more | show all | set size limit...

```
Projector[[1, 1]] // Simplify
```

```
$Aborted
```

```
HurSaveData["data1.m", "FootST", "ShankSTCOM", "KneeST", "ThighSTCOM",
 "Hip", "TorsoCOM", "ThighSWCOM", "KneeSW", "ShankSWCOM", "FootSW", "dyn1",
 "dyn2", "dyn3", "dyn4", "dyn5", "stepLength", "stepHeight", "verticalVel",
 "horizontalVel", "JacFootSW", "COMWhole", "LinearMomentumWholeBody",
 "LinearMomentumRateWholeBody", "JacWhole", "NJacWhole", "invans"]
```

```
HurGetRelativeDCM[a, n] // MatrixForm // Simplify
HurGetRelativeDCM[b, a] // MatrixForm // Simplify
HurGetRelativeDCM[c, b] // MatrixForm // Simplify
HurGetRelativeDCM[d, c] // MatrixForm // Simplify
HurGetRelativeDCM[e, d] // MatrixForm // Simplify
HurGetRelativeDCM[e, n] // MatrixForm // Simplify
```

$$\begin{pmatrix} \text{Cos}[q1[t]] & -\text{Sin}[q1[t]] & 0 \\ \text{Sin}[q1[t]] & \text{Cos}[q1[t]] & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \text{Cos}[q1[t]-q2[t]] & \text{Sin}[q1[t]-q2[t]] & 0 \\ -\text{Sin}[q1[t]-q2[t]] & \text{Cos}[q1[t]-q2[t]] & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \text{Cos}[q2[t]-q3[t]] & \text{Sin}[q2[t]-q3[t]] & 0 \\ -\text{Sin}[q2[t]-q3[t]] & \text{Cos}[q2[t]-q3[t]] & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \text{Cos}[q3[t]-q4[t]] & \text{Sin}[q3[t]-q4[t]] & 0 \\ -\text{Sin}[q3[t]-q4[t]] & \text{Cos}[q3[t]-q4[t]] & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \text{Cos}[q4[t]-q5[t]] & \text{Sin}[q4[t]-q5[t]] & 0 \\ -\text{Sin}[q4[t]-q5[t]] & \text{Cos}[q4[t]-q5[t]] & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \text{Cos}[q5[t]] & -\text{Sin}[q5[t]] & 0 \\ \text{Sin}[q5[t]] & \text{Cos}[q5[t]] & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

```
Transpose[HurGetJacobian[KneeST, a, n]].HurList2Column[
     {-f1, -f2, -f3, -tau1, -tau2, -tau3}] + Transpose[HurGetJacobian[KneeST, b, n]].
    HurList2Column[{f1, f2, f3, tau1, tau2, tau3}] // Simplify // MatrixForm
```

$$\begin{pmatrix} -\text{tau3} \\ \text{tau3} \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

```
Transpose[HurGetJacobian[KneeSW, d, n]].HurList2Column[
     {-f1, -f2, -f3, -tau1, -tau2, -tau3}] + Transpose[HurGetJacobian[KneeSW, e, n]].
    HurList2Column[{f1, f2, f3, tau1, tau2, tau3}] // Simplify // MatrixForm
```

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ -\text{tau3} \\ \text{tau3} \end{pmatrix}$$

Stance Knee Locking

```
ArrayFlatten[
 {{HurGlobalMMatrix, -HurList2Column[{-1, 1, 0, 0, 0}]}, {{{1, -1, 0, 0, 0}}, 0}}]
HurToJulia[%]
```

$$\left\{\left\{\text{Is}+2\,\text{lsa}\,\text{lsb}\left(\text{mb}+\text{ms}+2\,\text{mt}\right)+\text{lsb}^2\left(\text{mb}+\text{ms}+2\,\text{mt}\right)+\text{lsa}^2\left(\text{mb}+2\left(\text{ms}+\text{mt}\right)\right),\right.\right.$$
