

```
Quit[];
```

```
SetDirectory[NotebookDirectory[]];
```

```
<< HurToolbox.m
```

HurToolbox for modeling and analysis of multibody systems 1.0.0.

HurToolbox mainly uses vector manipulation (vectors, dyadics).

Coordinates and matrix representation of the dyadics are also available.

Available methods: Newton-Euler

Method, Euler-Lagrange Method, Hamiltonian Method, Kane's Method.

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Email questions, comments, or concerns to pilwonhur@tamu.edu.

```
HurInitialize[]
```

```
HurLoadData["data_dynamicsA_abs.m"]
```

```
HurDefineRF[a, b, c, d]
```

```
HurDefineGeneralizedCoordinates[q1[t], q2[t]]
```

```
HurDefineDCM[a, q1[t], {0, 0, 1}]
```

```
HurDefineDCM[b, q2[t], {0, 0, 1}]
```

```
HurDefineDCM[d, -gamma, {0, 0, 1}]
```

```
FOOTST = 0;
```

```
COMA = FOOTST + 1a a2; (*COM of stance leg*)
```

```
COMB = FOOTST + (1a + 1b) a2 - 1b b2; (*COM of swing leg*)
```

```
COMC = FOOTST + (1a + 1b) a2; (*Hip*)
```

```
HIP = COMC;
```

```
FOOTSW = FOOTST + (1a + 1b) a2 - (1a + 1b) b2; (*Swing foot position*)
```

```
HurDefineCOMPos[a, COMA];
```

```
HurDefineCOMPos[b, COMB];
```

```
HurDefineCOMPos[c, COMC];
```

```
HurKinematics[]
```

```

HurGlobalCOMVel
HurGlobalCOMAcc
HurGlobalAngularVel
HurGlobalAngularAcc

{0, -a1 la q1'[t],
 - (la + lb) (b1 Cos[q1[t] - q2[t]] + b2 Sin[q1[t] - q2[t]]) q1'[t] + b1 lb q2'[t],
 - (la + lb) (c1 Cos[q1[t]] + c2 Sin[q1[t]]) q1'[t], 0}

{0, -la (a2 q1'[t]^2 + a1 q1''[t]),
 b2 (- (la + lb) Cos[q1[t] - q2[t]] q1'[t]^2 + lb q2'[t]^2 - (la + lb) Sin[q1[t] - q2[t]] q1''[t]) +
 b1 ((la + lb) Sin[q1[t] - q2[t]] q1'[t]^2 - (la + lb) Cos[q1[t] - q2[t]] q1''[t] + lb q2''[t]),
 - (la + lb) ((c2 Cos[q1[t]] - c1 Sin[q1[t]]) q1'[t]^2 + (c1 Cos[q1[t]] + c2 Sin[q1[t]]) q1''[t]),
 0}

{0, n3 q1'[t], n3 q2'[t], 0, 0}

{0, a3 q1''[t], b3 q2''[t], 0, 0}

HurDefineMass[a, m];
HurDefineMass[b, m];
HurDefineMass[c, mh];
HurDefineInertia[a, {0, 0, 0, 0, 0, Iz}];
HurDefineInertia[b, {0, 0, 0, 0, 0, Iz}];

HurDefineVertical[n2];

Jaf = HurGetJacobian[FOOTST, a, n]
Jah = HurGetJacobian[HIP, a, n]
Jbh = HurGetJacobian[HIP, b, n]

{{0, 0}, {0, 0}, {0, 0}, {0, 0}, {0, 0}, {1, 0}}

{{-(la + lb) Cos[q1[t]], 0}, {-(la + lb) Sin[q1[t]], 0}, {0, 0}, {0, 0}, {0, 0}, {1, 0}}

{{-(la + lb) Cos[q1[t]], 0}, {-(la + lb) Sin[q1[t]], 0}, {0, 0}, {0, 0}, {0, 0}, {0, 1}}

Transpose[Jaf].HurList2Column[{0, 0, 0, 0, 0, tau1}] +
  Transpose[Jah].HurList2Column[{0, 0, 0, 0, 0, -tau2}] +
  Transpose[Jbh].HurList2Column[{0, 0, 0, 0, 0, tau2}] // MatrixForm

( tau1 - tau2
  tau2 )

HurDefineNonConservativeForces[tau1 - tau2, tau2]

{tau1 - tau2, tau2}

HurELEquation[]

{-tau1 + tau2 - 2 g la m Sin[q1[t]] - g lb m Sin[q1[t]] -
 g la mh Sin[q1[t]] - g lb mh Sin[q1[t]] - lb (la + lb) m Sin[q1[t] - q2[t]] q2'[t]^2 +
 (Iz + 2 la lb (m + mh) + lb^2 (m + mh) + la^2 (2 m + mh)) q1''[t] -
 la lb m Cos[q1[t] - q2[t]] q2''[t] - lb^2 m Cos[q1[t] - q2[t]] q2''[t],
 -tau2 + g lb m Sin[q2[t]] + lb (la + lb) m Sin[q1[t] - q2[t]] q1'[t]^2 -
 lb (la + lb) m Cos[q1[t] - q2[t]] q1''[t] + Iz q2''[t] + lb^2 m q2''[t]}

```

**HurGlobalMMatrix // MatrixForm**

$$\begin{pmatrix} I_z + 2 l_a l_b (m + m_h) + l_b^2 (m + m_h) + l_a^2 (2 m + m_h) & -l_b (l_a + l_b) m \cos[q_1[t] - q_2[t]] \\ -l_b (l_a + l_b) m \cos[q_1[t] - q_2[t]] & I_z + l_b^2 m \end{pmatrix}$$

