Numerical Optimisation

Project 1; Phase 1

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Auxiliary Percentage: 100%

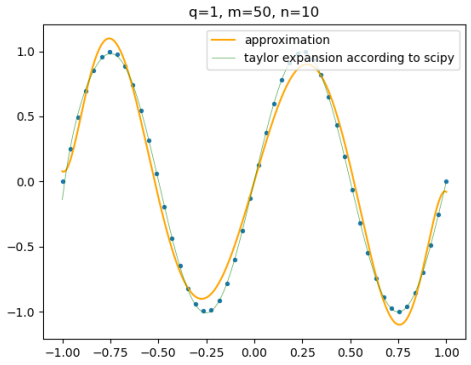
For tasks 1-5 I used the steepest descent method. To find the step length α_k I used the backtracking line search as described in the lecture (Algorithm 3.1). For this, I used $\rho = 0.9$ and varying c.

(i) Tasks 1-5 are solved using steepest descent, 6-10 using Newton's method.

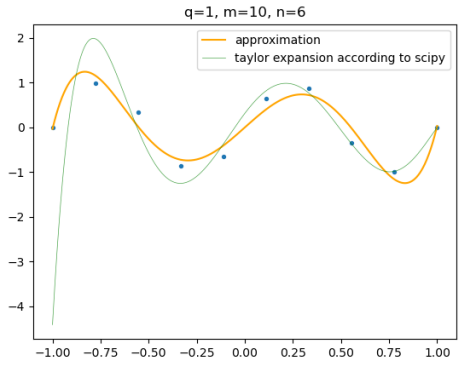
task	problem	$x \in \mathbb{R} : x$ loc. min.	$\tilde{x}$	$\|f(\tilde{x})\|$	$\|\tilde{x} - x^*\|$	iter.
1	$(x+1)^3 - (x+1)^2$	$\{ -\frac{1}{3} \}$	-0.33333306704602184	0.1481	2.662873e-07	5
2	$(x-9)(3x^3 - 73x^2 + 591x - 1617)$	$\{ \approx 9.69562 \}$	9.69562077	10.3963	1.770741e-09	9
3	$-e^{-(x-\frac{1}{2})^2}$	$\{ \frac{1}{2} \}$	0.50000003904933	1.0000	3.904933e-08	5
4	$e^{-(x^2-(x-1)^4)}$	$\{ 2 \}$	1.9999981757365604	0.0498	1.824263e-06	21
5	$e^{-((x-0.4)^4+(x-0.45)^2)}$	$\{ \approx 0.450254 \}$	0.6502538	0.99999369	2.537953e-04	6
6	$(x+1)^3 - (x+1)^2$	$\{ -\frac{1}{3} \}$	-0.33333331784628467	0.1481	1.548705e-08	4
7	$(x-9)(3x^3 - 73x^2 + 591x - 1617)$	$\{ \approx 9.69562 \}$	9.69562077	of 10.3963	6.424299e-11	10
8	$-e^{-(x-\frac{1}{2})^2}$	$\{ \frac{1}{2} \}$	0.49999999999999994	1.0000	5.876376e-17	6
9	$e^{-(x^2-(x-1)^4)}$	$\{ 2 \}$	2.0000000000068527	0.0498	6.852816e-12	10
10	$e^{-((x-0.4)^4+(x-0.45)^2)}$	$\{ \approx 0.450254 \}$	0.45025418489603697	1.0000	1.848960e-08	5

(ii) Tasks 11-15 are solved using steepest descent, 16-20 using Newton's method.

