

```
Quit[];
```

```
SetDirectory[NotebookDirectory[]];
```

```
<< HurToolbox.m
```

HurToolbox for modeling and analysis of multibody systems 1.0.0.

HurToolbox mainly uses vector manipulation (vectors, dyadics).

Coordinates and matrix representation of the dyadics are also available.

Available methods: Newton-Euler

Method, Euler-Lagrange Method, Hamiltonian Method, Kane's Method.

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Email questions, comments, or concerns to pilwonhur@tamu.edu.

```
HurInitialize[]
```

```
HurLoadData["data_dynamicsA_abs.m"]
```

```
HurDefineRF[a, b, c, d]
```

```
HurDefineGeneralizedCoordinates[q1[t], q2[t]]
```

```
HurDefineDCM[a, q1[t], {0, 0, 1}]
```

```
HurDefineDCM[b, q2[t], {0, 0, 1}]
```

```
HurDefineDCM[d, -gamma, {0, 0, 1}]
```

```
FOOTST = 0;
```

```
COMA = FOOTST + 1a a2; (*COM of stance leg*)
```

```
COMB = FOOTST + (1a + 1b) a2 - 1b b2; (*COM of swing leg*)
```

```
COMC = FOOTST + (1a + 1b) a2; (*Hip*)
```

```
HIP = COMC;
```

```
FOOTSW = FOOTST + (1a + 1b) a2 - (1a + 1b) b2; (*Swing foot position*)
```

```
HurDefineCOMPos[a, COMA];
```

```
HurDefineCOMPos[b, COMB];
```

```
HurDefineCOMPos[c, COMC];
```

```
HurKinematics[]
```

```
HurGlobalCOMVel
```

```
HurGlobalCOMAcc
```

```
HurGlobalAngularVel
```

```
HurGlobalAngularAcc
```

```
{0, -a1 1a q1'[t],
```

```
- (1a + 1b) (b1 Cos[q1[t] - q2[t]] + b2 Sin[q1[t] - q2[t]]) q1'[t] + b1 1b q2'[t],
```

```
- (1a + 1b) (c1 Cos[q1[t]] + c2 Sin[q1[t]]) q1'[t], 0}
```

```
{0, -1a (a2 q1'[t]^2 + a1 q1''[t]),
```

```
b2 (- (1a + 1b) Cos[q1[t] - q2[t]] q1'[t]^2 + 1b q2'[t]^2 - (1a + 1b) Sin[q1[t] - q2[t]] q1''[t]) +
```

```
b1 ((1a + 1b) Sin[q1[t] - q2[t]] q1'[t]^2 - (1a + 1b) Cos[q1[t] - q2[t]] q1''[t] + 1b q2''[t]),
```

```
- (1a + 1b) ((c2 Cos[q1[t]] - c1 Sin[q1[t]]) q1'[t]^2 +
```

```
(c1 Cos[q1[t]] + c2 Sin[q1[t]]) q1''[t]), 0}
```

```
{0, n3 q1'[t], n3 q2'[t], 0, 0}
```

```
{0, a3 q1''[t], b3 q2''[t], 0, 0}
```

```

HurDefineMass[a, m];
HurDefineMass[b, m];
HurDefineMass[c, mh];
HurDefineInertia[a, {0, 0, 0, 0, 0, Iz}];
HurDefineInertia[b, {0, 0, 0, 0, 0, Iz}];

HurDefineVertical[n2];

Jaf = HurGetJacobian[FOOTST, a, n]
Jah = HurGetJacobian[HIP, a, n]
Jbh = HurGetJacobian[HIP, b, n]
{{0, 0}, {0, 0}, {0, 0}, {0, 0}, {0, 0}, {1, 0}}
{{-(1a + 1b) Cos[q1[t]], 0}, {-(1a + 1b) Sin[q1[t]], 0}, {0, 0}, {0, 0}, {0, 0}, {1, 0}}
{{-(1a + 1b) Cos[q1[t]], 0}, {-(1a + 1b) Sin[q1[t]], 0}, {0, 0}, {0, 0}, {0, 0}, {0, 1}}

Transpose[Jaf].HurList2Column[{0, 0, 0, 0, 0, tau1}] +
  Transpose[Jah].HurList2Column[{0, 0, 0, 0, 0, -tau2}] +
  Transpose[Jbh].HurList2Column[{0, 0, 0, 0, 0, tau2}] // MatrixForm
( tau1 - tau2 )
  tau2

HurDefineNonConservativeForces[tau1 - tau2, tau2]
{tau1 - tau2, tau2}

HurELEquation[]
{-tau1 + tau2 - 2 g 1a m Sin[q1[t]] - g 1b m Sin[q1[t]] -
  g 1a mh Sin[q1[t]] - g 1b mh Sin[q1[t]] - 1b (1a + 1b) m Sin[q1[t] - q2[t]] q2'[t]^2 +
  (Iz + 2 1a 1b (m + mh) + 1b^2 (m + mh) + 1a^2 (2 m + mh)) q1''[t] -
  1a 1b m Cos[q1[t] - q2[t]] q2''[t] - 1b^2 m Cos[q1[t] - q2[t]] q2''[t],
  -tau2 + g 1b m Sin[q2[t]] + 1b (1a + 1b) m Sin[q1[t] - q2[t]] q1'[t]^2 -
  1b (1a + 1b) m Cos[q1[t] - q2[t]] q1''[t] + Iz q2''[t] + 1b^2 m q2''[t]}

HurGlobalMMatrix // MatrixForm
( Iz + 2 1a 1b (m + mh) + 1b^2 (m + mh) + 1a^2 (2 m + mh)   -1b (1a + 1b) m Cos[q1[t] - q2[t]] )
  -1b (1a + 1b) m Cos[q1[t] - q2[t]]                               Iz + 1b^2 m

HurGlobalCMatrix // MatrixForm
( 0 -1b (1a + 1b) m Sin[q1[t] - q2[t]] q2'[t] )
  1b (1a + 1b) m Sin[q1[t] - q2[t]] q1'[t] 0

HurGlobalGVector // MatrixForm
( -g (1b (m + mh) + 1a (2 m + mh)) Sin[q1[t]] )
  g 1b m Sin[q2[t]]

