Digital Signal Processing Tutorial

Group 23

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Exercise 1

We have three complex numbers given:

$$c_1 = -5 + 3j \qquad\qquad c_2 = \frac{\sqrt{2}}{2} e^{-\frac{3\pi j}{4}} \qquad\qquad c_3 = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} j$$

First, let's use Euler's formula to simplify c_2:

$$c_2 = \frac{\sqrt{2}}{2} e^{-\frac{3\pi j}{4}} = \frac{\sqrt{2}}{2} \left(\cos\left(\frac{3\pi}{4}\right) + j \sin\left(\frac{3\pi}{4}\right) \right) = \frac{\sqrt{2}}{2} \left(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} j \right) = -\frac{1}{2} + \frac{1}{2} j$$

With this, let us now compute the following numbers:

$$c_4 = c_1 + c_2 \quad = -5 - \frac{1}{2} + 3j + \frac{1}{2} j = -\frac{11}{2} + \frac{7}{2} j$$

$$c_5 = c_1 \cdot c_2 \quad = (-5 + 3j) \cdot \left(-\frac{1}{2} + \frac{1}{2} j \right) = \frac{5}{2} - \frac{5}{2} j - \frac{3}{2} j - \frac{3}{2} = 1 - 4j$$

$$c_6 = |c_3|^2 \quad = \left(\sqrt{ \left(\frac{1}{\sqrt{2}} \right)^2 + \left(\frac{1}{\sqrt{2}} \right)^2 } \right)^2 = \left(\sqrt{ \frac{1}{2} + \frac{1}{2} } \right)^2 = \left(\sqrt{1} \right)^2 = 1$$

$$c_7 = \arg(c_3) \quad = \text{atan2}\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right) = \arctan\left(\frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} \right) = \arctan(1) = \frac{\pi}{4}$$

$$c_8 = \frac{c_1}{c_2} \quad = \frac{-5 + 3j}{\frac{-1+1j}{2}} = \frac{-10 + 3j}{-1 + 1j} = \frac{(-1 - 1j)(-10 + 3j)}{(-1 - 1j)(-1 + 1j)} = \frac{10 - 3j - 10j + 3}{1 - 1j + 1j + 1} = \frac{13}{2} - \frac{13}{2} j$$

$$c_9 = c_1 \cdot c_1^* \quad = (-5 + 3j)(-5 - 3j) = 25 + 15j - 15j + 9 = 34$$