$$\left(\text{lsa}+\text{lsb}\right)\left(\text{ltb}\left(\text{mb}+\text{ms}+\text{mt}\right)+\text{lta}\left(\text{mb}+\text{ms}+2\,\text{mt}\right)\right)\text{Cos}[\text{q1}[\text{t}]-\text{q2}[\text{t}]],$$
$$\frac{1}{2}\,\text{lb}\left(\text{lsa}+\text{lsb}\right)\text{mb}\,\text{Cos}[\text{q1}[\text{t}]-\text{q3}[\text{t}]],-\left(\text{lsa}+\text{lsb}\right)\left(\text{lta}\,\text{ms}+\text{ltb}\left(\text{ms}+\text{mt}\right)\right)$$
$$\text{Cos}[\text{q1}[\text{t}]-\text{q4}[\text{t}]],-\text{lsb}\left(\text{lsa}+\text{lsb}\right)\text{ms}\,\text{Cos}[\text{q1}[\text{t}]-\text{q5}[\text{t}]],1\Big\},$$
$$\left\{\left(\text{lsa}+\text{lsb}\right)\left(\text{ltb}\left(\text{mb}+\text{ms}+\text{mt}\right)+\text{lta}\left(\text{mb}+\text{ms}+2\,\text{mt}\right)\right)\text{Cos}[\text{q1}[\text{t}]-\text{q2}[\text{t}]],\right.$$
$$\text{It}+2\,\text{lta}\,\text{ltb}\left(\text{mb}+\text{ms}+\text{mt}\right)+\text{ltb}^2\left(\text{mb}+\text{ms}+\text{mt}\right)+\text{lta}^2\left(\text{mb}+\text{ms}+2\,\text{mt}\right),$$
$$\frac{1}{2}\,\text{lb}\left(\text{lta}+\text{ltb}\right)\text{mb}\,\text{Cos}[\text{q2}[\text{t}]-\text{q3}[\text{t}]],$$
$$-\left(\text{lta}+\text{ltb}\right)\left(\text{lta}\,\text{ms}+\text{ltb}\left(\text{ms}+\text{mt}\right)\right)\text{Cos}[\text{q2}[\text{t}]-\text{q4}[\text{t}]],$$
$$-\text{lsb}\left(\text{lta}+\text{ltb}\right)\text{ms}\,\text{Cos}[\text{q2}[\text{t}]-\text{q5}[\text{t}]],-1\Big\},\left\{\frac{1}{2}\,\text{lb}\left(\text{lsa}+\text{lsb}\right)\text{mb}\,\text{Cos}[\text{q1}[\text{t}]-\text{q3}[\text{t}]],\right.$$
$$\frac{1}{2}\,\text{lb}\left(\text{lta}+\text{ltb}\right)\text{mb}\,\text{Cos}[\text{q2}[\text{t}]-\text{q3}[\text{t}]],\text{Ib}+\frac{\text{lb}^2\,\text{mb}}{4},0,0,0\Big\},$$
$$\left\{-\left(\text{lsa}+\text{lsb}\right)\left(\text{lta}\,\text{ms}+\text{ltb}\left(\text{ms}+\text{mt}\right)\right)\text{Cos}[\text{q1}[\text{t}]-\text{q4}[\text{t}]],\right.$$
$$-\left(\text{lta}+\text{ltb}\right)\left(\text{lta}\,\text{ms}+\text{ltb}\left(\text{ms}+\text{mt}\right)\right)\text{Cos}[\text{q2}[\text{t}]-\text{q4}[\text{t}]],0,$$
$$\text{It}+\text{lta}^2\,\text{ms}+2\,\text{lta}\,\text{ltb}\,\text{ms}+\text{ltb}^2\left(\text{ms}+\text{mt}\right),\text{lsb}\left(\text{lta}+\text{ltb}\right)\text{ms}\,\text{Cos}[\text{q4}[\text{t}]-\text{q5}[\text{t}]],0\Big\},$$
$$\left\{-\text{lsb}\left(\text{lsa}+\text{lsb}\right)\text{ms}\,\text{Cos}[\text{q1}[\text{t}]-\text{q5}[\text{t}]],-\text{lsb}\left(\text{lta}+\text{ltb}\right)\text{ms}\,\text{Cos}[\text{q2}[\text{t}]-\text{q5}[\text{t}]],\right.$$
$$0,\text{lsb}\left(\text{lta}+\text{ltb}\right)\text{ms}\,\text{Cos}[\text{q4}[\text{t}]-\text{q5}[\text{t}]],\text{Is}+\text{lsb}^2\,\text{ms},0\Big\},\{1,-1,0,0,0,0\}\Big\}$$