```

invsol = Flatten[HurELInverse[]]

$$\begin{aligned} \{q1''[t] \rightarrow & - \left((lb (la + lb) m \cos[q1[t] - q2[t]] \right. \\ & \left. (-\tau a u 2 + g lb m \sin[q2[t]] + lb (la + lb) m \sin[q1[t] - q2[t]] q1'[t]^2) + \right. \\ & \left. (Iz + lb^2 m) (-\tau a u 1 + \tau a u 2 - g (lb (m + mh) + la (2 m + mh)) \sin[q1[t]] - \right. \\ & \left. lb (la + lb) m \sin[q1[t] - q2[t]] q2'[t]^2) \right) / \\ & \left((Iz + lb^2 m) (Iz + 2 la lb (m + mh) + lb^2 (m + mh) + la^2 (2 m + mh)) - \right. \\ & \left. lb^2 (la + lb)^2 m^2 \cos[q1[t] - q2[t]]^2 \right), \\ q2''[t] \rightarrow & \left(2 Iz \tau a u 2 + 4 la^2 m \tau a u 2 + 4 la lb m \tau a u 2 + 2 lb^2 m \tau a u 2 + 2 la^2 mh \tau a u 2 + \right. \\ & 4 la lb mh \tau a u 2 + 2 lb^2 mh \tau a u 2 + 2 la lb m \tau a u 1 \cos[q1[t] - q2[t]] + \\ & 2 lb^2 m \tau a u 1 \cos[q1[t] - q2[t]] - 2 la lb m \tau a u 2 \cos[q1[t] - q2[t]] - \\ & 2 lb^2 m \tau a u 2 \cos[q1[t] - q2[t]] + 2 g la^2 lb m^2 \sin[2 q1[t] - q2[t]] + \\ & 3 g la lb^2 m^2 \sin[2 q1[t] - q2[t]] + g lb^3 m^2 \sin[2 q1[t] - q2[t]] + \\ & g la^2 lb m mh \sin[2 q1[t] - q2[t]] + 2 g la lb^2 m mh \sin[2 q1[t] - q2[t]] + \\ & g lb^3 m mh \sin[2 q1[t] - q2[t]] - 2 g Iz lb m \sin[q2[t]] - \\ & 2 g la^2 lb m^2 \sin[q2[t]] - g la lb^2 m^2 \sin[q2[t]] - g lb^3 m^2 \sin[q2[t]] - \\ & g la^2 lb m mh \sin[q2[t]] - 2 g la lb^2 m mh \sin[q2[t]] - g lb^3 m mh \sin[q2[t]] - \\ & 2 lb (la + lb) m (Iz + 2 la lb (m + mh) + lb^2 (m + mh) + la^2 (2 m + mh)) \\ & \sin[q1[t] - q2[t]] q1'[t]^2 + lb^2 (la + lb)^2 m^2 \sin[2 (q1[t] - q2[t])] q2'[t]^2 \Big) / \\ & \left(2 Iz^2 + 2 Iz (2 la lb (m + mh) + la^2 (2 m + mh) + lb^2 (2 m + mh)) + \right. \\ & lb^2 m (2 la lb (m + 2 mh) + lb^2 (m + 2 mh) + la^2 (3 m + 2 mh)) - \\ & \left. lb^2 (la + lb)^2 m^2 \cos[2 (q1[t] - q2[t])] \right) \} \end{aligned}$$

q1dd = q1''[t] /. invsol

HurToJulia[q1dd]

$$\begin{aligned} & - \left((lb (la + lb) m \cos[q1[t] - q2[t]] \right. \\ & \left. (-\tau a u 2 + g lb m \sin[q2[t]] + lb (la + lb) m \sin[q1[t] - q2[t]] q1'[t]^2) + \right. \\ & \left. (Iz + lb^2 m) (-\tau a u 1 + \tau a u 2 - g (lb (m + mh) + la (2 m + mh)) \sin[q1[t]] - \right. \\ & \left. lb (la + lb) m \sin[q1[t] - q2[t]] q2'[t]^2) \right) / \\ & \left((Iz + lb^2 m) (Iz + 2 la lb (m + mh) + lb^2 (m + mh) + la^2 (2 m + mh)) - \right. \\ & \left. lb^2 (la + lb)^2 m^2 \cos[q1[t] - q2[t]]^2 \right) \\ & -1 * ((Iz + (lb)^(2) * m) * (Iz + (2 * la * lb * (m + mh) + (lb)^(2) * (m + mh) + la)^(2) * (2 * m + mh))) + -1 * \\ & (lb)^(2) * ((la + lb)^(2) * (m)^(2) * (\cos((q1 + -1 * q2)))^(2)))^(2) * (-1 * (lb * (la + lb) * m * \cos((q1 + -1 * q2))) * (-1 * \tau a u 2 + (g * lb * m * \sin(q2) + lb * (la + lb) * m * \sin((q1 + -1 * q2))) * (q1d)^(2))) + (Iz \\ & + (lb)^(2) * m) * (-1 * \tau a u 1 + (\tau a u 2 + (-1 * g * (lb * (m + mh) + la * (2 * m + mh)) * \sin(q1) + -1 * lb * (la + lb) * \\ & m * \sin((q1 + -1 * q2))) * (q2d)^(2)))) \end{aligned}$$

```
q2dd = q2''[t] /. invsol
```

```
HurToJulia[q2dd]
```

