Problem 2

The distribution function is given by

$$F(x) = \begin{cases} \frac{(x-2)^2}{4}, & 2 \le x \le 3 \\ -\frac{x^2}{12} + x - 2, & 3 \le x \le 6 \end{cases}$$

The inverse to which is given by

$$F^{-1}(y) = \begin{cases} 2(\sqrt{y} + 1), & 0 \le y \le \frac{1}{4} \\ 6 - 2\sqrt{3(1-y)}, & \frac{1}{4} \le y \le 1 \end{cases}$$

1000 samples generated with inverse transform algorithm yield the following result.

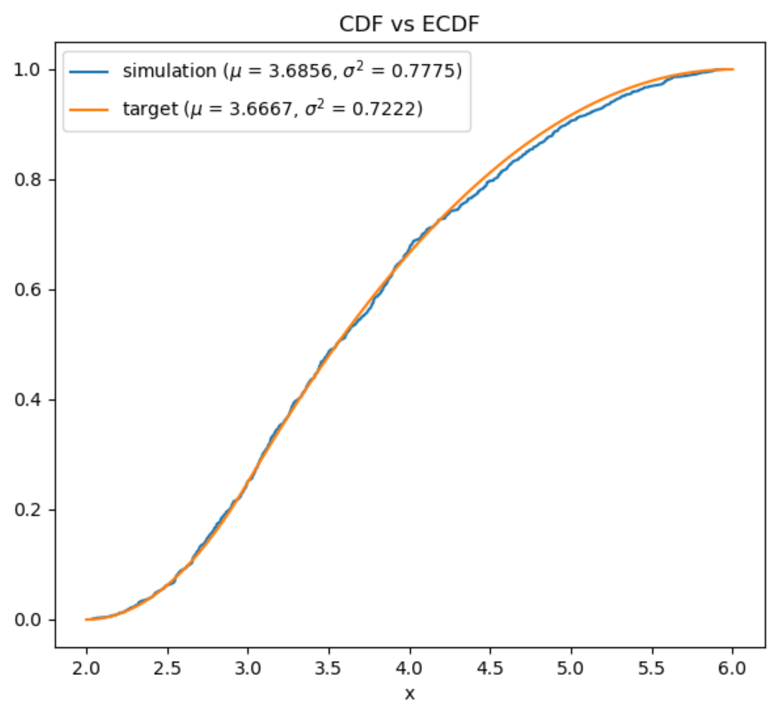

Figure 1: distribution comparison

Problem 4

$$F^{-1}(y) = \left[-\frac{\log(1-y)}{\alpha} \right]^{\frac{1}{\beta}}$$

The pdf is given by

$$f(x) = \alpha\beta\, x^{\beta-1} \exp(-\alpha x^{\beta})$$

The mean is $\frac{1}{\sqrt[\beta]{\alpha}}\Gamma\left(1+\frac{1}{\beta}\right)$ and the variance is $\frac{1}{\sqrt[\beta]{\alpha^2}}\left[\Gamma\left(2+\frac{1}{\beta}\right) - \Gamma\left(1+\frac{1}{\beta}\right)^2\right]$. 1000 samples generated with inverse transform algorithm yield the following result.

