

Problems

1. to what extent does $\text{grad } f$, near a critical pt., depend on the metric flow
 2. topological Rohlin's thm.
 3. "geometric" proof of wk. mixing
 \Rightarrow mixing for full set of T
 4. class. of sing. by local properties of grad. of flow
 5. Homogeneous spaces
 - a. implications between
 1. unique ergodicity
 2. minimality
 3. $h_u = 0$ and ergodic
 - b. simple or semi-simple case
 1. which one-param. subgps are G^u 's for ergodic affine
 2. try a.
 - c. relate dyn. properties to repn.
- try $m(u)$ field
- in first
- d. $K \Rightarrow$ Bernoulli
- e. wk. mixing + center s.s. \Rightarrow Bern.
- f. ergodic \Rightarrow unique meas. m.s. $h(u)$

6. zeta fun. for Ax. A

a. top. identification + } try. (and
conj. inv. of $\mathbb{Z}(0)$) } 2 first

b. $\mathbb{Z}(s)$ meromorphic for C^∞ flow

c. connection with Laplacian vs.
geod. results; automorphic forms

d. Anosov actions

7. Structure of basic sets

a. classification via (R, A)

b. local Ax. A \Rightarrow embeddable

c. can. coord. \Rightarrow "
(cpl. abel. quadrate \mathbb{R} 's)

d. phantom homotopy GPS.
- shift equiv. of induced maps!

e. dim Σ ; when a mfd.

8. Non Ax. A examples - Newhouse,
Abraham-Smale, Simon, Lorenz,
billiards

a. axiomatic description

b. stat. properties

c. $\forall \varepsilon > 0$ \exists horseshoe inside
with $h(f|_E) \geq h(f) - \varepsilon$.

d. stat properties of Lebesgue in
particular

- e. any specification-type property
9. unique ergodicity of W^u for partially Anosov diffeo
10. Stat. + dynamics of transforms of $[0, 1]$ - "nonexp." β -exp. like examples
11. Nonalgebraic Anosov diffeo class. 3-dim Anosov flows, var. neg. curv. surface conj. to const. curv?
12. $h(f) \geq \log |\lambda|$
 - a. diffeo.
 - b. \mathbb{P}^2 finite + hyp.
 - c. Ax. A with cycles
13. In Parry's "conj. to linear" paper, what are properties of constructed measure? Does this work for equiv. states too?
14. Suspensions of diffeos. - generally not a const. times sdy? - stronger statement is what. For Ax. A attractors?

15. Renewal thm. for dependent r.v.'s
- a. derive via motivation of Ax, A flows mixingness
 - b. how fast is the mixing for Ax, A flows

16. Brownian motion or diffusion giving a flow

17. symbolic dynamics for billiards

18. Interpret $\log \tilde{f}(x)$ as a potential function? — Kolm. idea on surfaces neg. curv.

19. Can you construct some Banach space so that h_μ is an eigenvalue of some can. operator

20. Obs. systems in stat. mech.
— top dyn. formulation?

21. ϕ_t C-dense. Unique α which
~~is $\phi_1^{-1} \cup \dots \cup \phi_{t-1}^{-1}$ max entropy?~~
 $\nu \phi_1^{-1} \cup \dots \Rightarrow \phi_t^{-1} \cup \dots$ ~~partition closed~~

22. canonical embedding of
Ax. A Σ_i

23. canonical C^0 perturb. of
Anosov diffeo to O -dim Σ_i 's
with same entropy

24. bifurcation of Ax. A in
terms of symbols

25. coding S.S. finite type
with same entropy

26. entropy in Hamiltonian
case? for P.D.E.'s? Relation
to ODE's?

27. Construct 2 dim. Hamilt.
diffeo. with ergodic set
of pos. meas.

28. Kupka-Smale + $h(f) > 0$,
 \Rightarrow homoclinic pt.

29. find Ax. A inf. attractor
in some ODE on R^3 (quadratic)

30. Fixing M . What is possible behaviors of good flow for all Riem. metric.
e.g. for some $\pi(M) = 0$
does some good flow have entropy 0

31. Anosov diffeo.
a. hyp. on $H_1(M)$
b. fixed pt.
c. $\Omega = M$

32. Is even

32. Classify s.s. with specification

33. Define $P(\mathcal{Q})$, equal state for certain words \mathcal{Q} .

34. μ equal state some cl_u of $\Sigma_N \Rightarrow h_u > 0$?

35. ergodic non hyp. autom of T^n not quotient of ss. finite type? not specification?