**HurGlobalCMatrix // MatrixForm**

$$\begin{pmatrix} 0 & -l_b (l_a + l_b) m \sin[q_1[t] - q_2[t]] q_2'[t] \\ l_b (l_a + l_b) m \sin[q_1[t] - q_2[t]] q_1'[t] & 0 \end{pmatrix}$$

**HurGlobalGVector // MatrixForm**

$$\begin{pmatrix} -g (l_b (m + m_h) + l_a (2 m + m_h)) \sin[q_1[t]] \\ g l_b m \sin[q_2[t]] \end{pmatrix}$$

**invsol = Flatten[HurELInverse[]]**

$$\begin{aligned} \{q_1''[t] \rightarrow & -\left( (l_b (l_a + l_b) m \cos[q_1[t] - q_2[t]] \right. \\ & \left. (-\tau_{u2} + g l_b m \sin[q_2[t]] + l_b (l_a + l_b) m \sin[q_1[t] - q_2[t]] q_1'[t]^2) + \right. \\ & \left. (I_z + l_b^2 m) (-\tau_{u1} + \tau_{u2} - g (l_b (m + m_h) + l_a (2 m + m_h)) \sin[q_1[t]] - \right. \\ & \left. l_b (l_a + l_b) m \sin[q_1[t] - q_2[t]] q_2'[t]^2) \right) / \\ & \left( (I_z + l_b^2 m) (I_z + 2 l_a l_b (m + m_h) + l_b^2 (m + m_h) + l_a^2 (2 m + m_h)) - \right. \\ & \left. l_b^2 (l_a + l_b)^2 m^2 \cos[q_1[t] - q_2[t]]^2 \right), \\ q_2''[t] \rightarrow & \left( 2 I_z \tau_{u2} + 4 l_a^2 m \tau_{u2} + 4 l_a l_b m \tau_{u2} + 2 l_b^2 m \tau_{u2} + 2 l_a^2 m_h \tau_{u2} + \right. \\ & 4 l_a l_b m_h \tau_{u2} + 2 l_b^2 m_h \tau_{u2} + 2 l_a l_b m \tau_{u1} \cos[q_1[t] - q_2[t]] + \\ & 2 l_b^2 m \tau_{u1} \cos[q_1[t] - q_2[t]] - 2 l_a l_b m \tau_{u2} \cos[q_1[t] - q_2[t]] - \\ & 2 l_b^2 m \tau_{u2} \cos[q_1[t] - q_2[t]] + 2 g l_a^2 l_b m^2 \sin[2 q_1[t] - q_2[t]] + \\ & 3 g l_a l_b^2 m^2 \sin[2 q_1[t] - q_2[t]] + g l_b^3 m^2 \sin[2 q_1[t] - q_2[t]] + \\ & g l_a^2 l_b m m_h \sin[2 q_1[t] - q_2[t]] + 2 g l_a l_b^2 m m_h \sin[2 q_1[t] - q_2[t]] + \\ & g l_b^3 m m_h \sin[2 q_1[t] - q_2[t]] - 2 g I_z l_b m \sin[q_2[t]] - \\ & 2 g l_a^2 l_b m^2 \sin[q_2[t]] - g l_a l_b^2 m^2 \sin[q_2[t]] - g l_b^3 m^2 \sin[q_2[t]] - \\ & g l_a^2 l_b m m_h \sin[q_2[t]] - 2 g l_a l_b^2 m m_h \sin[q_2[t]] - g l_b^3 m m_h \sin[q_2[t]] - \\ & 2 l_b (l_a + l_b) m (I_z + 2 l_a l_b (m + m_h) + l_b^2 (m + m_h) + l_a^2 (2 m + m_h)) \\ & \left. \sin[q_1[t] - q_2[t]] q_1'[t]^2 + l_b^2 (l_a + l_b)^2 m^2 \sin[2 (q_1[t] - q_2[t])] q_2'[t]^2 \right) / \\ & \left( 2 I_z^2 + 2 I_z (2 l_a l_b (m + m_h) + l_a^2 (2 m + m_h) + l_b^2 (2 m + m_h)) + \right. \\ & l_b^2 m (2 l_a l_b (m + 2 m_h) + l_b^2 (m + 2 m_h) + l_a^2 (3 m + 2 m_h)) - \\ & \left. l_b^2 (l_a + l_b)^2 m^2 \cos[2 (q_1[t] - q_2[t])] \right) \} \end{aligned}$$

```
q1dd = q1''[t] /. invsol
```

```
HurToJulia[q1dd]
```

$$\begin{aligned}
& - \left( (lb (la + lb) m \cos[q_1[t] - q_2[t]] \right. \\
& \quad \left( -\tau_2 + g lb m \sin[q_2[t]] + lb (la + lb) m \sin[q_1[t] - q_2[t]] q_1'[t]^2 \right) + \\
& \quad (Iz + lb^2 m) (-\tau_1 + \tau_2 - g (lb (m + mh) + la (2m + mh)) \sin[q_1[t]] - \\
& \quad lb (la + lb) m \sin[q_1[t] - q_2[t]] q_2'[t]^2) \Big) / \\
& \left( (Iz + lb^2 m) (Iz + 2 la lb (m + mh) + lb^2 (m + mh) + la^2 (2m + mh)) - \right. \\
& \quad \left. lb^2 (la + lb)^2 m^2 \cos[q_1[t] - q_2[t]]^2 \right) \\
& -1 * ((Iz + (lb)^2 * m) * (Iz + (2 * la * lb * (m + mh) + (lb)^2 * (m + mh) + (la)^2 * (2 * m + mh))) + -1 * (lb \\
& )^2 * ((la + lb))^2 * (m)^2 * (\cos((q_1 - 1 * q_2)))^2) )^(-1) * (lb * (la + lb) * m * \cos((q_1 - 1 * \\
& q_2)) * (-1 * \tau_2 + (g * lb * m * \sin(q_2) + lb * (la + lb) * m * \sin((q_1 - 1 * q_2)) * (q_1 d)^2) + (Iz + (lb)^2 \\
& * m) * (-1 * \tau_1 + (\tau_2 + (-1 * g * (lb * (m + mh) + la * (2 * m + mh)) * \sin(q_1) + -1 * lb * (la + lb) * m * \sin((q_1 - 1 \\
& * q_2)) * (q_2 d)^2)))
\end{aligned}$$

```
q2dd = q2''[t] /. invsol
```

```
HurToJulia[q2dd]
```

