

Digital Signal Processing SS 2024 – Exercise 5

Digital Signal Processing Tutorial

Group 23

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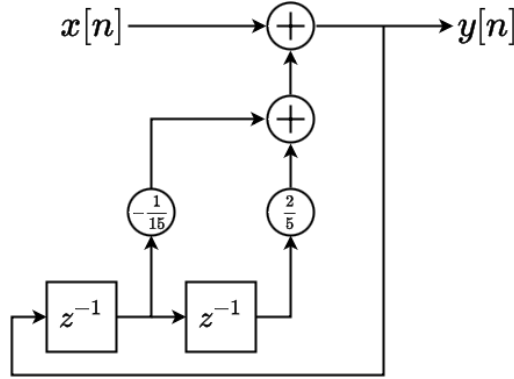
Exercise 1

Exercise 2

We are given the difference equation

$$y[n] = x[n] - \frac{1}{15}y[n-1] + \frac{2}{5}y[n-2].$$

a) We sketch the corresponding block diagram:



- b) The system is an IIR Filter. This is because in the difference equation we have non-zero a coefficients, i.e., it is a recursive filter.
- c) To find the transfer function, we first need to transform the system to the z -domain. For the transformation we use the linearity of the z -transformation. We assume that the signal is causal, i.e., $x[n] = y[n] = 0$ for $n < 0$.

$$y[n] = x[n] - \frac{1}{15}y[n-1] + \frac{2}{5}y[n-2] \quad (1)$$

$$\circ \quad (2)$$

$$Y(z) = X(z) - \frac{1}{15}z^{-1}Y(z) + \frac{2}{5}z^{-2}Y(z) \quad \Longleftrightarrow \quad (3)$$

$$Y(z) \left(1 + \frac{1}{15}z^{-1} - \frac{2}{5}z^{-2} \right) = X(z) \quad \Longleftrightarrow \quad (4)$$

$$Y(z) = \left(1 + \frac{1}{15}z^{-1} - \frac{2}{5}z^{-2} \right)^{-1} X(z) \quad (5)$$

We can see that the transfer function $H(z) = \frac{Y(z)}{X(z)}$ is given by

$$H(z) = \left(1 + \frac{1}{15}z^{-1} - \frac{2}{5}z^{-2} \right)^{-1} \quad (6)$$

$$= \frac{1}{1 + \frac{1}{15}z^{-1} - \frac{2}{5}z^{-2}} \quad (7)$$

$$= \frac{1}{1 + \frac{1}{15}z^{-1} - \frac{2}{5}z^{-2}} \frac{z^2}{z^2} \quad (8)$$

$$= \frac{z^2}{z^2 + \frac{1}{15}z - \frac{2}{5}} \quad (9)$$

$$(10)$$

- d) The roots of the numerator are clearly 0. The roots of the denominator can be determined from $z^2 + \frac{1}{15}z - \frac{2}{5} = 0$:

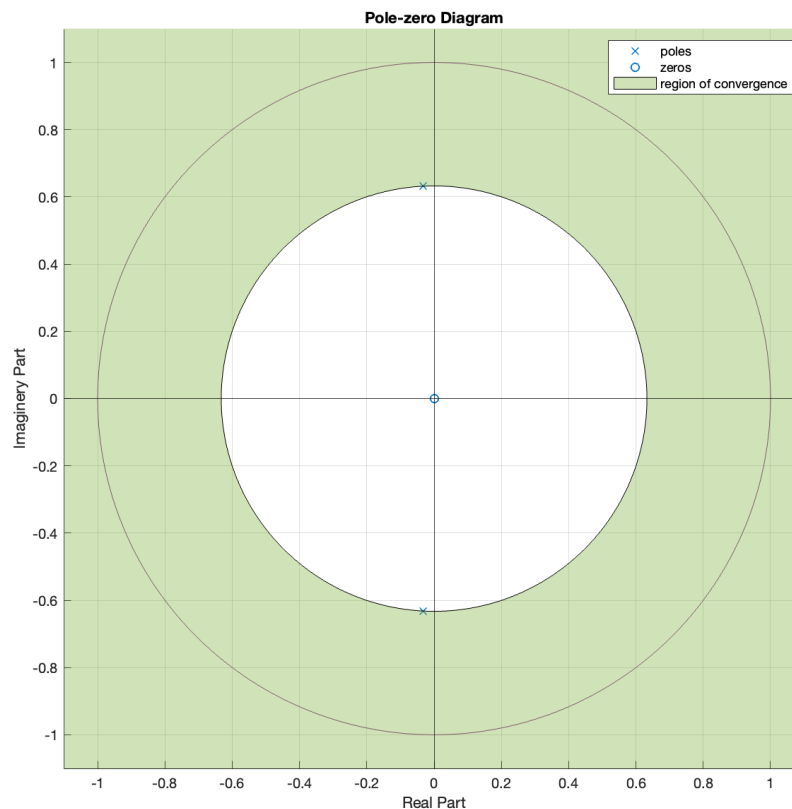
$$z_{1,2} = -\frac{1}{30} \pm \sqrt{\frac{1}{1125} - \frac{8}{20}} \quad (11)$$

$$= -\frac{1}{30} \pm j\sqrt{\frac{8980}{22500}} \quad (12)$$

$$= -\frac{1}{30} \pm j\frac{\sqrt{8980}}{150} \quad (13)$$

$$\approx -0.0333 \pm j0.0574 \quad (14)$$

You can see a sketch of the pole-zero map:



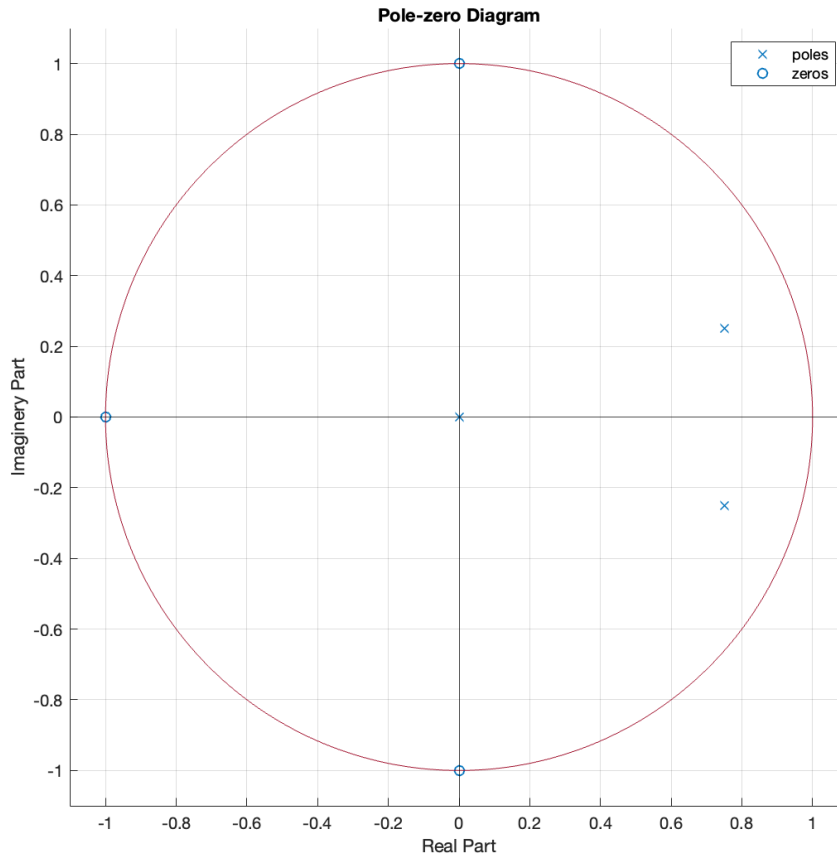
- e) The system is stable. This can be seen by the region of convergence including the unit circle.

Exercise 3

We are given a set of poles and zeros:

- Zeros: $N_1 = -1$, $N_2 = j$, $N_3 = -j$
- Poles: $P_1 = 0$, $P_2 = 0.75 + j0.25$, $P_3 = 0.27 - 0.25j$

- a) This filter has only real coefficients, since when we express the transfer function in the pole-zero representation then we can see that the numerator and denominator both contain a complex number and their corresponding conjugate, meaning that when we multiply it out we will be left with only real coefficients.
- b) You can see a sketch of the pole-zero map:



- c) The transfer function can be obtained by writing it first in pole-zero representation and then multiplying out the clauses :

$$H(z) = b_0 z^{M-N} \frac{(z - N_1)(z - N_2)(z - N_3)}{(z - P_1)(z - P_2)(z - P_3)} \quad (15)$$