```
[(Is+(2*lsa*lsb*(mb+(ms+2*mt))+((lsb)^(2)*(mb+(ms+2*mt))+(lsa)^(2)*(mb+2*(ms+mt)))
  ))  (lsa+lsb)*(ltb*(mb+(ms+mt))+lta*(mb+(ms+2*mt)))*cos((q1+-1*q2))
  1/2*lb*(lsa+lsb)*mb*cos((q1+-1*q3))
  -1*(lsa+lsb)*(lta*ms+ltb*(ms+mt))*cos((q1+-1*q4))
  -1*lsb*(lsa+lsb)*ms*cos((q1+-1*q5))
  1;(lsa+lsb)*(ltb*(mb+(ms+mt))+lta*(mb+(ms+2*mt)))*cos((q1+-1*q2))
  (It+(2*lta*ltb*(mb+(ms+mt))+((ltb)^(2)*(mb+(ms+mt))+(lta)^(2)*(mb+(ms+2*mt)))))
  1/2*lb*(lta+ltb)*mb*cos((q2+-1*q3))
  -1*(lta+ltb)*(lta*ms+ltb*(ms+mt))*cos((q2+-1*q4))
  -1*lsb*(lta+ltb)*ms*cos((q2+-1*q5))  -1;1/2*lb*(lsa+lsb)*mb*cos((q1+-1*q3))
  1/2*lb*(lta+ltb)*mb*cos((q2+-1*q3))  (Ib+1/4*(lb)^(2)*mb)
  0 0 0;-1*(lsa+lsb)*(lta*ms+ltb*(ms+mt))*cos((q1+-1*q4))
  -1*(lta+ltb)*(lta*ms+ltb*(ms+mt))*cos((q2+-1*q4))  0
  (It+((lta)^(2)*ms+(2*lta*ltb*ms+(ltb)^(2)*(ms+mt))))
  lsb*(lta+ltb)*ms*cos((q4+-1*q5))  0;-1*lsb*(lsa+lsb)*ms*cos((q1+-1*q5))
  -1*lsb*(lta+ltb)*ms*cos((q2+-1*q5))  0
  lsb*(lta+ltb)*ms*cos((q4+-1*q5))  (Is+(lsb)^(2)*ms)  0;1 -1 0 0 0 0]
```

```
ArrayFlatten[
 {{HurGlobalMMatrix, -HurList2Column[{0, 0, 0, -1, 1}]}, {{{0, 0, 0, 1, -1}}, 0}}]
HurToJulia[%]
```

$$\left\{\left\{\text{Is}+2\,\text{lsa}\,\text{lsb}\left(\text{mb}+\text{ms}+2\,\text{mt}\right)+\text{lsb}^2\left(\text{mb}+\text{ms}+2\,\text{mt}\right)+\text{lsa}^2\left(\text{mb}+2\left(\text{ms}+\text{mt}\right)\right),\right.\right.$$
$$\left(\text{lsa}+\text{lsb}\right)\left(\text{ltb}\left(\text{mb}+\text{ms}+\text{mt}\right)+\text{lta}\left(\text{mb}+\text{ms}+2\,\text{mt}\right)\right)\text{Cos}[\text{q1}[\text{t}]-\text{q2}[\text{t}]],$$
$$\frac{1}{2}\,\text{lb}\left(\text{lsa}+\text{lsb}\right)\text{mb}\,\text{Cos}[\text{q1}[\text{t}]-\text{q3}[\text{t}]],-\left(\text{lsa}+\text{lsb}\right)\left(\text{lta}\,\text{ms}+\text{ltb}\left(\text{ms}+\text{mt}\right)\right)$$
$$\text{Cos}[\text{q1}[\text{t}]-\text{q4}[\text{t}]],-\text{lsb}\left(\text{lsa}+\text{lsb}\right)\text{ms}\,\text{Cos}[\text{q1}[\text{t}]-\text{q5}[\text{t}]],0\},$$
$$\left\{\left(\text{lsa}+\text{lsb}\right)\left(\text{ltb}\left(\text{mb}+\text{ms}+\text{mt}\right)+\text{lta}\left(\text{mb}+\text{ms}+2\,\text{mt}\right)\right)\text{Cos}[\text{q1}[\text{t}]-\text{q2}[\text{t}]],\right.$$
$$\text{It}+2\,\text{lta}\,\text{ltb}\left(\text{mb}+\text{ms}+\text{mt}\right)+\text{ltb}^2\left(\text{mb}+\text{ms}+\text{mt}\right)+\text{lta}^2\left(\text{mb}+\text{ms}+2\,\text{mt}\right),$$
$$\frac{1}{2}\,\text{lb}\left(\text{lta}+\text{ltb}\right)\text{mb}\,\text{Cos}[\text{q2}[\text{t}]-\text{q3}[\text{t}]],-\left(\text{lta}+\text{ltb}\right)\left(\text{lta}\,\text{ms}+\text{ltb}\left(\text{ms}+\text{mt}\right)\right)$$