$$\begin{aligned}
& (2 Iz \tau_2 + 4 la^2 m \tau_2 + 4 la lb m \tau_2 + 2 lb^2 m \tau_2 + 2 la^2 mh \tau_2 + 4 la lb mh \tau_2 + \\
& \quad 2 lb^2 mh \tau_2 + 2 la lb m \tau_1 \cos[q_1[t] - q_2[t]] + 2 lb^2 m \tau_1 \cos[q_1[t] - q_2[t]] - \\
& \quad 2 la lb m \tau_2 \cos[q_1[t] - q_2[t]] - 2 lb^2 m \tau_2 \cos[q_1[t] - q_2[t]] + \\
& \quad 2 g la^2 lb m^2 \sin[2 q_1[t] - q_2[t]] + 3 g la lb^2 m^2 \sin[2 q_1[t] - q_2[t]] + \\
& \quad g lb^3 m^2 \sin[2 q_1[t] - q_2[t]] + g la^2 lb m mh \sin[2 q_1[t] - q_2[t]] + \\
& \quad 2 g la lb^2 m mh \sin[2 q_1[t] - q_2[t]] + g lb^3 m mh \sin[2 q_1[t] - q_2[t]] - 2 g Iz lb m \sin[q_2[t]] - \\
& \quad 2 g la^2 lb m^2 \sin[q_2[t]] - g la lb^2 m^2 \sin[q_2[t]] - g lb^3 m^2 \sin[q_2[t]] - \\
& \quad g la^2 lb m mh \sin[q_2[t]] - 2 g la lb^2 m mh \sin[q_2[t]] - g lb^3 m mh \sin[q_2[t]] - \\
& \quad 2 lb (la + lb) m (Iz + 2 la lb (m + mh) + lb^2 (m + mh) + la^2 (2m + mh)) \sin[q_1[t] - q_2[t]] q_1'[t]^2 + \\
& \quad lb^2 (la + lb)^2 m^2 \sin[2 (q_1[t] - q_2[t])] q_2'[t]^2) / \\
& (2 Iz^2 + 2 Iz (2 la lb (m + mh) + la^2 (2m + mh) + lb^2 (2m + mh)) + lb^2 m \\
& \quad (2 la lb (m + 2mh) + lb^2 (m + 2mh) + la^2 (3m + 2mh)) - lb^2 (la + lb)^2 m^2 \cos[2 (q_1[t] - q_2[t])]) \\
& ((2 * (Iz)^2 + (2 * Iz * (2 * la * lb * (m + mh) + (la)^2 * (2 * m + mh) + (lb)^2 * (2 * m + mh))) + ((lb)^2 * m \\
& * (2 * la * lb * (m + 2 * mh) + (lb)^2 * (m + 2 * mh) + (la)^2 * (3 * m + 2 * mh))) + -1 * (lb)^2 * ((la + lb))^2 * \\
& (m)^2 * \cos(2 * (q_1 - 1 * q_2))))^(-1) * (2 * Iz * \tau_2 + (4 * (la)^2 * m * \tau_2 + (4 * la * lb * m * \tau_2 \\
& + (2 * (lb)^2 * m * \tau_2 + (2 * (la)^2 * mh * \tau_2 + (4 * la * lb * mh * \tau_2 + (2 * (lb)^2 * mh * \tau_2 + (2 * la * \\
& lb * m * \tau_1 * \cos((q_1 - 1 * q_2)) + (2 * (lb)^2 * m * \tau_1 * \cos((q_1 - 1 * q_2)) + (-2 * la * lb * m * \tau_2 * \cos(( \\
& q_1 - 1 * q_2)) + (-2 * (lb)^2 * m * \tau_2 * \cos((q_1 - 1 * q_2)) + (2 * g * (la)^2 * lb * (m)^2 * \sin((2 * q_1 - 1 * \\
& q_2)) + (3 * g * la * (lb)^2 * (m)^2 * \sin((2 * q_1 - 1 * q_2)) + (g * (lb)^3 * (m)^2 * \sin((2 * q_1 - 1 * \\
& q_2)) + (g * (la)^2 * lb * m * mh * \sin((2 * q_1 - 1 * q_2)) + (2 * g * la * (lb)^2 * m * mh * \sin((2 * q_1 - 1 * q_2)) + \\
& (g * (lb)^3 * m * mh * \sin((2 * q_1 - 1 * q_2)) + (-2 * g * Iz * lb * m * \sin(q_2) + (-2 * g * (la)^2 * lb * (m)^2 * \\
& \sin(q_2) + (-1 * g * la * (lb)^2 * (m)^2 * \sin(q_2) + (-1 * g * (lb)^3 * (m)^2 * \sin(q_2) + (-1 * g * (la) \\
& ^2 * lb * m * mh * \sin(q_2) + (-2 * g * la * (lb)^2 * m * mh * \sin(q_2) + (-1 * g * (lb)^3 * m * mh * \sin(q_2) + (-2 \\
& * lb * (la + lb) * m * (Iz + (2 * la * lb * (m + mh) + (lb)^2 * (m + mh) + (la)^2 * (2 * m + mh)))) * \sin((q_1 - 1 * \\
& q_2)) * (q_1 d)^2 + (lb)^2 * ((la + lb))^2 * (m)^2 * \sin(2 * (q_1 - 1 * q_2)) * (q_2 d)^2))))))
\end{aligned}$$

```
JacQ1dd = Grad[q1dd, {q1[t], q2[t], q1'[t], q2'[t]}] // Simplify
```

```
JacQ2dd = Grad[q2dd, {q1[t], q2[t], q1'[t], q2'[t]}] // Simplify
```

