

```

In[63]:= Quit[];

In[1]:= SetDirectory[NotebookDirectory[]];
<< HurToolbox.m

HurToolbox for modeling and analysis of multibody systems 1.0.0.
HurToolbox mainly uses vector manipulation (vectors, dyadics).
Coordinates and matrix representation of the dyadics are also available.
Available methods: Newton-Euler
    Method, Euler-Lagrange Method, Hamiltonian Method, Kane's Method.
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Email questions, comments, or concerns to pilwonhur@tamu.edu.

In[3]:= HurInitialize[]

HurLoadData["data_impactA_abs.m"]

In[4]:= HurDefineRF[a, b, c, d]
HurDefineGeneralizedCoordinates[q1[t], q2[t], q3[t], q4[t]]
(*q3, q4 are the x, y position of stance foot*)
HurDefineDCM[a, q1[t], {0, 0, 1}]
HurDefineDCM[b, q2[t], {0, 0, 1}]
HurDefineDCM[d, -gamma, {0, 0, 1}]

In[9]:= FOOTST = q3[t] n1 + q4[t] n2;
COMA = FOOTST + la a2; (*COM of stance leg*)
COMB = FOOTST + (la + lb) a2 - lb b2; (*COM of swing leg*)
COMC = FOOTST + (la + lb) a2; (*Hip*)
HIP = COMC;
FOOTSW = FOOTST + (la + lb) a2 - (la + lb) b2; (*Swing foot position*)

In[13]:= HurDefineCOMPos[a, COMA];
HurDefineCOMPos[b, COMB];
HurDefineCOMPos[c, COMC];

In[16]:= HurKinematics[]

```

```
In[17]:= HurGlobalCOMVel
HurGlobalCOMAcc
HurGlobalAngularVel
HurGlobalAngularAcc
```

```
Out[17]= {0,
  -a1 l1 q1'[t] + (a1 Cos[q1[t]] - a2 Sin[q1[t]]) q3'[t] + (a2 Cos[q1[t]] + a1 Sin[q1[t]]) q4'[t],
  - (l1 + l2) (b1 Cos[q1[t] - q2[t]] + b2 Sin[q1[t] - q2[t]]) q1'[t] + b1 l2 q2'[t] +
    b1 Cos[q2[t]] q3'[t] - b2 Sin[q2[t]] q3'[t] + b2 Cos[q2[t]] q4'[t] + b1 Sin[q2[t]] q4'[t],
  - (l1 + l2) (c1 Cos[q1[t]] + c2 Sin[q1[t]]) q1'[t] + c1 q3'[t] + c2 q4'[t], 0}
```

```
Out[18]= {0, -a2 l1 q1'[t]^2 - a1 l1 q1''[t] + a1 Cos[q1[t]] q3''[t] -
  a2 Sin[q1[t]] q3''[t] + a2 Cos[q1[t]] q4''[t] + a1 Sin[q1[t]] q4''[t],
  b2 (- (l1 + l2) Cos[q1[t] - q2[t]] q1'[t]^2 + l2 q2'[t]^2 - l1 Sin[q1[t] - q2[t]] q1''[t] -
    l2 Sin[q1[t] - q2[t]] q1''[t] - Sin[q2[t]] q3''[t] + Cos[q2[t]] q4''[t]) +
  b1 ((l1 + l2) Sin[q1[t] - q2[t]] q1'[t]^2 - (l1 + l2) Cos[q1[t] - q2[t]] q1''[t] +
    l2 q2''[t] + Cos[q2[t]] q3''[t] + Sin[q2[t]] q4''[t]),
  - (l1 + l2) (c2 Cos[q1[t]] - c1 Sin[q1[t]]) q1'[t]^2 -
  (l1 + l2) (c1 Cos[q1[t]] + c2 Sin[q1[t]]) q1''[t] + c1 q3''[t] + c2 q4''[t], 0}
```

```
Out[19]= {0, n3 q1'[t], n3 q2'[t], 0, 0}
```

```
Out[20]= {0, a3 q1''[t], b3 q2''[t], 0, 0}
```

```
In[21]:= HurDefineMass[a, m];
HurDefineMass[b, m];
HurDefineMass[c, mh];
HurDefineInertia[a, {0, 0, 0, 0, 0, Iz}];
HurDefineInertia[b, {0, 0, 0, 0, 0, Iz}];
```

```
In[26]:= HurDefineVertical[n2];
```

```
In[27]:= Jaf = HurGetJacobian[FOOTST, a, n]
Jah = HurGetJacobian[HIP, a, n]
Jbh = HurGetJacobian[HIP, b, n]
```

```
Out[27]= {{0, 0, 1, 0}, {0, 0, 0, 1}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {1, 0, 0, 0}}
```

```
Out[28]= {{-(l1 + l2) Cos[q1[t]], 0, 1, 0}, {-(l1 + l2) Sin[q1[t]], 0, 0, 1},
  {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {1, 0, 0, 0}}
```

```
Out[29]= {{-(l1 + l2) Cos[q1[t]], 0, 1, 0}, {-(l1 + l2) Sin[q1[t]], 0, 0, 1},
  {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 1, 0, 0}}
```

```
In[30]:= Transpose[Jaf].HurList2Column[{0, 0, 0, 0, 0, tau1}] +
  Transpose[Jah].HurList2Column[{0, 0, 0, 0, 0, -tau2}] +
  Transpose[Jbh].HurList2Column[{0, 0, 0, 0, 0, tau2}] // MatrixForm
```

```
Out[30]//MatrixForm=
```

$$\begin{pmatrix} \tau_1 - \tau_2 \\ \tau_2 \\ 0 \\ 0 \end{pmatrix}$$

```
In[31]:= HurDefineNonConservativeForces[tau1 - tau2, tau2, 0, 0]
```

```
Out[31]:= {tau1 - tau2, tau2, 0, 0}
```

```
In[32]:= HurELEquation[]
```

```
Out[32]:= {-tau1 + tau2 - 2 g l a m Sin[q1[t]] - g l b m Sin[q1[t]] -  
  g l a m h Sin[q1[t]] - g l b m h Sin[q1[t]] - l b (l a + l b) m Sin[q1[t] - q2[t]] q2'[t]^2 +  
  (Iz + 2 l a l b (m + m h) + l b^2 (m + m h) + l a^2 (2 m + m h)) q1''[t] - l a l b m Cos[q1[t] - q2[t]] q2''[t] -  
  l b^2 m Cos[q1[t] - q2[t]] q2''[t] - 2 l a m Cos[q1[t]] q3''[t] - l b m Cos[q1[t]] q3''[t] -  
  l a m h Cos[q1[t]] q3''[t] - l b m h Cos[q1[t]] q3''[t] - 2 l a m Sin[q1[t]] q4''[t] -  
  l b m Sin[q1[t]] q4''[t] - l a m h Sin[q1[t]] q4''[t] - l b m h Sin[q1[t]] q4''[t],  
 -tau2 + g l b m Sin[q2[t]] + l b (l a + l b) m Sin[q1[t] - q2[t]] q1'[t]^2 -  
  l b (l a + l b) m Cos[q1[t] - q2[t]] q1''[t] + Iz q2''[t] +  
  l b^2 m q2''[t] + l b m Cos[q2[t]] q3''[t] + l b m Sin[q2[t]] q4''[t],  
 (l b (m + m h) + l a (2 m + m h)) Sin[q1[t]] q1'[t]^2 - l b m Sin[q2[t]] q2'[t]^2 -  
  2 l a m Cos[q1[t]] q1''[t] - l b m Cos[q1[t]] q1''[t] - l a m h Cos[q1[t]] q1''[t] -  
  l b m h Cos[q1[t]] q1''[t] + l b m Cos[q2[t]] q2''[t] + 2 m q3''[t] + m h q3''[t],  
 2 g m + g m h - (l b (m + m h) + l a (2 m + m h)) Cos[q1[t]] q1'[t]^2 + l b m Cos[q2[t]] q2'[t]^2 -  
  2 l a m Sin[q1[t]] q1''[t] - l b m Sin[q1[t]] q1''[t] - l a m h Sin[q1[t]] q1''[t] -  
  l b m h Sin[q1[t]] q1''[t] + l b m Sin[q2[t]] q2''[t] + 2 m q4''[t] + m h q4''[t]}
```

```
In[33]:= HurGlobalMMatrix // MatrixForm
```

```
Out[33]//MatrixForm=
```

$$\begin{pmatrix} Iz + 2 l a l b (m + m h) + l b^2 (m + m h) + l a^2 (2 m + m h) & -l b (l a + l b) m \cos[q_1[t] - q_2[t]] & -l b (m + m h) \\ -l b (l a + l b) m \cos[q_1[t] - q_2[t]] & Iz + l b^2 m & 0 \\ -l b (m + m h) + l a (2 m + m h) \cos[q_1[t]] & l b m \cos[q_2[t]] & 0 \\ -l b (m + m h) + l a (2 m + m h) \sin[q_1[t]] & l b m \sin[q_2[t]] & 0 \end{pmatrix}$$

```
In[34]:= HurGlobalCMatrix // MatrixForm
```

```
Out[34]//MatrixForm=
```

$$\begin{pmatrix} 0 & -l b (l a + l b) m \sin[q_1[t] - q_2[t]] q_2'[t] & 0 & 0 \\ l b (l a + l b) m \sin[q_1[t] - q_2[t]] q_1'[t] & 0 & 0 & 0 \\ (l b (m + m h) + l a (2 m + m h)) \sin[q_1[t]] q_1'[t] & -l b m \sin[q_2[t]] q_2'[t] & 0 & 0 \\ -(l b (m + m h) + l a (2 m + m h)) \cos[q_1[t]] q_1'[t] & l b m \cos[q_2[t]] q_2'[t] & 0 & 0 \end{pmatrix}$$

```
In[35]:= HurGlobalGVector // MatrixForm
```

```
Out[35]//MatrixForm=
```

$$\begin{pmatrix} -g (l b (m + m h) + l a (2 m + m h)) \sin[q_1[t]] \\ g l b m \sin[q_2[t]] \\ 0 \\ g (2 m + m h) \end{pmatrix}$$

```
In[36]:= FOOTSW
```

```
Out[36]:= a2 (l a + l b) - b2 (l a + l b) + n1 q3[t] + n2 q4[t]
```

```
In[37]:= Jbfoot = HurGetJacobian[FOOTSW, b, n][[1 ;; 2, ;;]]
```

```
Out[37]:= {{-(l a + l b) Cos[q1[t]], (l a + l b) Cos[q2[t]], 1, 0},  
  {-(l a + l b) Sin[q1[t]], (l a + l b) Sin[q2[t]], 0, 1}}
```

```
In[38]:= ImpactLHS = ArrayFlatten[
  {{HurGlobalMMatrix, -Transpose[Jbfoot]}, {Jbfoot, Table[0, {i, 2}, {j, 2}]}}];
ImpactLHS // MatrixForm
ArrayFlatten[{{HurGlobalMMatrix, -Transpose[Jbfoot]}}] // MatrixForm
Jbfoot // MatrixForm
```

Out[39]//MatrixForm=

$$\begin{pmatrix} Iz + 2 la lb (m + mh) + lb^2 (m + mh) + la^2 (2 m + mh) & -lb (la + lb) m \cos[q1[t] - q2[t]] & - (lb (m + mh) - lb (la + lb) m \cos[q1[t] - q2[t]]) \\ - (lb (m + mh) + la (2 m + mh)) \cos[q1[t]] & lb m \cos[q2[t]] & Iz + lb^2 m \\ - (lb (m + mh) + la (2 m + mh)) \sin[q1[t]] & lb m \sin[q2[t]] & lb m \cos[q2[t]] \\ - (la + lb) \cos[q1[t]] & (la + lb) \cos[q2[t]] & lb m \sin[q2[t]] \\ - (la + lb) \sin[q1[t]] & (la + lb) \sin[q2[t]] & \end{pmatrix} \quad 11$$

Out[40]//MatrixForm=

$$\begin{pmatrix} Iz + 2 la lb (m + mh) + lb^2 (m + mh) + la^2 (2 m + mh) & -lb (la + lb) m \cos[q1[t] - q2[t]] & - (lb (m + mh) - lb (la + lb) m \cos[q1[t] - q2[t]]) \\ - (lb (m + mh) + la (2 m + mh)) \cos[q1[t]] & lb m \cos[q2[t]] & Iz + lb^2 m \\ - (lb (m + mh) + la (2 m + mh)) \sin[q1[t]] & lb m \sin[q2[t]] & lb m \cos[q2[t]] \\ - (la + lb) \cos[q1[t]] & (la + lb) \cos[q2[t]] & lb m \sin[q2[t]] \\ - (la + lb) \sin[q1[t]] & (la + lb) \sin[q2[t]] & \end{pmatrix} \quad 11$$

