Homework 6

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Exercise 13

```
x <- c(13,14,12,11,16,16,17,20,20,21)
n <- length(x)

m <- mean(x)
s <- sd(x)

print(sprintf("mean %.3f, std %.3f", m, s))
```

```
## [1] "mean 16.000, std 3.528"
```

```
ref_val <- 14
a <- 0.05
b <- 0.2
```

Since we want to know whether there is a difference, we use a two-sided one-sample t-test.

```
ua <- qnorm(1-a/2)
ub <- qnorm(1-b)
cat(sprintf("u(1-a/2) %.3f, u(1-b) %.3f", ua, ub), "\n")
```

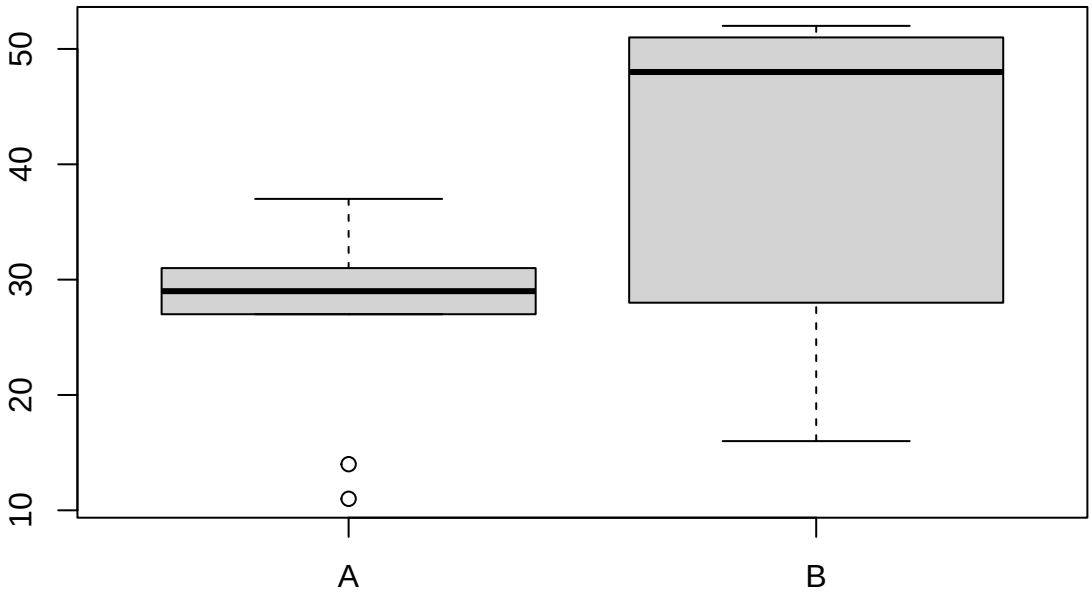
```
## u(1-a/2) 1.960, u(1-b) 0.842
```

```
d <- m - ref_val

n <- (ua+ub)^2 * s^2 / d^2
cat(sprintf("We need %s samples.", ceiling(n)))
```

```
## We need 25 samples.
```

Exercise 14

```
A <- sort(c(11,37,32,27,29,29,27,31,14))
B <- sort(c(52,38,51,48,49,52,28,17,16))

boxplot(A, B, names=c("A", "B"))
```



Check for normality

First we check if our data is normally distributed. For this, we use a Lilliefors test.

```
ma <- mean(A)
sa <- sd(A)
S <- 1:length(A) / length(A)
S_shift <- c(0, 1:(length(A)-1)/length(A))
F0 <- pnorm(A, ma, sa)

test_statistic <- max(max(abs(S-F0)), max(abs(S_shift-F0)))
critical_value <- 0.249
cat(sprintf("Test statistic %s", test_statistic), "\n")
```