$$\text{Cos}[\text{q2}[\text{t}]-\text{q4}[\text{t}]],-\text{lsb}\left(\text{lta}+\text{ltb}\right)\text{ms}\,\text{Cos}[\text{q2}[\text{t}]-\text{q5}[\text{t}]],0\},$$
$$\left\{\frac{1}{2}\,\text{lb}\left(\text{lsa}+\text{lsb}\right)\text{mb}\,\text{Cos}[\text{q1}[\text{t}]-\text{q3}[\text{t}]],\ \frac{1}{2}\,\text{lb}\left(\text{lta}+\text{ltb}\right)\text{mb}\,\text{Cos}[\text{q2}[\text{t}]-\text{q3}[\text{t}]],\right.$$
$$\left.\text{Ib}+\frac{\text{lb}^2\,\text{mb}}{4},0,0,0\right\},\left\{-\left(\text{lsa}+\text{lsb}\right)\left(\text{lta}\,\text{ms}+\text{ltb}\left(\text{ms}+\text{mt}\right)\right)\text{Cos}[\text{q1}[\text{t}]-\text{q4}[\text{t}]],\right.$$
$$-\left(\text{lta}+\text{ltb}\right)\left(\text{lta}\,\text{ms}+\text{ltb}\left(\text{ms}+\text{mt}\right)\right)\text{Cos}[\text{q2}[\text{t}]-\text{q4}[\text{t}]],0,$$
$$\text{It}+\text{lta}^2\,\text{ms}+2\,\text{lta}\,\text{ltb}\,\text{ms}+\text{ltb}^2\left(\text{ms}+\text{mt}\right),\text{lsb}\left(\text{lta}+\text{ltb}\right)\text{ms}\,\text{Cos}[\text{q4}[\text{t}]-\text{q5}[\text{t}]],1\},$$
$$\left\{-\text{lsb}\left(\text{lsa}+\text{lsb}\right)\text{ms}\,\text{Cos}[\text{q1}[\text{t}]-\text{q5}[\text{t}]],-\text{lsb}\left(\text{lta}+\text{ltb}\right)\text{ms}\,\text{Cos}[\text{q2}[\text{t}]-\text{q5}[\text{t}]],\right.$$
$$\left.0,\text{lsb}\left(\text{lta}+\text{ltb}\right)\text{ms}\,\text{Cos}[\text{q4}[\text{t}]-\text{q5}[\text{t}]],\text{Is}+\text{lsb}^2\,\text{ms},-1\right\},\{0,0,0,1,-1,0\}\right\}$$

```
[((Is+(2*lsa*lsb*(mb+(ms+2*mt))+((lsb)^(2)*(mb+(ms+2*mt))+(lsa)^(2)*(mb+2*(ms+mt)))
  ))  (lsa+lsb)*(ltb*(mb+(ms+mt))+lta*(mb+(ms+2*mt)))*cos((q1+-1*q2))
  1/2*lb*(lsa+lsb)*mb*cos((q1+-1*q3))
  -1*(lsa+lsb)*(lta*ms+ltb*(ms+mt))*cos((q1+-1*q4))
  -1*lsb*(lsa+lsb)*ms*cos((q1+-1*q5))
  0;(lsa+lsb)*(ltb*(mb+(ms+mt))+lta*(mb+(ms+2*mt)))*cos((q1+-1*q2))
  (It+(2*lta*ltb*(mb+(ms+mt))+((ltb)^(2)*(mb+(ms+mt))+(lta)^(2)*(mb+(ms+2*mt)))))
  1/2*lb*(lta+ltb)*mb*cos((q2+-1*q3))
  -1*(lta+ltb)*(lta*ms+ltb*(ms+mt))*cos((q2+-1*q4))
  -1*lsb*(lta+ltb)*ms*cos((q2+-1*q5))  0;1/2*lb*(lsa+lsb)*mb*cos((q1+-1*q3))
  1/2*lb*(lta+ltb)*mb*cos((q2+-1*q3))  (Ib+1/4*(lb)^(2)*mb)
  0 0 0;-1*(lsa+lsb)*(lta*ms+ltb*(ms+mt))*cos((q1+-1*q4))
  -1*(lta+ltb)*(lta*ms+ltb*(ms+mt))*cos((q2+-1*q4))  0
  (It+((lta)^(2)*ms+(2*lta*ltb*ms+(ltb)^(2)*(ms+mt))))
  lsb*(lta+ltb)*ms*cos((q4+-1*q5))  1;-1*lsb*(lsa+lsb)*ms*cos((q1+-1*q5))
  -1*lsb*(lta+ltb)*ms*cos((q2+-1*q5))  0
  lsb*(lta+ltb)*ms*cos((q4+-1*q5))  (Is+(lsb)^(2)*ms)  -1;0 0 0 1 -1 0]
```

```
KneeLockMat = ArrayFlatten[
  {{HurGlobalMMatrix, -HurList2Column[{-1, 1, 0, 0, 0}]}, {{{1, -1, 0, 0, 0}}, 0}}]
```

$$\left\{\left\{\text{Is}+2\,\text{lsa}\,\text{lsb}\left(\text{mb}+\text{ms}+2\,\text{mt}\right)+\text{lsb}^2\left(\text{mb}+\text{ms}+2\,\text{mt}\right)+\text{lsa}^2\left(\text{mb}+2\left(\text{ms}+\text{mt}\right)\right),\right.\right.$$