Out[41]//MatrixForm=

$$\begin{pmatrix} - (la + lb) \cos[q1[t]] & (la + lb) \cos[q2[t]] & 1 & 0 \\ - (la + lb) \sin[q1[t]] & (la + lb) \sin[q2[t]] & 0 & 1 \end{pmatrix}$$

```
In[42]:= HurToJulia[ImpactLHS]
```

```
Out[42]= [ (Iz + (2*la*lb*(m+mh) + (lb)^(2)*(m+mh) + (la)^(2)*(2*m+mh))) )
  -1*lb*(la+lb)*m*cos((q1+-1*q2)) -1*(lb*(m+mh)+la*(2*m+mh))*cos(q1)
  -1*(lb*(m+mh)+la*(2*m+mh))*sin(q1) (la+lb)*cos(q1)
  (la+lb)*sin(q1); -1*lb*(la+lb)*m*cos((q1+-1*q2))
  (Iz + (lb)^(2)*m) lb*m*cos(q2) lb*m*sin(q2) -1*(la+lb)*cos(q2)
  -1*(la+lb)*sin(q2); -1*(lb*(m+mh)+la*(2*m+mh))*cos(q1)
  lb*m*cos(q2) (2*m+mh) 0 -1 0; -1*(lb*(m+mh)+la*(2*m+mh))*sin(q1)
  lb*m*sin(q2) 0 (2*m+mh) 0 -1; -1*(la+lb)*cos(q1) (la+lb)*cos(q2)
  1 0 0 0; -1*(la+lb)*sin(q1) (la+lb)*sin(q2) 0 1 0 0]
```

```
In[43]:= HurGlobalMMatrix.HurList2Column[{q1'[t], q2'[t], 0, 0}]
HurToJulia[%]
```

```
Out[43]= { { (Iz + 2 la lb (m + mh) + lb^2 (m + mh) + la^2 (2 m + mh)) q1'[t] -
  lb (la + lb) m Cos[q1[t] - q2[t]] q2'[t] },
  { -lb (la + lb) m Cos[q1[t] - q2[t]] q1'[t] + (Iz + lb^2 m) q2'[t] },
  { - (lb (m + mh) + la (2 m + mh)) Cos[q1[t]] q1'[t] + lb m Cos[q2[t]] q2'[t] },
  { - (lb (m + mh) + la (2 m + mh)) Sin[q1[t]] q1'[t] + lb m Sin[q2[t]] q2'[t] } }
```

```
Out[44]= [ ((Iz + (2*la*lb*(m+mh) + (lb)^(2)*(m+mh) + (la)^(2)*(2*m+mh))) ) *q1d+-1*lb*(la+lb)*m*cos((
  q1+-1*q2))*q2d); (-1*lb*(la+lb)*m*cos((q1+-1*q2))*q1d+(Iz+(lb)^(2)*m)*q2d); (-1*(lb*(
  m+mh)+la*(2*m+mh))*cos(q1)*q1d+lb*m*cos(q2)*q2d); (-1*(lb*(m+mh)+la*(2*m+mh))*sin(q1)
  *q1d+lb*m*sin(q2)*q2d) ]
```

Expressions for impact dynamics constraints

```
In[45]:= HurGlobalMMatrix // MatrixForm
```

```
Out[45]//MatrixForm=
```

$$\begin{pmatrix} Iz + 2 la lb (m + mh) + lb^2 (m + mh) + la^2 (2 m + mh) & -lb (la + lb) m \cos[q1[t] - q2[t]] & - (lb (m + mh) - lb (la + lb) m \cos[q1[t] - q2[t]]) & Iz + lb^2 m \\ - (lb (m + mh) + la (2 m + mh)) \cos[q1[t]] & lb m \cos[q2[t]] & \\ - (lb (m + mh) + la (2 m + mh)) \sin[q1[t]] & lb m \sin[q2[t]] & \end{pmatrix} \quad 11$$

```
In[46]:= HurGlobalMMatrix
```

```
Out[46]= {{Iz + 2 la lb (m + mh) + lb^2 (m + mh) + la^2 (2 m + mh), -lb (la + lb) m Cos[q1[t] - q2[t]],  
- (lb (m + mh) + la (2 m + mh)) Cos[q1[t]], - (lb (m + mh) + la (2 m + mh)) Sin[q1[t]]},  
{-lb (la + lb) m Cos[q1[t] - q2[t]], Iz + lb^2 m, lb m Cos[q2[t]], lb m Sin[q2[t]]},  
{- (lb (m + mh) + la (2 m + mh)) Cos[q1[t]], lb m Cos[q2[t]], 2 m + mh, 0},  
{- (lb (m + mh) + la (2 m + mh)) Sin[q1[t]], lb m Sin[q2[t]], 0, 2 m + mh}}
```

```
In[47]:= impDynConst1 = ArrayFlatten[{{HurGlobalMMatrix, -Transpose[Jbfoot]}}].  
HurList2Column[{q1dp, q2dp, q3dp, q4dp, F1, F2}] -  
HurGlobalMMatrix.HurList2Column[{q1'[t], q2'[t], 0, 0}] // Simplify
```

```
Out[47]= {{(Iz + 2 la lb (m + mh) + lb^2 (m + mh) + la^2 (2 m + mh)) q1dp + F1 (la + lb) Cos[q1[t]] -  
(lb (m + mh) + la (2 m + mh)) q3dp Cos[q1[t]] - lb (la + lb) m q2dp Cos[q1[t] - q2[t]] +  
F2 (la + lb) Sin[q1[t]] - (lb (m + mh) + la (2 m + mh)) q4dp Sin[q1[t]] -  
(Iz + 2 la lb (m + mh) + lb^2 (m + mh) + la^2 (2 m + mh)) q1'[t] +  
lb (la + lb) m Cos[q1[t] - q2[t]] q2'[t]},  
{(Iz + lb^2 m) q2dp - lb (la + lb) m q1dp Cos[q1[t] - q2[t]] - F1 (la + lb) Cos[q2[t]] +  
lb m q3dp Cos[q2[t]] - F2 (la + lb) Sin[q2[t]] + lb m q4dp Sin[q2[t]] +  
lb (la + lb) m Cos[q1[t] - q2[t]] q1'[t] - (Iz + lb^2 m) q2'[t]},  
{-F1 + (2 m + mh) q3dp - (lb (m + mh) + la (2 m + mh)) q1dp Cos[q1[t]] + lb m q2dp Cos[q2[t]] +  
(lb (m + mh) + la (2 m + mh)) Cos[q1[t]] q1'[t] - lb m Cos[q2[t]] q2'[t]},  
{-F2 + (2 m + mh) q4dp - (lb (m + mh) + la (2 m + mh)) q1dp Sin[q1[t]] + lb m q2dp Sin[q2[t]] +  
(lb (m + mh) + la (2 m + mh)) Sin[q1[t]] q1'[t] - lb m Sin[q2[t]] q2'[t]}}
```

```
In[48]:= impDynConst2 = Jbfoot.HurList2Column[{q1dp, q2dp, q3dp, q4dp}]
```

```
Out[48]= {{q3dp - (la + lb) q1dp Cos[q1[t]] + (la + lb) q2dp Cos[q2[t]]},  
{q4dp - (la + lb) q1dp Sin[q1[t]] + (la + lb) q2dp Sin[q2[t]]}}
```

```
In[49]:= impDynConst1[[1]][[1]]  
HurToJulia[%]
```

```
Out[49]= (Iz + 2 la lb (m + mh) + lb^2 (m + mh) + la^2 (2 m + mh)) q1dp + F1 (la + lb) Cos[q1[t]] -  
(lb (m + mh) + la (2 m + mh)) q3dp Cos[q1[t]] - lb (la + lb) m q2dp Cos[q1[t] - q2[t]] +  
F2 (la + lb) Sin[q1[t]] - (lb (m + mh) + la (2 m + mh)) q4dp Sin[q1[t]] -  
(Iz + 2 la lb (m + mh) + lb^2 (m + mh) + la^2 (2 m + mh)) q1'[t] +  
lb (la + lb) m Cos[q1[t] - q2[t]] q2'[t]
```

```
Out[50]= ((Iz + (2*la*lb*(m+mh) + ((lb)^(2)*(m+mh) + (la)^(2)*(2*m+mh))))*q1dp + (F1*(la+lb)*cos(q1) +  
-1*(lb*(m+mh) + la*(2*m+mh))*q3dp*cos(q1) + (-1*lb*(la+lb)*m*q2dp*cos((q1+(-1*q2)) + (F2*(  
la+lb)*sin(q1) + (-1*(lb*(m+mh) + la*(2*m+mh))*q4dp*sin(q1) + (-1*(Iz + (2*la*lb*(m+mh) + ((  
lb)^(2)*(m+mh) + (la)^(2)*(2*m+mh))))*q1d+lb*(la+lb)*m*cos((q1+(-1*q2))*q2d))))))
```

```

In[51]:= impDynConst1[[2]][[1]]
HurToJulia[%]

Out[51]= (Iz + lb^2 m) q2dp - lb (1a + lb) m q1dp Cos[q1[t] - q2[t]] -
F1 (1a + lb) Cos[q2[t]] + lb m q3dp Cos[q2[t]] - F2 (1a + lb) Sin[q2[t]] +
lb m q4dp Sin[q2[t]] + lb (1a + lb) m Cos[q1[t] - q2[t]] q1'[t] - (Iz + lb^2 m) q2'[t]

Out[52]= ((Iz + (lb)^(2) * m) * q2dp + (-1 * lb * (1a + lb) * m * q1dp * cos((q1 - 1 * q2)) + (-1 * F1 * (1a + lb) * cos(q2) + (lb
* m * q3dp * cos(q2) + (-1 * F2 * (1a + lb) * sin(q2) + (lb * m * q4dp * sin(q2) + (lb * (1a + lb) * m * cos((q1 - 1 *
q2)) * q1d - 1 * (Iz + (lb)^(2) * m) * q2d))))))

In[53]:= impDynConst1[[3]][[1]]
HurToJulia[%]

Out[53]= -F1 + (2 m + mh) q3dp - (lb (m + mh) + 1a (2 m + mh)) q1dp Cos[q1[t]] + lb m q2dp Cos[q2[t]] +
(lb (m + mh) + 1a (2 m + mh)) Cos[q1[t]] q1'[t] - lb m Cos[q2[t]] q2'[t]

Out[54]= (-1 * F1 + ((2 * m + mh) * q3dp + (-1 * (lb * (m + mh) + 1a * (2 * m + mh)) * q1dp * cos(q1) + (lb * m * q2dp * cos(q2) + ((lb
* (m + mh) + 1a * (2 * m + mh)) * cos(q1) * q1d - 1 * lb * m * cos(q2) * q2d))))))

In[55]:= impDynConst1[[4]][[1]]
HurToJulia[%]

Out[55]= -F2 + (2 m + mh) q4dp - (lb (m + mh) + 1a (2 m + mh)) q1dp Sin[q1[t]] + lb m q2dp Sin[q2[t]] +
(lb (m + mh) + 1a (2 m + mh)) Sin[q1[t]] q1'[t] - lb m Sin[q2[t]] q2'[t]

Out[56]= (-1 * F2 + ((2 * m + mh) * q4dp + (-1 * (lb * (m + mh) + 1a * (2 * m + mh)) * q1dp * sin(q1) + (lb * m * q2dp * sin(q2) + ((lb
* (m + mh) + 1a * (2 * m + mh)) * sin(q1) * q1d - 1 * lb * m * sin(q2) * q2d))))))

In[57]:= impDynConst2[[1]][[1]]
HurToJulia[%]

Out[57]= q3dp - (1a + lb) q1dp Cos[q1[t]] + (1a + lb) q2dp Cos[q2[t]]

Out[58]= (q3dp + (-1 * (1a + lb) * q1dp * cos(q1) + (1a + lb) * q2dp * cos(q2)))

In[59]:= impDynConst2[[2]][[1]]
HurToJulia[%]

Out[59]= q4dp - (1a + lb) q1dp Sin[q1[t]] + (1a + lb) q2dp Sin[q2[t]]

Out[60]= (q4dp + (-1 * (1a + lb) * q1dp * sin(q1) + (1a + lb) * q2dp * sin(q2)))

In[61]:= temp = Inverse[HurGlobalMMatrix].Transpose[Jbfoot] // Simplify