$$\begin{aligned}
& \left\{ \left( - \left( (Iz + lb^2 m) (Iz + 2 la lb (m + mh) + lb^2 (m + mh) + la^2 (2 m + mh)) - \right. \right. \right. \\
& \quad \left. \left. \left. lb^2 (la + lb)^2 m^2 \cos[q1[t] - q2[t]]^2 \right) \right. \right. \\
& \quad \left( lb^2 (la + lb)^2 m^2 \cos[q1[t] - q2[t]]^2 q1'[t]^2 - lb (la + lb) m \sin[q1[t] - q2[t]] \right. \\
& \quad \left. \left. (-\tau u2 + g lb m \sin[q2[t]] + lb (la + lb) m \sin[q1[t] - q2[t]] q1'[t]^2) + \right. \right. \\
& \quad \left. \left. (Iz + lb^2 m) (-g (lb (m + mh) + la (2 m + mh)) \cos[q1[t]] - \right. \right. \\
& \quad \left. \left. lb (la + lb) m \cos[q1[t] - q2[t]] q2'[t]^2) \right) \right. \\
& \quad \left. 2 lb^2 (la + lb)^2 m^2 \cos[q1[t] - q2[t]] \sin[q1[t] - q2[t]] (lb (la + lb) m \cos[q1[t] - q2[t]] \right. \\
& \quad \left. \left. (-\tau u2 + g lb m \sin[q2[t]] + lb (la + lb) m \sin[q1[t] - q2[t]] q1'[t]^2) + \right. \right. \\
& \quad \left. \left. (Iz + lb^2 m) (-\tau u1 + \tau u2 - g (lb (m + mh) + la (2 m + mh)) \sin[q1[t]] - \right. \right. \\
& \quad \left. \left. lb (la + lb) m \sin[q1[t] - q2[t]] q2'[t]^2) \right) \right) \Big/ \\
& \quad \left( (Iz + lb^2 m) (Iz + 2 la lb (m + mh) + lb^2 (m + mh) + la^2 (2 m + mh)) - \right. \\
& \quad \left. lb^2 (la + lb)^2 m^2 \cos[q1[t] - q2[t]]^2 \right), \\
& \quad \left( lb (la + lb) m \left( - \left( (Iz + lb^2 m) (Iz + 2 la lb (m + mh) + lb^2 (m + mh) + la^2 (2 m + mh)) - lb^2 (la + lb)^2 \right. \right. \right. \\
& \quad \left. \left. \left. m^2 \cos[q1[t] - q2[t]]^2 \right) (g lb m \cos[q1[t] - 2 q2[t]] - \tau u2 \sin[q1[t] - q2[t]] - \right. \right. \right. \\
& \quad \left. \left. \left. lb (la + lb) m \cos[2 (q1[t] - q2[t])] q1'[t]^2 + (Iz + lb^2 m) \cos[q1[t] - q2[t]] q2'[t]^2 \right) - \right. \right. \\
& \quad \left. \left. 2 lb (la + lb) m \cos[q1[t] - q2[t]] \sin[q1[t] - q2[t]] (lb (la + lb) m \cos[q1[t] - q2[t]] \right. \right. \\
& \quad \left. \left. \left. (-\tau u2 + g lb m \sin[q2[t]] + lb (la + lb) m \sin[q1[t] - q2[t]] q1'[t]^2) + \right. \right. \right. \\
& \quad \left. \left. \left. (Iz + lb^2 m) (-\tau u1 + \tau u2 - g (lb (m + mh) + la (2 m + mh)) \sin[q1[t]] - \right. \right. \right. \\
& \quad \left. \left. \left. lb (la + lb) m \sin[q1[t] - q2[t]] q2'[t]^2) \right) \right) \Big/ \right. \\
& \quad \left( (Iz + lb^2 m) (Iz + 2 la lb (m + mh) + lb^2 (m + mh) + la^2 (2 m + mh)) - \right. \\
& \quad \left. lb^2 (la + lb)^2 m^2 \cos[q1[t] - q2[t]]^2 \right), \\
& \quad - \left( \left( 2 lb^2 (la + lb)^2 m^2 \cos[q1[t] - q2[t]] \sin[q1[t] - q2[t]] q1'[t] \right) \Big/ \right. \\
& \quad \left( (Iz + lb^2 m) (Iz + 2 la lb (m + mh) + lb^2 (m + mh) + la^2 (2 m + mh)) - \right. \\
& \quad \left. \left. lb^2 (la + lb)^2 m^2 \cos[q1[t] - q2[t]]^2 \right) \right), \\
& \quad \left( 2 lb (la + lb) m (Iz + lb^2 m) \sin[q1[t] - q2[t]] q2'[t] \right) \Big/ \\
& \quad \left( (Iz + lb^2 m) (Iz + 2 la lb (m + mh) + lb^2 (m + mh) + la^2 (2 m + mh)) - \right. \\
& \quad \left. \left. lb^2 (la + lb)^2 m^2 \cos[q1[t] - q2[t]]^2 \right) \right\} \\
& \left\{ 2 lb (la + lb) m \right. \\
& \quad \left( \left( 2 Iz^2 + 2 Iz (2 la lb (m + mh) + la^2 (2 m + mh) + lb^2 (2 m + mh)) + lb^2 (2 la lb (m + 2 mh) + \right. \right. \right. \\
& \quad \left. \left. \left. lb^2 (m + 2 mh) + la^2 (3 m + 2 mh)) - lb^2 (la + lb)^2 m^2 \cos[2 (q1[t] - q2[t])] \right) \right) \right. \\
& \quad \left( 2 g la m \cos[2 q1[t] - q2[t]] + g lb m \cos[2 q1[t] - q2[t]] + g la mh \cos[2 q1[t] - q2[t]] + \right. \\
& \quad \left. g lb mh \cos[2 q1[t] - q2[t]] - \tau u1 \sin[q1[t] - q2[t]] + \tau u2 \sin[q1[t] - q2[t]] - \right. \\
& \quad \left. (Iz + 2 la lb (m + mh) + lb^2 (m + mh) + la^2 (2 m + mh)) \cos[q1[t] - q2[t]] q1'[t]^2 + \right. \\
& \quad \left. lb (la + lb) m \cos[2 (q1[t] - q2[t])] q2'[t]^2 \right) - \\
& \quad \left. lb (la + lb) m \sin[2 (q1[t] - q2[t])] \left( 2 Iz \tau u2 + 4 la^2 m \tau u2 + 4 la lb m \tau u2 + 2 lb^2 m \tau u2 + \right. \right. \\
& \quad \left. \left. 2 la^2 mh \tau u2 + 4 la lb mh \tau u2 + 2 lb^2 mh \tau u2 + 2 la lb m \tau u1 \cos[q1[t] - q2[t]] + \right. \right. \\
& \quad \left. \left. 2 lb^2 m \tau u1 \cos[q1[t] - q2[t]] - 2 la lb m \tau u2 \cos[q1[t] - q2[t]] - \right. \right. \\
& \quad \left. \left. 2 lb^2 m \tau u2 \cos[q1[t] - q2[t]] + 2 g la^2 lb m^2 \sin[2 q1[t] - q2[t]] + \right. \right. \\
& \quad \left. \left. 3 g la lb^2 m^2 \sin[2 q1[t] - q2[t]] + g lb^3 m^2 \sin[2 q1[t] - q2[t]] + \right. \right. \\
& \quad \left. \left. g la^2 lb m mh \sin[2 q1[t] - q2[t]] + 2 g la lb^2 m mh \sin[2 q1[t] - q2[t]] \right) + \right.
\end{aligned}$$