$$\begin{aligned} & \left(2 I_z \tau u^2 + 4 l a^2 m \tau u^2 + 4 l a l b m \tau u^2 + 2 l b^2 m \tau u^2 + 2 l a^2 m h \tau u^2 + 4 l a l b m h \tau u^2 + \right. \\ & \quad 2 l b^2 m h \tau u^2 + 2 l a l b m \tau u^1 \cos[q_1[t] - q_2[t]] + 2 l b^2 m \tau u^1 \cos[q_1[t] - q_2[t]] - \\ & \quad 2 l a l b m \tau u^2 \cos[q_1[t] - q_2[t]] - 2 l b^2 m \tau u^2 \cos[q_1[t] - q_2[t]] + \\ & \quad 2 g l a^2 l b m^2 \sin[2 q_1[t] - q_2[t]] + 3 g l a l b^2 m^2 \sin[2 q_1[t] - q_2[t]] + \\ & \quad g l b^3 m^2 \sin[2 q_1[t] - q_2[t]] + g l a^2 l b m h \sin[2 q_1[t] - q_2[t]] + \\ & \quad 2 g l a l b^2 m h \sin[2 q_1[t] - q_2[t]] + g l b^3 m h \sin[2 q_1[t] - q_2[t]] - \\ & \quad 2 g I_z l b m \sin[q_2[t]] - 2 g l a^2 l b m^2 \sin[q_2[t]] - g l a l b^2 m^2 \sin[q_2[t]] - \\ & \quad g l b^3 m^2 \sin[q_2[t]] - g l a^2 l b m h \sin[q_2[t]] - 2 g l a l b^2 m h \sin[q_2[t]] - \\ & \quad g l b^3 m h \sin[q_2[t]] - 2 l b (l a + l b) m (I_z + 2 l a l b (m + m h) + l b^2 (m + m h) + l a^2 (2 m + m h)) \\ & \quad \left. \sin[q_1[t] - q_2[t]] q_1'[t]^2 + l b^2 (l a + l b)^2 m^2 \sin[2 (q_1[t] - q_2[t])] q_2'[t]^2 \right) / \\ & \left(2 I_z^2 + 2 I_z (2 l a l b (m + m h) + l a^2 (2 m + m h) + l b^2 (2 m + m h)) + \right. \\ & \quad l b^2 m (2 l a l b (m + 2 m h) + l b^2 (m + 2 m h) + l a^2 (3 m + 2 m h)) - \\ & \quad \left. l b^2 (l a + l b)^2 m^2 \cos[2 (q_1[t] - q_2[t])] \right) \\ & \left((2 * (I_z)^{(2)} + (2 * I_z * (2 * l a * l b * (m + m h) + ((l a)^{(2)} * (2 * m + m h) + (l b)^{(2)} * (2 * m + m h)))) + ((l b)^{(2)} * \right. \\ & \quad m * (2 * l a * l b * (m + 2 * m h) + ((l b)^{(2)} * (m + 2 * m h) + (l a)^{(2)} * (3 * m + 2 * m h)))) + (-1 * (l b)^{(2)} * ((l a + \\ & \quad l b))^{(2)} * (m)^{(2)} * \cos(2 * (q_1 - 1 * q_2))))))^{(-1)} * (2 * I_z * \tau u^2 + (4 * (l a)^{(2)} * m * \tau u^2 + (4 * l a * \\ & \quad l b * m * \tau u^2 + (2 * (l b)^{(2)} * m * \tau u^2 + (2 * (l a)^{(2)} * m h * \tau u^2 + (4 * l a * l b * m h * \tau u^2 + (2 * (l b)^{(2)} * m h * \\ & \quad \tau u^2 + (2 * l a * l b * m * \tau u^1 * \cos((q_1 - 1 * q_2)) + (2 * (l b)^{(2)} * m * \tau u^1 * \cos((q_1 - 1 * q_2)) + (-2 * l a * l b \\ & \quad * m * \tau u^2 * \cos((q_1 - 1 * q_2)) + (-2 * (l b)^{(2)} * m * \tau u^2 * \cos((q_1 - 1 * q_2)) + (2 * g * (l a)^{(2)} * l b * (m) \\ & \quad ^{(2)} * \sin((2 * q_1 - 1 * q_2)) + (3 * g * l a * (l b)^{(2)} * (m)^{(2)} * \sin((2 * q_1 - 1 * q_2)) + (g * (l b)^{(3)} * (m) \\ & \quad)^{(2)} * \sin((2 * q_1 - 1 * q_2)) + (g * (l a)^{(2)} * l b * m * m h * \sin((2 * q_1 - 1 * q_2)) + (2 * g * l a * (l b)^{(2)} * m \\ & \quad * m h * \sin((2 * q_1 - 1 * q_2)) + (g * (l b)^{(3)} * m * m h * \sin((2 * q_1 - 1 * q_2)) + (-2 * g * I_z * l b * m * \sin(q_2) + \\ & \quad -2 * g * (l a)^{(2)} * l b * (m)^{(2)} * \sin(q_2) + (-1 * g * l a * (l b)^{(2)} * (m)^{(2)} * \sin(q_2) + (-1 * g * (l b)^{(3)} \\ & \quad) * (m)^{(2)} * \sin(q_2) + (-1 * g * (l a)^{(2)} * l b * m * m h * \sin(q_2) + (-2 * g * l a * (l b)^{(2)} * m * m h * \sin(q_2) + \\ & \quad (-1 * g * (l b)^{(3)} * m * m h * \sin(q_2) + (-2 * l b * (l a + l b) * m * (I_z + (2 * l a * l b * (m + m h) + ((l b)^{(2)} * (m + m h) \\ & \quad) + (l a)^{(2)} * (2 * m + m h)))) * \sin((q_1 - 1 * q_2)) * (q_1 d)^{(2)} + (l b)^{(2)} * ((l a + l b))^{(2)} * (m)^{(2)} * \\ & \quad \sin(2 * (q_1 - 1 * q_2)) * (q_2 d)^{(2)})))))))))))))))))))))) \end{aligned}$$

```
JacQ1dd = Grad[q1dd, {q1[t], q2[t], q1'[t], q2'[t]}] // Simplify
```

```
JacQ2dd = Grad[q2dd, {q1[t], q2[t], q1'[t], q2'[t]}] // Simplify
```