```

[illegible]

```
In[62]:= coef = temp.Inverse[Jbfoot.temp].Jbfoot // Simplify
```

```
Out[62]= { { (1b (1a + 1b) m (-2 Iz - 2 1a^2 m - 1a 1b m - 1b^2 m - 1a^2 mh - 2 1a 1b mh -
      1b^2 mh + (1a + 1b) (1b (m + mh) + 1a (2 m + mh)) Cos[2 (q1[t] - q2[t])])) /
      (-2 Iz^2 - 2 Iz (2 1a 1b (m + mh) + 1a^2 (2 m + mh) + 1b^2 (2 m + mh)) -
      1b^2 m (2 1a 1b (m + 2 mh) + 1b^2 (m + 2 mh) + 1a^2 (3 m + 2 mh)) +
      1b^2 (1a + 1b)^2 m^2 Cos[2 (q1[t] - q2[t])]) ,
      - ((2 1b (1a + 1b) m (-Iz + 1a 1b m) Cos[q1[t] - q2[t]]) /
      (-2 Iz^2 - 2 Iz (2 1a 1b (m + mh) + 1a^2 (2 m + mh) + 1b^2 (2 m + mh)) -
      1b^2 m (2 1a 1b (m + 2 mh) + 1b^2 (m + 2 mh) + 1a^2 (3 m + 2 mh)) +
      1b^2 (1a + 1b)^2 m^2 Cos[2 (q1[t] - q2[t])]) ) ,
      (1b m ((2 Iz + 1b^2 (m + mh) + 1a^2 (2 m + mh) + 1a 1b (m + 2 mh)) Cos[q1[t]] -
      (1a + 1b) (1b (m + mh) + 1a (2 m + mh)) Cos[q1[t] - 2 q2[t]])) /
      (-2 Iz^2 - 2 Iz (2 1a 1b (m + mh) + 1a^2 (2 m + mh) + 1b^2 (2 m + mh)) -
      1b^2 m (2 1a 1b (m + 2 mh) + 1b^2 (m + 2 mh) + 1a^2 (3 m + 2 mh)) +
      1b^2 (1a + 1b)^2 m^2 Cos[2 (q1[t] - q2[t])]) ,
      (1b m ((2 Iz + 1b^2 (m + mh) + 1a^2 (2 m + mh) + 1a 1b (m + 2 mh)) Sin[q1[t]] +
      (1a + 1b) (1b (m + mh) + 1a (2 m + mh)) Sin[q1[t] - 2 q2[t]])) /
      (-2 Iz^2 - 2 Iz (2 1a 1b (m + mh) + 1a^2 (2 m + mh) + 1b^2 (2 m + mh)) -
      1b^2 m (2 1a 1b (m + 2 mh) + 1b^2 (m + 2 mh) + 1a^2 (3 m + 2 mh)) +
      1b^2 (1a + 1b)^2 m^2 Cos[2 (q1[t] - q2[t])]) } ,
      { (2 (1a + 1b) (1b^2 m (1b mh + 1a (m + mh)) + Iz (1b (m + mh) + 1a (2 m + mh))) Cos[q1[t] - q2[t]] /
      (-2 Iz^2 - 2 Iz (2 1a 1b (m + mh) + 1a^2 (2 m + mh) + 1b^2 (2 m + mh)) -
      1b^2 m (2 1a 1b (m + 2 mh) + 1b^2 (m + 2 mh) + 1a^2 (3 m + 2 mh)) +
      1b^2 (1a + 1b)^2 m^2 Cos[2 (q1[t] - q2[t])]) ,
      ((1a + 1b) (-1b^2 m (3 1a m + 1b m + 2 1a mh + 2 1b mh) - 2 Iz (1b (m + mh) + 1a (2 m + mh)) +
      1b^2 (1a + 1b) m^2 Cos[2 (q1[t] - q2[t])])) /
      (-2 Iz^2 - 2 Iz (2 1a 1b (m + mh) + 1a^2 (2 m + mh) + 1b^2 (2 m + mh)) -
      1b^2 m (2 1a 1b (m + 2 mh) + 1b^2 (m + 2 mh) + 1a^2 (3 m + 2 mh)) +
      1b^2 (1a + 1b)^2 m^2 Cos[2 (q1[t] - q2[t])]) , (-1b^2 (1a + 1b) m^2 Cos[2 q1[t] - q2[t]] +
      (1b^2 m (3 1a m + 1b m + 2 1a mh + 2 1b mh) + 2 Iz (1b (m + mh) + 1a (2 m + mh))) Cos[q2[t]] /
      (2 Iz^2 + 2 Iz (2 1a 1b (m + mh) + 1a^2 (2 m + mh) + 1b^2 (2 m + mh)) +
      1b^2 m (2 1a 1b (m + 2 mh) + 1b^2 (m + 2 mh) + 1a^2 (3 m + 2 mh)) -
      1b^2 (1a + 1b)^2 m^2 Cos[2 (q1[t] - q2[t])]) , (-1b^2 (1a + 1b) m^2 Sin[2 q1[t] - q2[t]] +
      (1b^2 m (3 1a m + 1b m + 2 1a mh + 2 1b mh) + 2 Iz (1b (m + mh) + 1a (2 m + mh))) Sin[q2[t]] /
      (2 Iz^2 + 2 Iz (2 1a 1b (m + mh) + 1a^2 (2 m + mh) + 1b^2 (2 m + mh)) +
      1b^2 m (2 1a 1b (m + 2 mh) + 1b^2 (m + 2 mh) + 1a^2 (3 m + 2 mh)) -
      1b^2 (1a + 1b)^2 m^2 Cos[2 (q1[t] - q2[t])]) } ,
      { - ((1a + 1b) (2 (Iz - 1a 1b m) (2 Iz + 1b^2 (m + mh) + 1a^2 (2 m + mh) + 1a 1b (m + 2 mh)) Cos[q1[t]] +
      (1a + 1b) (- (2 Iz (1b (m + mh) + 1a (2 m + mh)) + 1b m (1b^2 mh - 1a^2 (2 m + mh)))) Cos[
```



$$\begin{aligned}
& \left( q1[t] - 2 q2[t] \right) + lb \left( la + lb \right) m \left( lb mh + la \left( 2 m + mh \right) \right) \cos \left[ 3 q1[t] - 2 q2[t] \right] \Big) \Big) / \\
& \left( 4 Iz^2 + 4 Iz \left( 2 la lb \left( m + mh \right) + la^2 \left( 2 m + mh \right) + lb^2 \left( 2 m + mh \right) \right) + \right. \\
& \quad \left. 2 lb^2 m \left( 2 la lb \left( m + 2 mh \right) + lb^2 \left( m + 2 mh \right) + la^2 \left( 3 m + 2 mh \right) \right) - \right. \\
& \quad \left. 2 lb^2 \left( la + lb \right)^2 m^2 \cos \left[ 2 \left( q1[t] - q2[t] \right) \right] \right) \Big), - \left( \left( la + lb \right) \left( - Iz + la lb m \right) \right. \\
& \quad \left( lb \left( la + lb \right) m \cos \left[ 2 q1[t] - q2[t] \right] + \left( - 2 Iz + \left( la - lb \right) lb m \right) \cos \left[ q2[t] \right] \right) \Big) / \\
& \left( - 2 Iz^2 - 2 Iz \left( 2 la lb \left( m + mh \right) + la^2 \left( 2 m + mh \right) + lb^2 \left( 2 m + mh \right) \right) - lb^2 m \left( 2 la lb \left( m + 2 mh \right) + \right. \right. \\
& \quad \left. \left. lb^2 \left( m + 2 mh \right) + la^2 \left( 3 m + 2 mh \right) \right) + lb^2 \left( la + lb \right)^2 m^2 \cos \left[ 2 \left( q1[t] - q2[t] \right) \right] \right) \Big), \\
& \left( 4 Iz^2 + 4 Iz la^2 m + 4 Iz lb^2 m - 2 la^3 lb m^2 - 2 la lb^3 m^2 + 2 Iz la^2 mh + 4 Iz la lb mh + \right. \\
& \quad \left. 2 Iz lb^2 mh - la^3 lb m mh - la^2 lb^2 m mh + la lb^3 m mh + lb^4 m mh - \right. \\
& \quad lb \left( la + lb \right) m \left( 2 Iz + 2 la lb mh + lb^2 mh + la^2 \left( 2 m + mh \right) \right) \cos \left[ 2 q1[t] \right] + \\
& \quad lb \left( la + lb \right)^2 m \left( lb mh + la \left( 2 m + mh \right) \right) \cos \left[ 2 \left( q1[t] - q2[t] \right) \right] - \\
& \quad 4 Iz la^2 m \cos \left[ 2 q2[t] \right] - 6 Iz la lb m \cos \left[ 2 q2[t] \right] - 2 Iz lb^2 m \cos \left[ 2 q2[t] \right] + \\
& \quad 2 la^3 lb m^2 \cos \left[ 2 q2[t] \right] + 2 la^2 lb^2 m^2 \cos \left[ 2 q2[t] \right] - 2 Iz la^2 mh \cos \left[ 2 q2[t] \right] - \\
& \quad 4 Iz la lb mh \cos \left[ 2 q2[t] \right] - 2 Iz lb^2 mh \cos \left[ 2 q2[t] \right] + la^3 lb m mh \cos \left[ 2 q2[t] \right] + \\
& \quad \left. la^2 lb^2 m mh \cos \left[ 2 q2[t] \right] - la lb^3 m mh \cos \left[ 2 q2[t] \right] - lb^4 m mh \cos \left[ 2 q2[t] \right] \right) \Big) / \\
& \left( 4 Iz^2 + 4 Iz \left( 2 la lb \left( m + mh \right) + la^2 \left( 2 m + mh \right) + lb^2 \left( 2 m + mh \right) \right) + \right. \\
& \quad \left. 2 lb^2 m \left( 2 la lb \left( m + 2 mh \right) + lb^2 \left( m + 2 mh \right) + la^2 \left( 3 m + 2 mh \right) \right) - \right. \\
& \quad \left. 2 lb^2 \left( la + lb \right)^2 m^2 \cos \left[ 2 \left( q1[t] - q2[t] \right) \right] \right) \Big), \\
& - \left( \left( la + lb \right) \left( lb m \left( 2 Iz + 2 la lb mh + lb^2 mh + la^2 \left( 2 m + mh \right) \right) \sin \left[ 2 q1[t] \right] + \right. \right. \\
& \quad \left. lb \left( la + lb \right) m \left( lb mh + la \left( 2 m + mh \right) \right) \sin \left[ 2 \left( q1[t] - q2[t] \right) \right] + \right. \\
& \quad \left. \left( 2 Iz \left( lb \left( m + mh \right) + la \left( 2 m + mh \right) \right) + lb m \left( lb^2 mh - la^2 \left( 2 m + mh \right) \right) \right) \sin \left[ 2 q2[t] \right] \right) \Big) / \\
& \left( 4 Iz^2 + 4 Iz \left( 2 la lb \left( m + mh \right) + la^2 \left( 2 m + mh \right) + lb^2 \left( 2 m + mh \right) \right) + 2 lb^2 m \left( 2 la lb \left( m + 2 mh \right) + \right. \right. \\
& \quad \left. \left. lb^2 \left( m + 2 mh \right) + la^2 \left( 3 m + 2 mh \right) \right) - 2 lb^2 \left( la + lb \right)^2 m^2 \cos \left[ 2 \left( q1[t] - q2[t] \right) \right] \right) \Big) \Big), \\
& \left\{ - \left( \left( la + lb \right) \left( 2 \left( Iz - la lb m \right) \left( 2 Iz + lb^2 \left( m + mh \right) + la^2 \left( 2 m + mh \right) + la lb \left( m + 2 mh \right) \right) \sin \left[ q1[t] \right] + \right. \right. \right. \\
& \quad \left( la + lb \right) \left( \left( 2 Iz \left( lb \left( m + mh \right) + la \left( 2 m + mh \right) \right) + lb m \left( lb^2 mh - la^2 \left( 2 m + mh \right) \right) \right) \sin \left[ \right. \right. \\
& \quad \left. \left. q1[t] - 2 q2[t] \right] + lb \left( la + lb \right) m \left( lb mh + la \left( 2 m + mh \right) \right) \sin \left[ 3 q1[t] - 2 q2[t] \right] \right) \Big) \Big) / \\
& \left( 4 Iz^2 + 4 Iz \left( 2 la lb \left( m + mh \right) + la^2 \left( 2 m + mh \right) + lb^2 \left( 2 m + mh \right) \right) + \right. \\
& \quad \left. 2 lb^2 m \left( 2 la lb \left( m + 2 mh \right) + lb^2 \left( m + 2 mh \right) + la^2 \left( 3 m + 2 mh \right) \right) - \right. \\
& \quad \left. 2 lb^2 \left( la + lb \right)^2 m^2 \cos \left[ 2 \left( q1[t] - q2[t] \right) \right] \right) \Big), - \left( \left( la + lb \right) \left( - Iz + la lb m \right) \right. \\
& \quad \left( lb \left( la + lb \right) m \sin \left[ 2 q1[t] - q2[t] \right] + \left( - 2 Iz + \left( la - lb \right) lb m \right) \sin \left[ q2[t] \right] \right) \Big) / \\
& \left( - 2 Iz^2 - 2 Iz \left( 2 la lb \left( m + mh \right) + la^2 \left( 2 m + mh \right) + lb^2 \left( 2 m + mh \right) \right) - lb^2 m \left( 2 la lb \left( m + 2 mh \right) + \right. \right. \\
& \quad \left. \left. lb^2 \left( m + 2 mh \right) + la^2 \left( 3 m + 2 mh \right) \right) + lb^2 \left( la + lb \right)^2 m^2 \cos \left[ 2 \left( q1[t] - q2[t] \right) \right] \right) \Big), \\
& - \left( \left( la + lb \right) \left( lb m \left( 2 Iz + 2 la lb mh + lb^2 mh + la^2 \left( 2 m + mh \right) \right) \sin \left[ 2 q1[t] \right] - \right. \right. \\
& \quad \left. lb \left( la + lb \right) m \left( lb mh + la \left( 2 m + mh \right) \right) \sin \left[ 2 \left( q1[t] - q2[t] \right) \right] + \right. \\
& \quad \left. \left( 2 Iz \left( lb \left( m + mh \right) + la \left( 2 m + mh \right) \right) + lb m \left( lb^2 mh - la^2 \left( 2 m + mh \right) \right) \right) \sin \left[ 2 q2[t] \right] \right) \Big) / \\
& \left( 4 Iz^2 + 4 Iz \left( 2 la lb \left( m + mh \right) + la^2 \left( 2 m + mh \right) + lb^2 \left( 2 m + mh \right) \right) + \right. \\
& \quad \left. 2 lb^2 m \left( 2 la lb \left( m + 2 mh \right) + lb^2 \left( m + 2 mh \right) + la^2 \left( 3 m + 2 mh \right) \right) - \right. \\
& \quad \left. 2 lb^2 \left( la + lb \right)^2 m^2 \cos \left[ 2 \left( q1[t] - q2[t] \right) \right] \right) \Big), \\
& \left( 4 Iz^2 + 4 Iz la^2 m + 4 Iz lb^2 m - 2 la^3 lb m^2 - 2 la lb^3 m^2 + 2 Iz la^2 mh + 4 Iz la lb mh + \right.
\end{aligned}$$