$$\left(\text{lsa}+\text{lsb}\right)\left(\text{ltb}\left(\text{mb}+\text{ms}+\text{mt}\right)+\text{lta}\left(\text{mb}+\text{ms}+2\,\text{mt}\right)\right)\text{Cos}[\text{q1}[\text{t}]-\text{q2}[\text{t}]],$$
$$\frac{1}{2}\,\text{lb}\left(\text{lsa}+\text{lsb}\right)\text{mb}\,\text{Cos}[\text{q1}[\text{t}]-\text{q3}[\text{t}]],-\left(\text{lsa}+\text{lsb}\right)\left(\text{lta}\,\text{ms}+\text{ltb}\left(\text{ms}+\text{mt}\right)\right)$$
$$\text{Cos}[\text{q1}[\text{t}]-\text{q4}[\text{t}]],-\text{lsb}\left(\text{lsa}+\text{lsb}\right)\text{ms}\,\text{Cos}[\text{q1}[\text{t}]-\text{q5}[\text{t}]],1\},$$
$$\left\{\left(\text{lsa}+\text{lsb}\right)\left(\text{ltb}\left(\text{mb}+\text{ms}+\text{mt}\right)+\text{lta}\left(\text{mb}+\text{ms}+2\,\text{mt}\right)\right)\text{Cos}[\text{q1}[\text{t}]-\text{q2}[\text{t}]],\right.$$
$$\text{It}+2\,\text{lta}\,\text{ltb}\left(\text{mb}+\text{ms}+\text{mt}\right)+\text{ltb}^2\left(\text{mb}+\text{ms}+\text{mt}\right)+\text{lta}^2\left(\text{mb}+\text{ms}+2\,\text{mt}\right),$$
$$\frac{1}{2}\,\text{lb}\left(\text{lta}+\text{ltb}\right)\text{mb}\,\text{Cos}[\text{q2}[\text{t}]-\text{q3}[\text{t}]],$$
$$-\left(\text{lta}+\text{ltb}\right)\left(\text{lta}\,\text{ms}+\text{ltb}\left(\text{ms}+\text{mt}\right)\right)\text{Cos}[\text{q2}[\text{t}]-\text{q4}[\text{t}]],$$
$$-\text{lsb}\left(\text{lta}+\text{ltb}\right)\text{ms}\,\text{Cos}[\text{q2}[\text{t}]-\text{q5}[\text{t}]],-1\},\left\{\frac{1}{2}\,\text{lb}\left(\text{lsa}+\text{lsb}\right)\text{mb}\,\text{Cos}[\text{q1}[\text{t}]-\text{q3}[\text{t}]],\right.$$
$$\frac{1}{2}\,\text{lb}\left(\text{lta}+\text{ltb}\right)\text{mb}\,\text{Cos}[\text{q2}[\text{t}]-\text{q3}[\text{t}]],\text{Ib}+\frac{\text{lb}^2\,\text{mb}}{4},0,0,0\},$$
$$\left\{-\left(\text{lsa}+\text{lsb}\right)\left(\text{lta}\,\text{ms}+\text{ltb}\left(\text{ms}+\text{mt}\right)\right)\text{Cos}[\text{q1}[\text{t}]-\text{q4}[\text{t}]],\right.$$
$$-\left(\text{lta}+\text{ltb}\right)\left(\text{lta}\,\text{ms}+\text{ltb}\left(\text{ms}+\text{mt}\right)\right)\text{Cos}[\text{q2}[\text{t}]-\text{q4}[\text{t}]],0,$$
$$\text{It}+\text{lta}^2\,\text{ms}+2\,\text{lta}\,\text{ltb}\,\text{ms}+\text{ltb}^2\left(\text{ms}+\text{mt}\right),\text{lsb}\left(\text{lta}+\text{ltb}\right)\text{ms}\,\text{Cos}[\text{q4}[\text{t}]-\text{q5}[\text{t}]],0\},$$
$$\left\{-\text{lsb}\left(\text{lsa}+\text{lsb}\right)\text{ms}\,\text{Cos}[\text{q1}[\text{t}]-\text{q5}[\text{t}]],-\text{lsb}\left(\text{lta}+\text{ltb}\right)\text{ms}\,\text{Cos}[\text{q2}[\text{t}]-\text{q5}[\text{t}]],\right.$$
$$0,\text{lsb}\left(\text{lta}+\text{ltb}\right)\text{ms}\,\text{Cos}[\text{q4}[\text{t}]-\text{q5}[\text{t}]],\text{Is}+\text{lsb}^2\,\text{ms},0\},\{1,-1,0,0,0,0\}\}$$