$$\begin{aligned}
& \left( \begin{aligned}
& g lb^3 m mh \sin[2 q1[t] - q2[t]] - 2 g Iz lb m \sin[q2[t]] - \\
& 2 g la^2 lb m^2 \sin[q2[t]] - g la lb^2 m^2 \sin[q2[t]] - g lb^3 m^2 \sin[q2[t]] - \\
& g la^2 lb m mh \sin[q2[t]] - 2 g la lb^2 m mh \sin[q2[t]] - g lb^3 m mh \sin[q2[t]] - \\
& 2 lb (la + lb) m (Iz + 2 la lb (m + mh) + lb^2 (m + mh) + la^2 (2 m + mh)) \\
& \sin[q1[t] - q2[t]] q1'[t]^2 + lb^2 (la + lb)^2 m^2 \sin[2 (q1[t] - q2[t])] q2'[t]^2 \Big) \Big) / \\
& \left( \begin{aligned}
& 2 Iz^2 + 2 Iz (2 la lb (m + mh) + la^2 (2 m + mh) + lb^2 (2 m + mh)) + \\
& lb^2 m (2 la lb (m + 2 mh) + lb^2 (m + 2 mh) + la^2 (3 m + 2 mh)) - \\
& lb^2 (la + lb)^2 m^2 \cos[2 (q1[t] - q2[t])] \Big)^2, \\
& lb m \Big( \begin{aligned}
& 2 Iz^2 + 2 Iz (2 la lb (m + mh) + la^2 (2 m + mh) + lb^2 (2 m + mh)) + \\
& lb^2 m (2 la lb (m + 2 mh) + lb^2 (m + 2 mh) + la^2 (3 m + 2 mh)) - \\
& lb^2 (la + lb)^2 m^2 \cos[2 (q1[t] - q2[t])] \Big) \\
& \left( \begin{aligned}
& - 2 g la^2 m \cos[2 q1[t] - q2[t]] - 3 g la lb m \cos[2 q1[t] - q2[t]] - \\
& g lb^2 m \cos[2 q1[t] - q2[t]] - g la^2 mh \cos[2 q1[t] - q2[t]] - \\
& 2 g la lb mh \cos[2 q1[t] - q2[t]] - g lb^2 mh \cos[2 q1[t] - q2[t]] - 2 g Iz \cos[q2[t]] - \\
& 2 g la^2 m \cos[q2[t]] - g la lb m \cos[q2[t]] - g lb^2 m \cos[q2[t]] - g la^2 mh \cos[q2[t]] - \\
& 2 g la lb mh \cos[q2[t]] - g lb^2 mh \cos[q2[t]] + 2 la \tau1 \sin[q1[t] - q2[t]] + \\
& 2 lb \tau1 \sin[q1[t] - q2[t]] - 2 la \tau2 \sin[q1[t] - q2[t]] - 2 lb \tau2 \\
& \sin[q1[t] - q2[t]] + 2 (la + lb) (Iz + 2 la lb (m + mh) + lb^2 (m + mh) + la^2 (2 m + mh)) \\
& \cos[q1[t] - q2[t]] q1'[t]^2 - 2 lb (la + lb)^2 m \cos[2 (q1[t] - q2[t])] q2'[t]^2 \Big) + \\
& 2 lb (la + lb)^2 m \sin[2 (q1[t] - q2[t])] \Big( 2 Iz \tau2 + 4 la^2 m \tau2 + 4 la lb m \tau2 + \\
& 2 lb^2 m \tau2 + 2 la^2 mh \tau2 + 4 la lb mh \tau2 + 2 lb^2 mh \tau2 + \\
& 2 la lb m \tau1 \cos[q1[t] - q2[t]] + 2 lb^2 m \tau1 \cos[q1[t] - q2[t]] - \\
& 2 la lb m \tau2 \cos[q1[t] - q2[t]] - 2 lb^2 m \tau2 \cos[q1[t] - q2[t]] + \\
& 2 g la^2 lb m^2 \sin[2 q1[t] - q2[t]] + 3 g la lb^2 m^2 \sin[2 q1[t] - q2[t]] + \\
& g lb^3 m^2 \sin[2 q1[t] - q2[t]] + g la^2 lb m mh \sin[2 q1[t] - q2[t]] + 2 g la lb^2 m mh \\
& \sin[2 q1[t] - q2[t]] + g lb^3 m mh \sin[2 q1[t] - q2[t]] - 2 g Iz lb m \sin[q2[t]] - \\
& 2 g la^2 lb m^2 \sin[q2[t]] - g la lb^2 m^2 \sin[q2[t]] - g lb^3 m^2 \sin[q2[t]] - \\
& g la^2 lb m mh \sin[q2[t]] - 2 g la lb^2 m mh \sin[q2[t]] - g lb^3 m mh \sin[q2[t]] - \\
& 2 lb (la + lb) m (Iz + 2 la lb (m + mh) + lb^2 (m + mh) + la^2 (2 m + mh)) \\
& \sin[q1[t] - q2[t]] q1'[t]^2 + lb^2 (la + lb)^2 m^2 \sin[2 (q1[t] - q2[t])] q2'[t]^2 \Big) \Big) \Big) / \\
& \left( \begin{aligned}
& 2 Iz^2 + 2 Iz (2 la lb (m + mh) + la^2 (2 m + mh) + lb^2 (2 m + mh)) + \\
& lb^2 m (2 la lb (m + 2 mh) + lb^2 (m + 2 mh) + la^2 (3 m + 2 mh)) - \\
& lb^2 (la + lb)^2 m^2 \cos[2 (q1[t] - q2[t])] \Big)^2, \\
& - \left( \begin{aligned}
& 4 lb (la + lb) m (Iz + 2 la lb (m + mh) + lb^2 (m + mh) + la^2 (2 m + mh)) \\
& \sin[q1[t] - q2[t]] q1'[t] \Big) \Big) / \\
& \left( \begin{aligned}
& 2 Iz^2 + 2 Iz (2 la lb (m + mh) + la^2 (2 m + mh) + lb^2 (2 m + mh)) + \\
& lb^2 m (2 la lb (m + 2 mh) + lb^2 (m + 2 mh) + la^2 (3 m + 2 mh)) - \\
& lb^2 (la + lb)^2 m^2 \cos[2 (q1[t] - q2[t])] \Big) \Big), \\
& \left( 2 lb^2 (la + lb)^2 m^2 \sin[2 (q1[t] - q2[t])] q2'[t] \Big) / \\
& \left( \begin{aligned}
& 2 Iz^2 + \\
& 2 Iz (2 la lb (m + mh) + la^2 (2 m + mh) + lb^2 (2 m + mh)) + \\
& lb^2 m (2 la lb (m + 2 mh) + lb^2 (m + 2 mh) + la^2 (3 m + 2 mh)) -
\end{aligned} \right)
\end{aligned} \right)
\end{aligned}$$