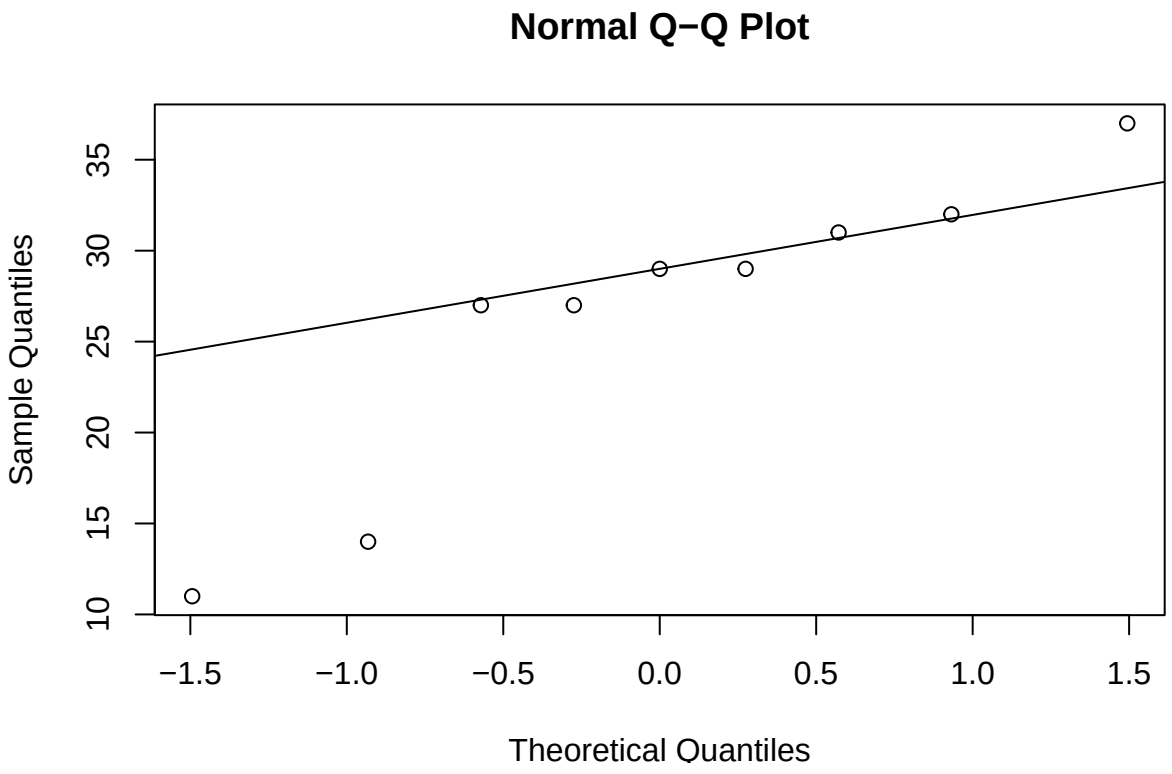
```
## Test statistic 0.309253452744006
```

```
cat(sprintf("The null hypothesis %s be rejected.",
            if (test_statistic > critical_value) "can" else "cannot"))
```

```
## The null hypothesis can be rejected.
```

Since `A` is not normally distributed we forgo checking `B` and immediately go over to using a non parametric test. I am not sure if this is correct, since we do have ties in both groups and therefore the KS test should maybe not be used. We can also a QQ-plot:

```
qqnorm(A)
qqline(A)
```



Test

We use a Mann-Whitney-U Test. We need to use the exact MWU test because we have ties

```
#install.packages("coin")
library(coin)
```

```
## Loading required package: survival
```

```
data<-c(A,B)
group<-as.factor(c(rep("A",length(A)),rep("B",length(B))))
wilcox_test(data~group,distribution = "exact")
```

```
##
##  Exact Wilcoxon-Mann-Whitney Test
##
## data:  data by group (A, B)
## Z = -1.9014, p-value = 0.05895
## alternative hypothesis: true mu is not equal to 0
```

This indicates that we cannot reject the null hypothesis.

Exercise 15

```
A <- c(54.5,60.4,85.6,78.2,120.6,121)
B <- c(55.5,69.6,86.7,81.6,116.5,115)
```

We know that normality of the data cannot be assumed. Therefore, we use a Wilcoxon Test for paired data.

Manual Wilcoxon Test

```
d <- A - B

df <- data.frame(difference=d, rank=rank(abs(d)))
print(df)
```

```
##   difference rank
## 1       -1.0    1
## 2       -9.2    6
## 3       -1.1    2
## 4       -3.4    3
## 5        4.1    4
## 6        6.0    5
```

```
df <- df[df$difference!=0,]
pos_rank_sum <- sum(df$rank[df$difference>0])
neg_rank_sum <- sum(df$rank[df$difference<0])
cat(sprintf("positive rank sum %s", pos_rank_sum), "\n")
```

```
## positive rank sum 9
```

```
cat(sprintf("negative rank sum %s", neg_rank_sum), "\n")
```

```
## negative rank sum 12
```

```
test_statistic <- min(pos_rank_sum, neg_rank_sum)
cat(sprintf("test statistic %s", test_statistic), "\n")
```

```
## test statistic 9
```

```
critical_value <- 0

cat(sprintf("The null hypothesis %s be rejected.",
            if (test_statistic <= critical_value) "can" else "cannot"))
```

```
## The null hypothesis cannot be rejected.
```

Implemented Test

```
wilcox.test(A,B,paired=TRUE,correct=FALSE)
```

```
##
##  Wilcoxon signed rank exact test
##
## data:  A and B
## V = 9, p-value = 0.8438
## alternative hypothesis: true location shift is not equal to 0
```

```r
wilcoxsign_test(A~B,distribution = "exact",zero.method="Wilcoxon")
```

```
##
##  Exact Wilcoxon Signed-Rank Test
##
## data:  y by x (pos, neg)
##   stratified by block
## Z = -0.31449, p-value = 0.8438
## alternative hypothesis: true mu is not equal to 0
```

```r
wilcoxsign_test(A~B,distribution = "asymptotic",zero.method="Wilcoxon")
```

```
##
##  Asymptotic Wilcoxon Signed-Rank Test
##
## data:  y by x (pos, neg)
##   stratified by block
## Z = -0.31449, p-value = 0.7532
## alternative hypothesis: true mu is not equal to 0
```

```r
wilcoxsign_test(A~B,distribution = approximate(nresample = 10000),zero.method="Wilcoxon")
```

```
##
##  Approximative Wilcoxon Signed-Rank Test
##
## data:  y by x (pos, neg)
##   stratified by block
## Z = -0.31449, p-value = 0.8479
## alternative hypothesis: true mu is not equal to 0
```