$$\begin{aligned}
& 2 I_z l b^2 m h - l a^3 l b m m h - l a^2 l b^2 m m h + l a l b^3 m m h + l b^4 m m h + \\
& l b (l a + l b) m (2 I_z + 2 l a l b m h + l b^2 m h + l a^2 (2 m + m h)) \cos[2 q_1[t]] + \\
& l b (l a + l b)^2 m (l b m h + l a (2 m + m h)) \cos[2 (q_1[t] - q_2[t])] + \\
& 4 I_z l a^2 m \cos[2 q_2[t]] + 6 I_z l a l b m \cos[2 q_2[t]] + 2 I_z l b^2 m \cos[2 q_2[t]] - \\
& 2 l a^3 l b m^2 \cos[2 q_2[t]] - 2 l a^2 l b^2 m^2 \cos[2 q_2[t]] + 2 I_z l a^2 m h \cos[2 q_2[t]] + \\
& 4 I_z l a l b m h \cos[2 q_2[t]] + 2 I_z l b^2 m h \cos[2 q_2[t]] - l a^3 l b m m h \cos[2 q_2[t]] - \\
& l a^2 l b^2 m m h \cos[2 q_2[t]] + l a l b^3 m m h \cos[2 q_2[t]] + l b^4 m m h \cos[2 q_2[t]] \Big) / \\
& \left( 4 I_z^2 + 4 I_z (2 l a l b (m + m h) + l a^2 (2 m + m h) + l b^2 (2 m + m h)) + \right. \\
& \quad 2 l b^2 m (2 l a l b (m + 2 m h) + l b^2 (m + 2 m h) + l a^2 (3 m + 2 m h)) - \\
& \quad \left. 2 l b^2 (l a + l b)^2 m^2 \cos[2 (q_1[t] - q_2[t])] \right) \Big\} \Big\}
\end{aligned}$$

In[63]:= **Dimensions[coef]**

Out[63]= {4, 4}

In[64]:= **LinOp = IdentityMatrix[4] - coef // Simplify**

Out[64]= 
$$\begin{aligned}
& \left\{ \left\{ 1 - (l b (l a + l b) m (-2 I_z - 2 l a^2 m - l a l b m - l b^2 m - l a^2 m h - 2 l a l b m h - \right. \right. \\
& \quad \left. l b^2 m h + (l a + l b) (l b (m + m h) + l a (2 m + m h)) \cos[2 (q_1[t] - q_2[t])]) \right) / \\
& \quad \left( -2 I_z^2 - 2 I_z (2 l a l b (m + m h) + l a^2 (2 m + m h) + l b^2 (2 m + m h)) - \right. \\
& \quad \left. l b^2 m (2 l a l b (m + 2 m h) + l b^2 (m + 2 m h) + l a^2 (3 m + 2 m h)) + \right. \\
& \quad \left. l b^2 (l a + l b)^2 m^2 \cos[2 (q_1[t] - q_2[t])] \right), \\
& \quad (2 l b (l a + l b) m (-I_z + l a l b m) \cos[q_1[t] - q_2[t]]) / \\
& \quad \left( -2 I_z^2 - 2 I_z (2 l a l b (m + m h) + l a^2 (2 m + m h) + l b^2 (2 m + m h)) - \right. \\
& \quad \left. l b^2 m (2 l a l b (m + 2 m h) + l b^2 (m + 2 m h) + l a^2 (3 m + 2 m h)) + \right. \\
& \quad \left. l b^2 (l a + l b)^2 m^2 \cos[2 (q_1[t] - q_2[t])] \right), \\
& \quad (l b m (- (2 I_z + l b^2 (m + m h) + l a^2 (2 m + m h) + l a l b (m + 2 m h)) \cos[q_1[t]] + \\
& \quad (l a + l b) (l b (m + m h) + l a (2 m + m h)) \cos[q_1[t] - 2 q_2[t]]) / \\
& \quad \left( -2 I_z^2 - 2 I_z (2 l a l b (m + m h) + l a^2 (2 m + m h) + l b^2 (2 m + m h)) - \right. \\
& \quad \left. l b^2 m (2 l a l b (m + 2 m h) + l b^2 (m + 2 m h) + l a^2 (3 m + 2 m h)) + \right. \\
& \quad \left. l b^2 (l a + l b)^2 m^2 \cos[2 (q_1[t] - q_2[t])] \right), \\
& \quad - \left( (l b m ((2 I_z + l b^2 (m + m h) + l a^2 (2 m + m h) + l a l b (m + 2 m h)) \sin[q_1[t]] + \right. \\
& \quad (l a + l b) (l b (m + m h) + l a (2 m + m h)) \sin[q_1[t] - 2 q_2[t]]) / \\
& \quad \left( -2 I_z^2 - 2 I_z (2 l a l b (m + m h) + l a^2 (2 m + m h) + l b^2 (2 m + m h)) - l b^2 m (2 l a l b (m + 2 m h) + \right. \\
& \quad \left. l b^2 (m + 2 m h) + l a^2 (3 m + 2 m h)) + l b^2 (l a + l b)^2 m^2 \cos[2 (q_1[t] - q_2[t])] \right) \Big\}, \\
& \quad \left\{ - \left( (2 (l a + l b) (l b^2 m (l b m h + l a (m + m h)) + I_z (l b (m + m h) + l a (2 m + m h))) \cos[q_1[t] - q_2[t]] \right) / \right. \\
& \quad \left( -2 I_z^2 - 2 I_z (2 l a l b (m + m h) + l a^2 (2 m + m h) + l b^2 (2 m + m h)) - \right. \\
& \quad \left. l b^2 m (2 l a l b (m + 2 m h) + l b^2 (m + 2 m h) + l a^2 (3 m + 2 m h)) + \right. \\
& \quad \left. l b^2 (l a + l b)^2 m^2 \cos[2 (q_1[t] - q_2[t])] \right) \Big\}, \\
& \quad (2 (I_z - l a l b m) (I_z + l b^2 m)) / (2 I_z^2 + 2 I_z (2 l a l b (m + m h) + l a^2 (2 m + m h) + l b^2 (2 m + m h)) + \\
& \quad l b^2 m (2 l a l b (m + 2 m h) + l b^2 (m + 2 m h) + l a^2 (3 m + 2 m h)) - \\
& \quad \left. l b^2 (l a + l b)^2 m^2 \cos[2 (q_1[t] - q_2[t])]) \right), (l b^2 (l a + l b) m^2 \cos[2 q_1[t] - q_2[t]] -
\end{aligned}$$

$$\begin{aligned}
& \left( lb^2 m (3 la m + lb m + 2 la mh + 2 lb mh) + 2 Iz (lb (m + mh) + la (2 m + mh)) \right) \cos[q_2[t]] \Big/ \\
& \left( 2 Iz^2 + 2 Iz (2 la lb (m + mh) + la^2 (2 m + mh) + lb^2 (2 m + mh)) + \right. \\
& \quad lb^2 m (2 la lb (m + 2 mh) + lb^2 (m + 2 mh) + la^2 (3 m + 2 mh)) - \\
& \quad \left. lb^2 (la + lb)^2 m^2 \cos[2 (q_1[t] - q_2[t])] \right), (lb^2 (la + lb) m^2 \sin[2 q_1[t] - q_2[t]] - \\
& \quad (lb^2 m (3 la m + lb m + 2 la mh + 2 lb mh) + 2 Iz (lb (m + mh) + la (2 m + mh))) \sin[q_2[t]] \Big/ \\
& \quad \left( 2 Iz^2 + 2 Iz (2 la lb (m + mh) + la^2 (2 m + mh) + lb^2 (2 m + mh)) + \right. \\
& \quad \quad lb^2 m (2 la lb (m + 2 mh) + lb^2 (m + 2 mh) + la^2 (3 m + 2 mh)) - \\
& \quad \quad \left. lb^2 (la + lb)^2 m^2 \cos[2 (q_1[t] - q_2[t])] \right) \Big\}, \\
& \left\{ \left( (la + lb) (2 (Iz - la lb m) (2 Iz + lb^2 (m + mh) + la^2 (2 m + mh) + la lb (m + 2 mh)) \cos[q_1[t]] + \right. \right. \\
& \quad (la + lb) (- (2 Iz (lb (m + mh) + la (2 m + mh)) + lb m (lb^2 mh - la^2 (2 m + mh))) \cos[ \\
& \quad \quad q_1[t] - 2 q_2[t]] + lb (la + lb) m (lb mh + la (2 m + mh)) \cos[3 q_1[t] - 2 q_2[t]]) \Big) \Big/ \\
& \quad \left( 4 Iz^2 + 4 Iz (2 la lb (m + mh) + la^2 (2 m + mh) + lb^2 (2 m + mh)) + \right. \\
& \quad \quad 2 lb^2 m (2 la lb (m + 2 mh) + lb^2 (m + 2 mh) + la^2 (3 m + 2 mh)) - \\
& \quad \quad \left. 2 lb^2 (la + lb)^2 m^2 \cos[2 (q_1[t] - q_2[t])] \right), ((la + lb) (-Iz + la lb m) \\
& \quad (lb (la + lb) m \cos[2 q_1[t] - q_2[t]] + (-2 Iz + (la - lb) lb m) \cos[q_2[t]]) \Big) \Big/ \\
& \quad \left( -2 Iz^2 - 2 Iz (2 la lb (m + mh) + la^2 (2 m + mh) + lb^2 (2 m + mh)) - \right. \\
& \quad \quad lb^2 m (2 la lb (m + 2 mh) + lb^2 (m + 2 mh) + la^2 (3 m + 2 mh)) + \\
& \quad \quad \left. lb^2 (la + lb)^2 m^2 \cos[2 (q_1[t] - q_2[t])] \right), \\
& \left( (la + lb) (4 Iz la m + 4 Iz lb m + 2 la^2 lb m^2 + 4 la lb^2 m^2 + 2 lb^3 m^2 + 2 Iz la mh + 2 Iz lb mh + \right. \\
& \quad la^2 lb m mh + 4 la lb^2 m mh + 3 lb^3 m mh + lb m (2 Iz + 2 la lb mh + lb^2 mh + la^2 (2 m + mh)) \\
& \quad \cos[2 q_1[t]] - lb (la + lb)^2 m (2 m + mh) \cos[2 (q_1[t] - q_2[t])] + 4 Iz la m \cos[2 q_2[t]] + \\
& \quad 2 Iz lb m \cos[2 q_2[t]] - 2 la^2 lb m^2 \cos[2 q_2[t]] + 2 Iz la mh \cos[2 q_2[t]] + \\
& \quad \left. 2 Iz lb mh \cos[2 q_2[t]] - la^2 lb m mh \cos[2 q_2[t]] + lb^3 m mh \cos[2 q_2[t]]) \Big) \Big/ \\
& \quad \left( 4 Iz^2 + 4 Iz (2 la lb (m + mh) + la^2 (2 m + mh) + lb^2 (2 m + mh)) + \right. \\
& \quad \quad 2 lb^2 m (2 la lb (m + 2 mh) + lb^2 (m + 2 mh) + la^2 (3 m + 2 mh)) - \\
& \quad \quad \left. 2 lb^2 (la + lb)^2 m^2 \cos[2 (q_1[t] - q_2[t])] \right), \\
& \left( (la + lb) (lb m (2 Iz + 2 la lb mh + lb^2 mh + la^2 (2 m + mh)) \sin[2 q_1[t]] + \right. \\
& \quad lb (la + lb) m (lb mh + la (2 m + mh)) \sin[2 (q_1[t] - q_2[t])] + \\
& \quad \quad (2 Iz (lb (m + mh) + la (2 m + mh)) + lb m (lb^2 mh - la^2 (2 m + mh))) \sin[2 q_2[t]]) \Big) \Big/ \\
& \quad \left( 4 Iz^2 + 4 Iz (2 la lb (m + mh) + la^2 (2 m + mh) + lb^2 (2 m + mh)) + \right. \\
& \quad \quad 2 lb^2 m (2 la lb (m + 2 mh) + lb^2 (m + 2 mh) + la^2 (3 m + 2 mh)) - \\
& \quad \quad \left. 2 lb^2 (la + lb)^2 m^2 \cos[2 (q_1[t] - q_2[t])] \right) \Big\}, \\
& \left\{ \left( (la + lb) (2 (Iz - la lb m) (2 Iz + lb^2 (m + mh) + la^2 (2 m + mh) + la lb (m + 2 mh)) \sin[q_1[t]] + \right. \right. \\
& \quad (la + lb) ((2 Iz (lb (m + mh) + la (2 m + mh)) + lb m (lb^2 mh - la^2 (2 m + mh))) \sin[ \\
& \quad \quad q_1[t] - 2 q_2[t]] + lb (la + lb) m (lb mh + la (2 m + mh)) \sin[3 q_1[t] - 2 q_2[t]]) \Big) \Big/ \\
& \quad \left( 4 Iz^2 + 4 Iz (2 la lb (m + mh) + la^2 (2 m + mh) + lb^2 (2 m + mh)) + \right. \\
& \quad \quad 2 lb^2 m (2 la lb (m + 2 mh) + lb^2 (m + 2 mh) + la^2 (3 m + 2 mh)) - \\
& \quad \quad \left. 2 lb^2 (la + lb)^2 m^2 \cos[2 (q_1[t] - q_2[t])] \right), ((la + lb) (-Iz + la lb m) \\
& \quad (lb (la + lb) m \sin[2 q_1[t] - q_2[t]] + (-2 Iz + (la - lb) lb m) \sin[q_2[t]]) \Big) \Big/
\end{aligned}$$