```
KneeLockMat /. {q1[t] → 0.1, q2[t] → 0.05, q3[t] → 0.0, q4[t] → -0.05, q5[t] → -0.1}
MatrixRank[%]
```

$$\left\{\left\{\text{Is}+2\,\text{lsa}\,\text{lsb}\left(\text{mb}+\text{ms}+2\,\text{mt}\right)+\text{lsb}^2\left(\text{mb}+\text{ms}+2\,\text{mt}\right)+\text{lsa}^2\left(\text{mb}+2\left(\text{ms}+\text{mt}\right)\right),\right.\right.$$
$$0.99875\left(\text{lsa}+\text{lsb}\right)\left(\text{ltb}\left(\text{mb}+\text{ms}+\text{mt}\right)+\text{lta}\left(\text{mb}+\text{ms}+2\,\text{mt}\right)\right),0.497502\,\text{lb}\left(\text{lsa}+\text{lsb}\right)\text{mb},$$
$$-0.988771\left(\text{lsa}+\text{lsb}\right)\left(\text{lta}\,\text{ms}+\text{ltb}\left(\text{ms}+\text{mt}\right)\right),-0.980067\,\text{lsb}\left(\text{lsa}+\text{lsb}\right)\text{ms},1\},$$
$$\left\{0.99875\left(\text{lsa}+\text{lsb}\right)\left(\text{ltb}\left(\text{mb}+\text{ms}+\text{mt}\right)+\text{lta}\left(\text{mb}+\text{ms}+2\,\text{mt}\right)\right),\right.$$
$$\text{It}+2\,\text{lta}\,\text{ltb}\left(\text{mb}+\text{ms}+\text{mt}\right)+\text{ltb}^2\left(\text{mb}+\text{ms}+\text{mt}\right)+\text{lta}^2\left(\text{mb}+\text{ms}+2\,\text{mt}\right),$$
$$0.499375\,\text{lb}\left(\text{lta}+\text{ltb}\right)\text{mb},-0.995004\left(\text{lta}+\text{ltb}\right)\left(\text{lta}\,\text{ms}+\text{ltb}\left(\text{ms}+\text{mt}\right)\right),$$
$$-0.988771\,\text{lsb}\left(\text{lta}+\text{ltb}\right)\text{ms},-1\},$$
$$\left\{0.497502\,\text{lb}\left(\text{lsa}+\text{lsb}\right)\text{mb},0.499375\,\text{lb}\left(\text{lta}+\text{ltb}\right)\text{mb},\text{Ib}+\frac{\text{lb}^2\,\text{mb}}{4},0,0,0\right\},$$
$$\left\{-0.988771\left(\text{lsa}+\text{lsb}\right)\left(\text{lta}\,\text{ms}+\text{ltb}\left(\text{ms}+\text{mt}\right)\right),\right.$$
$$-0.995004\left(\text{lta}+\text{ltb}\right)\left(\text{lta}\,\text{ms}+\text{ltb}\left(\text{ms}+\text{mt}\right)\right),0,$$
$$\text{It}+\text{lta}^2\,\text{ms}+2\,\text{lta}\,\text{ltb}\,\text{ms}+\text{ltb}^2\left(\text{ms}+\text{mt}\right),0.99875\,\text{lsb}\left(\text{lta}+\text{ltb}\right)\text{ms},0\},$$
$$\left\{-0.980067\,\text{lsb}\left(\text{lsa}+\text{lsb}\right)\text{ms},-0.988771\,\text{lsb}\left(\text{lta}+\text{ltb}\right)\text{ms},0,\right.$$
$$0.99875\,\text{lsb}\left(\text{lta}+\text{ltb}\right)\text{ms},\text{Is}+\text{lsb}^2\,\text{ms},0\},\{1,-1,0,0,0,0\}\}$$

6

```
HurGlobalRF
```

{n, a, b, c, d, e}

```
COMWholeX = HurUnifyTriadsCoord[COMWhole, n][[1]];
COMWholeY = HurUnifyTriadsCoord[COMWhole, n][[2]];
COMWholeX // MatrixForm
COMWholeY // MatrixForm
```