$$\begin{aligned}
& \left\{ \left(- \left((Iz + lb^2 m) (Iz + 2 la lb (m + mh) + lb^2 (m + mh) + la^2 (2 m + mh)) - \right. \right. \right. \\
& \quad \left. \left. \left. lb^2 (la + lb)^2 m^2 \cos[q1[t] - q2[t]]^2 \right) \right. \right. \\
& \quad \left(lb^2 (la + lb)^2 m^2 \cos[q1[t] - q2[t]]^2 q1'[t]^2 - lb (la + lb) m \sin[q1[t] - q2[t]] \right. \\
& \quad \left. (-\tau u2 + g lb m \sin[q2[t]] + lb (la + lb) m \sin[q1[t] - q2[t]] q1'[t]^2) + \right. \\
& \quad \left. (Iz + lb^2 m) (-g (lb (m + mh) + la (2 m + mh)) \cos[q1[t]] - \right. \\
& \quad \left. lb (la + lb) m \cos[q1[t] - q2[t]] q2'[t]^2) \right) + \\
& \quad 2 lb^2 (la + lb)^2 m^2 \cos[q1[t] - q2[t]] \sin[q1[t] - q2[t]] \\
& \quad (lb (la + lb) m \cos[q1[t] - q2[t]] \\
& \quad (-\tau u2 + g lb m \sin[q2[t]] + lb (la + lb) m \sin[q1[t] - q2[t]] q1'[t]^2) + \\
& \quad (Iz + lb^2 m) (-\tau u1 + \tau u2 - g (lb (m + mh) + la (2 m + mh)) \sin[q1[t]] - \\
& \quad lb (la + lb) m \sin[q1[t] - q2[t]] q2'[t]^2) \bigg) \bigg) / \\
& \quad \left((Iz + lb^2 m) (Iz + 2 la lb (m + mh) + lb^2 (m + mh) + la^2 (2 m + mh)) - \right. \\
& \quad \left. lb^2 (la + lb)^2 m^2 \cos[q1[t] - q2[t]]^2 \right)^2, \\
& \quad \left(lb (la + lb) m \left(- \left((Iz + lb^2 m) (Iz + 2 la lb (m + mh) + lb^2 (m + mh) + la^2 (2 m + mh)) - \right. \right. \right. \\
& \quad \left. \left. \left. lb^2 (la + lb)^2 m^2 \cos[q1[t] - q2[t]]^2 \right) \right. \right. \\
& \quad \left(g lb m \cos[q1[t] - 2 q2[t]] - \tau u2 \sin[q1[t] - q2[t]] - lb (la + lb) m \right. \\
& \quad \left. \cos[2 (q1[t] - q2[t])] \right) q1'[t]^2 + (Iz + lb^2 m) \cos[q1[t] - q2[t]] q2'[t]^2) - 2 lb \\
& \quad (la + lb) m \cos[q1[t] - q2[t]] \sin[q1[t] - q2[t]] (lb (la + lb) m \cos[q1[t] - q2[t]] \\
& \quad (-\tau u2 + g lb m \sin[q2[t]] + lb (la + lb) m \sin[q1[t] - q2[t]] q1'[t]^2) + \\
& \quad (Iz + lb^2 m) (-\tau u1 + \tau u2 - g (lb (m + mh) + la (2 m + mh)) \sin[q1[t]] - \\
& \quad lb (la + lb) m \sin[q1[t] - q2[t]] q2'[t]^2) \bigg) \bigg) / \\
& \quad \left((Iz + lb^2 m) (Iz + 2 la lb (m + mh) + lb^2 (m + mh) + la^2 (2 m + mh)) - \right. \\
& \quad \left. lb^2 (la + lb)^2 m^2 \cos[q1[t] - q2[t]]^2 \right)^2, \\
& \quad - \left(\left(2 lb^2 (la + lb)^2 m^2 \cos[q1[t] - q2[t]] \sin[q1[t] - q2[t]] q1'[t] \right) / \right. \\
& \quad \left((Iz + lb^2 m) (Iz + 2 la lb (m + mh) + lb^2 (m + mh) + la^2 (2 m + mh)) - \right. \\
& \quad \left. lb^2 (la + lb)^2 m^2 \cos[q1[t] - q2[t]]^2 \right) \bigg), \\
& \quad \left(2 lb (la + lb) m (Iz + lb^2 m) \sin[q1[t] - q2[t]] q2'[t] \right) / \\
& \quad \left((Iz + lb^2 m) (Iz + 2 la lb (m + mh) + lb^2 (m + mh) + la^2 (2 m + mh)) - \right. \\
& \quad \left. lb^2 (la + lb)^2 m^2 \cos[q1[t] - q2[t]]^2 \right) \bigg) \bigg\} \\
& \left\{ 2 lb (la + lb) m \right. \\
& \quad \left(\left(2 Iz^2 + 2 Iz (2 la lb (m + mh) + la^2 (2 m + mh) + lb^2 (2 m + mh)) + lb^2 m (2 la lb (m + 2 mh) + \right. \right. \\
& \quad \left. \left. lb^2 (m + 2 mh) + la^2 (3 m + 2 mh)) - lb^2 (la + lb)^2 m^2 \cos[2 (q1[t] - q2[t])] \right) \right) \\
& \quad (2 g la m \cos[2 q1[t] - q2[t]] + g lb m \cos[2 q1[t] - q2[t]] + \\
& \quad g la mh \cos[2 q1[t] - q2[t]] + g lb mh \cos[2 q1[t] - q2[t]] - \\
& \quad \tau u1 \sin[q1[t] - q2[t]] + \tau u2 \sin[q1[t] - q2[t]] - \\
& \quad (Iz + 2 la lb (m + mh) + lb^2 (m + mh) + la^2 (2 m + mh)) \cos[q1[t] - q2[t]] q1'[t]^2 + \\
& \quad lb (la + lb) m \cos[2 (q1[t] - q2[t])] q2'[t]^2) - lb (la + lb) m \\
& \quad \sin[2 (q1[t] - q2[t])] \left(2 Iz \tau u2 + 4 la^2 m \tau u2 + 4 la lb m \tau u2 + 2 lb^2 m \tau u2 + \right. \\
& \quad 2 la^2 mh \tau u2 + 4 la lb mh \tau u2 + 2 lb^2 mh \tau u2 + 2 la lb m \tau u1 \cos[q1[t] - q2[t]] + \\
& \quad 2 lb^2 m \tau u1 \cos[q1[t] - q2[t]] - 2 la lb m \tau u2 \cos[q1[t] - q2[t]] - \\
& \quad 2 lb^2 m \tau u2 \cos[q1[t] - q2[t]] + 2 g la^2 lb m^2 \sin[2 q1[t] - q2[t]] + \\
& \quad 3 g la lb^2 m^2 \sin[2 q1[t] - q2[t]] + g lb^3 m^2 \sin[2 q1[t] - q2[t]] + \\
& \quad g la^2 lb m mh \sin[2 q1[t] - q2[t]] + 2 g la lb^2 m mh \sin[2 q1[t] - q2[t]] + \\
& \quad g lb^3 m mh \sin[2 q1[t] - q2[t]] - 2 g Iz lb m \sin[q2[t]] \bigg) -
\end{aligned}$$