$$\begin{aligned}
& \left( -2 I_z^2 - 2 I_z (2 l_a l_b (m + mh) + l_a^2 (2 m + mh) + l_b^2 (2 m + mh)) - \right. \\
& \quad l_b^2 m (2 l_a l_b (m + 2 mh) + l_b^2 (m + 2 mh) + l_a^2 (3 m + 2 mh)) + \\
& \quad \left. l_b^2 (l_a + l_b)^2 m^2 \cos[2 (q_1[t] - q_2[t])] \right), \\
& \left( (l_a + l_b) (l_b m (2 I_z + 2 l_a l_b mh + l_b^2 mh + l_a^2 (2 m + mh)) \sin[2 q_1[t]] - \right. \\
& \quad l_b (l_a + l_b) m (l_b mh + l_a (2 m + mh)) \sin[2 (q_1[t] - q_2[t])] + \\
& \quad \left. (2 I_z (l_b (m + mh) + l_a (2 m + mh)) + l_b m (l_b^2 mh - l_a^2 (2 m + mh))) \sin[2 q_2[t]] \right) \Big/ \\
& \left( 4 I_z^2 + 4 I_z (2 l_a l_b (m + mh) + l_a^2 (2 m + mh) + l_b^2 (2 m + mh)) + \right. \\
& \quad 2 l_b^2 m (2 l_a l_b (m + 2 mh) + l_b^2 (m + 2 mh) + l_a^2 (3 m + 2 mh)) - \\
& \quad \left. 2 l_b^2 (l_a + l_b)^2 m^2 \cos[2 (q_1[t] - q_2[t])] \right), \\
& - \left( (l_a + l_b) (-4 I_z l_a m - 4 I_z l_b m - 2 l_a^2 l_b m^2 - 4 l_a l_b^2 m^2 - 2 l_b^3 m^2 - 2 I_z l_a mh - 2 I_z l_b mh - \right. \\
& \quad l_a^2 l_b m mh - 4 l_a l_b^2 m mh - 3 l_b^3 m mh + l_b m (2 I_z + 2 l_a l_b mh + l_b^2 mh + l_a^2 (2 m + mh)) \cos[ \\
& \quad 2 q_1[t]] + l_b (l_a + l_b)^2 m (2 m + mh) \cos[2 (q_1[t] - q_2[t])] + 4 I_z l_a m \cos[2 q_2[t]] + \\
& \quad 2 I_z l_b m \cos[2 q_2[t]] - 2 l_a^2 l_b m^2 \cos[2 q_2[t]] + 2 I_z l_a mh \cos[2 q_2[t]] + \\
& \quad \left. 2 I_z l_b mh \cos[2 q_2[t]] - l_a^2 l_b m mh \cos[2 q_2[t]] + l_b^3 m mh \cos[2 q_2[t]] \right) \Big/ \\
& \left( 4 I_z^2 + 4 I_z (2 l_a l_b (m + mh) + l_a^2 (2 m + mh) + l_b^2 (2 m + mh)) + 2 l_b^2 m (2 l_a l_b (m + 2 mh) + \right. \\
& \quad \left. l_b^2 (m + 2 mh) + l_a^2 (3 m + 2 mh)) - 2 l_b^2 (l_a + l_b)^2 m^2 \cos[2 (q_1[t] - q_2[t])] \right) \Big\}
\end{aligned}$$

In[65]:= **qdotplus = LinOp.HurList2Column[{q1dminus, q2dminus, 0, 0}] // Simplify**

$$\begin{aligned}
\text{Out[65]} = & \left\{ \left\{ \left( 2 lb (1a + lb) m (-Iz + 1a lb m) q2dminus \cos[q1[t] - q2[t]] - \right. \right. \right. \\
& q1dminus (2 Iz^2 + lb m (-1a^2 lb mh + 1a lb^2 mh + lb^3 mh - 1a^3 (2 m + mh)) + \\
& 2 Iz (lb^2 (m + mh) + 1a^2 (2 m + mh) + 1a lb (m + 2 mh)) + \\
& \left. \left. \left. lb (1a + lb)^2 m (lb mh + 1a (2 m + mh)) \cos[2 (q1[t] - q2[t])] \right) \right) \right\} / \\
& \left( -2 Iz^2 - 2 Iz (2 1a lb (m + mh) + 1a^2 (2 m + mh) + lb^2 (2 m + mh)) - \right. \\
& lb^2 m (2 1a lb (m + 2 mh) + lb^2 (m + 2 mh) + 1a^2 (3 m + 2 mh)) + \\
& \left. \left. \left. lb^2 (1a + lb)^2 m^2 \cos[2 (q1[t] - q2[t])] \right) \right) \right\}, \\
& \left\{ \left( 2 ((Iz - 1a lb m) (Iz + lb^2 m) q2dminus + (1a + lb) (lb^2 m (lb mh + 1a (m + mh)) + \right. \right. \right. \\
& Iz (lb (m + mh) + 1a (2 m + mh))) q1dminus \cos[q1[t] - q2[t]]) / \\
& \left( 2 Iz^2 + 2 Iz (2 1a lb (m + mh) + 1a^2 (2 m + mh) + lb^2 (2 m + mh)) + \right. \\
& lb^2 m (2 1a lb (m + 2 mh) + lb^2 (m + 2 mh) + 1a^2 (3 m + 2 mh)) - \\
& \left. \left. \left. lb^2 (1a + lb)^2 m^2 \cos[2 (q1[t] - q2[t])] \right) \right) \right\}, \{ (1a + lb) \\
& \left( (q1dminus (2 (Iz - 1a lb m) (2 Iz + lb^2 (m + mh) + 1a^2 (2 m + mh) + 1a lb (m + 2 mh)) \cos[q1[t]] + \right. \\
& (1a + lb) (- (2 Iz (lb (m + mh) + 1a (2 m + mh)) + lb m (lb^2 mh - 1a^2 (2 m + mh))) \cos[q1[t] \\
& \left. \left. \left. - 2 q2[t]] + lb (1a + lb) m (lb mh + 1a (2 m + mh)) \cos[3 q1[t] - 2 q2[t]] \right) \right) \right) / \\
& \left( 4 Iz^2 + 4 Iz (2 1a lb (m + mh) + 1a^2 (2 m + mh) + lb^2 (2 m + mh)) + \right. \\
& 2 lb^2 m (2 1a lb (m + 2 mh) + lb^2 (m + 2 mh) + 1a^2 (3 m + 2 mh)) - \\
& \left. \left. \left. 2 lb^2 (1a + lb)^2 m^2 \cos[2 (q1[t] - q2[t])] \right) \right) + ((-Iz + 1a lb m) q2dminus \right. \\
& (lb (1a + lb) m \cos[2 q1[t] - q2[t]] + (-2 Iz + (1a - lb) lb m) \cos[q2[t]]) / \\
& \left( -2 Iz^2 - 2 Iz (2 1a lb (m + mh) + 1a^2 (2 m + mh) + lb^2 (2 m + mh)) - \right. \\
& lb^2 m (2 1a lb (m + 2 mh) + lb^2 (m + 2 mh) + 1a^2 (3 m + 2 mh)) + \\
& \left. \left. \left. lb^2 (1a + lb)^2 m^2 \cos[2 (q1[t] - q2[t])] \right) \right) \right\}, \{ (1a + lb) \\
& \left( (q1dminus (2 (Iz - 1a lb m) (2 Iz + lb^2 (m + mh) + 1a^2 (2 m + mh) + 1a lb (m + 2 mh)) \sin[q1[t]] + \right. \\
& (1a + lb) ((2 Iz (lb (m + mh) + 1a (2 m + mh)) + lb m (lb^2 mh - 1a^2 (2 m + mh))) \sin[q1[t] - \\
& \left. \left. \left. 2 q2[t]] + lb (1a + lb) m (lb mh + 1a (2 m + mh)) \sin[3 q1[t] - 2 q2[t]] \right) \right) \right) / \\
& \left( 4 Iz^2 + 4 Iz (2 1a lb (m + mh) + 1a^2 (2 m + mh) + lb^2 (2 m + mh)) + \right. \\
& 2 lb^2 m (2 1a lb (m + 2 mh) + lb^2 (m + 2 mh) + 1a^2 (3 m + 2 mh)) - \\
& \left. \left. \left. 2 lb^2 (1a + lb)^2 m^2 \cos[2 (q1[t] - q2[t])] \right) \right) + ((-Iz + 1a lb m) q2dminus \right. \\
& (lb (1a + lb) m \sin[2 q1[t] - q2[t]] + (-2 Iz + (1a - lb) lb m) \sin[q2[t]]) / \\
& \left( -2 Iz^2 - 2 Iz (2 1a lb (m + mh) + 1a^2 (2 m + mh) + lb^2 (2 m + mh)) - \right. \\
& lb^2 m (2 1a lb (m + 2 mh) + lb^2 (m + 2 mh) + 1a^2 (3 m + 2 mh)) + \\
& \left. \left. \left. lb^2 (1a + lb)^2 m^2 \cos[2 (q1[t] - q2[t])] \right) \right) \right\} \} \}
\end{aligned}$$

Julia Expression for Constraints  
Dynamics at collocation points

Flatten[qdotplus][[1]]

