In[1]:=
```
Quit[];
```

In[1]:=
```
SetDirectory[NotebookDirectory[]]
<< HurToolbox.m
```

Out[1]= D:\Pilwon\Dropbox\TAMU\Courses\MEEN404\Fall2020\experiment1\proposal\graded

```
HurToolbox for modeling and analysis of multibody systems 2.0.5.
HurToolbox mainly uses vector manipulation (vectors, dyadics).
Coordinates and matrix representation of the dyadics are also available.
Available methods: Newton-Euler
   Method, Euler-Lagrange Method, Hamiltonian Method, Kane's Method.
```

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In[3]:=
```
(*To use n, i,j,k*)
(*HurInitialize[]*)
(*To use user-defined Newtonian RF and i,j,k*)
HurInitialize[rf0]
(*To use user-defined Newtonian RF and its own x,y,z*)
(*HurInitialize[rf0,"xyz"]*)

(*HurLoadData["data1.m"]*)
```

We have 7 links. If a,b,c,d,e,f,g are used, we have problems with g which is the gravitational acceleration. So, let's use rf1..

In[4]:=
```
HurDefineRF[rf1, rf2, rf3]
HurDefineGeneralizedCoordinates[q1[t]]
HurDefineDCM[rf1, q1[t], {0, 0, 1}]
```

In[7]:=
```
COM3 = -L1 i1;
COM2 = L2 i1;
COM1 = COM3 - (L1 + L2) / 2 i1 // Simplify
```

Out[9]= $-\frac{1}{2}\ \text{i1}\ (3\ \text{L1} + \text{L2})$

In[10]:=
```
HurDefineCOMPos[rf1, COM1]
HurDefineCOMPos[rf2, COM2]
HurDefineCOMPos[rf3, COM3]
```

In[13]:=
```
HurKinematics[]
```

In[14]:=
```
HurDefineMass[rf1, M1]
HurDefineMass[rf2, M2]
HurDefineMass[rf3, M3]
HurDefineInertia[rf1, {0, 0, 0, 0, 0, I1}]
```

In[18]:=
```
HurDefineVertical[j0]
```

In[19]:=
```
HurResetForces[]
HurResetMoments[]
```

```
(*HurDefineForces[rf1,Rx i0+Ry j0+Rx k0,-COM1]
 HurDefineForces[rf1,-M1 g j0,0]
 HurDefineForces[rf1,F2x i0+F2y j0+F2z k0,-COM1+COM2]
 HurDefineForces[rf1,F3x i0+F3y j0+F3z k0,-COM1+COM3]
 HurDefineMoments[rf1,M1x i0+M1y j0+M1z k0]
 HurDefineForces[rf2,-F2x i0-F2y j0-F2z k0,0]
 HurDefineForces[rf2,-M2 g j0,0]
 HurDefineMoments[rf2,M2x i0+M2y j0+M2z k0]
 HurDefineMoments[rf1,-M2x i0-M2y j0-M2z k0]
 HurDefineForces[rf3,-F3x i0-F3y j0-F3z k0,0]
 HurDefineForces[rf3,-M3 g j0,0]
 HurDefineMoments[rf3,M3x i0+M3y j0+M3z k0]
 HurDefineMoments[rf1,-M3x i0-M3y j0-M3z k0]*)
```

In[21]:=
```
HurDefineForces[rf1, Rx i0 + Ry j0, -COM1]
HurDefineForces[rf1, -M1 g j0, 0]
HurDefineForces[rf1, F2x i1 + F2y j1, -COM1 + COM2]
HurDefineForces[rf1, F3x i1 + F3y j1, -COM1 + COM3]
HurDefineMoments[rf1, M1z k0]
HurDefineForces[rf2, -F2x i1 - F2y j1, 0]
HurDefineForces[rf2, -M2 g j0, 0]
(*HurDefineMoments[rf2,M2z k0]
 HurDefineMoments[rf1,-M2z k0]*)
HurDefineForces[rf3, -F3x i1 - F3y j1, 0]
HurDefineForces[rf3, -M3 g j0, 0]
(*HurDefineMoments[rf3,M3z k0]
 HurDefineMoments[rf1,-M3z k0]*)
```

