Microeconomic Theory: TA Session

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About this TA Section

Some logistics:

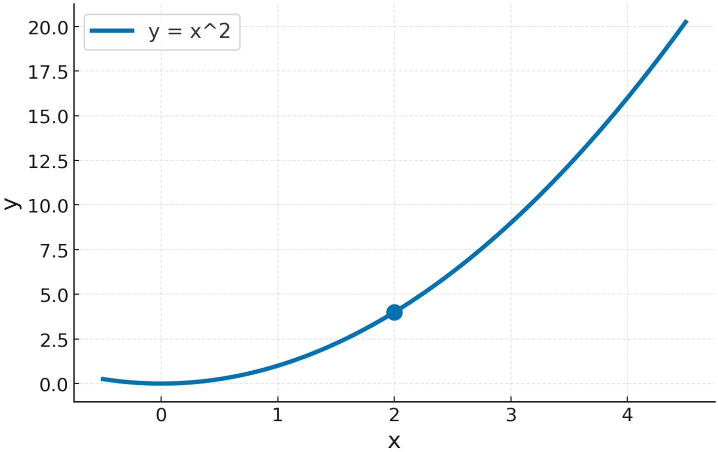
- Fridays, 9:00-9:50am, Krieger 180
- What we'll do: review class material; go over quizzes and/or assignments; answer any questions; do practice problems...
- Slides will be made available after each session on Github: https://github.com/pindawang/ElementsMicro_Fall24
- Attendance will be randomly taken for TA sessions

My office hour:

- In-person: Wednesdays, 4:30-5:30pm, Wyman Park W601A
- Zoom: by appointment
- My email: pwang66@jhu.edu

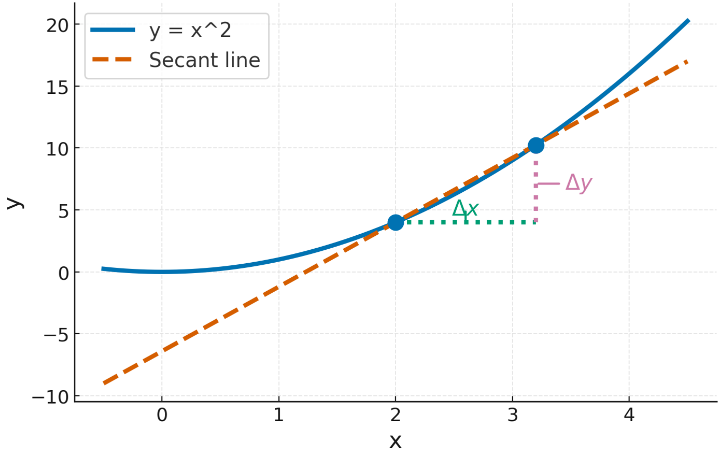
Review: First-order derivative

What is the rate of change of the curve $y = x^2$ at $x = 2$?



Review: First-order derivative

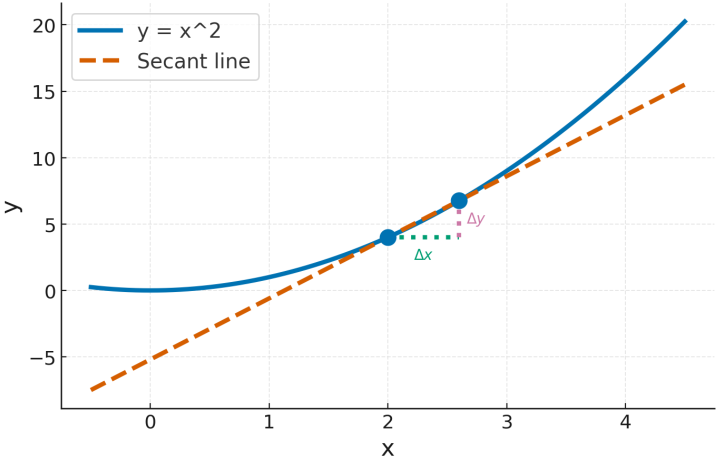
An intuitive answer would be $\Delta y / \Delta x$ at $x = 2$...



...But this is imprecise!

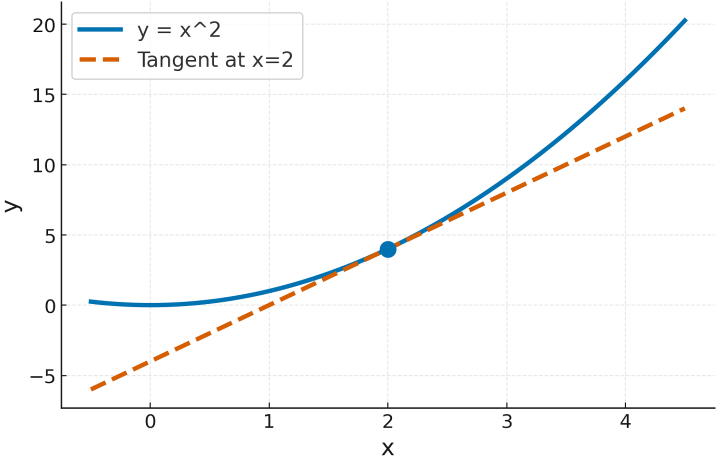
Review: First-order derivative

To make it more precise, we can decrease Δx...



Review: First-order derivative

...until Δx is close to zero



Review: First-order derivative

The (first-order) derivative of a function $y = f(x)$ at the point $x = x_0$ is its rate of change at $x = x_0$.

