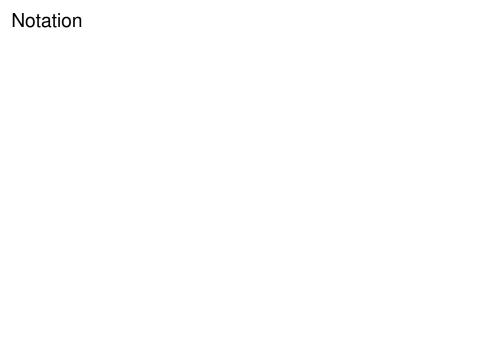
Character sheaves and Hecke algebras

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joint work with K. Vilonen, partly with M. Grinberg



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$$\mathcal{N}_i = \mathfrak{g}_i \cap \mathcal{N}$$

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Here

$$\mathfrak{F}: \operatorname{Perv}_K(\mathfrak{g}_{-1})_{\mathbb{C}^*\text{-conic}} o \operatorname{Perv}_K(\mathfrak{g}_1)_{\mathbb{C}^*\text{-conic}}$$

is the Fourier transform (we identify $\mathfrak{g}_{-1}^* \cong \mathfrak{g}_1$).

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Our goal Describe the character sheaves in $\operatorname{Char}_K(\mathfrak{g}_1)$ explicitly.

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The bijection

$$\operatorname{SPerv}_K(\mathcal{N}_{-1}) \xrightarrow{\mathfrak{F}} \operatorname{Char}_K(\mathfrak{g}_1)$$

can be viewed as an analogue of Springer correspondence/Lusztig's generalized Springer correspondence for $\mathbb{Z}/m\mathbb{Z}$ -graded Lie algebras.

Ungraded case/Springer correspondence

Recall Grothendieck-Springer resolution

$$\widetilde{\mathcal{N}} = G \times^B \mathfrak{n} \longrightarrow \widetilde{\mathfrak{g}} = G \times^B \mathfrak{b}$$
 $\downarrow^{\check{\pi}}$
 $\mathcal{N} \longrightarrow \mathfrak{g}$

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 $\widetilde{\mathfrak{F}}(\pi_* \mathbb{C}_{\widetilde{\mathcal{N}}}) \qquad \cong \qquad \check{\pi}_* \mathbb{C}_{\widetilde{\mathfrak{g}}}[-]$
 $\parallel \qquad \qquad \parallel \qquad \qquad \parallel$
 $\mathfrak{F}(\oplus \operatorname{IC}(\mathcal{O}, \mathcal{L}) \otimes V_{\mathcal{O}, \mathcal{L}}) \qquad \cong \qquad \bigoplus_{\rho \in \hat{W_G}} \operatorname{IC}(\mathfrak{g}^{rs}, \mathcal{M}_{\rho}) \otimes V_{\rho}$

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$$\mathfrak{F}(\pi_* \mathbb{C}_{\widetilde{\mathcal{N}}}) & \cong & \check{\pi}_* \mathbb{C}_{\widetilde{\mathfrak{g}}}[-] \\ \parallel & & \parallel \\ \mathfrak{F}(\oplus \operatorname{IC}(\mathcal{O}, \mathcal{L}) \otimes V_{\mathcal{O}, \mathcal{L}}) & \cong & \bigoplus_{\rho \in \hat{W_G}} \operatorname{IC}(\mathfrak{g}^{rs}, \mathcal{M}_{\rho}) \otimes V_{\rho} \end{array}$$

$$\Rightarrow \mathfrak{F}\operatorname{IC}(\mathcal{O},\mathcal{L}) \cong \operatorname{IC}(\mathfrak{g}^{rs},\mathcal{M}_{\rho})$$
 (Springer correspondence)

$\mathbb{Z}/m\mathbb{Z}$ -graded Lie algebras

Remark: In our context, an analogue of Grothendieck-Springer resolution does not seem to exist.

Cohomology of Hessenberg varieties (of regular semisimple elements) occuring in our situation is more complicated.

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Definition: A character sheaf is called *cuspidal* if it does not arise from parabolic induction of character sheaves in $\operatorname{Char}_{L_K} \mathfrak{l}_1$ for θ -stable Levi subgroups L contained in proper θ -stable parabolic subgroups.

