Digital Signal Processing SS 2024 – Exercise 2

Digital Signal Processing Tutorial

Group 23

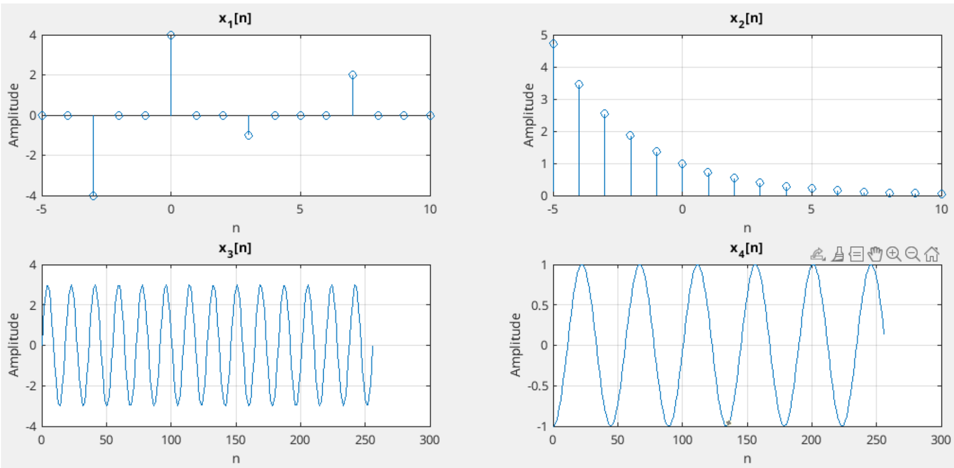
Aaron Zettler, 12105021

Pascal Pilz, 12111234

Exercise 1

(a) We plot the discrete time signals in Matlab

$$
\begin{aligned}
x_1[n] &= -4\delta[n+3] + 4\delta[n] - \delta[n-3] + 2\delta[n-7] && \text{for } -5 \leq n \leq 10 \\
x_2[n] &= e^{-0.31n} && \text{for } -5 \leq n \leq 10 \\
x_3[n] &= 3\sin\left(2\pi\frac{3.5}{64}n\right) && \text{for } 0 \leq n \leq 256 \\
x_4[n] &= -\cos\left(\frac{9}{64}n\right) && \text{for } 0 \leq n \leq 256
\end{aligned}
$$



(b) The normalized angular frequency Ω: For $x_3[n]$: $\Omega_{x3} = 0.3436$ and for $x_4[n]$: $\Omega_{x4} = 0.1406$:

(c) The signals $x_3[n]$ and $x_4[n]$ are periodic with fundamental periods $T_{x3} = 18.2857$ and $T_{x4} = 44.6804$, respectively.

(d) The powers for the periodic signals $x_3[n]$ and $x_4[n]$ are $P_{x3} = 4.5658$ and $P_{x4} = 0.4925$.

(e) The energies according to $W = \sum_{n=-\infty}^{\infty} |x[n]|^2$ for these time-limited signals $W_{x1} = 37$, $W_{x2} = 48.0394$, $W_{x3} = 1152$ and $W_{x4} = 128.9565$.

(f) All signal powers of (d) and signal energies of (e) in a single table.

Signal	Power (P)	Energy (W)
$x_1[n]$	X	37
$x_2[n]$	X	48.0394
$x_3[n]$	4.5658	1152
$x_4[n]$	0.4925	128.9565

Exercise 2

The signal $x[n] = (3, -1, 2, 0, 1)$ at sample times $n = (0, 1, 2, 3, 4)$ is the input to an LTI system with impulse response $h[n] = (2, 3, 4, 1)$ at sample times $n = (0, 1, 2, 3)$.

(a) We calculate the length of the output signal $y[n]$:

1) We know (DSP_04 page 15):
   If $x[n]$ and $h[n]$ have finite lengths N_x and N_h respectively,
   then the length of the output signal $y[n]$ is given by $N_y = N_x + N_h - 1$.

2) So with $N_x = 5$ and $N_h = 4$ we get: $N_y = 5 + 4 - 1 = 8$.

