Learning from data

Classification

February 3, 2020

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Leads us to the classification setting.

What are we up to?

- n individuals – each is associated with certain quantitative measurements called the predictors or features, say the p measurements of i-th individual are given as $(x_{i1}, x_{i2}, \ldots, x_{ip})$

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- n individuals – each is associated with certain quantitative measurements called the predictors or features, say the p measurements of i-th individual are given as $(x_{i1}, x_{i2}, \dots, x_{ip})$
- Each individual is also categorised using a qualitative variable: dependent variable
- Goal: given predictor measurements $(x_1, x_2, \dots, x_p)$ of certain individual, we wish to model y, the category this individual belongs to – Classification problem

A mathematician's retort

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- Modelling a function which takes only finitely many values is not difficult. For example, if y takes only two values 0 or 1, then we can set:

$$y = 1_{\{(x_1,\ldots,x_p)\in B\}}$$

or, in general, if y takes J distinct values $(a_1, \ldots, a_J)$ then we can set

$$y = a_1 \, 1_{\{(x_1,\ldots,x_p)\in B_1\}} + \cdots + a_J \, 1_{\{(x_1,\ldots,x_p)\in B_J\}}$$

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Issues with the proposed solution:

- y could be a qualitative variables, and thus the above method would fail.
- How does one incorporate statistical error (noise)?

Binary classifier: logistic regression

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Writing X for the variable “Credit.Amount”, the binary classifier is based on modelling:

$$\mathbb{P}(Y = 1|X = x)$$

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In other words,

$$p(x) = \frac{e^{\beta_0 + \beta_1 x}}{1 + e^{\beta_0 + \beta_1 x}}$$

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Method: Maximum likelihood estimation

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Let $\mathcal{Y}_0$ be the subset of training dataset for which $y_i = 0$, and similarly, $\mathcal{Y}_1$ be the subset of training dataset for which $y_i = 1$.

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The likelihood for this training sample is:

$$L(\beta_0, \beta_1) = \prod_{i \in \mathcal{Y}_1} p(x_i) \prod_{i \in \mathcal{Y}_0} [1 - p(x_i)]$$

The maximum likelihood estimates are those $\widehat{\beta_0}$ and $\widehat{\beta_1}$, which maximise $L(\beta_0, \beta_1)$

Maximum likelihood estimation

Finding those $\widehat{\beta_0}$ and $\widehat{\beta_1}$, which maximise

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Remark: Likelihood estimation is a very general approach, applicable to almost every setting of estimation. Least squares estimation in linear regression is a special case of likelihood estimation.

Interpretation of $\widehat{\beta_1}$: A unit increase in x, changes the odds by a factor of $e^{\widehat{\beta_1}}$.

Prediction

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$$\mathbb{P}(Y = 1 | X = x_0) = p(x_0) = \frac{e^{\widehat{\beta_0} + \widehat{\beta_1} x_i}}{1 + e^{\widehat{\beta_0} + \widehat{\beta_1} x_i}}$$

- If $p(x_0) > 0.5$ (say), then we classify x_0 into the category $Y = 1$, else into the category $Y = 0$.

Note: The thresholding at 0.5 is our choice. If we have some prior information then we may want to tweak this a little.

Testing: statistical significance of the model

The hypothesis of interest:

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\begin{aligned}
H_0 &: \beta_1 = 0 \\
H_1 &: \beta_1 \neq 0
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- The estimators are tested using the standard Wald's Z-test.
- Or, one could also use the *likelihood ratio (LR)* test.

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Testing: goodness of the model

- The testing phase includes predicting the classification for given set of x_i's, for which the true classification is already know.
- We count the number of 1's which are correctly identified as 1, and the number of 1's which are incorrectly identified as 0.
- Similarly, we count the number of 0's which are correctly identified as 0, and the number of 0's which are incorrectly identified as 1.

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We apply the standard 2-sample t-test.