HurToJulia[%]

$$\begin{aligned} & \left( 2 \, l_b \, (1a + l_b) \, m \, (-Iz + 1a \, l_b \, m) \, q2dminus \, \cos[q2[t]] + \right. \\ & \quad q1dminus \left( 2 \, Iz^2 + 2 \, Iz \, (2 \, 1a \, l_b \, (m + mh) + 1a^2 \, (2 \, m + mh) + l_b^2 \, (2 \, m + mh)) + \right. \\ & \quad \quad 1b^2 \, m \, (2 \, 1a \, l_b \, (m + 2 \, mh) + l_b^2 \, (m + 2 \, mh) + 1a^2 \, (3 \, m + 2 \, mh)) - \\ & \quad \quad 4 \, l_b \, (1a + l_b) \, m \, \cos\left[\frac{q2[t]}{2}\right]^2 \, (Iz + 2 \, 1a^2 \, m + 2 \, 1a \, l_b \, m + l_b^2 \, m + 1a^2 \, mh + 2 \, 1a \, l_b \, mh + l_b^2 \, mh - \\ & \quad \quad \quad (1a + l_b) \, (l_b \, (m + mh) + 1a \, (2 \, m + mh)) \, \cos[q2[t]]) - l_b^2 \, (1a + l_b)^2 \, m^2 \, \cos[2 \, q2[t]] \left. \right) \Bigg) / \\ & \left( 2 \, Iz^2 + 2 \, Iz \, (2 \, 1a \, l_b \, (m + mh) + 1a^2 \, (2 \, m + mh) + l_b^2 \, (2 \, m + mh)) + \right. \\ & \quad \quad 1b^2 \, m \, (2 \, 1a \, l_b \, (m + 2 \, mh) + l_b^2 \, (m + 2 \, mh) + 1a^2 \, (3 \, m + 2 \, mh)) - \\ & \quad \quad \quad l_b^2 \, (1a + l_b)^2 \, m^2 \, \cos[2 \, q2[t]] \left. \right) \\ & \left( (2 * (Iz)^(2) + (2 * Iz * (2 * 1a * l_b * (m + mh) + ((1a)^(2) * (2 * m + mh) + (l_b)^(2) * (2 * m + mh)))) + ((l_b)^(2) * m * \right. \\ & \quad * (2 * 1a * l_b * (m + 2 * mh) + ((l_b)^(2) * (m + 2 * mh) + (1a)^(2) * (3 * m + 2 * mh)))) + -1 * (l_b)^(2) * ((1a + l_b))^(2) * (m)^(2) * \cos(2 * q2) \left. \right) \Bigg)^{-1} * (2 * l_b * (1a + l_b) * m * (-1 * Iz + 1a * l_b * m) * q2dminus * \cos(q2) + \\ & \quad q1dminus * (2 * (Iz)^(2) + (2 * Iz * (2 * 1a * l_b * (m + mh) + ((1a)^(2) * (2 * m + mh) + (l_b)^(2) * (2 * m + mh)))) + ((l_b)^(2) * m * (2 * 1a * l_b * (m + 2 * mh) + ((l_b)^(2) * (m + 2 * mh) + (1a)^(2) * (3 * m + 2 * mh)))) + (-4 * l_b * (1a + l_b) * m * (\cos(1/2 * q2))^(2) * (Iz + (2 * (1a)^(2) * m + (2 * 1a * l_b * m + ((l_b)^(2) * m + ((1a)^(2) * mh + (2 * 1a * l_b * mh + ((l_b)^(2) * mh + -1 * (1a + l_b) * (l_b * (m + mh) + 1a * (2 * m + mh)) * \cos(q2) \left. \right) \Bigg) \Bigg) \Bigg)^{-1} * (l_b)^(2) * ((1a + l_b))^(2) * (m)^(2) * \cos(2 * q2) \left. \right) \Bigg) \Bigg) \end{aligned}$$

Flatten[qdotplus][[2]]

HurToJulia[%]

$$\begin{aligned} & \left( 2 \left( -2 \, (1a + l_b) \, (l_b \, mh + 1a \, (2 \, m + mh)) \right. \right. \\ & \quad \quad q1dminus \, \cos\left[\frac{q2[t]}{2}\right]^2 \, (Iz - 1a \, l_b \, m + l_b \, (1a + l_b) \, m \, \cos[q2[t]]) + \\ & \quad \quad \quad (Iz - 1a \, l_b \, m) \, q2dminus \, (Iz + l_b^2 \, m + l_b \, (1a + l_b) \, m \, \cos[q2[t]]) \left. \right) \Bigg) / \\ & \left( 2 \, Iz^2 + 2 \, Iz \, (2 \, 1a \, l_b \, (m + mh) + 1a^2 \, (2 \, m + mh) + l_b^2 \, (2 \, m + mh)) + \right. \\ & \quad \quad 1b^2 \, m \, (2 \, 1a \, l_b \, (m + 2 \, mh) + l_b^2 \, (m + 2 \, mh) + 1a^2 \, (3 \, m + 2 \, mh)) - l_b^2 \, (1a + l_b)^2 \, m^2 \, \cos[2 \, q2[t]] \left. \right) \\ & 2 * (-2 * (1a + l_b) * (l_b * mh + 1a * (2 * m + mh)) * q1dminus * (\cos(1/2 * q2))^(2) * (Iz + (-1 * 1a * l_b * m + l_b * (1a + l_b) * m * \cos(q2))) + (Iz + -1 * 1a * l_b * m) * q2dminus * (Iz + ((l_b)^(2) * m + l_b * (1a + l_b) * m * \cos(q2)))) * ((2 * (Iz)^(2) + (2 * Iz * (2 * 1a * l_b * (m + mh) + ((1a)^(2) * (2 * m + mh) + (l_b)^(2) * (2 * m + mh)))) + ((l_b)^(2) * m * (2 * 1a * l_b * (m + 2 * mh) + ((l_b)^(2) * (m + 2 * mh) + (1a)^(2) * (3 * m + 2 * mh)))) + -1 * (l_b)^(2) * ((1a + l_b))^(2) * (m)^(2) * \cos(2 * q2) \left. \right) \Bigg)^{-1} \end{aligned}$$