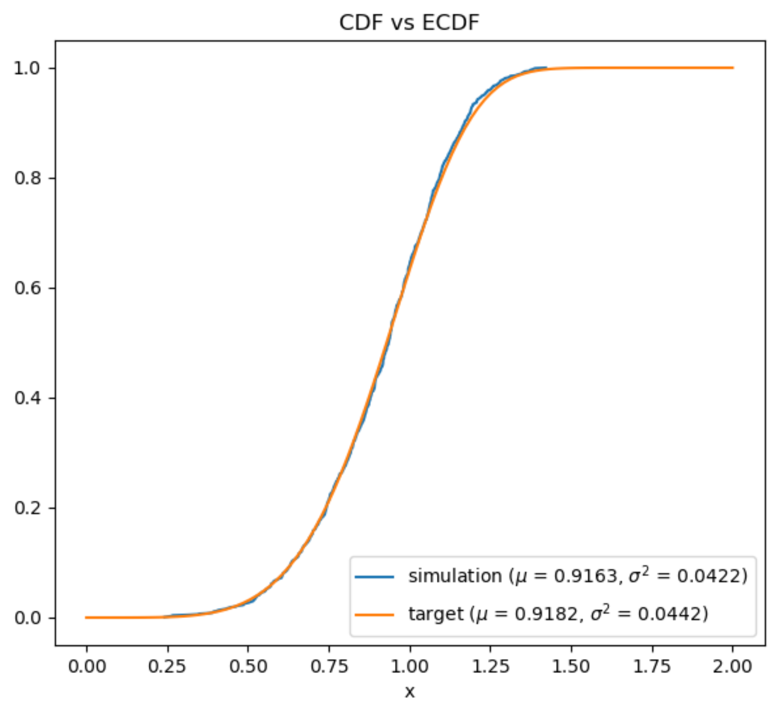

Figure 2: distribution comparison for $\alpha = 1$, $\beta = 5$

Problem 6

The inverse of the cdf is given by

$$F^{-1}(y) = -\log(1 - cy), \quad c = 1 - e^{-0.05}$$

Mean of the target distribution is given by,

$$\frac{1 - 1.05e^{-0.05}}{1 - e^{-0.05}}$$

1000 samples generated with inverse transform algorithm yield the following result.

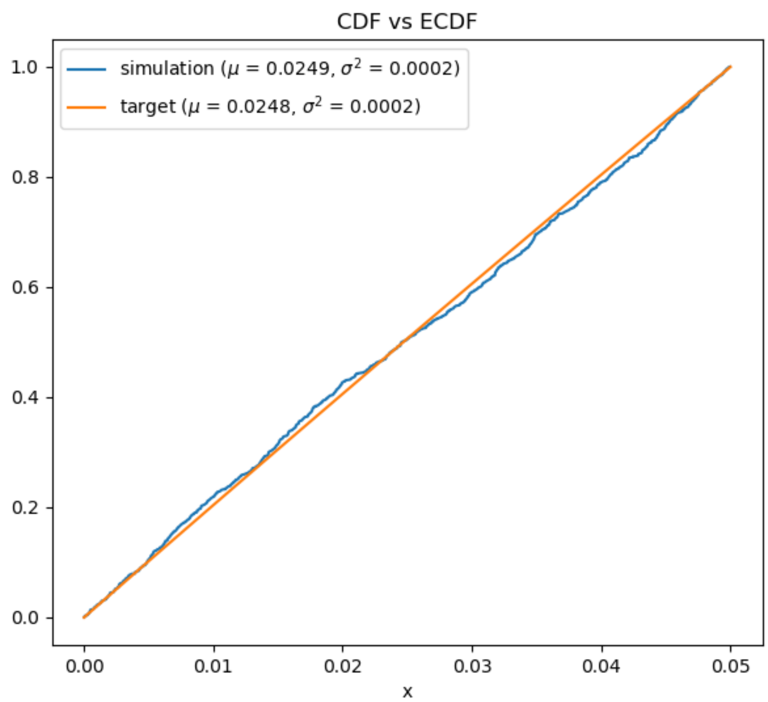

Figure 3: distribution comparison

Problem 8(a)

The inverse of $F_i(x)$ is given by

$$F_i^{-1}(y) = y^{\frac{1}{2i-1}}, \quad i = 1, 2, 3$$

1000 samples generated with composition method yield the following result.

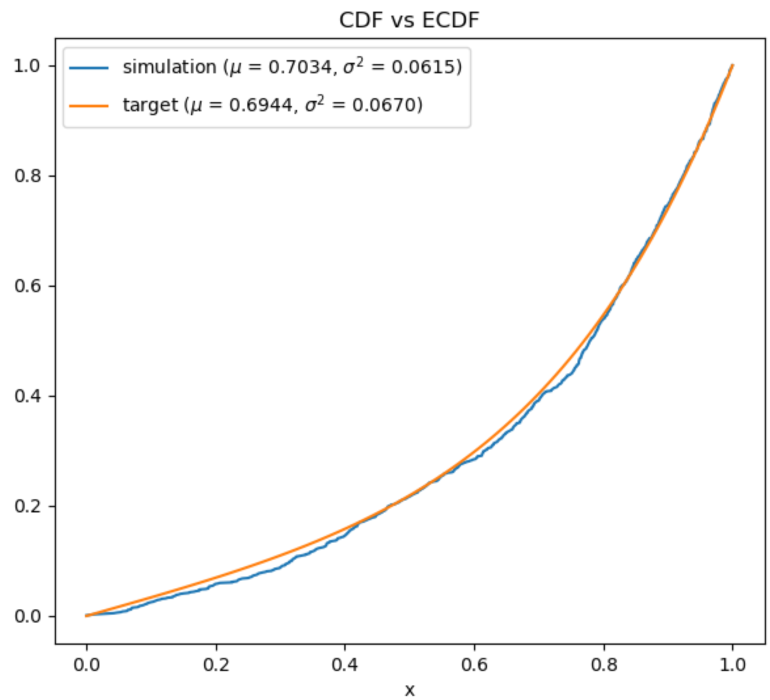

Figure 4: distribution comparison

Problem 8(b)

Our setup is as follows.

$$F_1(x) = 1 - e^{-2x}, \;\; 0 < x < \infty$$
$$F_2(x) = \begin{cases} x, & 0 < x < 1 \\ 1, & 1 \le x < \infty \end{cases}$$
$$F(x) = \frac{1}{3}F_1(x) + \frac{2}{3}F_2(x)$$

1000 samples generated with composition method yield the following result.