$$\frac{1}{mb + 2\,ms + 2\,mt}\Big(-lsa\,ms\,\text{Sin}[q1[t]] + mt\,\big(-(lsa + lsb)\,\text{Sin}[q1[t]] - lta\,\text{Sin}[q2[t]]\big) + mb\,\Big(-(lsa + lsb)\,\text{Sin}[q1[t]] - (lta + ltb)\,\text{Sin}[q2[t]] - \frac{1}{2}\,lb\,\text{Sin}[q3[t]]\Big) + mt\,\big(-(lsa + lsb)\,\text{Sin}[q1[t]] - (lta + ltb)\,\text{Sin}[q2[t]] + ltb\,\text{Sin}[q4[t]]\big) + ms\,\big(-(lsa + lsb)\,\text{Sin}[q1[t]] - (lta + ltb)\,\text{Sin}[q2[t]] + (lta + ltb)\,\text{Sin}[q4[t]] + lsb\,\text{Sin}[q5[t]]\big)\Big)$$

$$\frac{1}{mb + 2\,ms + 2\,mt}\Big(lsa\,ms\,\text{Cos}[q1[t]] + mt\,\big((lsa + lsb)\,\text{Cos}[q1[t]] + lta\,\text{Cos}[q2[t]]\big) + mb\,\Big((lsa + lsb)\,\text{Cos}[q1[t]] + (lta + ltb)\,\text{Cos}[q2[t]] + \frac{1}{2}\,lb\,\text{Cos}[q3[t]]\Big) + mt\,\big((lsa + lsb)\,\text{Cos}[q1[t]] + (lta + ltb)\,\text{Cos}[q2[t]] - ltb\,\text{Cos}[q4[t]]\big) + ms\,\big((lsa + lsb)\,\text{Cos}[q1[t]] + (lta + ltb)\,\text{Cos}[q2[t]] - (lta + ltb)\,\text{Cos}[q4[t]] - lsb\,\text{Cos}[q5[t]]\big)\Big)$$

```
VelCOMWholeX = D[COMWholeX, t] // Simplify
VelCOMWholeY = D[COMWholeY, t] // Simplify

HurToJulia[VelCOMWholeY];

y1 = VelCOMWholeX;
y1d = D[y1, t] // Simplify
```

$$\frac{1}{2\,(mb + 2\,(ms + mt))}\Big(2\,(lsb\,(mb + ms + 2\,mt) + lsa\,(mb + 2\,(ms + mt)))\,\text{Sin}[q1[t]]\,q1'[t]^2 + 2\,(ltb\,(mb + ms + mt) + lta\,(mb + ms + 2\,mt))\,\text{Sin}[q2[t]]\,q2'[t]^2 + lb\,mb\,\text{Sin}[q3[t]]\,q3'[t]^2 - 2\,lta\,ms\,\text{Sin}[q4[t]]\,q4'[t]^2 - 2\,ltb\,ms\,\text{Sin}[q4[t]]\,q4'[t]^2 - 2\,ltb\,mt\,\text{Sin}[q4[t]]\,q4'[t]^2 - 2\,lsb\,ms\,\text{Sin}[q5[t]]\,q5'[t]^2 - 2\,(lsb\,(mb + ms + 2\,mt) + lsa\,(mb + 2\,(ms + mt)))\,\text{Cos}[q1[t]]\,q1''[t] - 2\,(ltb\,(mb + ms + mt) + lta\,(mb + ms + 2\,mt))\,\text{Cos}[q2[t]]\,q2''[t] - lb\,mb\,\text{Cos}[q3[t]]\,q3''[t] + 2\,lta\,ms\,\text{Cos}[q4[t]]\,q4''[t] + 2\,ltb\,ms\,\text{Cos}[q4[t]]\,q4''[t] + 2\,ltb\,mt\,\text{Cos}[q4[t]]\,q4''[t] + 2\,lsb\,ms\,\text{Cos}[q5[t]]\,q5''[t]\Big)$$

```
Grad[y1d, D[HurGlobalGeneralizedCoordinates, t, t]]
```

$$\Big\{-\big((lsb\,(mb + ms + 2\,mt) + lsa\,(mb + 2\,(ms + mt)))\,\text{Cos}[q1[t]]\big)\,/\,(mb + 2\,(ms + mt))),\ -\big((ltb\,(mb + ms + mt) + lta\,(mb + ms + 2\,mt))\,\text{Cos}[q2[t]]\big)\,/\,(mb + 2\,(ms + mt))),\ -\frac{lb\,mb\,\text{Cos}[q3[t]]}{2\,(mb + 2\,(ms + mt))},\ (2\,lta\,ms\,\text{Cos}[q4[t]] + 2\,ltb\,ms\,\text{Cos}[q4[t]] + 2\,ltb\,mt\,\text{Cos}[q4[t]])\,/\,(2\,(mb + 2\,(ms + mt)))),\ \frac{lsb\,ms\,\text{Cos}[q5[t]]}{mb + 2\,(ms + mt)}\Big\}$$

```
q1dd = q1''[t] /. invans;
q2dd = q2''[t] /. invans;
q3dd = q3''[t] /. invans;
q4dd = q4''[t] /. invans;
y1dtemp = y1d /. invans;
```

```
HurSaveData["data2.m", "FootST", "ShankSTCOM", "KneeST", "ThighSTCOM",
 "Hip", "TorsoCOM", "ThighSWCOM", "KneeSW", "ShankSWCOM", "FootSW", "dyn1",
 "dyn2", "dyn3", "dyn4", "dyn5", "stepLength", "stepHeight", "verticalVel",
