Homework 2 (due April 20 @ 6:00pm)

1. Show that:
    a) An M/M/1 queue has lower L than an M/M/2 queue with the same ρ.
    b) An M/M/2 queue has lower L than two independent M/M/1 queues with the same service rate, but each one getting half of the arrivals.

2. For an M/M/1 queue:
    a) derive the variance of the number of customers in the system in steady-state.
    b) find $E\left[T_q \mid T_q > 0\right]$, that is, the expected time one must wait in the queue, given that one must wait at all.

    Repeat parts a) & b) for an M/M/c queue.

3. For an M/M/c queue, show that:
    a) $P\left[T \le t \middle| T_q > 0\right] = \frac{c(1-\rho)}{c(1-\rho)-1}\left(1 - e^{-\mu t}\right) - \frac{1}{c(1-\rho)-1}\left(1 - e^{-(c\mu-\lambda)t}\right)$
    b) $W(t) = \frac{c(1-\rho)-W_q(0)}{c(1-\rho)-1}\left(1 - e^{-\mu t}\right) - \frac{1-W_q(0)}{c(1-\rho)-1}\left(1 - e^{-(c\mu-\lambda)t}\right)$
    c) Use this to verify that $W = \frac{1}{\mu} + \left(\frac{r^c}{c!(c\mu)(1-\rho)^2}\right) p_0$

4. Compute the waiting time distribution $W(t)$ for an M/M/c/K queue.

5. Show that Erlang's formula can be recursively computed as:

$$B(c,r) = \begin{cases} \frac{rB(c-1,r)}{c+rB(c-1,r)}, & c \ge 1 \\ 1, & c = 0 \end{cases}$$

6. For the finite-source queue model (repairman model) show that:
    a) $q_n(M) = p_n(M-1)$
    b) For the model with Y spares, $q_n(M) = p_n(Y-1)$

7. Problem 2.41 from the reference book.