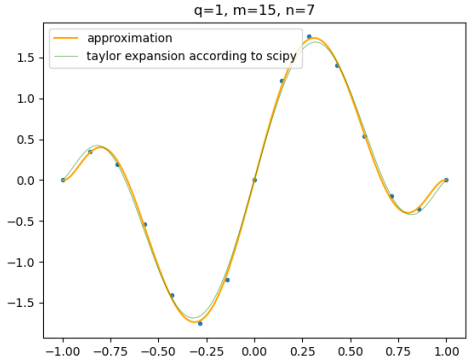
task	target function g	interval $[-q, q]$	number points m	degree n	stop.crit.	iter.
11	$\sin(2\pi t)$	$[-1, 1]$	50	10	1e-3	iter
12	$\sin(2\pi t)$	$[-1, 1]$	10	6	1e-3	2830
13	$\sin(2\pi t) + \sin(\pi t)$	$[-1, 1]$	15	7	1e-3	36047
14	$\sin(2\pi t) + \sin(\pi t) + \sin(5\pi t)$	$[-1, 1]$	20	10	1e-2	6315
15	$\sin(2\pi t) + e^t \cos(3\pi t)$	$[-1, 1]$	30	15	1e-2	66998
16	$\sin(2\pi t)$	$[-1, 1]$	20	7	1e-6	18
17	$\sin(2\pi t) + \sin(\pi t)$	$[-1, 1]$	100	2	1e-6	21
18	$\sin(2\pi t) + \sin(\pi t)$	$[-1, 1]$	50	10	1e-6	19
19	$\sin(2\pi t) + \sin(\pi t) + \sin(5\pi t)$	$[-1, 1]$	50	15	1e-6	20
20	$\sin(2\pi t) + e^t \cos(3\pi t)$	$[-1, 1]$	50	10	1e-6	21