$$lb^2 (la + lb)^2 m^2 \cos[2 (q1[t] - q2[t])] \}$$

```
HurSaveData["data_dynamicsA_abs.m", "invsol", "q1dd", "q2dd",
  "JacQ1dd", "JacQ2dd", "FOOTST", "COMA", "COMB", "COMC", "HIP", "FOOTSW"]
```

Julia expression for dynamic constraints

```
HurToJulia[q1dd]
```

```
-1*((-1*(Iz+(lb)^(2)*m+lb*(la+lb)*m*cos(q2)))^(2)+(Iz+(lb)^(2)*m)*(2*Iz+((la)^(2)*
m+((la+lb)^(2)*mh+(m*(lb+(la+lb)*cos(q2)))^(2)+(la+lb)^(2)*m*(sin(q2))^(2))))
)^(2))*(-1*(Iz+lb*(lb+(la+lb)*cos(q2)))*(-1*tau2+(-1*g*lb*m*sin((q1+q2))+lb*(la+
lb)*m*sin(q2)*(q1d)^(2)))+-1*(Iz+(lb)^(2)*m)*(tau1+(2*g*la*m*sin(q1)+(g*lb*m*sin(q1)
+(g*la*mh*sin(q1)+(g*lb*mh*sin(q1)+(g*lb*m*sin((q1+q2)))+(2*lb*(la+lb)*m*sin(q2)*q1d*
q2d+lb*(la+lb)*m*sin(q2)*(q2d)^(2))))))
```

```
HurToJulia[q2dd]
```

```
((2*(Iz)^(2)+(2*Iz*(2*la*lb*(m+mh)+(la)^(2)*(2*m+mh)+(lb)^(2)*(2*m+mh)))+(lb)^(2)*m
*(2*la*lb*(m+2*mh)+(lb)^(2)*(m+2*mh)+(la)^(2)*(3*m+2*mh)))+-1*(lb)^(2)*((la+lb)^(
2)*(m)^(2)*cos(2*q2))))^(2))*(-2*Iz*tau1+(-2*(lb)^(2)*m*tau1+(4*Iz*tau2+(4*(la)^(2)
)*m*tau2+(4*la*lb*m*tau2+(4*(lb)^(2)*m*tau2+(2*(la)^(2)*mh*tau2+(4*la*lb*mh*tau2+(2*
(lb)^(2)*mh*tau2+(-2*la*lb*m*tau1*cos(q2)+(-2*(lb)^(2)*m*tau1*cos(q2)+(4*la*lb*m*
tau2*cos(q2)+(4*(lb)^(2)*m*tau2*cos(q2)+(-4*g*Iz*la*m*sin(q1)+(-2*g*Iz*lb*m*sin(q1)+
(-3*g*la*(lb)^(2)*(m)^(2)*sin(q1)+(-1*g*(lb)^(3)*(m)^(2)*sin(q1)+(-2*g*Iz*la*mh*sin
(q1)+(-2*g*Iz*lb*mh*sin(q1)+(-2*g*la*(lb)^(2)*m*mh*sin(q1)+(-2*g*(lb)^(3)*m*mh*sin(
q1)+(2*g*Iz*lb*m*cos(q2)*sin(q1)+(-2*g*la*(lb)^(2)*(m)^(2)*cos(q2)*sin(q1)+(g*la*(lb)
)^(2)*(m)^(2)*(cos(q2))^(2)*sin(q1)+(g*(lb)^(3)*(m)^(2)*(cos(q2))^(2)*sin(q1)+(2*g*
Iz*lb*m*cos(q1)*sin(q2)+(4*g*(la)^(2)*lb*(m)^(2)*cos(q1)*sin(q2)+(4*g*la*(lb)^(2)*(m)
)^(2)*cos(q1)*sin(q2)+(2*g*(lb)^(3)*(m)^(2)*cos(q1)*sin(q2)+(2*g*(la)^(2)*lb*m*mh*
cos(q1)*sin(q2)+(4*g*la*(lb)^(2)*m*mh*cos(q1)*sin(q2)+(2*g*(lb)^(3)*m*mh*cos(q1)*sin
(q2)+(-1*g*la*(lb)^(2)*(m)^(2)*sin(q1)*(sin(q2))^(2)+(-1*g*(lb)^(3)*(m)^(2)*sin(q1)
*(sin(q2))^(2)+(g*la*(lb)^(2)*(m)^(2)*cos(q1)*sin(2*q2)+(g*(lb)^(3)*(m)^(2)*cos(q1)