$$\begin{aligned}
& \left(\begin{aligned}
& \tilde{2} g l a^2 l b m^2 \sin[q_2[t]] - g l a l b^2 m^2 \sin[q_2[t]] - g l b^3 m^2 \sin[q_2[t]] - \\
& g l a^2 l b m m h \sin[q_2[t]] - 2 g l a l b^2 m m h \sin[q_2[t]] - g l b^3 m m h \sin[q_2[t]] - \\
& 2 l b (l a + l b) m (I z + 2 l a l b (m + m h) + l b^2 (m + m h) + l a^2 (2 m + m h)) \\
& \sin[q_1[t] - q_2[t]] q_1'[t]^2 + l b^2 (l a + l b)^2 m^2 \sin[2 (q_1[t] - q_2[t])] q_2'[t]^2 \Big) \Big) / \\
& \left(\begin{aligned}
& 2 I z^2 + 2 I z (2 l a l b (m + m h) + l a^2 (2 m + m h) + l b^2 (2 m + m h)) + \\
& l b^2 m (2 l a l b (m + 2 m h) + l b^2 (m + 2 m h) + l a^2 (3 m + 2 m h)) - \\
& l b^2 (l a + l b)^2 m^2 \cos[2 (q_1[t] - q_2[t])] \Big)^2, \\
& \left(\begin{aligned}
& l b m \left(\begin{aligned}
& 2 I z^2 + 2 I z (2 l a l b (m + m h) + l a^2 (2 m + m h) + l b^2 (2 m + m h)) + \\
& l b^2 m (2 l a l b (m + 2 m h) + l b^2 (m + 2 m h) + l a^2 (3 m + 2 m h)) - \\
& l b^2 (l a + l b)^2 m^2 \cos[2 (q_1[t] - q_2[t])] \Big) \right) \\
& \left(- 2 g l a^2 m \cos[2 q_1[t] - q_2[t]] - 3 g l a l b m \cos[2 q_1[t] - q_2[t]] - \\
& g l b^2 m \cos[2 q_1[t] - q_2[t]] - g l a^2 m h \cos[2 q_1[t] - q_2[t]] - \\
& 2 g l a l b m h \cos[2 q_1[t] - q_2[t]] - g l b^2 m h \cos[2 q_1[t] - q_2[t]] - \\
& 2 g I z \cos[q_2[t]] - 2 g l a^2 m \cos[q_2[t]] - g l a l b m \cos[q_2[t]] - \\
& g l b^2 m \cos[q_2[t]] - g l a^2 m h \cos[q_2[t]] - 2 g l a l b m h \cos[q_2[t]] - \\
& g l b^2 m h \cos[q_2[t]] + 2 l a \tau_1 \sin[q_1[t] - q_2[t]] + 2 l b \tau_1 \sin[q_1[t] - q_2[t]] - \\
& 2 l a \tau_2 \sin[q_1[t] - q_2[t]] - 2 l b \tau_2 \sin[q_1[t] - q_2[t]] + 2 (l a + l b) \\
& (I z + 2 l a l b (m + m h) + l b^2 (m + m h) + l a^2 (2 m + m h)) \cos[q_1[t] - q_2[t]] q_1'[t]^2 - \\
& 2 l b (l a + l b)^2 m \cos[2 (q_1[t] - q_2[t])] q_2'[t]^2 \Big) + 2 l b (l a + l b)^2 m \\
& \sin[2 (q_1[t] - q_2[t])] \left(\begin{aligned}
& 2 I z \tau_2 + 4 l a^2 m \tau_2 + 4 l a l b m \tau_2 + 2 l b^2 m \tau_2 + \\
& 2 l a^2 m h \tau_2 + 4 l a l b m h \tau_2 + 2 l b^2 m h \tau_2 + 2 l a l b m \tau_1 \cos[q_1[t] - q_2[t]] + \\
& 2 l b^2 m \tau_1 \cos[q_1[t] - q_2[t]] - 2 l a l b m \tau_2 \cos[q_1[t] - q_2[t]] - \\
& 2 l b^2 m \tau_2 \cos[q_1[t] - q_2[t]] + 2 g l a^2 l b m^2 \sin[2 q_1[t] - q_2[t]] + \\
& 3 g l a l b^2 m^2 \sin[2 q_1[t] - q_2[t]] + g l b^3 m^2 \sin[2 q_1[t] - q_2[t]] + \\
& g l a^2 l b m m h \sin[2 q_1[t] - q_2[t]] + 2 g l a l b^2 m m h \sin[2 q_1[t] - q_2[t]] + \\
& g l b^3 m m h \sin[2 q_1[t] - q_2[t]] - 2 g I z l b m \sin[q_2[t]] - \\
& 2 g l a^2 l b m^2 \sin[q_2[t]] - g l a l b^2 m^2 \sin[q_2[t]] - g l b^3 m^2 \sin[q_2[t]] - \\
& g l a^2 l b m m h \sin[q_2[t]] - 2 g l a l b^2 m m h \sin[q_2[t]] - g l b^3 m m h \sin[q_2[t]] - \\
& 2 l b (l a + l b) m (I z + 2 l a l b (m + m h) + l b^2 (m + m h) + l a^2 (2 m + m h)) \\
& \sin[q_1[t] - q_2[t]] q_1'[t]^2 + l b^2 (l a + l b)^2 m^2 \sin[2 (q_1[t] - q_2[t])] q_2'[t]^2 \Big) \Big) \Big) / \\
& \left(\begin{aligned}
& 2 I z^2 + 2 I z (2 l a l b (m + m h) + l a^2 (2 m + m h) + l b^2 (2 m + m h)) + \\
& l b^2 m (2 l a l b (m + 2 m h) + l b^2 (m + 2 m h) + l a^2 (3 m + 2 m h)) - \\
& l b^2 (l a + l b)^2 m^2 \cos[2 (q_1[t] - q_2[t])] \Big)^2, \\
& - \left(\begin{aligned}
& 4 l b (l a + l b) m (I z + 2 l a l b (m + m h) + l b^2 (m + m h) + l a^2 (2 m + m h)) \\
& \sin[q_1[t] - q_2[t]] q_1'[t] \Big) / \\
& \left(\begin{aligned}
& 2 I z^2 + 2 I z (2 l a l b (m + m h) + l a^2 (2 m + m h) + l b^2 (2 m + m h)) + \\
& l b^2 m (2 l a l b (m + 2 m h) + l b^2 (m + 2 m h) + l a^2 (3 m + 2 m h)) - \\
& l b^2 (l a + l b)^2 m^2 \cos[2 (q_1[t] - q_2[t])] \Big) \right), \\
& \left(\begin{aligned}
& 2 l b^2 (l a + l b)^2 m^2 \sin[2 (q_1[t] - q_2[t])] \\
& q_2'[t] \Big) / \\
& \left(\begin{aligned}
& 2 I z^2 + 2 I z (2 l a l b (m + m h) + l a^2 (2 m + m h) + l b^2 (2 m + m h)) + \\
& l b^2 m (2 l a l b (m + 2 m h) + l b^2 (m + 2 m h) + l a^2 (3 m + 2 m h)) - \\
& l b^2 (l a + l b)^2 m^2 \cos[2 (q_1[t] - q_2[t])] \Big) \right) \Big\}
\end{aligned}
\end{aligned}
\right)
\end{aligned}$$