In[30]:=
```
HurNEEquation[]
```

Out[30]=
```
{{}, {F2x + F3x + Rx Cos[q1[t]] + (-g M1 + Ry) Sin[q1[t]] - 1/2 (3 L1 + L2) M1 q1′[t]^2,
  F2y + F3y + (-g M1 + Ry) Cos[q1[t]] - Rx Sin[q1[t]] + 1/2 (3 L1 + L2) M1 q1″[t],
  0, 0, 0, 1/2 (3 F2y (L1 + L2) + F3y (L1 + L2) + 2 M1z +
      (3 L1 + L2) (Ry Cos[q1[t]] - Rx Sin[q1[t]]) - 2 I1 q1″[t])},
 {-F2x Cos[q1[t]] + F2y Sin[q1[t]] + L2 M2 Cos[q1[t]] q1′[t]^2 + L2 M2 Sin[q1[t]] q1″[t],
  -g M2 - F2y Cos[q1[t]] - F2x Sin[q1[t]] +
   L2 M2 Sin[q1[t]] q1′[t]^2 - L2 M2 Cos[q1[t]] q1″[t], 0, 0, 0, 0},
 {-F3x Cos[q1[t]] + F3y Sin[q1[t]] - L1 M3 (Cos[q1[t]] q1′[t]^2 + Sin[q1[t]] q1″[t]),
  -g M3 - F3y Cos[q1[t]] - F3x Sin[q1[t]] -
   L1 M3 Sin[q1[t]] q1′[t]^2 + L1 M3 Cos[q1[t]] q1″[t], 0, 0, 0, 0}}
```

In[58]:=
```
HurGlobalNEEquation // MatrixForm
```

Out[58]//MatrixForm=
```
( {F2x + F3x + (F2x + F3x + 2 Rx) Cos[q1[t]] + (F2y + F3y - 2 g M1 + 2 Ry) Sin[q1[t]] - 1/2 (3 L1 + L2) M
```

In[59]:= **HurGlobalNEEquation[[2]] // MatrixForm**

Out[59]//MatrixForm=

```
F2x + F3x + (F2x + F3x + 2 Rx) Cos[q1[t]] + (F2y + F3y - 2
F2y + F3y + (F2y + F3y - 2 g M1 + 2 Ry) Cos[q1[t]] - (F2x +
0
0
0
1/2 (3 F2y L1 + F3y L1 + 3 F2y L2 + F3y L2 + 4 M1z + (3 F2y (L1 + L2) + F3y (L1 + L2) + 2 (3 L1 + L2) Ry) C
```

In[60]:= **HurGlobalNEEquation[[3]] // MatrixForm**

Out[60]//MatrixForm=

```
-F2x - F2x Cos[q1[t]] + F2y Sin[q1[t]] + L2 M2 Cos[q1[t]] q1′[t]^2 + L2 M2 Sin[q1[t]] q1″[t
-F2y - 2 g M2 - F2y Cos[q1[t]] - F2x Sin[q1[t]] + L2 M2 Sin[q1[t]] q1′[t]^2 - L2 M2 Cos[q1[t]] q1
0
0
0
0
```

In[61]:= **HurGlobalNEEquation[[4]] // MatrixForm**

Out[61]//MatrixForm=

```
-F3x - F3x Cos[q1[t]] + F3y Sin[q1[t]] - L1 M3 (Cos[q1[t]] q1′[t]^2 + Sin[q1[t]] q1″[t])
-F3y - 2 g M3 - F3y Cos[q1[t]] - F3x Sin[q1[t]] - L1 M3 Sin[q1[t]] q1′[t]^2 + L1 M3 Cos[q1[t]] q1
0
0
0
0
```

