Homework 13

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Exercise 32

```r
A <- c(80, 77, 71, 72, 75)
B <- c(84, 74, 81, 80, 79)
C <- c(76, 76, 74, 73, 74)

n <- 5
k <- 3
```

Hypothesis

Since we want to know if there is a global difference over time, we formulate the hypothesis as such:

- H_0: $\mu_A = \mu_B = \mu_C$
- H_1: $\mu_A \neq \mu_B$ or $\mu_A \neq \mu_C$ or $\mu_B \neq \mu_C$

Statistical test

We perform a one-way repeated measures ANOVA (rANOVA). For this, we first check the assumption of sphericity (normality can already be assumed).

Sphericity

To test for sphericity via a Mauchly's test.

```r
# Building an appropriate Data-Frame for Mauchly's Test
patid <- as.factor(c(seq(1:5), seq(1:5), seq(1:5)))
time  <- c(rep("A", length(A)), rep("B", length(B)), rep("C", length(C)))
bpm   <- c(A, B, C)
df    <- data.frame(patid, time, bpm)
View(df)

# Mauchly's Test of Sphericity
bpm_matrix <- matrix(bpm, nrow=5, ncol=3)
mauchly.test(lm(bpm_matrix~1), X=~ 1)
```

```
##
##  Mauchly's test of sphericity
##  Contrasts orthogonal to
##  ~1
##
##
## data:  SSD matrix from lm(formula = bpm_matrix ~ 1)
## W = 0.57903, p-value = 0.4406
```

As we can see, we can assume sphericity.

rANOVA

```r
m_grand <- mean(c(A, B, C))

# group means
m_A <- mean(A)
m_B <- mean(B)
m_C <- mean(C)

# patient means
m <- numeric(5)
for(i in 1:n) {
  m[i] <- mean(c(A[i], B[i], C[i]))
}

cat(sprintf("m_A: %.3f, m_B: %.3f, m_C: %.3f", m_A, m_B, m_C), "\n",
    "m:", sprintf("%.3f", m), "\n\n")
```

```
## m_A: 75.000, m_B: 79.600, m_C: 74.600
##  m: 80.000 75.667 75.333 75.000 76.000
```

```r
SS_cond   <- n * sum((m_A-m_grand)^2 + (m_B-m_grand)^2 + (m_C-m_grand)^2)
SS_object <- k * sum((m-m_grand)^2)
SS_within <- sum((A-m_A)^2 + (B-m_B)^2 + (C-m_C)^2)
SS_error  <- SS_within - SS_object

cat(sprintf("SS_cond: %.3f, SS_obbject: %.3f, SS_within: %.3f, SS_error: %.3f",
            SS_cond, SS_object, SS_within, SS_error), "\n")
```

```
## SS_cond: 77.200, SS_obbject: 50.267, SS_within: 114.400, SS_error: 64.133
```

```r
MS_cond   <- SS_cond / (k-1)
MS_object <- SS_object / (n-1)
MS_within <- SS_within / (k*(n-1))
MS_error  <- SS_error / ((n-1)*(k-1))

cat(sprintf("MS_cond: %.3f, MS_obbject: %.3f, MS_within: %.3f, MS_error: %.3f",
            MS_cond, MS_object, MS_within, MS_error), "\n\n")
```

```
## MS_cond: 38.600, MS_obbject: 12.567, MS_within: 9.533, MS_error: 8.017
```

```r
test_statistic <- MS_cond / MS_error
critical_value <- qf(0.95, k-1, (n-1)*(k-1))

cat(sprintf("test statistic: %.3f; critical value: %.3f" ,
            test_statistic, critical_value), "\n",
    sprintf("We can see that we %s reject the null hypothesis.",
            if (abs(test_statistic) > critical_value) "can" else "cannot"))
```

```
## test statistic: 4.815; critical value: 4.459
##  We can see that we can reject the null hypothesis.
```