HurSaveData["data_dynamicsA_abs.m", "invsol", "q1dd", "q2dd",
"JacQ1dd", "JacQ2dd", "FOOTST", "COMA", "COMB", "COMC", "HIP", "FOOTSW"]

Julia expression for dynamic constraints

HurToJulia[q1dd]

```
-1*( (-1*( (Iz+(lb)^(2)*m+lb*(la+lb)*m*cos(q2)))^(2)+(Iz+(lb)^(2)*m)*(2*Iz+(la)^(2)*m+((la+lb)^(2)*mh+(m*(lb+(la+lb)*cos(q2)))^(2)+(la+lb)^(2)*m*(sin(q2))^(2))))))^( -1)*(-1*(Iz+lb*m*(lb+(la+lb)*cos(q2)))*(-1*tau2+(-1*g*lb*m*sin((q1+q2))+lb*(la+lb)*m*sin(q2)*(q1d)^(2)))+-1*(Iz+(lb)^(2)*m)*(tau1+(2*g*la*m*sin(q1)+(g*lb*m*sin(q1)+(g*la*mh*sin(q1)+(g*lb*mh*sin(q1)+(g*lb*m*sin((q1+q2)))+(2*lb*(la+lb)*m*sin(q2)*q1d*q2d+lb*(la+lb)*m*sin(q2)*(q2d)^(2)))))))))
```

