Homework 4

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Exercise 8

```r
A <- c(7.2,7.7,8.0,8.1,8.3,8.4,8.4,8.5,8.6,8.7,9.1,9.1,9.1,9.8,10.1,10.3)
B <- c(8.1,9.2,10.0,10.4,10.6,10.9,11.1,11.9,12.0,12.1)

n <- length(A)
m <- length(B)

alpha <- 0.05

mean_A <- sum(A)/n
mean_B <- sum(B)/m

s_A <- sqrt(sum(A^2)/n - mean_A^2)
s_B <- sqrt(sum(B^2)/m - mean_B^2)

cat(sprintf("sample variance A: %.3f, sample variance B: %.3f", s_A, s_B))
```

```
## sample variance A: 0.818, sample variance B: 1.218
```

F-test

We want to perform an F-test to see if the "true" variances differ.

For this, we formula the hypothesis:

- H_0: $\sigma_A^2 = \sigma_B^2$
- H_1: $\sigma_A^2 \neq \sigma_B^2$

With the test statistic $F(x, y) = s_x^2/s_y^2$ for $s_x^2 > s_y^2$ and the critical value $F_{n_x-1,n_y-1,1-\alpha}$.

We have that that $s_A < s_B$, thus we have the test statistic $F = s_B^2/s_A^2$.

```r
test_statistic <- s_B^2 / s_A^2
critical_value <- qf(1-alpha, m-1, n-1)

cat(sprintf("test statistic: %.3f, critical value: %.3f",
            test_statistic, critical_value))
```

```
## test statistic: 2.220, critical value: 2.588
```

```r
cat(sprintf("The null hypothesis %s be rejected. p-value %s",
            if (test_statistic > critical_value) "can" else "cannot",
            df(test_statistic, m-1, n-1)))
```

```
## The null hypothesis cannot be rejected. p-value 0.116071862962748
```

Levene-test

I choose to use the mean to calculate the spread, just to make things easier.

```
z_A <- abs(A - mean_A)
mean_z_A <- sum(z_A)/n

z_B <- abs(B - mean_B)
mean_z_B <- sum(z_B)/m

z <- sum(c(z_A, z_B)) / (n+m)

test_statistic <- ( (n+m-2) * (n*(mean_z_A - z)^2 + m*(mean_z_B - z)^2) ) /
  ( sum((z_A - mean_z_A)^2) + sum((z_B - mean_z_B)^2) )
critical_value <- qf(1-alpha, 1, n+m-2)

cat(sprintf("test statistic: %.3f, critical value: %.3f",
            test_statistic, critical_value))
```

```
## test statistic: 1.594, critical value: 4.260
```

```
cat(sprintf("The null hypothesis %s be rejected.",
            if (test_statistic > critical_value) "can" else "cannot" ))
```

```
## The null hypothesis cannot be rejected.
```

```
suppressMessages(library(car))
suppressWarnings(
cat(sprintf("p-value of: manual implementation %.3f, package 'car' %.3f",
            1 - pf(test_statistic, 1, n+m-2),
            leveneTest(c(A,B), c(rep(0, n), rep(1, m)), center="mean")$`Pr(>F)`[1]))
)
```

```
## p-value of: manual implementation 0.219, package 'car' 0.219
```

Test to verify if there is a difference between the "true means"

Since both F-test and Levene-test cannot reject the null hypothesis, we can assume homoscedasticity.

We will perform an independent two-sample t-test:

```
s_p <- sqrt( ((n-1)*s_A^2 + (m-1)*s_B^2) / (n+m-2) )

test_statistic <- abs(mean_A - mean_B) * sqrt(n*m/n+m) / s_p

critical_value <- qt(1-(alpha/2), n+m-2)

cat(sprintf("test statistic: %.3f, critical value: %.3f",
            test_statistic, critical_value))
```

```
## test statistic: 8.687, critical value: 2.064
```

```
cat(sprintf("The null hypothesis %s be rejected. p-value %s",
            if (test_statistic > critical_value) "can" else "cannot",
            dt(test_statistic, n+m-2)))
```

```
## The null hypothesis can be rejected. p-value 7.55120171274825e-09
```

Confidence interval for difference of "true means"

```
val <- qt(1-(alpha/2), n+m-2) * s_p * sqrt(1/n + 1/m)

cat(sprintf("The true difference of the means is with %.f%% in [%.3f, %.3f]",
            (1-alpha)*100, mean_A-mean_B - val, mean_A-mean_B + val))
```

```
## The true difference of the means is with 95% in [-2.739, -1.096]
```

Example 9

```
mean_A <- 22.13
mean_B <- 18.68

s_A <- 3.74
s_B <- 1.21

n <- 8
m <- 6
```

Homoscedasticity

First we conduct an F-test to see whether we can assume homoscedasticity.

```
test_statistic <- s_A^2 / s_B^2
critical_value <- qf(0.95, n-1, m-1)

cat(sprintf("test statistic: %.3f, critical value: %.3f",
            test_statistic, critical_value))
```

```
## test statistic: 9.554, critical value: 4.876
```

```
cat(sprintf("The null hypothesis %s be rejected. p-value %s",
            if (test_statistic > critical_value) "can" else "cannot",
            df(test_statistic, m-1, n-1)))
```

```
## The null hypothesis can be rejected. p-value 0.00150766926950449
```

Test for difference of means

As we can see, we cannot assume homoscedasticity. Therefore, we use Welch's two-sample t-test:

```
Z_A <- ( ((s_A^2)/n) / ((s_A^2)/n + (s_B^2)/m) )^2 / (n-1)
Z_B <- ( ((s_B^2)/n) / ((s_A^2)/n + (s_B^2)/m) )^2 / (m-1)
df <- 1 / (Z_A + Z_B)

cat(sprintf("Z_A %.5f, Z_B %.5f, df %.2f", Z_A, Z_B, df))
```

```
## Z_A 0.11001, Z_B 0.00169, df 8.95
```

```
test_statistic <- (mean_A - mean_B) / sqrt(s_A^2/n + s_B^2/m)
critical_value <- qt(1-(0.05/2), df)

cat(sprintf("test statistic: %.3f, critical value: %.3f",
            test_statistic, critical_value))
```

```
## test statistic: 2.444, critical value: 2.264
```

```
cat(sprintf("The null hypothesis %s be rejected. p-value %s",
            if (test_statistic > critical_value) "can" else "cannot",
            dt(test_statistic, df)))
```

```
## The null hypothesis can be rejected. p-value 0.0304819517434573
```