In[43]:= **HurDefineVariableList[Rx, Ry, F2x, F2y, F3x, F3y, M1z]**

In[44]:= **Sol = HurSolveNEInverse[]**

Out[44]=

$$\{\{Rx \to \frac{1}{2}(3\,L1\,M1 + L2\,M1 - 2\,L2\,M2 + 2\,L1\,M3)(Cos[q1[t]]\,q1'[t]^2 + Sin[q1[t]]\,q1''[t]),$$

$$Ry \to \frac{1}{2}(2\,g\,(M1 + M2 + M3) + (3\,L1\,M1 + L2\,M1 - 2\,L2\,M2 + 2\,L1\,M3)\,Sin[q1[t]]\,q1'[t]^2 - (3\,L1\,M1 + L2\,M1 - 2\,L2\,M2 + 2\,L1\,M3)\,Cos[q1[t]]\,q1''[t]),$$

$$F2x \to -g\,M2\,Sin[q1[t]] + L2\,M2\,q1'[t]^2,\ F2y \to -M2\,(g\,Cos[q1[t]] + L2\,q1''[t]),$$

$$F3x \to -M3\,(g\,Sin[q1[t]] + L1\,q1'[t]^2),\ F3y \to -g\,M3\,Cos[q1[t]] + L1\,M3\,q1''[t],$$

$$M1z \to \frac{1}{4}(-2\,g\,(3\,L1\,M1 + L2\,M1 - 2\,L2\,M2 + 2\,L1\,M3)\,Cos[q1[t]] + (4\,I1 + 6\,L1\,L2\,M1 + L2^2\,(M1 + 4\,M2) + L1^2\,(9\,M1 + 4\,M3))\,q1''[t])\}\}$$

```
In[64]:= Grad[Flatten[HurGlobalNEEquation], {Rx, Ry, F2x, F2y, F3x, F3y, M1z}]
         MatrixRank[%]
```

Out[64]=
$$\{\{2\,\mathrm{Cos}[q1[t]],\ 2\,\mathrm{Sin}[q1[t]],\ 1+\mathrm{Cos}[q1[t]],\ \mathrm{Sin}[q1[t]],\ 1+\mathrm{Cos}[q1[t]],\ \mathrm{Sin}[q1[t]],\ 0\},\ \{-2\,\mathrm{Sin}[q1[t]],\ 2\,\mathrm{Cos}[q1[t]],\ -\mathrm{Sin}[q1[t]],\ 1+\mathrm{Cos}[q1[t]],\ -\mathrm{Sin}[q1[t]],\ 1+\mathrm{Cos}[q1[t]],\ 0\},\ \{0,0,0,0,0,0,0\},\ \{0,0,0,0,0,0,0\},\ \{0,0,0,0,0,0,0\},\ \Big\{-\left(3\,L1+L2\right)\mathrm{Sin}[q1[t]],\ \left(3\,L1+L2\right)\mathrm{Cos}[q1[t]],\ -\frac{3}{2}\left(L1+L2\right)\mathrm{Sin}[q1[t]],\ \frac{1}{2}\left(3\,L1+3\,L2+3\left(L1+L2\right)\mathrm{Cos}[q1[t]]\right),\ -\frac{1}{2}\left(L1+L2\right)\mathrm{Sin}[q1[t]],\ \frac{1}{2}\left(L1+L2+\left(L1+L2\right)\mathrm{Cos}[q1[t]]\right),\ 2\Big\},\ \{0,0,-1-\mathrm{Cos}[q1[t]],\ \mathrm{Sin}[q1[t]],0,0,0\},\ \{0,0,-\mathrm{Sin}[q1[t]],\ -1-\mathrm{Cos}[q1[t]],0,0,0\},\ \{0,0,0,0,0,0,0\},\ \{0,0,0,0,0,0,0\},\ \{0,0,0,0,0,0,0\},\ \{0,0,0,0,0,0,0\},\ \{0,0,0,0,-1-\mathrm{Cos}[q1[t]],\ \mathrm{Sin}[q1[t]],0\},\ \{0,0,0,0,-\mathrm{Sin}[q1[t]],\ -1-\mathrm{Cos}[q1[t]],0\},\ \{0,0,0,0,0,0,0\},\ \{0,0,0,0,0,0,0\},\ \{0,0,0,0,0,0,0\},\ \{0,0,0,0,0,0,0\}\}$$