In[66]:= **A = {{0, 0, 1, 0}, {0, 0, 0, 1}}**

Out[66]= **{{0, 0, 1, 0}, {0, 0, 0, 1}}**

In[67]:= **tempA = A.Inverse[HurGlobalMMatrix] // Simplify**

Out[67]= 
$$\left\{ \left\{ \left( 2 I z (2 m + m h) (l b (m + m h) + l a (2 m + m h)) + l b^2 m (2 l a (2 m^2 + 3 m m h + m h^2) + l b (m^2 + 4 m m h + 2 m h^2)) \right) \cos[q_1[t]] - l b^3 m^3 \cos[q_1[t] - 2 q_2[t]] \right) / \left( 2 I z^2 (2 m + m h)^2 + 4 I z l b^2 m (2 m^2 + 3 m m h + m h^2) + l b^4 m^2 (m^2 + 4 m m h + 2 m h^2) - l b^4 m^4 \cos[2 (q_1[t] - q_2[t])] \right), \right. \\ \left. - \left( (l b m (l b m (l b (m + m h) + l a (2 m + m h)) \cos[2 q_1[t] - q_2[t]] - (2 I z (2 m + m h) + l b m (l b (m + m h) - l a (2 m + m h))) \cos[q_2[t]] \right) / \left( -2 I z^2 (2 m + m h)^2 - 4 I z l b^2 m (2 m^2 + 3 m m h + m h^2) - l b^4 m^2 (m^2 + 4 m m h + 2 m h^2) + l b^4 m^4 \cos[2 (q_1[t] - q_2[t])] \right) \right), \\ (4 I z^2 m + 4 I z l a^2 m^2 + 4 I z l a l b m^2 + 6 I z l b^2 m^2 + 2 l a^2 l b^2 m^3 + l a l b^3 m^3 + l b^4 m^3 + 2 I z^2 m h + 4 I z l a^2 m m h + 6 I z l a l b m m h + 6 I z l b^2 m m h + 3 l a^2 l b^2 m^2 m h + 4 l a l b^3 m^2 m h + 3 l b^4 m^2 m h + I z l a^2 m h^2 + 2 I z l a l b m h^2 + I z l b^2 m h^2 + l a^2 l b^2 m m h^2 + 2 l a l b^3 m m h^2 + l b^4 m m h^2 + (l b (m + m h) + l a (2 m + m h)) (l b^2 m (l b m h + l a (m + m h)) + I z (l b (m + m h) + l a (2 m + m h))) \cos[2 q_1[t]] - l b^3 (l a + l b) m^3 \cos[2 (q_1[t] - q_2[t])] + I z l b^2 m^2 \cos[2 q_2[t]] - l a l b^3 m^3 \cos[2 q_2[t]] \right) / \left( 2 I z^2 (2 m + m h)^2 + 4 I z l b^2 m (2 m^2 + 3 m m h + m h^2) + l b^4 m^2 (m^2 + 4 m m h + 2 m h^2) - l b^4 m^4 \cos[2 (q_1[t] - q_2[t])] \right), \\ ((l b (m + m h) + l a (2 m + m h)) (l b^2 m (l b m h + l a (m + m h)) + I z (l b (m + m h) + l a (2 m + m h))) \sin[2 q_1[t]] + l b^2 m^2 (I z - l a l b m) \sin[2 q_2[t]]) / \left( 2 I z^2 (2 m + m h)^2 + 4 I z l b^2 m (2 m^2 + 3 m m h + m h^2) + l b^4 m^2 (m^2 + 4 m m h + 2 m h^2) - l b^4 m^4 \cos[2 (q_1[t] - q_2[t])] \right) \}, \\ \left\{ \left( 2 I z (2 m + m h) (l b (m + m h) + l a (2 m + m h)) + l b^2 m (2 l a (2 m^2 + 3 m m h + m h^2) + l b (m^2 + 4 m m h + 2 m h^2)) \right) \sin[q_1[t]] + l b^3 m^3 \sin[q_1[t] - 2 q_2[t]] \right) / \left( 2 I z^2 (2 m + m h)^2 + 4 I z l b^2 m (2 m^2 + 3 m m h + m h^2) + l b^4 m^2 (m^2 + 4 m m h + 2 m h^2) - l b^4 m^4 \cos[2 (q_1[t] - q_2[t])] \right), \\ - \left( (l b m (l b m (l b (m + m h) + l a (2 m + m h)) \sin[2 q_1[t] - q_2[t]] - (2 I z (2 m + m h) + l b m (l b (m + m h) - l a (2 m + m h))) \sin[q_2[t]] \right) / \left( -2 I z^2 (2 m + m h)^2 - 4 I z l b^2 m (2 m^2 + 3 m m h + m h^2) - l b^4 m^2 (m^2 + 4 m m h + 2 m h^2) + l b^4 m^4 \cos[2 (q_1[t] - q_2[t])] \right) \right), \\ ((l b (m + m h) + l a (2 m + m h)) (l b^2 m (l b m h + l a (m + m h)) + I z (l b (m + m h) + l a (2 m + m h))) \sin[2 q_1[t]] + l b^2 m^2 (I z - l a l b m) \sin[2 q_2[t]]) / \left( 2 I z^2 (2 m + m h)^2 + 4 I z l b^2 m (2 m^2 + 3 m m h + m h^2) + l b^4 m^2 (m^2 + 4 m m h + 2 m h^2) - l b^4 m^4 \cos[2 (q_1[t] - q_2[t])] \right), \\ (4 I z^2 m + 4 I z l a^2 m^2 + 4 I z l a l b m^2 + 6 I z l b^2 m^2 + 2 l a^2 l b^2 m^3 + l a l b^3 m^3 + l b^4 m^3 + 2 I z^2 m h + 4 I z l a^2 m m h + 6 I z l a l b m m h + 6 I z l b^2 m m h + 3 l a^2 l b^2 m^2 m h + 4 l a l b^3 m^2 m h + 3 l b^4 m^2 m h + I z l a^2 m h^2 + 2 I z l a l b m h^2 + I z l b^2 m h^2 + l a^2 l b^2 m m h^2 + 2 l a l b^3 m m h^2 + l b^4 m m h^2 - (l b (m + m h) + l a (2 m + m h)) (l b^2 m (l b m h + l a (m + m h)) + I z (l b (m + m h) + l a (2 m + m h))) \cos[2 q_1[t]] - l b^3 (l a + l b) m^3 \cos[2 (q_1[t] - q_2[t])] - I z l b^2 m^2 \cos[2 q_2[t]] + l a l b^3 m^3 \cos[2 q_2[t]] \right) / \left( 2 I z^2 (2 m + m h)^2 + 4 I z l b^2 m (2 m^2 + 3 m m h + m h^2) + l b^4 m^2 (m^2 + 4 m m h + 2 m h^2) - l b^4 m^4 \cos[2 (q_1[t] - q_2[t])] \right) \} \}$$

```
In[68]:= tempB = Inverse[tempA.Transpose[A]].tempA // Simplify
```

```
Out[68]= { { ( (lb^2 m (3 la m + lb m + 2 la mh + 2 lb mh) + 2 Iz (lb (m + mh) + la (2 m + mh))) Cos[q1[t]] -
lb^2 (la + lb) m^2 Cos[q1[t] - 2 q2[t]]) /
(2 Iz^2 + 2 Iz (2 la lb (m + mh) + la^2 (2 m + mh) + lb^2 (2 m + mh)) +
lb^2 m (2 la lb (m + 2 mh) + lb^2 (m + 2 mh) + la^2 (3 m + 2 mh)) -
lb^2 (la + lb)^2 m^2 Cos[2 (q1[t] - q2[t])]) ,
- ( (lb m ((la + lb) (lb (m + mh) + la (2 m + mh)) Cos[2 q1[t] - q2[t]] -
(2 Iz + lb^2 (m + mh) + la^2 (2 m + mh) + la lb (m + 2 mh)) Cos[q2[t]])) /
(- 2 Iz^2 - 2 Iz (2 la lb (m + mh) + la^2 (2 m + mh) + lb^2 (2 m + mh)) -
lb^2 m (2 la lb (m + 2 mh) + lb^2 (m + 2 mh) + la^2 (3 m + 2 mh)) +
lb^2 (la + lb)^2 m^2 Cos[2 (q1[t] - q2[t])]) , 1, 0} ,
{ ( (lb^2 m (3 la m + lb m + 2 la mh + 2 lb mh) + 2 Iz (lb (m + mh) + la (2 m + mh))) Sin[q1[t]] +
lb^2 (la + lb) m^2 Sin[q1[t] - 2 q2[t]]) /
(2 Iz^2 + 2 Iz (2 la lb (m + mh) + la^2 (2 m + mh) + lb^2 (2 m + mh)) +
lb^2 m (2 la lb (m + 2 mh) + lb^2 (m + 2 mh) + la^2 (3 m + 2 mh)) -
lb^2 (la + lb)^2 m^2 Cos[2 (q1[t] - q2[t])]) ,
(lb m (- (la + lb) (lb (m + mh) + la (2 m + mh)) Sin[2 q1[t] - q2[t]] +
(2 Iz + lb^2 (m + mh) + la^2 (2 m + mh) + la lb (m + 2 mh)) Sin[q2[t]])) /
(- 2 Iz^2 - 2 Iz (2 la lb (m + mh) + la^2 (2 m + mh) + lb^2 (2 m + mh)) -
lb^2 m (2 la lb (m + 2 mh) + lb^2 (m + 2 mh) + la^2 (3 m + 2 mh)) +
lb^2 (la + lb)^2 m^2 Cos[2 (q1[t] - q2[t])]) , 0, 1} }
```



```
In[69]:= HurList2Column[HurGlobalELEquation] /. {q1'[t] → 0, q2'[t] → 0, q3'[t] → 0, q4'[t] → 0}
lambda = tempB.% // Simplify
```

```
Out[69]= { {-tau1 + tau2 - 2 g la m Sin[q1[t]] - g lb m Sin[q1[t]] -
  g la mh Sin[q1[t]] - g lb mh Sin[q1[t]] - lb (la + lb) m Sin[q1[t] - q2[t]] q2'[t]^2},
  {-tau2 + g lb m Sin[q2[t]] + lb (la + lb) m Sin[q1[t] - q2[t]] q1'[t]^2},
  {(lb (m + mh) + la (2 m + mh)) Sin[q1[t]] q1'[t]^2 - lb m Sin[q2[t]] q2'[t]^2},
  {2 g m + g mh - (lb (m + mh) + la (2 m + mh)) Cos[q1[t]] q1'[t]^2 + lb m Cos[q2[t]] q2'[t]^2}}
```

```
Out[70]= { { (lb (m + mh) + la (2 m + mh)) Sin[q1[t]] q1'[t]^2 -
  (lb m ((la + lb) (lb (m + mh) + la (2 m + mh)) Cos[2 q1[t] - q2[t]] -
    (2 Iz + lb^2 (m + mh) + la^2 (2 m + mh) + la lb (m + 2 mh)) Cos[q2[t]]
    (-tau2 + g lb m Sin[q2[t]] + lb (la + lb) m Sin[q1[t] - q2[t]] q1'[t]^2)) /
  (-2 Iz^2 - 2 Iz (2 la lb (m + mh) + la^2 (2 m + mh) + lb^2 (2 m + mh)) -
    lb^2 m (2 la lb (m + 2 mh) + lb^2 (m + 2 mh) + la^2 (3 m + 2 mh)) +
    lb^2 (la + lb)^2 m^2 Cos[2 (q1[t] - q2[t])]) - lb m Sin[q2[t]] q2'[t]^2 +
  ((lb^2 m (3 la m + lb m + 2 la mh + 2 lb mh) + 2 Iz (lb (m + mh) + la (2 m + mh))) Cos[q1[t]] -
    lb^2 (la + lb) m^2 Cos[q1[t] - 2 q2[t]]) (tau1 - tau2 +
    g (lb (m + mh) + la (2 m + mh)) Sin[q1[t]] + lb (la + lb) m Sin[q1[t] - q2[t]] q2'[t]^2)) /
  (-2 Iz^2 - 2 Iz (2 la lb (m + mh) + la^2 (2 m + mh) + lb^2 (2 m + mh)) -
    lb^2 m (2 la lb (m + 2 mh) + lb^2 (m + 2 mh) + la^2 (3 m + 2 mh)) +
    lb^2 (la + lb)^2 m^2 Cos[2 (q1[t] - q2[t])]) },
  { 2 g m + g mh - (lb (m + mh) + la (2 m + mh)) Cos[q1[t]] q1'[t]^2 +
    (lb m ((la + lb) (lb (m + mh) + la (2 m + mh)) Sin[2 q1[t] - q2[t]] +
      (2 Iz + lb^2 (m + mh) + la^2 (2 m + mh) + la lb (m + 2 mh)) Sin[q2[t]]
      (-tau2 + g lb m Sin[q2[t]] + lb (la + lb) m Sin[q1[t] - q2[t]] q1'[t]^2)) /
    (-2 Iz^2 - 2 Iz (2 la lb (m + mh) + la^2 (2 m + mh) + lb^2 (2 m + mh)) -
      lb^2 m (2 la lb (m + 2 mh) + lb^2 (m + 2 mh) + la^2 (3 m + 2 mh)) +
      lb^2 (la + lb)^2 m^2 Cos[2 (q1[t] - q2[t])]) + lb m Cos[q2[t]] q2'[t]^2 -
    ((lb^2 m (3 la m + lb m + 2 la mh + 2 lb mh) + 2 Iz (lb (m + mh) + la (2 m + mh))) Sin[q1[t]] +
      lb^2 (la + lb) m^2 Sin[q1[t] - 2 q2[t]]) (tau1 - tau2 +
      g (lb (m + mh) + la (2 m + mh)) Sin[q1[t]] + lb (la + lb) m Sin[q1[t] - q2[t]] q2'[t]^2)) /
    (2 Iz^2 + 2 Iz (2 la lb (m + mh) + la^2 (2 m + mh) + lb^2 (2 m + mh)) +
      lb^2 m (2 la lb (m + 2 mh) + lb^2 (m + 2 mh) + la^2 (3 m + 2 mh)) -
      lb^2 (la + lb)^2 m^2 Cos[2 (q1[t] - q2[t])]) } }
```

```
HurToJulia[lambda[[1, 1]]]
```

```
((lb*(m+mh)+la*(2*m+mh))*sin(q1)+lb*m*sin((q1+q2)))*(q1d)^(2)+(((2*(Iz)^(2)+(2*Iz*(2*la*lb*(m+mh)+((la)^(2)*(2*m+mh)+(lb)^(2)*(2*m+mh)))+(lb)^(2)*m*(2*la*lb*(m+2*mh)+((lb)^(2)*(m+2*mh)+(la)^(2)*(3*m+2*mh)))+-1*(lb)^(2)*((la+lb)^(2)*(m)^(2)*cos(2*q2))))^(2)+-1*(lb)^(2)*m*(3*la*m+(lb*m+(2*la*mh+2*lb*mh)))+2*Iz*(lb*(m+mh)+la*(2*m+mh))*cos(q1)+lb*m*(cos(q1)*(2*(Iz+-1*la*lb*m)*cos(q2)+lb*(la+lb)*m*cos(2*q2))+-2*(Iz+(2*(la)^(2)*m+(2*la*lb*m+((lb)^(2)*m+((la)^(2)*mh+(2*la*lb*mh+((lb)^(2)*mh+lb*(la+lb)*m*cos(q2)))))))*sin(q1)*sin(q2)))*(-1*tau2+(-1*g*lb*m*sin((q1+q2))+lb*(la+lb)*m*sin(q2)*(q1d)^(2)))+(2*lb*m*sin((q1+q2))*q1d*q2d+(lb*m*sin((q1+q2))*(q2d)^(2))+-1*((2*(Iz)^(2)+(2*Iz*(2*la*lb*(m+mh)+((la)^(2)*(2*m+mh)+(lb)^(2)*(2*m+mh)))+(lb)^(2)*m*(2*la*lb*(m+2*mh)+((lb)^(2)*(m+2*mh)+(la)^(2)*(3*m+2*mh)))+-1*(lb)^(2)*((la+lb)^(2)*(m)^(2)*cos(2*q2))))^(2)+-1*((lb)^(2)*m*(3*la*m+(lb*m+(2*la*mh+2*lb*mh)))+2*Iz*(lb*(m+mh)+la*(2*m+mh))*cos(q1)+-1*(lb)^(2)*(la+lb)*(m)^(2)*cos((q1+2*q2)))*(tau1+(2*g*la*m*sin(q1)+(g*lb*m*sin(q1)+(g*la*mh*sin(q1)+(g*lb*mh*sin(q1)+(g*lb*m*sin((q1+q2)))+(2*lb*(la+lb)*m*sin(q2))*q1d*q2d+lb*(la+lb)*m*sin(q2)*(q2d)^(2))))))
```

```
HurToJulia[lambda[[2, 1]]]
```

```
(2*g*m+(g*mh+-1*((lb*(m+mh)+la*(2*m+mh))*cos(q1)+lb*m*cos((q1+q2)))*(q1d)^(2)+(((2*(Iz)^(2)+(2*Iz*(2*la*lb*(m+mh)+((la)^(2)*(2*m+mh)+(lb)^(2)*(2*m+mh)))+(lb)^(2)*m*(2*la*lb*(m+2*mh)+((lb)^(2)*(m+2*mh)+(la)^(2)*(3*m+2*mh)))+-1*(lb)^(2)*((la+lb)^(2)*(m)^(2)*cos(2*q2))))^(2)+-1*(lb)^(2)*m*(3*la*m+(lb*m+(2*la*mh+2*lb*mh)))+2*Iz*(lb*(m+mh)+la*(2*m+mh))*cos(q2)+-1*(lb)^(2)*(la+lb)*(m)^(2)*cos(2*q2)))))*sin(q1)+2*lb*m*cos(q1)*(Iz+(2*(la)^(2)*m+(2*la*lb*m+((lb)^(2)*m+((la)^(2)*mh+(2*la*lb*mh+((lb)^(2)*mh+lb*(la+lb)*m*cos(q2)))))))*sin(q2))*(-1*tau2+(-1*g*lb*m*sin((q1+q2))+lb*(la+lb)*m*sin(q2)*(q1d)^(2)))+(-2*lb*m*cos((q1+q2))*q1d*q2d+(-1*lb*m*cos((q1+q2))*(q2d)^(2))+-1*((2*(Iz)^(2)+(2*Iz*(2*la*lb*(m+mh)+((la)^(2)*(2*m+mh)+(lb)^(2)*(2*m+mh)))+(lb)^(2)*m*(2*la*lb*(m+2*mh)+((lb)^(2)*(m+2*mh)+(la)^(2)*(3*m+2*mh)))+-1*(lb)^(2)*((la+lb)^(2)*(m)^(2)*cos(2*q2))))^(2)+-1*((lb)^(2)*m*(3*la*m+(lb*m+(2*la*mh+2*lb*mh)))+2*Iz*(lb*(m+mh)+la*(2*m+mh))*sin(q1)+-1*(lb)^(2)*(la+lb)*(m)^(2)*sin((q1+2*q2)))*(tau1+(2*g*la*m*sin(q1)+(g*lb*m*sin(q1)+(g*la*mh*sin(q1)+(g*lb*mh*sin(q1)+(g*lb*m*sin((q1+q2)))+(2*lb*(la+lb)*m*sin(q2))*q1d*q2d+lb*(la+lb)*m*sin(q2)*(q2d)^(2))))))
```

```
HurSaveData["data_impactA_rel1.m", "Jbfoot", "impDynConst1",  
"impDynConst2", "temp", "coef", "LinOp", "qdotplus", "lambda"]
```

