Homework 11

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Exercise 27

```
X <- c(10, 8, 3, 4, 1, 5, 9, 2, 6, 6)
Y <- c(7, 9, 5, 3, 2, 6, 10, 1, 4, 8)
```

Spearman-rank correlation coefficient

```
X_rank <- c(10, 8, 3, 4, 1, 5, 9, 2, 6.5, 6.5)
Y_rank <- c(7, 9, 5, 3, 2, 6, 10, 1, 4, 8)

X_rank_sd <- sd(X_rank)
Y_rank_sd <- sd(Y_rank)

XY_rank_cov <- cov(X_rank, Y_rank)

rho_XY <- XY_rank_cov / (X_rank_sd * Y_rank_sd)

cat(sprintf("SD X rank: %.3f; SD Y rank: %.3f; COV X,Y rank: %.3f; Rho X,Y: %.3f",
            X_rank_sd, Y_rank_sd, XY_rank_cov, rho_XY))
```

```
## SD X rank: 3.018; SD Y rank: 3.028; COV X,Y rank: 7.611; Rho X,Y: 0.833
```

Test if the relationship is statistically significant

We perform the "Exact Test for No Linear Correlation", which just tests the correlation coefficient against 0.

```
n <- length(X)

test_statistic <- rho_XY * sqrt( (n-2) / (1 - rho_XY^2) )
critical_value <- qt(0.975, n-2)

cat(sprintf("test statistic: %.3f; critical value: %.3f"
            , test_statistic, critical_value), "\n",
    sprintf("We can see that we %s reject the null hypothesis.",
            if (abs(test_statistic) > critical_value) "can" else "cannot"))
```

```
## test statistic: 4.256; critical value: 2.306
##  We can see that we can reject the null hypothesis.
```

Exercise 28

In this examples we have correlated overlapping correlations coefficients, as such we use "Tests of Correlation Coefficients II".

```
r12 <- -0.205  # family social support - stress
r13 <-  0.285  # loneliness - stress
r23 <- -0.495  # family social support - loneliness
n <- 405

R_abs <- abs( 1 - r12^2 - r13^2 - r23^2 + 2 * r12 * r13 * r23 )
r_bar <- (r12 + r13) / 2
test_statistic <- (r12-r13) *
  sqrt( ((n-1) * (1+r23)) /
          ( 2 * ((n-1)/(n-3)) * R_abs + r_bar^2 * (1-r23)^3 ) )
critical_value <- qt(0.975, n-3)

cat(sprintf("test statistic: %.3f; critical value: %.3f"
            , test_statistic, critical_value), "\n",
    sprintf("We can see that we %s reject the null hypothesis.",
            if (abs(test_statistic) > critical_value) "can" else "cannot"))
```

```
## test statistic: -5.934; critical value: 1.966
##  We can see that we can reject the null hypothesis.
```

Exercise 29

```r
A <- c(2, 3, 5, 2)
B <- c(1, 4, 5, 3)
```

Kendall's Tau-b

```r
A_sort <- c(2, 2, 3, 5)
B_sort <- c(1, 3, 4, 5)
n <- length(A)

# 13 14 15, 34 35, 45
# +  +  +   +  +   +
# 31 34 35, 14 15, 45
# -  +  +   +  +   +

nc <- 5
nd <- 0

T <- 2 * (2-1) / 2
W <- 0

tau_b <- (nc - nd) /
  ( sqrt(0.5 * n * (n-1) - T) * sqrt(0.5 * n * (n-1) - W) )

cat(sprintf("Kendall Tau-b: %.3f", tau_b))
```

```
## Kendall Tau-b: 0.913
```

Test if correlation is significant

```r
v0 <- n * (n-1) * (2*n+5)
vt <- 2 * (2-1) * (2*2+5)
vw <- 0
v1 <- 0
v2 <- 0
v <- (v0 - vt - vw) / (18 + v1 + v2)

test_statistic <- (nc - nd) / sqrt(v)
critical_value <- qnorm(0.975)

cat(sprintf("test statistic: %.3f; critical value: %.3f"
            , test_statistic, critical_value), "\n",
    sprintf("We can see that we %s reject the null hypothesis.",
            if (abs(test_statistic) > critical_value) "can" else "cannot"))
```

```
## test statistic: 1.806; critical value: 1.960
##  We can see that we cannot reject the null hypothesis.
```