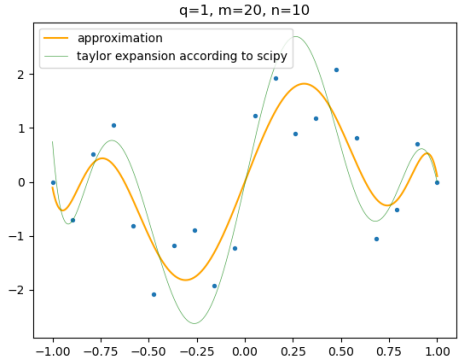
(a) task 11



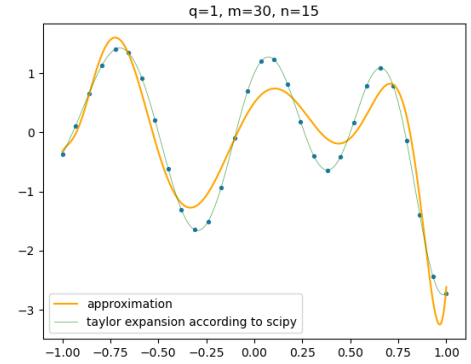
(b) task 12



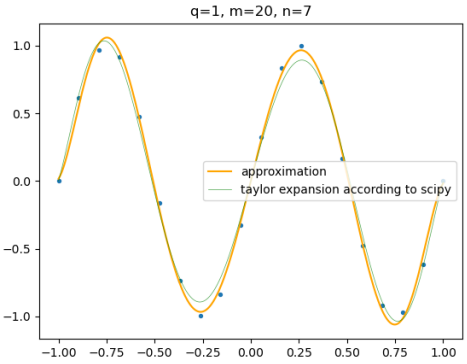
(a) task 13



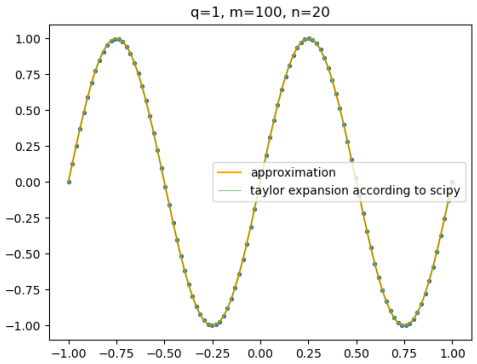
(b) task 14



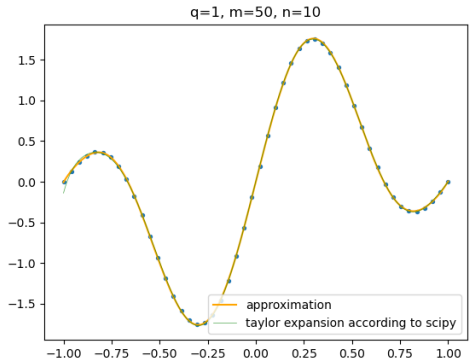
(a) task 15



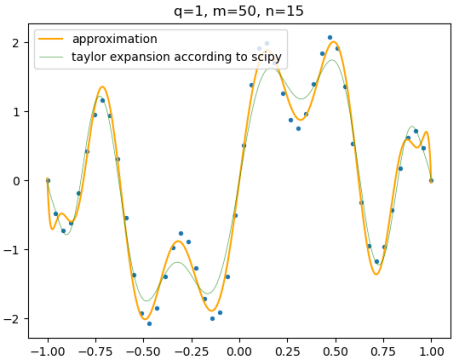
(b) task 16



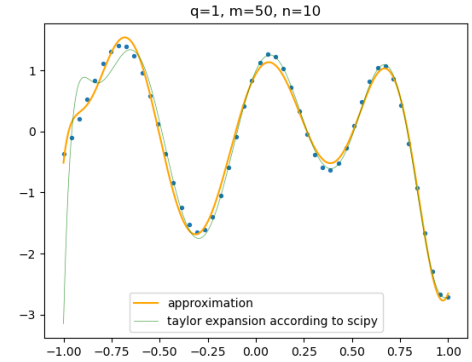
(a) task 17



(b) task 18



(a) task 19



(b) task 20

Figure 1: Problem (ii)

(iii) I changed the stopping criterion from $\|\nabla f_k\| < 10^{-6}$ to $\|\nabla f_k\| < 10^{-2}$, otherwise convergence would take too long (maybe even infinite due to numerical imprecision). You can see this with the condition numbers. Due to this the results are not very good. The eigenvalues are obtained using *numpy.linalg.eig*.