German Credit Database

The variables:

```
> names(G.credit)
 [1] "Creditability"                   "Account.Balance"
 [3] "Duration.of.Credit..month."      "Payment.Status.of.Previous.Credit"
 [5] "Purpose"                         "Credit.Amount"
 [7] "Value.Savings.Stocks"            "Length.of.current.employment"
 [9] "Instalment.per.cent"             "Sex...Marital.Status"
[11] "Guarantors"                      "Duration.in.Current.address"
[13] "Most.valuable.available.asset"   "Age..years."
[15] "Concurrent.Credits"              "Type.of.apartment"
[17] "No.of.Credits.at.this.Bank"      "Occupation"
[19] "No.of.dependents"                "Telephone"
[21] "Foreign.Worker"
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```

We are going to focus on the predictor “Credit.Amount” and the response as “Creditability”.

German Credit Database

First, let us test if there is indeed any effect of "Creditability" on the "Credit.Amount":

```
> t.test(Credit.Amount ~ Creditability, data=G.credit)

        Welch Two Sample t-test

data:  Credit.Amount by Creditability
t = 4.2643, df = 421.86, p-value = 2.477e-05
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
  513.5476 1391.8201
sample estimates:
mean in group 0 mean in group 1
       3938.127        2985.443
```

This is Welch's *t*-test which is a slight modification of the standard two sample *t*-test.

German Credit Database

Some more evidence of effect of “Creditability” on the “Credit.Amount”:

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> wilcox.test(Credit.Amount ~ Creditability, data=G.credit)

        Wilcoxon rank sum test with continuity correction

data:  Credit.Amount by Creditability
W = 116520, p-value = 0.005918
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alternative hypothesis: true location shift is not equal to 0
```

Conclusion: Indeed creditable group has a different distribution of “Credit.Amount” as compared to that of the non–creditable group. Implying that a binary classification indeed is a good idea.

Logistic regression: Creditability $\sim$ Credit.Amount

```
> library(gmodels)
> creditability.on.cramount <- glm(Creditability~Credit.Amount,data=G.credit,
+                                  family=binomial)
> summary(creditability.on.cramount)

Call:
glm(formula = Creditability ~ Credit.Amount, family = binomial,
    data = G.credit)

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-1.7022  -1.3674   0.7688   0.8269   1.4158

Coefficients:
                Estimate Std. Error z value Pr(>|z|)
(Intercept)    1.229e+00  1.083e-01  11.348  < 2e-16 ***
Credit.Amount -1.119e-04  2.355e-05  -4.751 2.02e-06 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

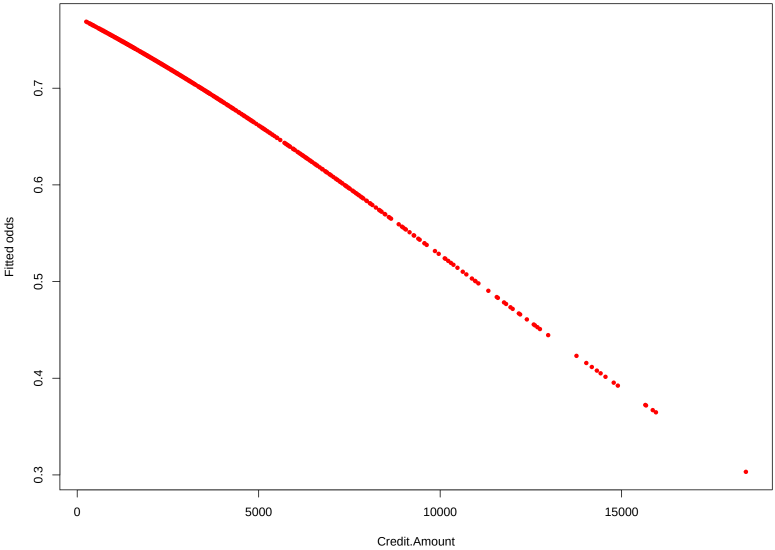
(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 1221.7  on 999  degrees of freedom
Residual deviance: 1199.1  on 998  degrees of freedom
AIC: 1203.1

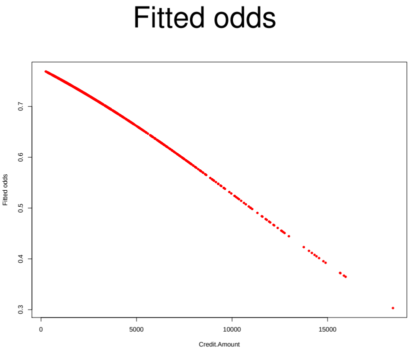
Number of Fisher Scoring iterations: 4
```