(b) We calculate the output signal $y[n]$ manually:

1) We know the formula for convolution (DSP_04 page 17): $y[n] = \sum_{i=0}^{\infty} x[i] \cdot h[n-i]$

2) Given $x[n] = (3, -1, 2, 0, 1)$ & $h[n] = (2, 3, 4, 1)$, we find the output signal $y[n]$ for each n:

$$
\begin{aligned}
&\text{For n=0:} && y[0] = x[0] \cdot h[0] = 3 \cdot 2 = 6 \\
&\text{For n=1:} && y[1] = x[0] \cdot h[1] + x[1] \cdot h[0] = 3 \cdot 3 + (-1) \cdot 2 = 7 \\
&\text{For n=2:} && y[2] = x[0] \cdot h[2] + x[1] \cdot h[1] + x[2] \cdot h[0] \\
& && = 3 \cdot 4 + (-1) \cdot 3 + 2 \cdot 2 = 12 - 3 + 4 = 13 \\
&\text{For n=3:} && y[3] = x[0] \cdot h[3] + x[1] \cdot h[2] + x[2] \cdot h[1] + x[3] \cdot h[0] \\
& && = 3 \cdot 1 + (-1) \cdot 4 + 2 \cdot 3 + 0 \cdot 2 = 3 - 4 + 6 + 0 = 5 \\
&\text{For n=4:} && y[4] = x[1] \cdot h[3] + x[2] \cdot h[2] + x[3] \cdot h[1] + x[4] \cdot h[0] \\
& && = (-1) \cdot 1 + 2 \cdot 4 + 0 \cdot 3 + 1 \cdot 2 = -1 + 8 + 0 + 2 = 9 \\
&\text{For n=5:} && y[5] = x[2] \cdot h[3] + x[3] \cdot h[2] + x[4] \cdot h[1] = 2 \cdot 1 + 0 \cdot 4 + 1 \cdot 3 = 2 + 0 + 3 = 5 \\
&\text{For n=6:} && y[6] = x[3] \cdot h[3] + x[4] \cdot h[2] = 0 \cdot 1 + 1 \cdot 4 = 0 + 4 = 4 \\
&\text{For n=7:} && y[7] = x[4] \cdot h[3] = 1 \cdot 1 = 1
\end{aligned}
$$

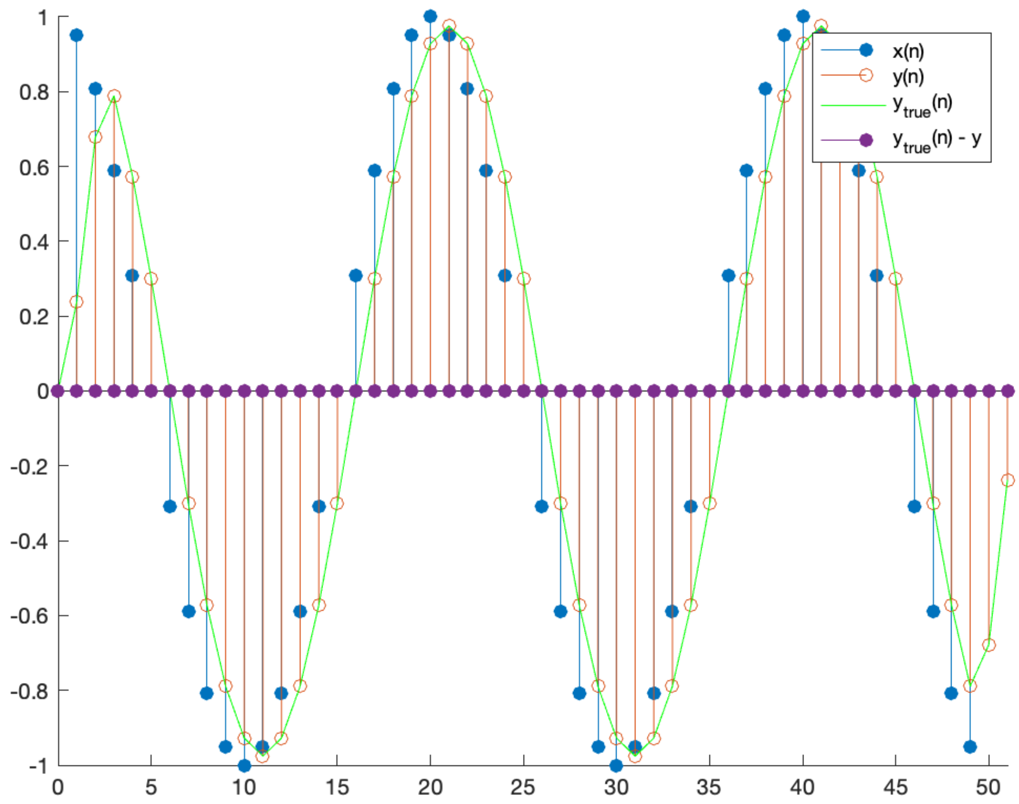
3) So, the output signal $y[n]$ is $(6, 7, 13, 5, 9, 5, 4, 1)$.

