Digital Signal Processing SS 2024 – Exercise 1

Digital Signal Processing Tutorial

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Exercise 1

We have three complex numbers given:

$$c_1 = -5 + 3j \qquad c_2 = \frac{\sqrt{2}}{2}e^{-\frac{3\pi j}{4}} \qquad c_3 = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}j$$

a) We do the calculations by hand. First, let's use Euler's formula to simplify c_2:

$$c_2 = \frac{\sqrt{2}}{2}e^{-\frac{3\pi j}{4}} = \frac{\sqrt{2}}{2}\left(\cos\left(\frac{3\pi}{4}\right) + j\sin\left(\frac{3\pi}{4}\right)\right) = \frac{\sqrt{2}}{2}\left(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}j\right) = -\frac{1}{2} + \frac{1}{2}j$$

With this, let us now compute the following numbers:

$$c_4 = c_1 + c_2 \quad = -5 - \frac{1}{2} + 3j + \frac{1}{2}j = -\frac{11}{2} + \frac{7}{2}j$$

$$c_5 = c_1 \cdot c_2 \quad = (-5+3j) \cdot \left(-\frac{1}{2} + \frac{1}{2}j\right) = \frac{5}{2} - \frac{5}{2}j - \frac{3}{2}j - \frac{3}{2} = 1 - 4j$$

$$c_6 = |c_3|^2 \quad = \left(\sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2}\right)^2 = \left(\sqrt{\frac{1}{2} + \frac{1}{2}}\right)^2 = \left(\sqrt{1}\right)^2 = 1$$

$$c_7 = \arg(c_3) \quad = \operatorname{atan2}\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = \arctan\left(\frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}}\right) = \arctan(1) = \frac{\pi}{4}$$

$$c_8 = \frac{c_1}{c_2} \quad = \frac{-5+3j}{\frac{-1+1j}{2}} = \frac{-10+6j}{-1+1j} = \frac{(-1-1j)(-10+6j)}{(-1-1j)(-1+1j)} = \frac{10-6j+10j+6}{1-1j+1j+1} = 8-2j$$

$$c_9 = c_1 \cdot c_1^* \quad = (-5+3j)(-5-3j) = 25 + 15j - 15j + 9 = 34$$

b) We check the result using MATLAB.

This is the code we use:

```
c1 = -5 + 3j
c2 = sqrt(2)/2 * exp((3j*pi)/4)
c3 = 1/sqrt(2) + 1j/sqrt(2)

c4 = c1 + c2
c5 = c1 * c2
```

```
c6 = abs(c3)^2
c7 = angle(c3)
c8 = c1 / c2
c9 = c1 * c1'
```

These are the results:

```
c1 = −5.0000 + 3.0000i
c2 = −0.5000 + 0.5000i
c3 = 0.7071 + 0.7071i
c4 = −5.5000 + 3.5000i
c5 = 1.0000 − 4.0000i
c6 = 1.0000
c7 = 0.7854
c8 = 8.0000 + 2.0000i
c9 = 34
```

Checks out. (Question: what is the best way to get this output from MATLAB to latex? The way I did it was copying it from the console, but that was rather cumbersome since I needed to remove the blank lines.)

Exercise 2

Exercise 3

Exercise 4

We need to find out whether

a) $y(t) = (x(t))^2$ b) $y(t) = x(t) \cdot \sin(\Omega_0 t)$

are linear and time invariant.

a) Let $y(t) = (x(t))^2$.

1) **Linearity.** It is easy to see that the system is not linear:

$$(\alpha x(t))^2 = \alpha^2 (x(t))^2 = \alpha^2 y(t) \neq \alpha y(t).$$

2) **Time invariance.** Let $y_1(t) = (x(t-\tau))^2$ be the system in which input is delayed, and let $y_2(t) = y(t-\tau)$ be the system in which the output is delayed. Since $y_1 = (x(t-\tau))^2 = y(t-\tau) = y_2$, we can see that the system is time invariant.

b) Let $y(t) = x(t) \cdot \sin(\Omega_0 t)$.

1) **Linearity.** Let $y_1(t) = (\alpha x_1(t) + \beta x_2(t)) \sin(\Omega_0 t)$, and let $y_2(t) = \alpha x_1(t) \sin(\Omega_0 t) + \beta x_2(t) \sin(\Omega_0 t)$. Then we can see that

$$y_1(t) = (\alpha x_1(t) + \beta x_2(t)) \sin(\Omega_0 t) = \alpha x_1(t) \sin(\Omega_0 t) + \beta x_2(t) \sin(\Omega_0 t) = y_2(t).$$

Therefore, the system is linear.

2) **Time invariance.** Let $y_1(t) = x(t-\tau) \sin(\Omega_0 t)$ be the system in which the input is delayed, and let $y_2(t) = y(t-\tau)$ be the system in which the output is delayed. Since $y_1 = x(t-\tau) \sin(\Omega_0 t) \neq x(t-\tau) \sin(\Omega_0 (t-\tau)) = y(t-\tau) = y_2$, we can see that the system in not time invariant.