*sin(2*q2)+(-2*lb*(la+lb)*m*(2*Iz+(2*(la)^(2)*m+(2*la*lb*m+(2*(lb)^(2)*m+((la)^(2)*
mh+(2*la*lb*mh+(lb)^(2)*mh+2*lb*(la+lb)*m*cos(q2)))))))*sin(q2)*(q1d)^(2)+(-4*lb*
(la+lb)*m*(Iz+(lb)^(2)*m+lb*(la+lb)*m*cos(q2))*sin(q2)*q1d*q2d+-2*lb*(la+lb)*m*(Iz
+((lb)^(2)*m+lb*(la+lb)*m*cos(q2))*sin(q2)*(q2d)^(2))))))))))))))))))))))))))
```

HIP

```
HurUnifyTriadsCoord[HIP, n]
```

```
a2 (la + lb)
```

```
{-(la + lb) Sin[q1[t]], (la + lb) Cos[q1[t]], 0, n}
```

```
HurUnifyTriadsCoord[F00TSW, n] // MatrixForm
```

```
HurUnifyTriadsCoord[F00TSW, d] // MatrixForm
```

$$\begin{pmatrix} (la+lb) (-\sin[q1[t]] + \sin[q2[t]]) \\ (la+lb) (\cos[q1[t]] - \cos[q2[t]]) \\ 0 \\ n \end{pmatrix}$$

$$\begin{pmatrix} (la+lb) (-\sin[\gamma + q1[t]] + \sin[\gamma + q2[t]]) \\ (la+lb) (\cos[\gamma + q1[t]] - \cos[\gamma + q2[t]]) \\ 0 \\ d \end{pmatrix}$$

```
HurToJulia[HurUnifyTriadsCoord[F00TSW, n][[1]]]
```

```
(la+lb)*(-1*sin(q1)+sin(q2))
```

```
HurToJulia[HurUnifyTriadsCoord[F00TSW, n][[2]]]
```

```
(la+lb)*(cos(q1)+-1*cos(q2))
```

```
HurToJulia[HurUnifyTriadsCoord[HIP, n][[1]]]
```

```
-1*(la+lb)*sin(q1)
```

```
HurToJulia[HurUnifyTriadsCoord[HIP, n][[2]]]
```

```
(la+lb)*cos(q1)
```

```
HurToJulia[HurUnifyTriadsCoord[F00TSW, d][[1]]]
```

```
(la+lb)*(-1*sin((gamma+q1))+sin((gamma+q2)))
```

```
HurToJulia[HurUnifyTriadsCoord[F00TSW, d][[2]]]
```

```
(la+lb)*(cos((gamma+q1))+-1*cos((gamma+q2)))
```

```
HurGlobalELEquation[[1]]
```

```
HurToJulia[%]
```

```
-tau1+tau2-2 g la m Sin[q1[t]]-g lb m Sin[q1[t]]-g la mh Sin[q1[t]]-
g lb mh Sin[q1[t]]-lb (la+lb) m Sin[q1[t]-q2[t]] q2'[t]^2+
(Iz+2 la lb (m+mh)+lb^2 (m+mh)+la^2 (2 m+mh)) q1''[t]-
la lb m Cos[q1[t]-q2[t]] q2''[t]-lb^2 m Cos[q1[t]-q2[t]] q2''[t]
```

```
(-1*tau1+(tau2+(-2*g*la*m*sin(q1)+(-1*g*lb*m*sin(q1)+(-1*g*la*mh*sin(q1)+(-1*g*lb*mh*
sin(q1)+(-1*lb*(la+lb)*m*sin((q1-1*q2))* (q2d)^2)+((Iz+(2*la*lb*(m+mh)+((lb)^(2)*
(m+mh)+(la)^(2)*(2*m+mh)))))*q1dd+(-1*la*lb*m*cos((q1-1*q2))*q2dd+-1*(lb)^(2)*m*cos(
(q1-1*q2))*q2dd))))))
```

```
Clear[q1dd, q2dd]
```



```
HurGlobalELEquation[[2]]
```

```
HurToJulia[%]
```