Out[65]= 7

```
In[45]:= F2x /. Flatten[Sol]
         F2y /. Flatten[Sol]
         F3x /. Flatten[Sol]
         F3y /. Flatten[Sol]
```

Out[45]= $-g\,M2\,\mathrm{Sin}[q1[t]]+L2\,M2\,q1'[t]^2$

Out[46]= $-M2\left(g\,\mathrm{Cos}[q1[t]]+L2\,q1''[t]\right)$

Out[47]= $-M3\left(g\,\mathrm{Sin}[q1[t]]+L1\,q1'[t]^2\right)$

Out[48]= $-g\,M3\,\mathrm{Cos}[q1[t]]+L1\,M3\,q1''[t]$

In[49]:=
```
HurDefineVariableList[Rx, Ry, F2x, F2y, F3x, F3y, q1''[t]]
Sol = HurSolveNEInverse[]
```

Out[50]=

$$\Big\{\Big\{Rx \to \Big(\big(3\,L1\,M1 + L2\,M1 - 2\,L2\,M2 + 2\,L1\,M3\big)\Big(2\,\big(2\,M1z + g\,\big(3\,L1\,M1 + L2\,M1 - 2\,L2\,M2 + 2\,L1\,M3\big)\,\mathrm{Cos}[q1[t]]\big)\,\mathrm{Sin}[q1[t]] + \big(4\,I1 + 6\,L1\,L2\,M1 + L2^2\,(M1 + 4\,M2) + L1^2\,(9\,M1 + 4\,M3)\big)\,\mathrm{Cos}[q1[t]]\,q1'[t]^2\Big)\Big) \Big/ \Big(2\,\big(4\,I1 + 6\,L1\,L2\,M1 + L2^2\,(M1 + 4\,M2) + L1^2\,(9\,M1 + 4\,M3)\big)\Big),$$

$$Ry \to \Big(-4\,M1z\,\big(3\,L1\,M1 + L2\,M1 - 2\,L2\,M2 + 2\,L1\,M3\big)\,\mathrm{Cos}[q1[t]] + g\,\Big(9\,L1^2\,M1^2 + 6\,L1\,L2\,M1^2 + L2^2\,M1^2 + 18\,L1^2\,M1\,M2 + 24\,L1\,L2\,M1\,M2 + 14\,L2^2\,M1\,M2 + 4\,L2^2\,M2^2 + 14\,L1^2\,M1\,M3 + 8\,L1\,L2\,M1\,M3 + 2\,L2^2\,M1\,M3 + 8\,L1^2\,M2\,M3 + 8\,L1\,L2\,M2\,M3 + 8\,L2^2\,M2\,M3 + 4\,L1^2\,M3^2 + 8\,I1\,(M1 + M2 + M3) - \big(3\,L1\,M1 + L2\,M1 - 2\,L2\,M2 + 2\,L1\,M3\big)^2\,\mathrm{Cos}[2\,q1[t]]\Big) + \big(3\,L1\,M1 + L2\,M1 - 2\,L2\,M2 + 2\,L1\,M3\big)\,\big(4\,I1 + 6\,L1\,L2\,M1 + L2^2\,(M1 + 4\,M2) + L1^2\,(9\,M1 + 4\,M3)\big)\,\mathrm{Sin}[q1[t]]\,q1'[t]^2\Big) \Big/ \Big(2\,\big(4\,I1 + 6\,L1\,L2\,M1 + L2^2\,(M1 + 4\,M2) + L1^2\,(9\,M1 + 4\,M3)\big)\Big),\ F2x \to M2\,\big(-g\,\mathrm{Sin}[q1[t]] + L2\,q1'[t]^2\big),$$