HurToJulia[q2dd]

```
((2*(Iz)^(2)+(2*Iz*(2*la*lb*(m+mh)+(la)^(2)*(2*m+mh)+(lb)^(2)*(2*m+mh)))+(lb)^(2)*m*(2*la*lb*(m+2*mh)+(lb)^(2)*(m+2*mh)+(la)^(2)*(3*m+2*mh)))+-1*(lb)^(2)*((la+lb)^(2)*(m)^(2)*cos(2*q2))))^( -1)*(-2*Iz*tau1+(-2*(lb)^(2)*m*tau1+(4*Iz*tau2+(4*(la)^(2)*m*tau2+(4*la*lb*m*tau2+(4*(lb)^(2)*m*tau2+(2*(la)^(2)*mh*tau2+(4*la*lb*mh*tau2+(2*(lb)^(2)*mh*tau2+(-2*la*lb*m*tau1*cos(q2)+(-2*(lb)^(2)*m*tau1*cos(q2)+(4*la*lb*m*tau2*cos(q2)+(4*(lb)^(2)*m*tau2*cos(q2)+(-4*g*Iz*la*m*sin(q1)+(-2*g*Iz*lb*m*sin(q1)+(-3*g*la*(lb)^(2)*(m)^(2)*sin(q1)+(-1*g*(lb)^(3)*(m)^(2)*sin(q1)+(-2*g*Iz*la*mh*sin(q1)+(-2*g*Iz*lb*mh*sin(q1)+(-2*g*la*(lb)^(2)*m*mh*sin(q1)+(-2*g*(lb)^(3)*m*mh*sin(q1)+(2*g*Iz*lb*m*cos(q2)*sin(q1)+(-2*g*la*(lb)^(2)*(m)^(2)*cos(q2)*sin(q1)+(g*la*(lb)^(2)*(m)^(2)*(cos(q2))^(2)*sin(q1)+(g*(lb)^(3)*(m)^(2)*(cos(q2))^(2)*sin(q1)+(2*g*Iz*lb*m*cos(q1)*sin(q2)+(4*g*(la)^(2)*lb*(m)^(2)*cos(q1)*sin(q2)+(4*g*la*(lb)^(2)*(m)^(2)*cos(q1)*sin(q2)+(2*g*(lb)^(3)*m*mh*cos(q1)*sin(q2)+(4*g*la*(lb)^(2)*m*mh*cos(q1)*sin(q2)+(2*g*(lb)^(3)*m*mh*cos(q1)*sin(q2)+(-1*g*la*(lb)^(2)*(m)^(2)*sin(q1)*(sin(q2))^(2)+(-1*g*(lb)^(3)*(m)^(2)*sin(q1)*(sin(q2))^(2)+(g*la*(lb)^(2)*(m)^(2)*cos(q1)*sin(2*q2)+(g*(lb)^(3)*(m)^(2)*cos(q1)*sin(2*q2)+(-2*lb*(la+lb)*m*(2*Iz+(2*(la)^(2)*m+(2*la*lb*m+(2*(lb)^(2)*m+(la)^(2)*mh+(2*la*lb*mh+(lb)^(2)*mh+2*lb*(la+lb)*m*cos(q2)))))))*sin(q2)*(q1d)^(2)+(-4*lb*(la+lb)*m*(Iz+(lb)^(2)*m+lb*(la+lb)*m*cos(q2)))*sin(q2)*q1d*q2d+-2*lb*(la+lb)*m*(Iz+(lb)^(2)*m+lb*(la+lb)*m*cos(q2)))*sin(q2)*(q2d)^(2))))))))))))))))))))))))))))))))))))))))
```

HIP

HurUnifyTriadsCoord[HIP, n]

a2 (la + lb)

{-(la + lb) Sin[q1[t]], (la + lb) Cos[q1[t]], 0, n}

HurUnifyTriadsCoord[FOOTSW, n] // MatrixForm

HurUnifyTriadsCoord[FOOTSW, d] // MatrixForm

$$\begin{pmatrix} (la + lb) (-\sin[q1[t]] + \sin[q2[t]]) \\ (la + lb) (\cos[q1[t]] - \cos[q2[t]]) \\ 0 \\ n \end{pmatrix}$$

$$\begin{pmatrix} (la + lb) (-\sin[\gamma + q1[t]] + \sin[\gamma + q2[t]]) \\ (la + lb) (\cos[\gamma + q1[t]] - \cos[\gamma + q2[t]]) \\ 0 \\ d \end{pmatrix}$$

HurToJulia[HurUnifyTriadsCoord[FOOTSW, n][[1]]]

(la+lb)*(-1*sin(q1)+sin(q2))

```
HurToJulia[HurUnifyTriadsCoord[FOOTSW, n][[2]]]
```

```
(la+lb)*(cos(q1)+-1*cos(q2))
```

```
HurToJulia[HurUnifyTriadsCoord[HIP, n][[1]]]
```

```
-1*(la+lb)*sin(q1)
```

```
HurToJulia[HurUnifyTriadsCoord[HIP, n][[2]]]
```

```
(la+lb)*cos(q1)
```

```
HurToJulia[HurUnifyTriadsCoord[FOOTSW, d][[1]]]
```

```
(la+lb)*(-1*sin(gamma+q1))+sin(gamma+q2))
```

```
HurToJulia[HurUnifyTriadsCoord[FOOTSW, d][[2]]]
```

```
(la+lb)*(cos(gamma+q1)+-1*cos(gamma+q2))
```

```
HurGlobalEEquation[[1]]
```

```
HurToJulia[%]
```

```
-tau1+tau2-2 g la m Sin[q1[t]]-g lb m Sin[q1[t]]-g la mh Sin[q1[t]]-  
g lb mh Sin[q1[t]]-lb (la+lb) m Sin[q1[t]-q2[t]] q2'[t]^2+  
(Iz+2 la lb (m+mh)+lb^2 (m+mh)+la^2 (2 m+mh)) q1''[t]-  
la lb m Cos[q1[t]-q2[t]] q2''[t]-lb^2 m Cos[q1[t]-q2[t]] q2''[t]
```

```
(-1*tau1+(tau2+(-2*g*la*m*sin(q1)+(-1*g*lb*m*sin(q1)+(-1*g*la*mh*sin(q1)+(-1*g*lb*  
mh*sin(q1)+(-1*lb*(la+lb)*m*sin((q1+-1*q2))* (q2d)^2)+(Iz+(2*la*lb*(m+mh)+(lb)  
^2)*(m+mh)+(la)^2*(2*m+mh)))))*q1dd+(-1*la*lb*m*cos((q1+-1*q2))*q2dd+-1*(lb)^  
2)*m*cos((q1+-1*q2))*q2dd))))))
```

```
Clear[q1dd, q2dd]
```

```
HurGlobalEEquation[[2]]
```

```
HurToJulia[%]
```

```
-tau2+g lb m Sin[q2[t]]+lb (la+lb) m Sin[q1[t]-q2[t]] q1'[t]^2-  
lb (la+lb) m Cos[q1[t]-q2[t]] q1''[t]+Iz q2''[t]+lb^2 m q2''[t]
```

```
(-1*tau2+(g*lb*m*sin(q2)+(lb*(la+lb)*m*sin((q1+-1*q2))* (q1d)^2)+(-1*lb*(la+lb)*m*  
cos((q1+-1*q2))*q1dd+(Iz*q2dd+(lb)^2)*m*q2dd))))))
```

```
HurGlobalEEquation // MatrixForm
```

