Homework 4 (due May 10 @ 6:00pm)

1. In an $M/M/c$ system, recall the definition of $N(t)$ as the number of customers in the system at time t after the last departure, and T the r.v. of inter-departure time.
   a) Show that $N(T)$ and T are independent
   b) Show that successive inter-departure times are mutually independent

2. Consider a two-node series queueing system (single server at each node) with $Poi(\lambda)$ arrivals and Exponential service times with rates μ_1, μ_2. The first node has infinite capacity. The second node has finite capacity of K customers (including the one in service): when K customers are in the second station, any subsequent arrival is *blocked* from the system (we assume that the first node knows when the second one is full, instantaneously).
   a) Compute the blocking probability
   b) Compute the expected number of customers and mean waiting time in the system

3. Write the balance equations for a Jackson network where each node has c_i servers, and verify that the steady-state distribution is given by:
$$p_{\bar{n}} = \prod_{i=1}^{k} \left(\frac{r_i^{n_i}}{a_i(n_i)} \cdot p_{0i} \right) \qquad (r_i \equiv \lambda_i/\mu_i),$$
   where $a_i(n_i) = \begin{cases} n_i! & (n_i < c_i) \\ c_i^{n_i - c_i} c_i! & (n_i \geq c_i) \end{cases}$ and p_{0i} is such that $\sum_{n_i=0}^{\infty} p_0 r_i^{n_i}/a_i(n_i) = 1$
   Repeat for a closed Jackson network.

4. Consider a cyclic queue with two nodes (unlimited servers per node & infinite capacity).
   a) Write the steady-state distribution
   b) Use this to derive the steady-state distribution for the machine repair problem (no spares)

5. For a closed Jackson network:
   a) Why is l included in the summation in the formula $D_l(N) = \sum_{i=1}^{k} v_i W_i(N)$ used in MVA?
   b) Verify and interpret the detailed balance equations:
$$p_i(n,\ N) = \frac{\lambda_i(N)}{\mu_i} p_i(n-1,\ N-1) \qquad (n, N \geq 1)$$

6. Problem 4.10 from reference book; show your computer code.

7. Problem 4.18 from reference book. Additionally, use Buzen algorithm to compute $G(35)$; show your computer code.