task	problem	$\{x \in \mathbb{R}^n : x \text{ loc. min.}\}$	$\bar{x}$	$\|f(\bar{x})\|$	$\|\bar{x} - x^*\|$	eig.val. $\lambda_1, \ldots, \lambda_n$ of Q	$\frac{\lambda_n - \lambda_1}{\lambda_n + \lambda_1}$	$\frac{\lambda_n}{\lambda_1}$	iter.
21	$n = 5$	{ (5, -120, 630, -1120, 630) }	(-2.3142, 31.4853, -62.0001, -35.2023, 85.1932)	115.48	1405.51	{ 1.5670, 0.2085, 0.0114, 3.0589e-4, 3.2879e-06 }	-0.9999	2.0981e-06	2038
22	$n = 8$	{ (6, -278, 2717, -9684, 12439, 1804, -15160, 8207) }	(0.8584, -25.0388, 91.8059, -12.0188, -101.0061, -98.9850, -1.1862, 175.4548)	244.9215	23572.6820	{ 1.6959, 0.2981, 0.0262, 0.0014, 5.4369e-05, 1.2943e-06, 1.7988e-08, 1.1115e-10 }	-0.9999	6.55412e-11	6885
23	$n = 12$	{ (-7, 331, -3375, 12062, -13459, -6647, 9962, 13193, 950, -14225, -14724, 16022) }	(2.7494, -28.2722, 23.9808, 65.6464, 30.9794, -32.4878, -84.1341, -103.1135, -82.2686, -21.9255, 74.0605, 200.5100)	279.9208	36484.6892	{ 1.7953, 0.3802, 0.0447, 0.0037, 0.0002, 1.1163e-05, 4.0823e-07, 1.1228e-08, 2.2519e-10, 3.1113e-12, 2.6488e-14, 1.0955e-16 }	-1.0	6.1023e-17	6880
24	$n = 20$	{ (7, -349, 3436, -11386, 10140, 8391, -3849, -10825, -8635, -1102, 6793, 11234, 10599, 5554, -2205, -9789, -14105, -12393, -1784, 20405) }	(2.5408, 37.9365, -83.6696, -40.7594, 41.9088, 92.5506, 97.5810, 67.2641, 17.4699, -37.4019, -86.4135, -122.1064, -139.8910, -137.3396, -113.5645, -68.7285, -3.6786, 80.3189, 181.7650, 299.0559)	493.5051	41324.4256	{ 1.9071, 0.4870, 0.0755, 0.0089, 0.0008, 7.0334e-05, 4.8305e-06, 2.8276e-07, 1.4139e-08, 6.0360e-10, 2.1928e-11, 6.7408e-13, 1.7382e-14, 3.7547e-16, 1.3662e-17, 9.7897e-18, 1.1364e-18, -3.3954e-18+1.9338e-18j, -3.3954e-18-1.9338e-18j), -9.6106e-18 }	-1.0	-5.0392e-18	11725
25	$n = 30$	{ (8, -272, 1979, -4326, 721, 3496, 2360, -279, -2436, -3272, -2835, -1537, 112, 1668, 2821, 3406, 3385, 2794, 1758, 431, -1010, -2376, -3491, -4184, -4299, -3694, -2260, 101, 3480, 7932) }	(0.9888, -27.0951, 109.843 , -37.9596, -115.0667, -95.9121, -27.9969, 46.8452, 104.7627, 136.2247, 140.5346 121.6452, 85.4868, 38.4203, -13.5812, -65.3118, -112.46, -151.6005, -180.1179, -196.1031, -198.2434, -185.7175, -158.1018, -115.2882, -57.4151, 15.1907, 102.0612, 202.6263, 316.2444, 442.2279)	828.3468	15980.9316	{ 1.9864, 0.5725, 0.1056, 0.0154, 0.0019, 0.0002, 1.9965e-05, 1.6986e-06, 1.2925e-07, 8.8280e-09, 5.4235e-10, 3.0008e-11, 1.4958e-12, 6.7143e-14, 2.7112e-15, 9.8549e-17, 1.2370e-17, 9.4688e-18, 8.0259e-18, 3.2087e-18, 2.8685e-18+1.0937e-18j, 2.8685e-18-1.0937e-18j, 9.7422e-19, -3.1024e-18, -4.3801e-18+3.4639e-18j, -4.3801e-18-3.4639e-18j, -5.9583e-18+1.4215e-18j, -5.9583e-18-1.4215e-18j, -7.5597e-18, -1.2380e-17 }	-0.9999	-6.2322e-18	25612

(iv) Tasks 26-30. The stopping criterion considers $\|\nabla f_k\|$.

task	problem	$\{(x,y) \in \mathbb{R}^2 : (x,y) \text{ loc. min.}\}$	$\tilde{x}$	$\|f(\tilde{x})\|$	$\|\tilde{x} - x^*\|$	stop crit.	iter.
26	$(10x_1 - 10x_2^2)^2 + (1 - x_0)^2$	{ (1,1) }	(1., 1.)	0.0	7.5e-01	1e-06	2
27	$(2x_1^2 - 8x_2)^2 + (x_1 - 1)^2$	{ (1, 1/4) }	(1., 0.25)	0.0	0.e+00	1e-06	2
28	$(10x_1^2 - 2.5x_2)^2 + (x_1 + 1)^2$	{ (-1, 2) }	(-1., 2.)	0.0	0.e+00	1e-18	2
29	$(10x_1 - 0.01x_2^2)^2 + (x_2 - 100)^2$	{ (10, 100) }	(10., 100.)	4.3e-30	0.e+00	1e-18	2
30	$(x_2 - x_1^2)^2 + (100 - x_1)^2$	{ (100, 10.000) }	(100., 10000.00000001)	3.5343e-21	1.1889e-08	1e-18	5