36. Geod. flow expansive
⇒ Anosov flow

37. Geod. flow h-expansive?

38. ~~g~~ $m = \text{Riem. metric } g$
with $g \text{ vol. } M = 1$.
What is g ?

$\{h(\text{geod. time } t) : g \in M\}$?
Relation to top. inv. of M

39. Ambrose-Kakutani thm. for
 R^n ~~flows~~ actions

40. Entropy of autom of C^*_alg .

41. Is ϕ_t acly. pt. for
h-diff $\rightarrow R$ for ϕ_t AxH flow?
(Anosov)

42. Is $h(\phi_t|E)$ diff. in E
for Hamiltonian case?
Any relation to class. or quant. mech?

43. Defn. Gibbs meas. for homeo. - relate to eq. state?
44. How big is \mathcal{E} ? ~~is~~ defn.
State some $\{ \in \mathcal{C}(\Lambda) \} \subset M_f(X)$
45. Any "local" invariants
(near fixed pt.) which are entropy-like
46. Eq. states for 1-diml quan. lattice systems w/o finite range
47. Any entropy-like inv for singularity of diff. map?
48. Suppose $F: \text{Cantor set } C \rightarrow R$
bdd, total var. Is there a homeo. $g: [0,1] \rightarrow$ and

diff $f: [0, 1] \rightarrow \mathbb{R} \times \mathbb{A}$
 $f = f \circ g | C$

49. Does min. or u.e. for diffeo
 $\Rightarrow h(f) = 0$ (try hom. case too)
- Is there a minimal diffeo
hom. to $(\mathbb{T}^1) \times \text{id.}$ on T^3
 - (Seifert) min. flow on S^3
50. Is there a transitive ergodic diffeo of S^2, D^2
51. Look for inv. meas. of
some standard foliations
52. Define \mathcal{R} (foliation). Does
 $h(\text{fol.}) > 0$ make sense?
53. Is the space of an expansive homeo always finite dim?
Is every minimal expansive homeo. on a \mathcal{C} -dim. space?

54. C-dense Ax. A flows
- speed of mixing
 - asympt. expressions for
per. orbits
 - $\pi_1(g)$ intrinsically ergodic
 - direct pf. of mixing of
meas.
 - analogue of " $h(f) \geq \log |\lambda|$ "
 - understand $\det(t - A)$ in vain
- rel. to $S(\tau)$
 - stability of C-density for
attractors
 - cond. on g so that $\Sigma_A(g)$
 C^∞ or anal. embeddable
as basic set
 - can a closed orbit of
Anosov flow be null-homotopic

55. Entropy of autom. amalgam
- gpc, slugs

56. horocycle flow ^{an} expansive flow?

57. $l(f^{nd})$ grows slowly with
"n" for many f and Ax. A different

58. Is Gutzwiller's example an Anosov flow?
59. Computer program for A_{x,A} attractors
60. Study flows $V(r) + \frac{1}{2}mv^2 = h$ for various $V(r)$. Stat. mech. literature Hénon-Heiles, Toda...
61. Rocklin-thm. for stable pseudo-gp. action
- ergodic thm. - avg. procedures
62. covering space for $\Sigma_A \rightarrow T^2$ curves. To $R^2 \rightarrow T^2$

63. C^r diff. which
are not C^{r+1} qualitatively.
Find. C^r diff. $f: V \rightarrow M$ and
 C^2 embedding $g: V \rightarrow N$
and \tilde{f} extending f to N ,
s.t. $\tilde{f} \circ C^2$ on $M \setminus f(N)$
qual not C^2 . (qual.
behavior of \tilde{f} due to
 $C^2 V$ but not exp. \tilde{f})

64. Entropy of gp. action.
There is no smooth $R^n(Z^n)$
action with pos. entropy
when $n > 1$. Is this true for
all lie gp's. (or Π) of
 $\dim > 1$? Nilpotent.

65. Foliation ergodic theory

- a. Ambrose-Kakutani - esp. R^n
- b. does mixing make any
sense - in category, / different
 C^∞ , analytic etc.
- c. average procedure difficulties
— ergodic thm., existence
inv. meas., ergodic decompos.,
u.e. + maf. corr.
— poly. growth enuf.
thus inv. inv.
- d. look at some specific foliations
- e. Plante stuff on connections
with homology
- f. positive entropy make sense

66. Central limit thm. ^{other} ~~the~~
that strong statistics near an
attractor of diff eqs.
67. correspondence principle
of quantum mechanics.
Investigate for some
simple mechanical systems.
^{Is it expensiveness related to ~~green~~?}
68. Electric circuits.
a. Analogue computer for
finding Ax. At examples
b. Is noise sometimes
due to hyp. set in the
dynamics?
69. Is $h \cdot \text{Diff}^r(M) \rightarrow R$
generically ct. - for
some r .
70. Classify all Anosov
systems or attractors
(which Ω_i can occur
as attractors).