Exercise 3

We have the impulse response of an LTI system $h[n] = (0.25, 0.5, 0.25)$ at sample indices $n = (0, 1, 2)$. The input signal is $x[n] = \cos\left(\frac{2\pi}{20}n\right)$ for $0 \leq n < 50$.

a) From the lecture: If $x[n]$ and $h[n]$ have finite lengths N_x and N_h, the length of the output signal results to $N_y = N_x + N_h - 1$. In our example we have $N_x = 50$ and $N_h = 3$, therefore we can calculate the length of the output signal L_y (N_y in the lecture) as $\underline{L_y = 52}$.

b) For the actual implementation, see code `Ex03.m`. Here it was important to explicitly define $x[n]$ to be 0 outside the given range.



c)

d) As we can see in the plot, the "true" y and our calculated y match up.

Exercise 4

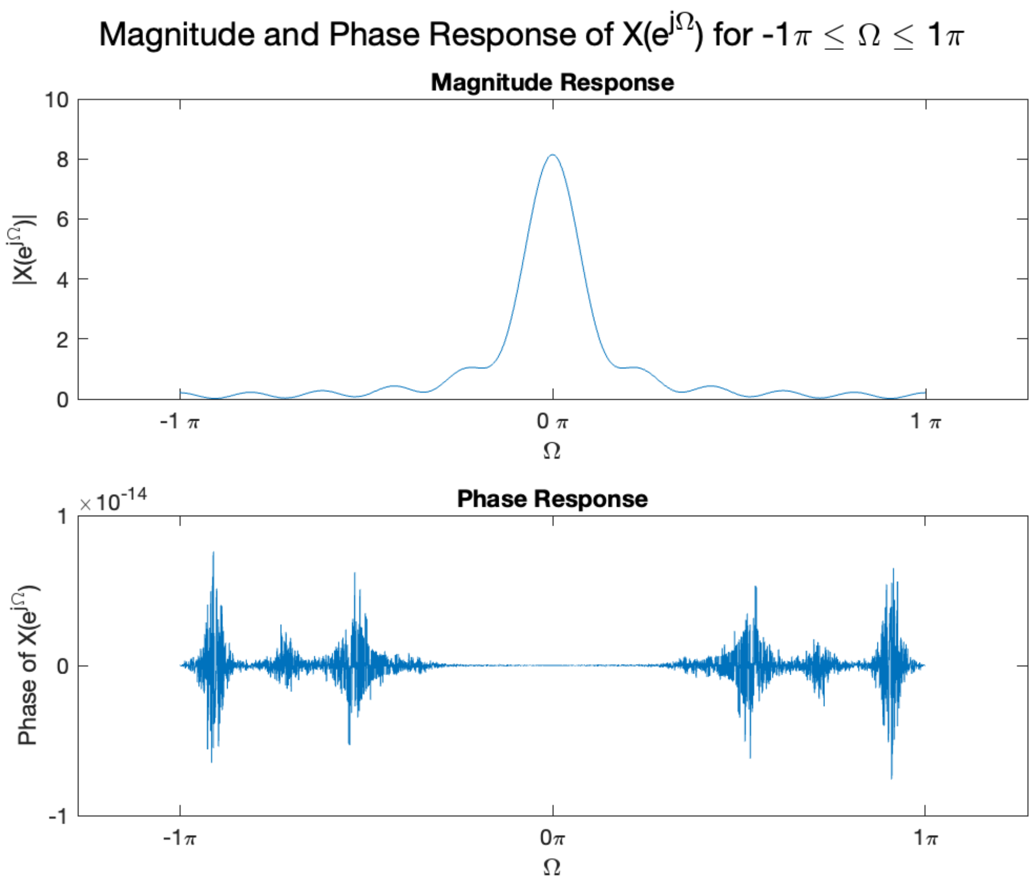
a) See `Ex04.m` for the full code, here I show only the function `dtft`:

```
function X = dtft(x, n, w)
    % DTFT Computes Discrete-time Fourier transform
    % @param    x: finite duration sequence over n
    % @param    n: sample position vector
    % @param    w: frquency location vector
    % @return   X: DTFT values computed at w frequencies

    X = x * exp(-1j .* n' * w);

end
```

