HurToolbox is a very powerful tool to derive the equations of motion for multibody systems in 3D spaces. Once reference frames, generalized coordinates, direction cosine matrices, center of mass (COM) of reference frames, and inertial properties, obtaining equations of motion via either NE or EL methods are immediate. The only reason that HurNEEquations[] or HurELEquations[] takes long (e.g., a few seconds for the coin problem, or 10-20 seconds for the same coin problem with additional generalized coordinates) is that “Simplify” process is performed. If the “Simplify” option is turned off[[1]](#footnote-1), the time spent on HurNEEquations[] or HurELEquations[] is less than a second. The derived equations of motion are stored in HurGlobalNEEquation, and HurGlobalELEquation for HurNEEquations[] and HurELEquations[], respectively.

A serious issue happens when we perform the inverse process. As we know, inverse process is needed to find the expression for the double time derivatives of the generalized coordinates, (). Knowing is important since the state space model requires them for the numerical simulation. Note that is directly related to the equations of motion whereas describe the current state of the system. In other words, tells us how the states will be updated for the next time step. Inverse process will enable us to get the from the equations of motion. Luckily, the equations of motion are linear[[2]](#footnote-2) in terms of unknown variables (e.g., , generalized forces). Therefore, finding these unknown variables should be straightforward. This is true when the “Simplify” option is turned off. Usually, the solution found in this way is very lengthy and includes complicated nonlinear terms of sine and cosine functions in the denominators. In most cases, these expressions can be simplified significantly. However, the simplification process requires significant computational power due to exponentially increasing complexity of the combinatorics problems. Usually, this complication is unavoidable and is limited by the computational power of the computers. Maybe, this will be resolved when the quantum computers prevail.

In this note, I will introduce two ways of workaround for this issue: i) simplify the equations more efficiently, and ii) solve the inverse problems numerically during the simulation.

1. “Simplify” option can be turned on and off by the following commands: HurTurnOffSimplify[], HurTurnOnSimplify[] [↑](#footnote-ref-1)
2. To be mathematically rigorous, equations of motion are affine in terms of and generalized forces. [↑](#footnote-ref-2)