71. Cancellation of S_2 's.
Simplest f in an isotopy class

72. Does $h(f) = 0$ for a cts. (or diff) flow on cpt. 2-manifold.

73. If f Anosov and $g \geq f$,
does $h(g) \geq h(f)$.

74. If f $\in A_x, A$ is there
an A_x, A g ^{conic} near f with
 $\dim S_2(g) = d$ + $h(g) = h(f)$,

75. Conditions on M to
admit Anosov f.

76. Does $h(f)$ have minimum
in isotopy class?

77. Conjugacy between
top. + measure ch.

- a. what ~~is~~ ϕ s.t. $h(f) = h(\phi)$
- b. entropy - conj. + equiv on Baire sets
 - what are the equiv. relations on homeo. or maps of S^1 , subshifts

78. Calculate h for CDE
on R^n , e.g. linear diff.

79. infinite measure space
particular.

80. Reddy examples of expansive
~~maps~~ diff. related to Anosov-
diff. Are expansive
diff. likely to be Anosov?

81. geometric pf. of u.e. for
inlet rot. S!

82. (Plante) codim 1 minimal
fol. has at most one
inv. measure.
83. Unstable foliations
of Anosov differ given by
nilpotent groups action
84. Invariant or approx. inv.
finite dim. subspaces for
Navier-Stokes eqn.
85. Codon frequencies via
efull-stable semi"potential"
86. alg. varieties - Weil conj.
cohomology - any entropy
here? any rel. to hom. eigenvalues?
87. If translation by gp. elt.
on G/P is minimal is that elt.
nilpotent in gp? (e.g. have 0. entropy)

88. Index Ω fixed pt. of diffeo removable by small perturbation (Hirsch).

89. For An-flow f on M . Try to get approx. its curves Ω_M by pseudo-orbits f_k , calculate $\#_M \dots$ as in Morse theory

90. If f An. on M and M contractible, what does $H^k(M) \cong H^k(\pi_1(M))$ tell you via f ? eigenvalue info? (see Hu p. 200-202)

91. Among C^1 "expanding" maps¹ on $[0,1]$ is f_b bts? (e.g.)

92. Among deg n poly. maps of $[0,1]^{\leq n}$, and Ax. A ones open+dense? bad ones stratified ~~by~~ set ...

(write Keane)

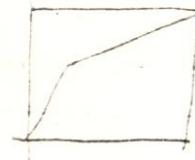
better bouncing disks

91. 3 particles on S^1 - how do you do it.

2 particles on T^2 different mass

92. a_1, a_2 integers > 0. $a_1 \neq a_2 \Rightarrow a_1, a_2 \geq d_{i+j}$ when $i, j \in \mathbb{Z}_{\geq 0}$. Can there be the next of symbolic seg lengths 1 to n ?

93 (Heimon)



$$f: [0,1]^2 \rightarrow$$

$$f(x) = f(x) + b$$

Is f_b ergodic w.r.t. left meas. (not inv.) we

$$f_b(A) = A \Rightarrow m(A) = 0 \text{ or } 1$$

94. Does every manifold M^n ($n \geq 3$) admit smooth Bernoulli flows?
 (full)
95. Mackov part. for alg. geom. examples
 (full)
96. LAM flow using per.
 flows on M without
 assuming $M = T^k$.
97. C^1 nonrigidification
 of T^2 preserving left invariant
98. Use in hsys, one with finite
 area instead of compact non-
 hyperbolic
99. (Dong) find an open partition
 in $\mathbb{R}^{1,2}$, not w. Bern.
 Find inv. of infinitistic codes

c. Symbolic dynamics and
 PDEs talk, Newhouse, locality

101. If a C^1 Anosov preserves
 smooth inv. μ , is μ an
 equal. stat. for $\# \log D^u(x)$.

102. Is the false zeta func
 of a basic set rational?
 Define false zeta func of
 a 1-flow basic set?

$$\tilde{\Xi}(s) = \prod_{\text{def}} (1 - \lambda_j e^{-s\ell(M)})$$

±1 according
 whether $Df|T^u_x$
 has even orientation
 2 related to $\#(c)$ related
 to mod 2 part of torsion

1. $\#(c) = \#(c_0)$
 2. $\#(c) = \#(c_1)$

103. How can you write

$$1+t^2+t^3 = \prod_{i=0}^{\infty} (1+t^{n_i})$$

in $\mathbb{Z}[[t]]$.