****interpretation****

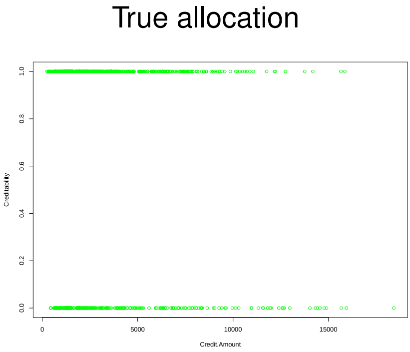
Predictions



Compare



Fitted odds
Fitted odds
0.3
0.4
0.5
0.6
0.7
0
5000
10000
15000
Credit.Amount



True allocation
Creditability
0.0
0.2
0.4
0.6
0.8
1.0
0
5000
10000
15000
Credit.Amount

Compare

Total Observations in Table: 1000

creditability.f	prediction.for.CrAm.c No	Yes	Row Total
No	20	280	300
	6.667%	93.333%	30.000%
Yes	9	691	700
	1.286%	98.714%	70.000%
Column Total	29	971	1000

Overal, the prediction seems to work fairly well, in that the classifier could correctly predict 71.1% times.

Specifically, the model seems to perform very well when predicting 1s, but it performs poorly in predicting the 0s.

Multiple logistic regression

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Say, we have measurements on p predictors or features, and we wish to classify each point into one or the other group (binary classification).

We shall again be interested in modelling:

$$p(x_1, \dots, x_p) = \mathbb{P}\left(Y = 1 | X_1 = x_1, \dots, X_p = x_p\right)$$

Once again, we propose:

$$\log\left(\frac{p(X_1,\ldots,X_p)}{1-p(X_1,\ldots,X_p)}\right)=\beta_0+\beta_1X_1+\cdots+\beta_pX_p$$

Once again, we propose:

$$\log\left(\frac{p(X_1,\ldots,X_p)}{1-p(X_1,\ldots,X_p)}\right) = \beta_0 + \beta_1 X_1 + \cdots + \beta_p X_p$$

Estimation: maximum likelihood estimation, i.e., we look for those values of $\beta_0, \beta_1, \ldots, \beta_p$ which maximise

$$L(\beta_0,\beta_1,\ldots,\beta_p) = \prod_{i\in\mathcal{Y}_1} p(x_{1,i},\ldots,x_{p,i}) \prod_{i\in\mathcal{Y}_0} \left[1 - p(x_{1,i},\ldots,x_{p,i})\right]$$

Few remarks:

- Interpretation of $\widehat{\beta}_i$ for $i = 1, \ldots, p$: Keeping all other variables constant, a unit increase in the i-th feature would change the odds by a factor of $e^{\widehat{\beta}_i}$.

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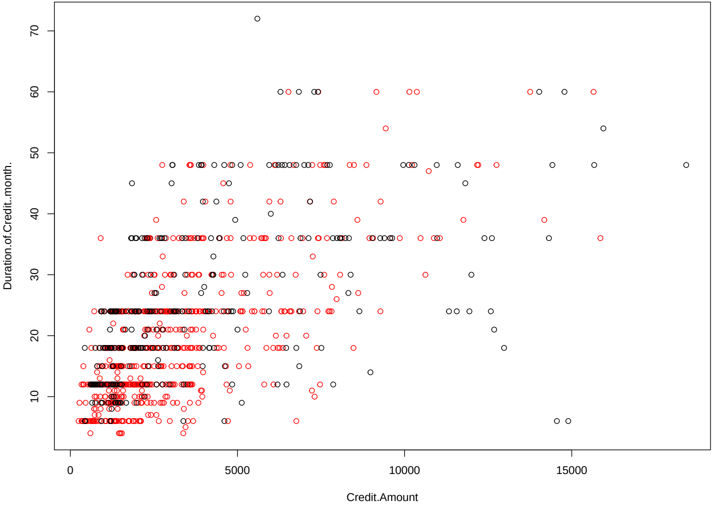
- Interpretation of $\widehat{\beta}_i$ for $i = 1, \ldots, p$: Keeping all other variables constant, a unit increase in the i-th feature would change the odds by a factor of $e^{\widehat{\beta}_i}$.
- It is desirable that different features give indeed different information $\Rightarrow$ multicollinearity. This is again tested by the same VIF

German Credit Database

The plot for the three variables looks like:



Multiple Logistic Regression: output

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> summary(creditability.on.cramdur)

Call:
glm(formula = Creditability ~ Credit.Amount + Duration.of.Credit..month.,
    family = binomial, data = G.credit)

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-1.8249  -1.2734   0.7164   0.8533   1.5020

Coefficients:
                            Estimate Std. Error z value Pr(>|z|)
(Intercept)                1.670e+00  1.466e-01  11.390  < 2e-16 ***
Credit.Amount             -2.300e-05  3.059e-05  -0.752    0.452
Duration.of.Credit..month. -3.412e-02  7.282e-03  -4.685  2.8e-06 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 1221.7  on 999  degrees of freedom
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****interpret****

Predictions

Total Observations in Table: 1000

	prediction.for.CrAmDur.c		
creditability.f	No	Yes	Row Total
No	37 12.333%	263 87.667%	300 30.000%
Yes	29 4.143%	671 95.857%	700 70.000%
Column Total	66	934	1000

Overall efficiency of the model has actually reduced a bit as compared to the logistic model with single predictor “Credit.Amount”.