That is to say, it is $\frac{f(x)-f(x_0)}{x-x_0}$ when x is "very close" to x_0

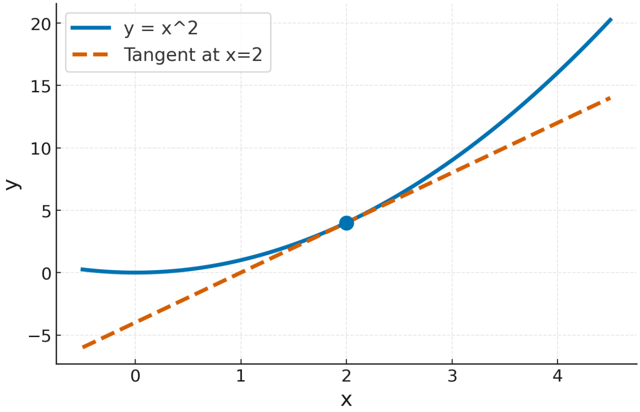
We often use y', $f'(x)$ or $\frac{dy}{dx}$ to represent the derivative of the function $y = f(x)$

In math jargon:

$$f'(x) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}$$

Review: First-order derivative

Graphically, the derivative of $f(x)$ at $x = x_0$ is the slope of the tangent line to the function at $x = x_0$



Here, the slope of the tangent line is 4, hence the derivative of $y = x^2$ at $x = 2$ equals 4

Review: First-order derivative

Frequently used derivatives:

Function	Derivative
$y = ax + b$	$y' = a$
$y = ax^2 + bx + c$	$y' = 2ax + b$
$y = \sqrt{x}$	$y' = 1/2\sqrt{x}$
$y = 1/x$	$y' = -1/x^2$
$y = x^a$	$y' = ax^{a-1}$
$y = a^x$	$y' = a^x \ln a$
$y = \ln x$	$y' = 1/x$

Review: First-order derivative

Similarly, we can calculate the derivative of a function of two or more variables

e.g. A utility function $U = 3x_1x_2$

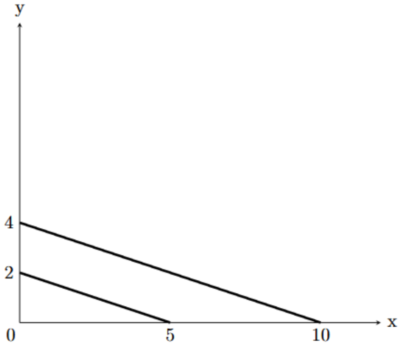
The (partial) derivative with regard to x_1: $\frac{\partial U}{\partial x_1} = 3x_2$

When calculating the derivative with regard to x_1, just treat x_2 as if it's a number

(You may also see others using U_{x_1} or just $\frac{dU}{dx_1}$ to represent the derivative with regard to x)

Utility functions and indifference curves

1. Draw the indifference curves that correspond to the utility function $U = 2x + 5y$. What is the relationship between the two goods?

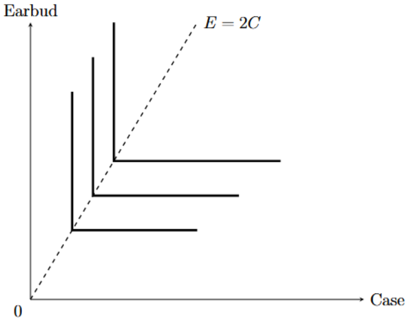


They're perfect substitutes.

Utility functions and indifference curves

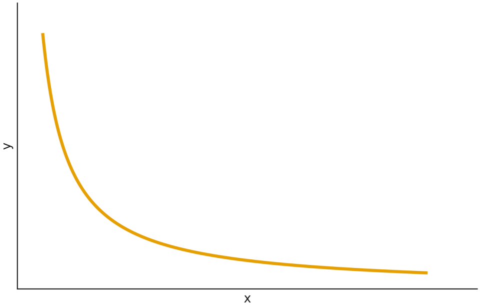
2. Earbuds and earbud cases are perfect complements: 1 earbud case always comes with 2 earbuds. What is the utility function for those two goods?

Answer: the utility function is $U = \min\{E, 2C\}$. The indifference curves look like this:



Utility functions and indifference curves

3. Indifference curves are usually bowed inward. Which of the following utility functions give rise to an indifference curve that is bowed inward?
   - A. $U = x^2 + y^2$
   - B. $U = x + 3y$
   - C. $U = 2xy$



Utility maximization

4. John has the following utility function for apples and bananas: $U = 3a^{0.5}b^{0.5}$. He has 20 dollars. One apple costs \$4, and one banana costs \$2. Solve for John's optimal bundle.

Answer: Don't be scared by the powers in the utility function. Taking the square of both sides, we get $U^2 = 9ab \Rightarrow b = \frac{U^2}{9a}$. (So, in some sense, you can think of $U = 3a^{0.5}b^{0.5}$ as equivalent to $U = ab$.) The derivative is $\frac{db}{da} = -\frac{U^2}{9a^2}$. The budget constraint is $4a + 2b = 20 \Rightarrow b = 10 - 2a$. The derivative is $\frac{db}{da} = -2$.
With $MRS = P_a/P_b$, we have $\frac{U^2}{9a^2} = 2$. Plugging in $b = \frac{U^2}{9a}$, we have $\frac{b}{a} = 2 \Rightarrow a = \frac{5}{2},\ b = 5$.