104. Note:

$$\frac{1}{1-t} = \prod_{n=0}^{\infty} (1+t^{2^n}) \text{ in } \mathbb{Z}[[t]]$$

Is there a ~~diff~~ map $D^2 \rightarrow D^2$ s.t.
no sinks or sources +
hyperbolic by this formula?
Other factorizations of $\frac{1}{1-t}$?

105. Sullivan's stacks of rings
problem

106. Is the multiplicity of 1
as an eigenvalue of A a
flow conjugacy inv. of Σ_A ?

How about $\prod (1-\lambda_i)$
 $\lambda_i \neq 1$ no

107. Embed auto. cpt gps as
basic sets

108. Let $\text{grp. } G$ be given by
generators S with relns.
Consider

$$V = \{x : x \in \prod_{i=0}^{\infty} (S \cup S^{-1}), x
 \text{ reduced or } \prod_{i=0}^{\infty}\}$$

What is V ? intrinsic properties?
Entropy = ? --

109. (Thom) Look at Markov part. on
 Γ^n when all λ_i distinct + real

110. Top. entropy of Krieger
map of alg. variety V

- related to $\dim V - \log$
(zeta func. rad. of conv.)

→ Relation to zeta func.
+ weil conjectures

III. central limit thm. for
 β -transform $x \rightarrow (\beta x)$

112. for hyp. attractor Λ of
 $\dim \Lambda = 1$, does $W^s(x) \cap \Lambda$
contain a disk of dimension
 $k = n - \dim W^u(x)$.

113. For a rot. fcn. $f(z)$
giving deg n map $z \rightarrow f(z)$
of S^2 , does this map
have entropy $\log n$?
($\geq \log n$ by Misiurewicz?)
 \leq a.e. by Guckenheimer

114. let $(I-A)$ as gp.

Invariant for weak fol.
 W^{uu} , lying in $\text{Hom}^{\text{int}}_m(M, S^1)$?

115. are analytic maps expansive?

116. For subshift Σ_A^+ ,
 A on stable torus of stable
 R^n is an invariant. Describe
it invariantly ($R^i = ?$, $A = ?$)

117. (Sullivan) Show $\det Df = 1$
for $f: M \rightarrow M$ (C^1 distal),
eigenvalue of $f_x \in T_S f_{xq}$
with g Hausd-Smale?

118. For $\Gamma \subset \text{SL}(2, \mathbb{R}) = G$ with
 G/Γ finite volume, describe
inv. measures of horocycle
fol. on G/Γ .

119. grad F for F analytic $R^n \rightarrow R$.
Stratification of orbits near
a singular point

120. entropy c for diff. $G = \mathbb{R}^n$ or \mathbb{Z}^n
action ($n \geq 2$). For nilpotent
 G ? general lie gp. G ?

121. Calculate h_μ for G/Γ finite
meas, not compact

~~1. 122. Homocycle flow L.B?~~
 (Katoe-O system)
~~+ O-entropy~~ \Rightarrow S conduct
 For building ~~then~~ does

122. Vibration thin. for-hu(?)

$$M = \bigcup_x N_x, \quad N_x \text{ manifolds}$$

$$f(N_x) = N_x, \text{ Then}$$

$$s(f_* \text{ on } M) \leq \sup_x (f_* \text{ on } N_x)$$

If $f|N_x$ isometry, does
 $s(f_* \text{ on } M) = 1$? Are all
 distal diffcs built up this
 way - i.e., extensions where
 homotopy works.

123. cancellation thin. for ...
 2 basic sets - analogue
 of cobordism theory? or?

124. well comp. for basic sets

125. Embedding alg. variety
 over E_p in a basic set.

126. Put orientation into S-phen.
 of flow? what should
 $S(0)$ be? ... $S(t)$ depends
 only on H_t of (M_0, M_{t-1}) .

127. Homocycle flow L.B?

128. R_1, R_2 suspensions under
 horndill form. isomorphic?
 good-flow case

129. $W(f)$ as rate of growth of
 eigenfun. for operator in
 momentum space - Fourier
 transform of Laplacian on M ?

130. Which surfaces + which homotopy classes of domains admit

- ① expansive homeo
- ② distal homeo

131. If f given by

its. for generic

(1) map $I \rightarrow I$

(cts. map)

132. $\dot{X} = Q(X)$ on \mathbb{R}^3 ,

Q quadratic. Is there a ~~soft~~ condition on coefficients which guarantees a homoclinic pt. (complicated)

attractor). E.g. like for
Reynolds no?