Cuspidal character sheaves

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All *non-nilpotent support* cuspidal character sheaves arise from a (generalised) nearby cycle construction.

They are associated to irreducible representations of Hecke algebras of complex reflection groups at roots of unity.

 Nilpotent support cuspidal character sheaves (biorbital) are rare, as in Lusztig's generalised Springer correspondence. They "come from classical cuspidals".

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 - They exist only when θ induces a GIT stable grading on \mathfrak{g} , that is, (G, K) is a split symmetric pair.
- We can view the nearby cycle construction as a replacement of the Grothendieck-Springer resolution.

The adjoint quotient

We assume that θ induces a grading where non-nilpotent support cuspidal character sheaves exist (these can be explicitly described, at least conjecturally for now).

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Let $\mathfrak{a} \subset \mathfrak{g}_1$ be a Cartan subspace. By Vinberg, we have

$$f:\mathfrak{g}_1\to\mathfrak{g}_1/\!/K\cong\mathfrak{a}/W$$

where $W \cong N_K(\mathfrak{a})/Z_K(\mathfrak{a})$. It is in general a complex reflection group.

Nearby cycle construction

Let $\bar{a} \in \mathfrak{a}^{rs}/W$. Then $X_{\bar{a}} = f^{-1}(a)$ is a regular semisimple K-orbit. We have

$$\pi_1^K(X_{\bar{a}}) \cong Z_K(\mathfrak{a})/Z_K(\mathfrak{a})^0 := I.$$

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Let $\chi \in \hat{I}$ and \mathcal{L}_{χ} the corresponding K-equivariant local system on $X_{\bar{a}}$. We form the nearby cycle sheaf

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Theorem (Grinberg) We have

$$\mathfrak{F}P_\chi\cong\mathrm{IC}(\mathfrak{g}_1^{rs},\mathcal{M}_\chi)$$

where \mathcal{M}_{χ} is given by a representation M_{χ} of $\pi_{1}^{K}(\mathfrak{g}_{1}^{rs})$.

The equivariant fundamental group $\pi_1^K(\mathfrak{g}_1^{rs})$

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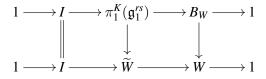
$$1 \longrightarrow I \longrightarrow \pi_1^K(\mathfrak{g}_1^{rs}) \longrightarrow B_W \longrightarrow 1$$

$$\downarrow \qquad \qquad \downarrow \qquad \qquad \downarrow$$

$$1 \longrightarrow I \longrightarrow \widetilde{W} \longrightarrow W \longrightarrow 1$$

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The top sequence splits via a Kostant slice. We write

$$\pi_1^K(\mathfrak{g}_1^{rs}) := \widetilde{B}_W \cong B_W \ltimes I.$$

Theorem (Grinberg-Vilonen-X.) We have

$$M_\chi\cong\mathbb{C}[\widetilde{B}_W]\otimes_{\mathbb{C}[\widetilde{B}_{W,\chi}^0]}\left(\mathcal{H}_{W_\chi^0}\otimes\mathbb{C}_\chi
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where $W_{\chi}^0 \subset \operatorname{Stab}_W \chi$ is a complex reflection subgroup and $\mathcal{H}_{W_{\chi}^0}$ is a cyclotomic Hecke algebra associated to W_{χ}^0 .

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- The above theorem holds in the setting of stable polar representations.

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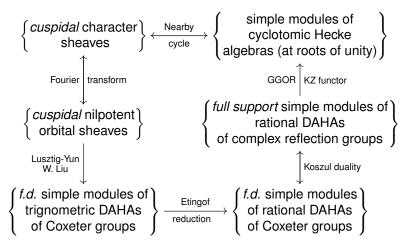
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Example: when m=2 and (G,K) split pair, W^0_χ is a Coxeter group and the Hecke relations for $\mathcal{H}_{W^0_\chi}$ are $(x-1)^2=0$. (Endoscopic interpretation as pointed out by C.C. Tsai.)



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Moreover, in classical types, the arrow *Koszul duality* coincides with the Koszul duality for blocks of cyclotomic rational DAHAs (type $G_{m,1,r}$) conjectured by Chuang-Miyachi, established by Rouquier-Shan-Varagnolo-Vasserot.

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Remark: The diagram also suggests Koszul duality between blocks of rational DAHAs of exceptional types.