## Contents

Our Go	al	i
Chapte	r 1. Like Programming, Mathematics has a Culture	1
Chapte	r 2. Polynomials	5
2.1	Polynomials, Java, and Definitions	5
2.2	A Little More Notation	13
2.3	Existence & Uniqueness	14
2.4	Realizing it in Code	22
2.5	Application: Sharing Secrets	24
2.6	Cultural Review	27
2.7	Exercises	27
2.8	Chapter Notes	31
Chapte	r 3. On Pace and Patience	35
Chapte	r 4. Sets	39
4.1	Sets, Functions, and Their -Jections	40
4.2	Clever Bijections and Counting	48
4.3	Proof by Induction and Contradiction	51
4.4	Application: Stable Marriages	54
4.5	Cultural Review	58
4.6	Exercises	59
4.7	Chapter Notes	61
Chapte	r 5. Variable Names, Overloading, and Your Brain	63
Chapter	r 6. Graphs	69
6.1	The Definition of a Graph	69
6.2	Graph Coloring	71
6.3	Register Allocation and Hardness	73
6.4	Planarity and the Euler Characteristic	75
6.5	Application: the Five Color Theorem	78

6.6	Approximate Coloring	33
6.7		35
6.8	Exercises	35
6.9	Chapter Notes	37
Chapter	7. The Many Subcultures of Mathematics 8	9
Chapter	8. Calculus with One Variable	5
8.1	Lines and Curves	96
8.2	Limits	0(
8.3	The Derivative	)7
8.4	Taylor Series	
8.5	Remainders	17
8.6	Application: Finding Roots	
8.7	Cultural Review	25
8.8	Exercises	25
Chapter	9. On Types and Tail Calls	9
Chapter	10. Linear Algebra	5
10.1	Linear Maps and Vector Spaces	36
10.2	Linear Maps, Formally This Time	ŀ1
10.3	The Basis and Linear Combinations	ŀ3
10.4	Dimension	ŀ7
10.5	Matrices	19
	Conjugations and Computations	5
	One Vector Space to Rule Them All	8
10.8	Geometry of Vector Spaces	9
	Application: Singular Value Decomposition	64
10.10	Cultural Review	19
10.11	Exercises	19
10.12	Chapter Notes	32
Chapter	11. Live and Learn Linear Algebra (Again)	5
Chapter	12. Eigenvectors and Eigenvalues	1
12.1	Eigenvalues of Graphs	)3
	Limiting the Scope: Symmetric Matrices	)5
12.3	Inner Products	8(
12.4	Orthonormal Bases	)2
12.5	Computing Eigenvalues	)5
12.6	The Spectral Theorem	)7
12.7	Application: Waves	)9
12.8	Cultural Review	25

12.9 Exercises	226
12.10 Chapter Notes	229
Chantan 12 Digan and Fammality	022
Chapter 13. Rigor and Formality	233
Chapter 14. Multivariable Calculus and Optimization	239
14.1 Generalizing the Derivative	239
14.2 Linear Approximations	
14.3 Vector-valued Functions and the Chain Rule	246
14.4 Computing the Total Derivative	248
14.5 The Geometry of the Gradient	251
14.6 Optimizing Multivariable Functions	
14.7 Gradient Descent: an Optimization Hammer	261
14.8 Gradients of Computation Graphs	262
14.9 Application: Automatic Differentiation and a Simple Neura	
14.10 Cultural Review	281
14.11 Exercises	281
14.12 Chapter Notes	284
Chapter 15. The Argument for Big-O Notation	291
Chapter 16. Groups	301
16.1 The Geometric Perspective	303
16.2 The Interface Perspective	
16.3 Homomorphisms: Structure Preserving Functions	
16.4 Building Blocks of Groups	312
16.5 Geometry as the Study of Groups	314
16.6 The Symmetry Group of the Poincaré Disk	
16.7 Application: Drawing Hyperbolic Tessellations	329
16.8 Cultural Review	345
16.9 Exercises	345
16.10 Chapter Notes	350
Chapter 17. A New Interface	353
Appendix A. Notation	363
Appendix B. A Summary of Proofs	365
B.1 Propositional and first-order logic	
B.2 Methods of proof	
B.3 How does one actually prove things?	
4 1 0 4 1 ID	0.50
Appendix C. Annotated Resources	373
C.1 Fundamentals and Foundations	373

C.2	Polynomials	374
C.3	Graph Theory and Combinatorics	375
C.4	Calculus and Analysis	375
C.5	Linear Algebra	376
C.6	Optimization	377
C.7	Abstract Algebra (Groups, etc.)	377
C.8	Topology	378
C.9	Computer Science, Theory, and Algorithms	378
C.10	Fun and Recreation	380
About t	he Author and Cover	381
Index		383