133. C^0 differ. of 2-diss.
preserving smooth w/
 $h_u > 0$ (ergodic
one): calculate
Tsallis entropy of Hamiltonian
upto 1st in Arnold-Ave. 2 >

134. $\frac{d}{dt} S_T \leq$ cpt. pos.

① entropy > 0

$\Rightarrow T$ cpt. ~~h-irr.~~

~~intelligent NC~~

② ent = 0 + ergodic \Rightarrow

N.

135. Hor. + good. for $\Gamma = \mathrm{SL}(3, \mathbb{Z})$.

- min. no. almost
- sym. dyn. (ell. frac?)

136. For most C^2 maps $f: [0, 1] \rightarrow [0, 1]$
if $x_0 \in$ hyp. sets $\Lambda \subset [0, 1]$
s.t. $f^k(x_0) > h(f) - \varepsilon$,

137. Ergodic smooth map of



Prod. A^2B of Dehn twists?

$$h(f) > 0 \quad = h(f) ?$$

138. Billiards on



What example where ergodic

139. Hyperfinite fol. is
hyperfinite? Is adding
w^{ws} on on $\Sigma_{[0, 1]}^+$ Borel
hyperfinite?

140. $g: G/\Gamma \rightarrow G/\Gamma$ alg. - from
auto. of G . Does $ng \Rightarrow h(f) \geq h(g)$

Entropy conj. on manifold

141. $g: G/\Gamma \rightarrow G/\Gamma$ auto.
Is $\mathrm{Car}(g \circ \pi)$ an alg. no?

142. Adding machine not an
inv. set for C^2 map of 2 disks?
what?

143. GR(a) analog. integer
for χ auto. of solvable
pp?

(144). $f: [0, 1] \rightarrow \mathbb{C}^2$, cut.
Yes. at nondy. Then $V_\varepsilon > 0$
i.e. \exists inv. expanding set
 X_ε s.t. $h(f(X_\varepsilon)) > h(f) - \varepsilon$.

(145). Does $h(f) = \log \lambda$ for
Thurston's quasi-Anosov f.

Yes

- (146. (a)) s.s. finite type have good quotient with fixed pt.
- (b) Bowen per. pt. $p \in S^1$. Is there a Markov part. with p in interior
- (c) If $\mathcal{S}^e + p$ does \mathcal{S}^e contain a per. orbit
- (d) 2 s.s. f.t. with same entropy \Rightarrow common good quotient
- (e) Σ_A, Σ_B aperiodic, $h(\tau_A) < h(\tau_B)$
does $\tau_A \sqcup \tau_B$ end in Σ_B

147. inv. distribution for good
(Anosov) flow — approximated?
(Kadar) by periodic orbit measures?

148. Kleinian pp. P limit set
— specification when
all are no parabolic or elliptic
elts.

149. (Handel) \mathbb{F} cross-section
unif. for all num. sets of flow
 $\rightarrow \mathbb{F}$ cross-section

150. Is there an expansive homeo.
of S^2 ?

151. $x \sim y$ on Σ_A^+ if

$$\tau^n x = \tau^m y \text{ some } n, m > 0.$$

Find inv. of (Σ_A^+, x) , top.

152. $f: M \rightarrow M$ C^∞ Anosov
 $f \in C^r$ and

$$f(x) = u(x) - u(Tx).$$

Does $u \in C^r$? (r ≥ 2)

153. $\phi_t: T_1 M \hookrightarrow$ Anosov
geod. flows and

$V: M \rightarrow \mathbb{R}$ s.t. $SV(\pi \phi_t x) = 0$
on every closed geod.

Does $V \equiv 0$?

154. Geod. flows on surfaces

$$h_u > 0 \text{ if } g \neq \mathbb{Z}^2$$

155. (Thurston-Sullivan?) Are all smooth actions of \mathbb{P}_g on S^1 (which are top conj. to a standard one)
diff. conj. to a standard one?

156. Kleinian gp. Γ .

h.d. $N(\Gamma) \leq 2$ if $N(\Gamma) \neq S^2$

157. On closure \bar{T} of Teichmüller
space. Cts. param.

$$\bar{T} \times \Sigma_A^+ \rightarrow S^2$$

s.t. image $(t, \Sigma_A^+) = N(\Gamma_t)$
h.d. (Γ_t) cts. in $t \in \bar{T}$?