$$F2y \to -\frac{M2\,\big(4\,L2\,M1z + g\,\big(4\,I1 + (L1 + L2)\,(9\,L1\,M1 + 3\,L2\,M1 + 4\,L1\,M3)\big)\,\mathrm{Cos}[q1[t]]\big)}{4\,I1 + 6\,L1\,L2\,M1 + L2^2\,(M1 + 4\,M2) + L1^2\,(9\,M1 + 4\,M3)},$$

$$F3x \to -M3\,\big(g\,\mathrm{Sin}[q1[t]] + L1\,q1'[t]^2\big),$$

$$F3y \to -\frac{M3\,\big(-4\,L1\,M1z + g\,\big(4\,I1 + (L1 + L2)\,(3\,L1\,M1 + L2\,M1 + 4\,L2\,M2)\big)\,\mathrm{Cos}[q1[t]]\big)}{4\,I1 + 6\,L1\,L2\,M1 + L2^2\,(M1 + 4\,M2) + L1^2\,(9\,M1 + 4\,M3)},$$

$$q1''[t] \to \frac{4\,M1z + 2\,g\,\big(3\,L1\,M1 + L2\,M1 - 2\,L2\,M2 + 2\,L1\,M3\big)\,\mathrm{Cos}[q1[t]]}{4\,I1 + 6\,L1\,L2\,M1 + L2^2\,(M1 + 4\,M2) + L1^2\,(9\,M1 + 4\,M3)}\Big\}\Big\}$$

In[51]:=
```
F2x /. Flatten[Sol] /. M1z → 0
F2y /. Flatten[Sol] /. M1z → 0
F3x /. Flatten[Sol] /. M1z → 0
F3y /. Flatten[Sol] /. M1z → 0
q1''[t] /. Flatten[Sol] /. M1z → 0
```

Out[51]= $M2\,\big(-g\,\mathrm{Sin}[q1[t]] + L2\,q1'[t]^2\big)$

Out[52]=

$$-\frac{g\,M2\,\big(4\,I1 + (L1 + L2)\,(9\,L1\,M1 + 3\,L2\,M1 + 4\,L1\,M3)\big)\,\mathrm{Cos}[q1[t]]}{4\,I1 + 6\,L1\,L2\,M1 + L2^2\,(M1 + 4\,M2) + L1^2\,(9\,M1 + 4\,M3)}$$

Out[53]= $-M3\,\big(g\,\mathrm{Sin}[q1[t]] + L1\,q1'[t]^2\big)$

Out[54]=

$$-\frac{g\,\big(4\,I1 + (L1 + L2)\,(3\,L1\,M1 + L2\,M1 + 4\,L2\,M2)\big)\,M3\,\mathrm{Cos}[q1[t]]}{4\,I1 + 6\,L1\,L2\,M1 + L2^2\,(M1 + 4\,M2) + L1^2\,(9\,M1 + 4\,M3)}$$

Out[55]=

$$\frac{2\,g\,\big(3\,L1\,M1 + L2\,M1 - 2\,L2\,M2 + 2\,L1\,M3\big)\,\mathrm{Cos}[q1[t]]}{4\,I1 + 6\,L1\,L2\,M1 + L2^2\,(M1 + 4\,M2) + L1^2\,(9\,M1 + 4\,M3)}$$

In[73]:=
```
HurUnifyTriads[COM1, rf0]
HurUnifyTriads[COM1, rf1]
```

Out[73]= $-\frac{1}{2}\,i0\,(3\,L1 + L2)\,\mathrm{Cos}[q1[t]] - \frac{1}{2}\,j0\,(3\,L1 + L2)\,\mathrm{Sin}[q1[t]]$

Out[74]= $\frac{1}{2}\,i1\,(-3\,L1 - L2)$