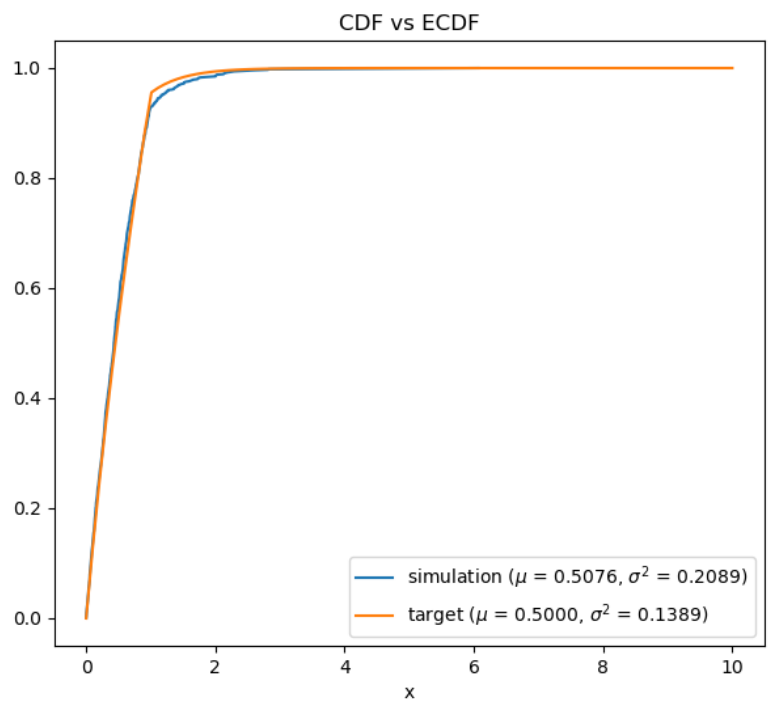

Figure 5: distribution comparison

Problem 8(c)

Our setup is as follows. The probability weights have been generated randomly.

$$\bar{\alpha} = (0.2099324, 0.23783094, 0.26164978, 0.00775745, 0.28282942)$$

1500 samples generated with composition method yield the following result.

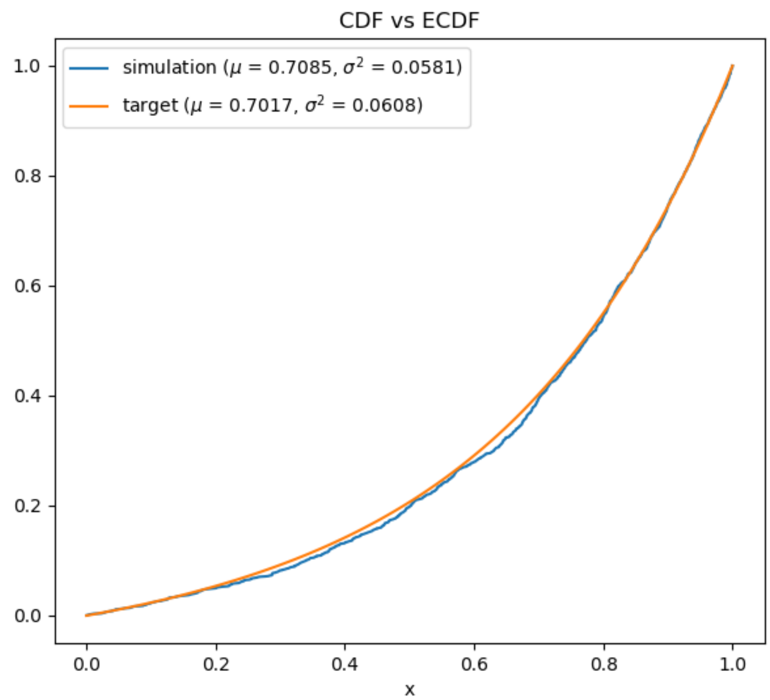

Figure 6: distribution comparison