For impact dynamics constraints without inversion

```

impactLHS = HurGlobalMMatrix.HurList2Column[{q1dp, q2dp, q3dp, q4dp}] -
Transpose[Jbfoot[[1 ;; 2, ;;]]].HurList2Column[{Fimp1, Fimp2}] // Simplify
{ {q2dp (Iz + lb2 m + lb (la + lb) m Cos[q2[t]]) +
  q1dp (2 Iz + 2 la2 m + 2 la lb m + 2 lb2 m + la2 mh + 2 la lb mh + lb2 mh + 2 lb (la + lb) m Cos[q2[t]]) +
  Fimp1 (la + lb) (Cos[q1[t]] + Cos[q1[t] + q2[t]]) +
  q3dp (- (lb (m + mh) + la (2 m + mh)) Cos[q1[t]] - lb m Cos[q1[t] + q2[t]]) +
  Fimp2 (la + lb) (Sin[q1[t]] + Sin[q1[t] + q2[t]]) +
  q4dp (- (lb (m + mh) + la (2 m + mh)) Sin[q1[t]] - lb m Sin[q1[t] + q2[t]]),
  { (Iz + lb2 m) q2dp + q1dp (Iz + lb2 m + lb (la + lb) m Cos[q2[t]]) +
    Fimp1 (la + lb) Cos[q1[t] + q2[t]] - lb m q3dp Cos[q1[t] + q2[t]] +
    Fimp2 (la + lb) Sin[q1[t] + q2[t]] - lb m q4dp Sin[q1[t] + q2[t]]},
  {- Fimp1 + 2 m q3dp + mh q3dp - (lb (m + mh) + la (2 m + mh)) q1dp Cos[q1[t]] -
    lb m (q1dp + q2dp) Cos[q1[t] + q2[t]]},
  {- Fimp2 + 2 m q4dp + mh q4dp - (lb (m + mh) + la (2 m + mh)) q1dp Sin[q1[t]] -
    lb m (q1dp + q2dp) Sin[q1[t] + q2[t]]} }

impactRHS = HurGlobalMMatrix.HurList2Column[{q1dm, q2dm, 0, 0}] // Simplify
{ {2 la2 m q1dm + 2 la lb m q1dm + 2 lb2 m q1dm + la2 mh q1dm + 2 la lb mh q1dm + lb2 mh q1dm +
  lb2 m q2dm + Iz (2 q1dm + q2dm) + lb (la + lb) m (2 q1dm + q2dm) Cos[q2[t]]},
  { (Iz + lb2 m) (q1dm + q2dm) + lb (la + lb) m q1dm Cos[q2[t]]},
  {- (lb (m + mh) + la (2 m + mh)) q1dm Cos[q1[t]] - lb m (q1dm + q2dm) Cos[q1[t] + q2[t]]},
  {- (lb (m + mh) + la (2 m + mh)) q1dm Sin[q1[t]] - lb m (q1dm + q2dm) Sin[q1[t] + q2[t]]} }

impactVel = Jbfoot[[1 ;; 2, ;;]].HurList2Column[{q1dp, q2dp, q3dp, q4dp}] // Simplify
{ {q3dp - (la + lb) q1dp Cos[q1[t]] - (la + lb) (q1dp + q2dp) Cos[q1[t] + q2[t]]},
  {q4dp - (la + lb) q1dp Sin[q1[t]] - (la + lb) (q1dp + q2dp) Sin[q1[t] + q2[t]]} }

impactLHS[[1, 1]]
HurToJulia[%]
q2dp (Iz + lb2 m + lb (la + lb) m Cos[q2[t]]) +
q1dp (2 Iz + 2 la2 m + 2 la lb m + 2 lb2 m + la2 mh + 2 la lb mh + lb2 mh + 2 lb (la + lb) m Cos[q2[t]]) +
Fimp1 (la + lb) (Cos[q1[t]] + Cos[q1[t] + q2[t]]) +
q3dp (- (lb (m + mh) + la (2 m + mh)) Cos[q1[t]] - lb m Cos[q1[t] + q2[t]]) +
Fimp2 (la + lb) (Sin[q1[t]] + Sin[q1[t] + q2[t]]) +
q4dp (- (lb (m + mh) + la (2 m + mh)) Sin[q1[t]] - lb m Sin[q1[t] + q2[t]])

(q2dp*(Iz + (lb)^(2)*m + lb*(la + lb)*m*cos(q2)) + (q1dp*(2*Iz + (2*(la)^(2)*m + (2*la*lb*m + (2*
(lb)^(2)*m + (la)^(2)*mh + (2*la*lb*mh + (lb)^(2)*mh + 2*lb*(la + lb)*m*cos(q2)))))) + (
Fimp1*(la + lb)*(cos(q1) + cos((q1 + q2))) + (q3dp*(-1*(lb*(m + mh) + la*(2*m + mh))*cos(q1) - 1*lb
*m*cos((q1 + q2))) + (Fimp2*(la + lb)*(sin(q1) + sin((q1 + q2))) + q4dp*(-1*(lb*(m + mh) + la*(2*m +
mh))*sin(q1) - 1*lb*m*sin((q1 + q2))))))

```

**impactLHS[[2, 1]]**

**HurToJulia[%]**

$$\begin{aligned} & (Iz + lb^2 m) q2dp + q1dp (Iz + lb^2 m + lb (la + lb) m \cos[q2[t]]) + \\ & Fimp1 (la + lb) \cos[q1[t] + q2[t]] - lb m q3dp \cos[q1[t] + q2[t]] + \\ & Fimp2 (la + lb) \sin[q1[t] + q2[t]] - lb m q4dp \sin[q1[t] + q2[t]] \\ & ((Iz + (lb)^{(2)} * m) * q2dp + (q1dp * (Iz + ((lb)^{(2)} * m + lb * (la + lb) * m * \cos(q2))) + (Fimp1 * (la + lb) * \cos( \\ & (q1 + q2)) + (-1 * lb * m * q3dp * \cos((q1 + q2)) + (Fimp2 * (la + lb) * \sin((q1 + q2)) + -1 * lb * m * q4dp * \sin((q1 \\ & + q2)))))) \end{aligned}$$

**impactLHS[[3, 1]]**

**HurToJulia[%]**

$$\begin{aligned} & -Fimp1 + 2 m q3dp + mh q3dp - \\ & (lb (m + mh) + la (2 m + mh)) q1dp \cos[q1[t]] - lb m (q1dp + q2dp) \cos[q1[t] + q2[t]] \\ & (-1 * Fimp1 + (2 * m * q3dp + (mh * q3dp + (-1 * (lb * (m + mh) + la * (2 * m + mh)) * q1dp * \cos(q1) - 1 * lb * m * (q1dp + \\ & q2dp) * \cos((q1 + q2)))))) \end{aligned}$$

**impactLHS[[4, 1]]**

**HurToJulia[%]**

$$\begin{aligned} & -Fimp2 + 2 m q4dp + mh q4dp - \\ & (lb (m + mh) + la (2 m + mh)) q1dp \sin[q1[t]] - lb m (q1dp + q2dp) \sin[q1[t] + q2[t]] \\ & (-1 * Fimp2 + (2 * m * q4dp + (mh * q4dp + (-1 * (lb * (m + mh) + la * (2 * m + mh)) * q1dp * \sin(q1) - 1 * lb * m * (q1dp + \\ & q2dp) * \sin((q1 + q2)))))) \end{aligned}$$

**impactRHS[[1, 1]]**

**HurToJulia[%]**

$$\begin{aligned} & 2 la^2 m q1dm + 2 la lb m q1dm + 2 lb^2 m q1dm + la^2 mh q1dm + 2 la lb mh q1dm + \\ & lb^2 mh q1dm + lb^2 m q2dm + Iz (2 q1dm + q2dm) + lb (la + lb) m (2 q1dm + q2dm) \cos[q2[t]] \\ & (2 * (la)^{(2)} * m * q1dm + (2 * la * lb * m * q1dm + (2 * (lb)^{(2)} * m * q1dm + ((la)^{(2)} * mh * q1dm + (2 * la * lb * mh * \\ & q1dm + ((lb)^{(2)} * mh * q1dm + ((lb)^{(2)} * m * q2dm + (Iz * (2 * q1dm + q2dm) + lb * (la + lb) * m * (2 * q1dm + q2dm) \\ & * \cos(q2)))))) \end{aligned}$$

**impactRHS[[2, 1]]**

**HurToJulia[%]**

$$\begin{aligned} & (Iz + lb^2 m) (q1dm + q2dm) + lb (la + lb) m q1dm \cos[q2[t]] \\ & ((Iz + (lb)^{(2)} * m) * (q1dm + q2dm) + lb * (la + lb) * m * q1dm * \cos(q2)) \end{aligned}$$

**impactRHS[[3, 1]]**

**HurToJulia[%]**

$$\begin{aligned} & - (lb (m + mh) + la (2 m + mh)) q1dm \cos[q1[t]] - lb m (q1dm + q2dm) \cos[q1[t] + q2[t]] \\ & (-1 * (lb * (m + mh) + la * (2 * m + mh)) * q1dm * \cos(q1) - 1 * lb * m * (q1dm + q2dm) * \cos((q1 + q2))) \end{aligned}$$

```
impactRHS[ [4, 1]]
```

```
HurToJulia[%]
```

$$- \left( lb (m + mh) + la (2m + mh) \right) q1dm \sin[q1[t]] - lbm (q1dm + q2dm) \sin[q1[t] + q2[t]]$$

$$(-1 * (lb * (m + mh) + la * (2 * m + mh)) * q1dm * \sin(q1) + -1 * lb * m * (q1dm + q2dm) * \sin((q1 + q2)))$$

```
impactVel[ [1, 1]]
```

```
HurToJulia[%]
```

$$q3dp - (la + lb) q1dp \cos[q1[t]] - (la + lb) (q1dp + q2dp) \cos[q1[t] + q2[t]]$$

$$(q3dp + (-1 * (la + lb) * q1dp * \cos(q1) + -1 * (la + lb) * (q1dp + q2dp) * \cos((q1 + q2))))$$

```
impactVel[ [2, 1]]
```

```
HurToJulia[%]
```

$$q4dp - (la + lb) q1dp \sin[q1[t]] - (la + lb) (q1dp + q2dp) \sin[q1[t] + q2[t]]$$

$$(q4dp + (-1 * (la + lb) * q1dp * \sin(q1) + -1 * (la + lb) * (q1dp + q2dp) * \sin((q1 + q2))))$$

```
HurSaveData["data_impactAAcc_rel.m", "Jbfoot", "impDynConst1", "impDynConst2", "temp",  
"coef", "LinOp", "qdotplus", "lambda", "impactLHS", "impactRHS", "impactVel"]
```

```
In[71]:= ELforGRF = HurGlobalELEquation /.
```

```
{q3[t] → 0, q4[t] → 0, q3'[t] → 0, q4'[t] → 0, q3''[t] → 0, q4''[t] → 0};
```

```
% //
```

```
MatrixForm
```

```
Out[72]//MatrixForm=
```

$$\begin{pmatrix} -\tau u1 + \tau u2 - 2 g la m \sin[q1[t]] - g lb m \sin[q1[t]] - g la mh \sin[q1[t]] - g lb mh \sin[q1[t]] - lb \\ -\tau u2 + g lb m \sin[q2[t]] + lb (la \\ (lb (m + mh) + la (2m + mh)) \sin[q1[t]] q1'[t]^2 - lb m \sin[q2[t]] q2' \\ 2 g m + g mh - (lb (m + mh) + la (2m + mh)) \cos[q1[t]] q1'[t]^2 + lb m \cos[q2[$$

```
In[73]:= lambda1 = ELforGRF[ [3]] // Simplify
```

```
lambda2 = ELforGRF[ [4]] // Simplify
```

```
Out[73]=
```

$$(lb (m + mh) + la (2m + mh)) \sin[q1[t]] q1'[t]^2 - lbm \sin[q2[t]] q2'[t]^2 - 2 la m \cos[q1[t]] q1''[t] - lbm \cos[q1[t]] q1''[t] - la mh \cos[q1[t]] q1''[t] - lb mh \cos[q1[t]] q1''[t] + lbm \cos[q2[t]] q2''[t]$$

```
Out[74]=
```

$$2 g m + g mh - (lb (m + mh) + la (2m + mh)) \cos[q1[t]] q1'[t]^2 + lbm \cos[q2[t]] q2'[t]^2 - 2 la m \sin[q1[t]] q1''[t] - lbm \sin[q1[t]] q1''[t] - la mh \sin[q1[t]] q1''[t] - lb mh \sin[q1[t]] q1''[t] + lbm \sin[q2[t]] q2''[t]$$

```
In[75]:= HurToJulia[lambda1]
```

```
Out[75]=
```

$$((lb * (m + mh) + la * (2 * m + mh)) * \sin(q1) * (q1d)^{(2)} + (-1 * lb * m * \sin(q2) * (q2d)^{(2)} + (-2 * la * m * \cos(q1) * q1dd + (-1 * lb * m * \cos(q1) * q1dd + (-1 * la * mh * \cos(q1) * q1dd + (-1 * lb * mh * \cos(q1) * q1dd + lb * m * \cos(q2) * q2dd))))))$$

```
In[76]:= HurToJulia[lambda2]
```

```
Out[76]=
```

$$(2 * g * m + (g * mh + (-1 * (lb * (m + mh) + la * (2 * m + mh)) * \cos(q1) * (q1d)^{(2)} + (lb * m * \cos(q2) * (q2d)^{(2)} + (-2 * la * m * \sin(q1) * q1dd + (-1 * lb * m * \sin(q1) * q1dd + (-1 * la * mh * \sin(q1) * q1dd + (-1 * lb * mh * \sin(q1) * q1dd + lb * m * \sin(q2) * q2dd))))))$$

```
HurSaveData["data_impactAAcc_abs.m", "Jbfoot", "impDynConst1",  
  "impDynConst2", "temp", "coef", "LinOp", "qdotplus", "lambda",  
  "lambda1", "lambda2", "impactLHS", "impactRHS", "impactVel"]
```