 "horizontalVel", "JacFootSW", "COMWhole", "LinearMomentumWholeBody",
 "LinearMomentumRateWholeBody", "JacWhole", "NJacWhole", "invans", "COMWholeX",
 "VelCOMWholeX", "y1d", "q1dd", "q2dd", "q3dd", "q4dd", "q5dd", "y1dtemp"]
```

```
HurToJulia[q1dd]
```

```
g21 = Grad[q1dd, {tau1, tau2, tau3, tau4, tau5}];
g22 = Grad[q2dd, {tau1, tau2, tau3, tau4, tau5}];
g23 = Grad[q3dd, {tau1, tau2, tau3, tau4, tau5}];
g24 = Grad[q4dd, {tau1, tau2, tau3, tau4, tau5}];
g1 = Grad[y1dtemp, {tau1, tau2, tau3, tau4, tau5}];
```

```
HurSaveData["data2.m", "FootST", "ShankSTCOM", "KneeST", "ThighSTCOM", "Hip",
 "TorsoCOM", "ThighSWCOM", "KneeSW", "ShankSWCOM", "FootSW", "dyn1", "dyn2", "dyn3",
 "dyn4", "dyn5", "stepLength", "stepHeight", "verticalVel", "horizontalVel",
 "JacFootSW", "COMWhole", "LinearMomentumWholeBody", "LinearMomentumRateWholeBody",
 "JacWhole", "NJacWhole", "invans", "COMWholeX", "VelCOMWholeX", "y1d", "q1dd",
 "q2dd", "q3dd", "q4dd", "q5dd", "y1dtemp", "g1", "g21", "g22", "g23", "g24"]
```

```
COMWholeX
```

```
COMWholeX
```

```
HurToJulia[y1dtemp];
```

```
g10 = y1dtemp /. {tau1 → 0, tau2 → 0, tau3 → 0, tau4 → 0, tau5 → 0};
g11 = D[y1dtemp, tau1];
g12 = D[y1dtemp, tau2];
g13 = D[y1dtemp, tau3];
g14 = D[y1dtemp, tau4];
g15 = D[y1dtemp, tau5];
```

```
HurToJulia[g11];
```

```
g210 = q1dd /. {tau1 → 0, tau2 → 0, tau3 → 0, tau4 → 0, tau5 → 0};
g211 = D[q1dd, tau1];
g212 = D[q1dd, tau2];
g213 = D[q1dd, tau3];
g214 = D[q1dd, tau4];
g215 = D[q1dd, tau5];

g220 = q2dd /. {tau1 → 0, tau2 → 0, tau3 → 0, tau4 → 0, tau5 → 0};
g221 = D[q2dd, tau1];
g222 = D[q2dd, tau2];
g223 = D[q2dd, tau3];
g224 = D[q2dd, tau4];
g225 = D[q2dd, tau5];

g230 = q3dd /. {tau1 → 0, tau2 → 0, tau3 → 0, tau4 → 0, tau5 → 0};
g231 = D[q3dd, tau1];
g232 = D[q3dd, tau2];
g233 = D[q3dd, tau3];
g234 = D[q3dd, tau4];
g235 = D[q3dd, tau5];

g240 = q4dd /. {tau1 → 0, tau2 → 0, tau3 → 0, tau4 → 0, tau5 → 0};
g241 = D[q4dd, tau1];
g242 = D[q4dd, tau2];
g243 = D[q4dd, tau3];
g244 = D[q4dd, tau4];
g245 = D[q4dd, tau5];

HurToJulia[g245];

HurSaveData["data4.m", "FootST", "ShankSTCOM", "KneeST", "ThighSTCOM", "Hip",
 "TorsoCOM", "ThighSWCOM", "KneeSW", "ShankSWCOM", "FootSW", "dyn1", "dyn2", "dyn3",
 "dyn4", "dyn5", "stepLength", "stepHeight", "verticalVel", "horizontalVel",
 "JacFootSW", "COMWhole", "LinearMomentumWholeBody", "LinearMomentumRateWholeBody",
 "JacWhole", "NJacWhole", "invans", "COMWholeX", "VelCOMWholeX", "y1d", "q1dd",
 "q2dd", "q3dd", "q4dd", "q5dd", "y1dtemp", "g1", "g21", "g22", "g23", "g24",
 "g10", "g11", "g12", "g13", "g14", "g15", "g210", "g211", "g212", "g213",
 "g214", "g215", "g220", "g221", "g222", "g223", "g224", "g225", "g230", "g231",
 "g232", "g233", "g234", "g235", "g240", "g241", "g242", "g243", "g244", "g245"]
```