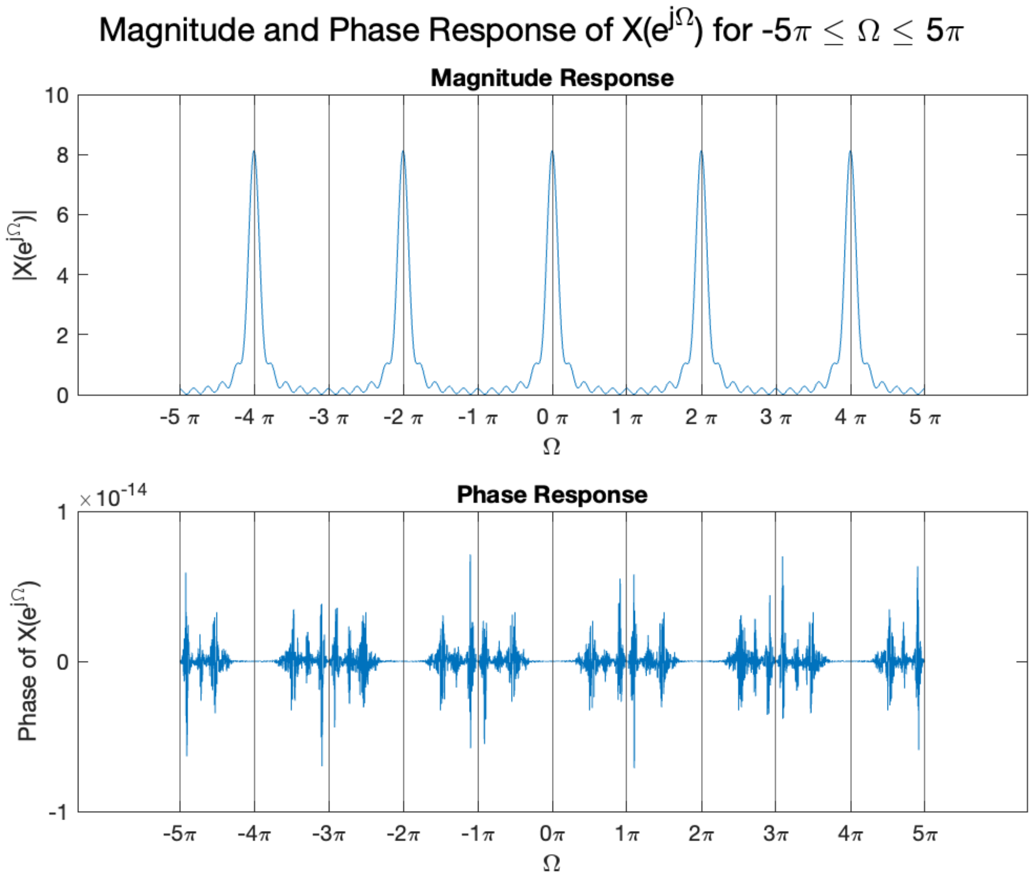
We call the function via `X = dtft(@x, n, w)`, where

- `x(n) = ((0.8).^abs(n)) .* (u(n+10) - u(n-11))` with `u(n) = 1.*(n>=0)`
- `n = -10:10`
- `w = linspace(Omega * -pi, Omega * pi, 1000)` for `Omega = 1`

b) What I can observe on the y-axis of the phase response plot is that the phase response for the settings described above is very small, in fact less than 10^{-14}.



c) If we change `Omega` from `Omega = 1` to `Omega = 5`, we can see that the magnitude response has peaks and troughs at multiples of π. We can also see that the phase response seems to be mirrored and flipped around the y-axis (graphic on next page).



Magnitude and Phase Response of X(e^jΩ) for -5π ≤ Ω ≤ 5π
Magnitude Response
|X(e^jΩ)|
10
8
6
4
2
0
-5 π
-4 π
-3 π
-2 π
-1 π
0 π
1 π
2 π
3 π
4 π
5 π
Ω
Phase Response
×10^-14
Phase of X(e^jΩ)
1
0
-1
-5π
-4π
-3π
-2π
-1π
0π
1π
2π
3π
4π
5π
Ω