Although the model still performs poorly in predicting the 0s, but it is better than the single predictor model.

However, the model’s efficiency is classifying correctly identifying the 1s has considerably reduced.

Comparing the two binary classifiers

Predicting creditability using "Credit.Amount"

Total Observations in Table: 1000

creditability.f	prediction.for.CrAm.c: No	Yes	Row Total
No	20	280	300
	6.667%	93.333%	30.000%
Yes	9	691	700
	1.286%	98.714%	70.000%
Column Total	29	971	1000

Predicting creditability using "Credit.Amount" and "Duration"

Total Observations in Table: 1000

creditability.f	prediction.for.CrAmDur.c: No	Yes	Row Total
No	37	263	300
	12.333%	87.667%	30.000%
Yes	29	671	700
	4.143%	95.857%	70.000%
Column Total	66	934	1000

Which one would we choose?

Comparing the two binary classifiers

Predicting creditability using "Credit.Amount"

```
Total Observations in Table:  1000

                | prediction.for.CrAm.c
creditability.f |        No |       Yes | Row Total |
----------------|-----------|-----------|-----------|
             No |        20 |       280 |       300 |
                |    6.667% |   93.333% |   30.000% |
----------------|-----------|-----------|-----------|
            Yes |         9 |       691 |       700 |
                |    1.286% |   98.714% |   70.000% |
----------------|-----------|-----------|-----------|
   Column Total |        29 |       971 |      1000 |
----------------|-----------|-----------|-----------|
```

Predicting creditability using "Credit.Amount" and "Duration"

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Total Observations in Table:  1000

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----------------|-----------|-----------|-----------|
            Yes |        29 |       671 |       700 |
                |    4.143% |   95.857% |   70.000% |
----------------|-----------|-----------|-----------|
   Column Total |        66 |       934 |      1000 |
----------------|-----------|-----------|-----------|
```

Which one would we choose?

Sensitivity: The ability to correctly identify the actual "positives"

Specificity: The ability to correctly identify the actual "negatives"

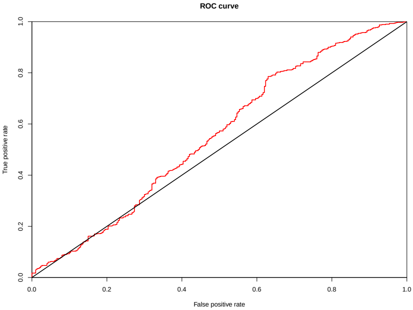
ROC curve and AUC

Receiver operating characteristic (ROC) curve, historically, originated from communications theory, as a simple tool to plot **true positive rate (sensitivity)** and **false positive rate (specificity)** for various values of the threshold.

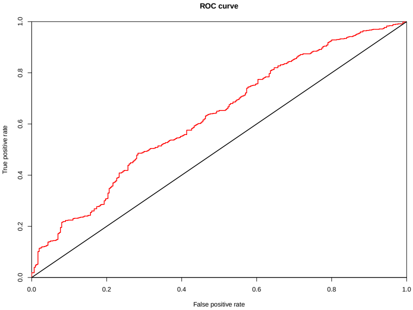
ROC curve and AUC

Receiver operating characteristic (ROC) curve, historically, originated from communications theory, as a simple tool to plot **true positive rate (sensitivity)** and **false positive rate (specificity)** for various values of the threshold.

ROC for predicting creditability using "Credit.Amount"



ROC for predicting creditability using "Credit.Amount" and "Duration"



ROC curve and AUC

ROC curve provides compelling visual depiction of the goodness of models.

However, it is often preferable to be able to quote a single, which is the **area under the curve (AUC)**.

This is the area under the ROC curve. A good model should have a high value of AUC (close to 1).

For the two models we considered, the AUC were:
simple logistic: 0.5548
multiple logistic: 0.6257

Another approach: KNN classifier