$$\begin{pmatrix} -\tau_1 - g l_a m \sin[q_1[t]] - g (l_a + l_b) m h \sin[q_1[t]] + g m (- (l_a + l_b) \sin[q_1[t]] - l_b \sin[q_1[t] - q_2[t]]) \\ -\tau_2 + g l_b m \sin[q_2[t]] + l_b (l_a + l_b) m \sin[q_1[t] - q_2[t]] q_1'[t]^2 - l_b (l_a + l_b) m \cos[q_1[t] - q_2[t]] q_1''[t] + I_z q_2''[t] + l_b^2 m q_2''[t] \end{pmatrix}$$

```
temp = HurGlobalEEquation[[2]] /. {
```

```
q1[t] -> q1, q2[t] -> q2, q1'[t] -> q1d, q2'[t] -> q2d, q1''[t] -> q1dd, q2''[t] -> q2dd}
```

```
Iz q2dd + lb^2 m q2dd - tau2 + q1dd (Iz + lb^2 m + lb (la + lb) m Cos[q2]) +  
lb (la + lb) m q1d^2 Sin[q2] - g lb m Sin[q1 + q2]
```

```
f[q1_, q2_, q1d_, q2d_, q1dd_, q2dd_] = temp
```

```
Iz q2dd + lb^2 m q2dd - tau2 + q1dd (Iz + lb^2 m + lb (la + lb) m Cos[q2]) +  
lb (la + lb) m q1d^2 Sin[q2] - g lb m Sin[q1 + q2]
```


f[1, 1, 1, 1, 1, 1]

$2 I z + 2 l b^2 m - \tau a u^2 + l b (l a + l b) m \cos[1] + l b (l a + l b) m \sin[1] - g l b m \sin[2]$

g[q1_, q2_] := q1 + q2

g[1, 2]

3

HurGlobalMMatrix

HurToMatlab[%]

$\left\{ \left\{ 2 I z + l a^2 m + (l a + l b)^2 m h + m \left((l b + (l a + l b) \cos[q_2[t]])^2 + (l a + l b)^2 \sin[q_2[t]]^2 \right), \right. \right.$
 $\left. I z + l b m (l b + (l a + l b) \cos[q_2[t]]) \right\}, \left\{ I z + l b^2 m + l b (l a + l b) m \cos[q_2[t]], I z + l b^2 m \right\} \right\}$
 $\left[(2 * I z + ((l a)^{(2)} * m + ((l a + l b)^{(2)} * m h + m * ((l b + (l a + l b) * \cos(q_2)))^{(2)} + ((l a + l b))^{(2)} * (\sin(q_2))^{(2)}))) , (I z + l b * m * (l b + (l a + l b) * \cos(q_2))) ; (I z + ((l b)^{(2)} * m + l b * (l a + l b) * m * \cos(q_2))) , (I z + (l b)^{(2)} * m)] \right]$

HurGlobalMMatrix

HurToJulia[%]

$\left\{ \left\{ I z + 2 l a l b (m + m h) + l b^2 (m + m h) + l a^2 (2 m + m h), -l b (l a + l b) m \cos[q_1[t] - q_2[t]] \right\}, \right.$
 $\left. \left\{ -l b (l a + l b) m \cos[q_1[t] - q_2[t]], I z + l b^2 m \right\} \right\}$
 $\left[(I z + (2 * l a * l b * (m + m h) + ((l b)^{(2)} * (m + m h) + (l a)^{(2)} * (2 * m + m h)))) \right.$
 $\left. -1 * l b * (l a + l b) * m * \cos((q_1 + -1 * q_2)) ; -1 * l b * (l a + l b) * m * \cos((q_1 + -1 * q_2)) (I z + (l b)^{(2)} * m) \right]$

HurGlobalCMatrix

HurToJulia[%]

$\left\{ \left\{ 0, -l b (l a + l b) m \sin[q_1[t] - q_2[t]] q_2'[t] \right\}, \left\{ l b (l a + l b) m \sin[q_1[t] - q_2[t]] q_1'[t], 0 \right\} \right\}$
 $\left[0 -1 * l b * (l a + l b) * m * \sin((q_1 + -1 * q_2)) * q_2 d ; l b * (l a + l b) * m * \sin((q_1 + -1 * q_2)) * q_1 d \ 0 \right]$

HurGlobalGVector

HurToJulia[HurList2Column[%]]

$\left\{ -g (l b (m + m h) + l a (2 m + m h)) \sin[q_1[t]], g l b m \sin[q_2[t]] \right\}$
 $\left[-1 * g * (l b * (m + m h) + l a * (2 * m + m h)) * \sin(q_1) ; g * l b * m * \sin(q_2) \right]$