$$-\tau_2 + g l_b m \sin[q_2[t]] + l_b (l_a + l_b) m \sin[q_1[t] - q_2[t]] q_1'[t]^2 - l_b (l_a + l_b) m \cos[q_1[t] - q_2[t]] q_1''[t] + I_z q_2''[t] + l_b^2 m q_2''[t]$$

$$(-1*\tau_2 + (g*l_b*m*\sin(q_2) + (l_b*(l_a+l_b)*m*\sin((q_1+-1*q_2)))*(q_1d)^{(2)} + (-1*l_b*(l_a+l_b)*m*\cos((q_1+-1*q_2))*q_1dd + (I_z*q_2dd + (l_b)^{(2)}*m*q_2dd))))$$

```
HurGlobalELEquation // MatrixForm
```

$$\begin{pmatrix} -\tau_1 - g l_a m \sin[q_1[t]] - g (l_a + l_b) m h \sin[q_1[t]] + g m (- (l_a + l_b) \sin[q_1[t]] - l_b \sin[q_1[t] + q_2[t]]) \\ -\tau_2 + g l_b m \sin[q_2[t]] + l_b (l_a + l_b) m \sin[q_1[t] - q_2[t]] q_1'[t]^2 - l_b (l_a + l_b) m \cos[q_1[t] - q_2[t]] q_1''[t] + I_z q_2''[t] + l_b^2 m q_2''[t] \end{pmatrix}$$

```
temp = HurGlobalELEquation[[2]] /. 
```

```
{q1[t] -> q1, q2[t] -> q2, q1'[t] -> q1d, q2'[t] -> q2d, q1''[t] -> q1dd, q2''[t] -> q2dd}
```

$$I_z q_2dd + l_b^2 m q_2dd - \tau_2 + q_1dd (I_z + l_b^2 m + l_b (l_a + l_b) m \cos[q_2]) + l_b (l_a + l_b) m q_1d^2 \sin[q_2] - g l_b m \sin[q_1 + q_2]$$

```
f[q1_, q2_, q1d_, q2d_, q1dd_, q2dd_] = temp
```

$$I_z q_2dd + l_b^2 m q_2dd - \tau_2 + q_1dd (I_z + l_b^2 m + l_b (l_a + l_b) m \cos[q_2]) + l_b (l_a + l_b) m q_1d^2 \sin[q_2] - g l_b m \sin[q_1 + q_2]$$

```
f[1, 1, 1, 1, 1, 1]
```

$$2 I_z + 2 l_b^2 m - \tau_2 + l_b (l_a + l_b) m \cos[1] + l_b (l_a + l_b) m \sin[1] - g l_b m \sin[2]$$

```
g[q1_, q2_] := q1 + q2
```

```
g[1, 2]
```

```
3
```

```
HurGlobalMMatrix
```

```
HurToMatlab[%]
```

$$\left\{ \left\{ 2 I_z + l_a^2 m + (l_a + l_b)^2 m h + m \left( (l_b + (l_a + l_b) \cos[q_2[t]])^2 + (l_a + l_b)^2 \sin[q_2[t]]^2 \right), I_z + l_b m (l_b + (l_a + l_b) \cos[q_2[t]]) \right\}, \left\{ I_z + l_b^2 m + l_b (l_a + l_b) m \cos[q_2[t]], I_z + l_b^2 m \right\} \right\}$$

$$[(2*I_z + ((l_a)^{(2)}*m + ((l_a+l_b))^{(2)}*mh + m*((l_b+(l_a+l_b)*\cos(q_2)))^{(2)} + ((l_a+l_b))^{(2)}*(\sin(q_2))^{(2)}))), (I_z+l_b*m*(l_b+(l_a+l_b)*\cos(q_2)))] ; (I_z + ((l_b)^{(2)}*m + l_b*(l_a+l_b)*m*\cos(q_2))), (I_z + (l_b)^{(2)}*m)]$$

```
HurGlobalMMatrix
```

```
HurToJulia[%]
```

$$\left\{ \left\{ I_z + 2 l_a l_b (m + m h) + l_b^2 (m + m h) + l_a^2 (2 m + m h), -l_b (l_a + l_b) m \cos[q_1[t] - q_2[t]] \right\}, \left\{ -l_b (l_a + l_b) m \cos[q_1[t] - q_2[t]], I_z + l_b^2 m \right\} \right\}$$

$$[(I_z + (2*l_a*l_b*(m+mh) + (l_b)^{(2)}*(m+mh) + (l_a)^{(2)}*(2*m+mh))), (-1*l_b*(l_a+l_b)*m*\cos((q_1+-1*q_2)) ; -1*l_b*(l_a+l_b)*m*\cos((q_1+-1*q_2)) (I_z + (l_b)^{(2)}*m)]$$

**HurGlobalMatrix**

**HurToJulia[%]**

$\{ \{0, -lb (la + lb) m \sin[q_1[t] - q_2[t]] q_2'[t]\}, \{lb (la + lb) m \sin[q_1[t] - q_2[t]] q_1'[t], 0\} \}$

$[0 \quad -1*lb*(la+lb)*m*\sin((q_1+-1*q_2))*q_2d; lb*(la+lb)*m*\sin((q_1+-1*q_2))*q_1d \quad 0]$

**HurGlobalGVector**

**HurToJulia[HurList2Column[%]]**

$\{-g (lb (m + mh) + la (2 m + mh)) \sin[q_1[t]], g lb m \sin[q_2[t]]\}$

$[-1*g*(lb*(m+mh)+la*(2*m+mh))*\sin(q_1); g*lb*m*\sin(q_2)]$