Homework 3

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Problem 1

Diagonal Dominance

Convergence criterion for both methods is that the input matrix be diagonally dominant. rand(N) + a*diag(randperm(N)) does not produce diagonally doiminant matrices for a = 1 and large N as entries of rand(N) are ≈ 0.1. For N = 5 the rowsum is ≈ 50. So rand(50) + diag(randperm(50)) produces matrix that is not diagonally dominant at about 5 rows. To rectify this we increase a and observe that the methods converge for a = 5 when N = 500. In the final draft of our code we set a = 10. We also observe that converge is faster with greater values of a.

Convergence rate

Plot reveal that Gauss-Seidel is faster than Jacobi by a factor greater than 1.5. Both methods take longer for small matrices than direct methods (e.g Gaussian elimination) but with larger matrices speed difference between iterative methods and direct methods becomes smaller.

Problem 2

Although Rayleigh iteration has cubic convergence, at each step it has to solve a linear system whereas power method only uses matrix multiplication. As a result Rayleigh iteration requires a longer computation time.

Problem 3

For larger condition number both methods require more iterations to converge to the desired tolerance among matrices of same size. Conjugate gradient requires lesser number of iterations than steepest descent to converge. For k = 6 steepest descent requires almost 100 times more iterations than conjugate gradient. From the plots it's also clear that during initial iterations relative residue is much larger for steepest descent than conjugate gradient. Since steepest descent takes too long to converge for k = 5 we stop at k = 4.

Problem 4

First observation here is that neither bicg nor gmres is suitable for solving Ax = b as they are designed to solve sparse systems and therefore produce considerable errors. Error plots show that bicg ang gmres closely follow each other for each k. Conjugate gradient is also not suitable for solving Mx = c because although M is symmetric it's not positive definite.