$$= b_0 \frac{(z + 1)(z + j)(z - j)}{z(0.75 + j0.25)(0.27 - 0.25j)} \quad (16)$$

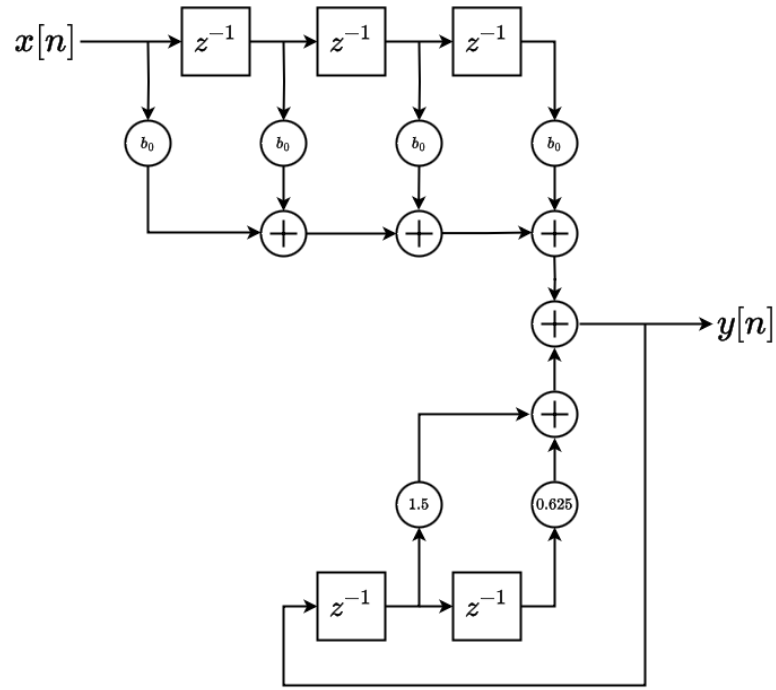
$$= b_0 \frac{z^3 + z^2 + z + 1}{z^3 - 1.5z^2 + 0.625z} \quad (17)$$

$$= \frac{b_0 z^3 + b_0 z^2 + b_0 z + b_0}{z^3 - 1.5z^2 + 0.625z} \quad (18)$$

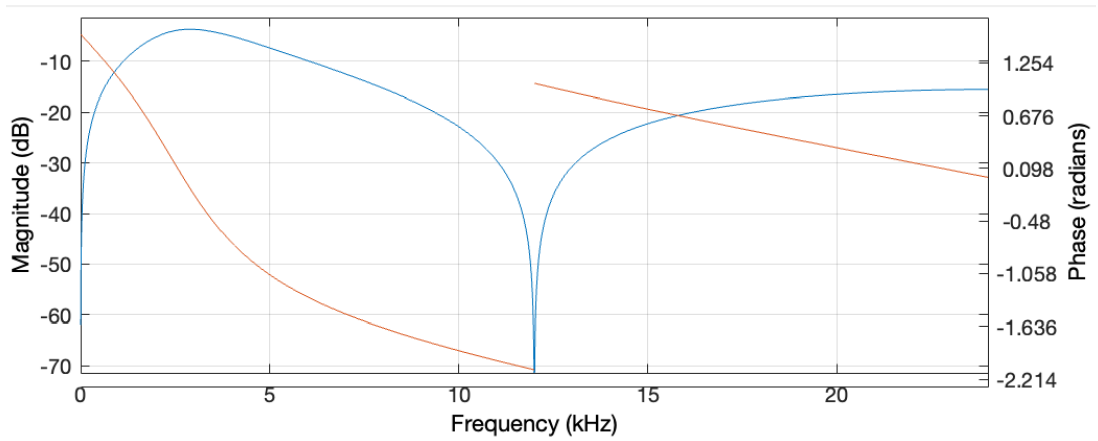
$$= \frac{b_0 + b_0 z^{-1} + b_0 z^{-2} + b_0 z^{-3}}{1 - 1.5z^{-1} + 0.625z^{-2}} \quad (19)$$

$$(20)$$

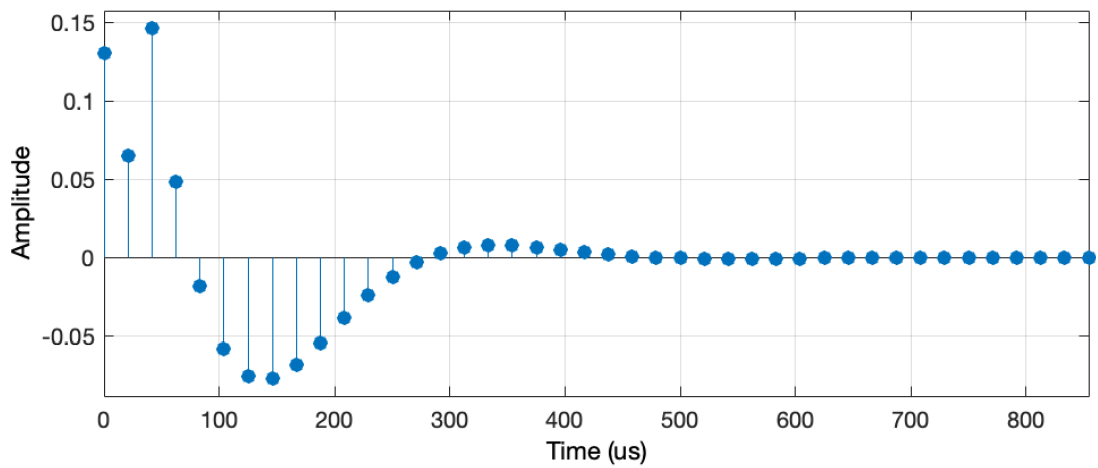
d) The block diagram of a direct-form-I implementation of the filter can be seen:



e) The magnitude and phase response can be seen:



f) The impulse response for $0 \leq n \leq 50$ can be seen:



Exercise 4