Microeconomic Theory: TA Session

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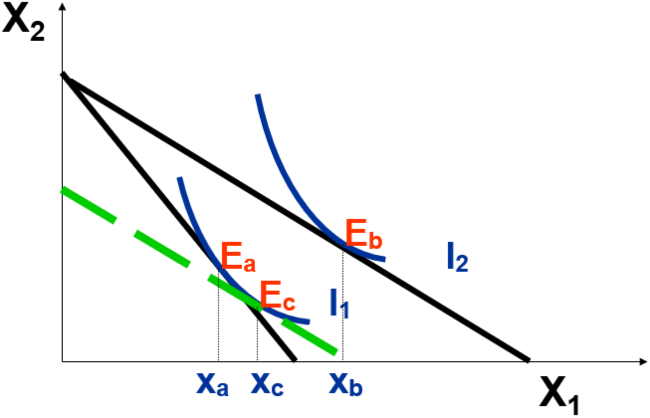
Hicksian decomposition of income and substitution effects



Slutsky decomposition of income and substitution effects

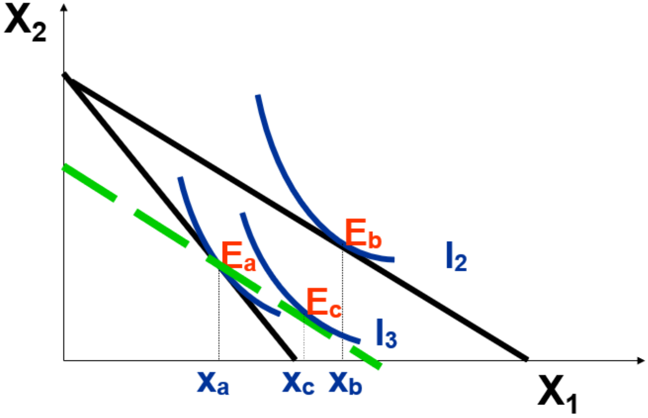
The Hicksian method calculates substitution effect by keeping utility level constant

Another way to think about substitution effect is to keep purchasing power constant:

- What is the change in demand when the consumer's income is adjusted so that, at the new prices, s/he can just afford to buy the original bundle?

This is the **Slutsky method**

Slutsky decomposition of income and substitution effects



Hicksian vs. Slutsky

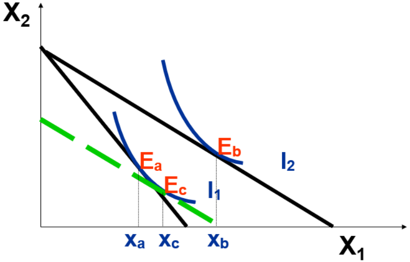


Figure: Hicksian

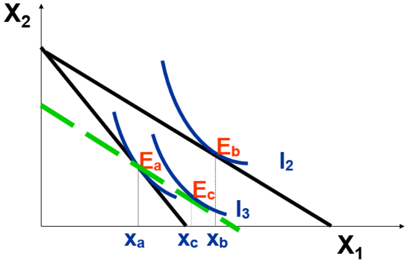


Figure: Slutsky

Consider an economy with two normal goods, and the price of x_1 falls. Which substitution effect is larger for x_1, Hicksian or Slutsky?
Answer: The Slutsky substitution effect is larger. Notice that the "interim" budget constraint (the green dashed line) for the Slutsky method lies further out than that of the Hicksian method. With normal goods, this means that Slutsky $x_c \geq$ Hicksian x_c.

Calculating the Slutsky effects

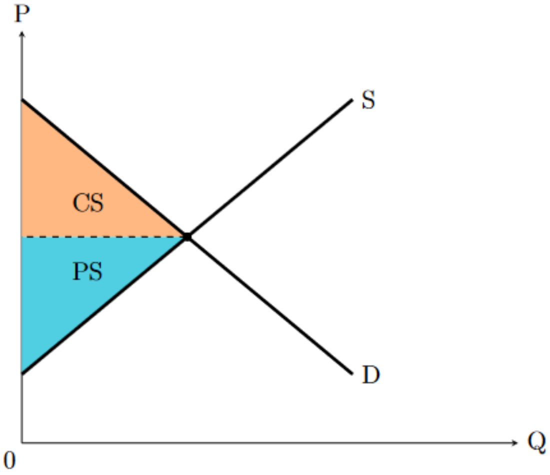
Julie's utility function for goods x and y is $U = xy + 4x$. He has \$44 dollars. Each unit of x costs \$1, and each unit of y costs \$1.

1. Calculate his optimal consumption bundle.
2. Now the price of good x rises to \$2. Calculate his optimal consumption bundle.
3. Calculate the Slutsky income and substitution effects of both goods resulting from this change.

Calculating the Slutsky effects - answer

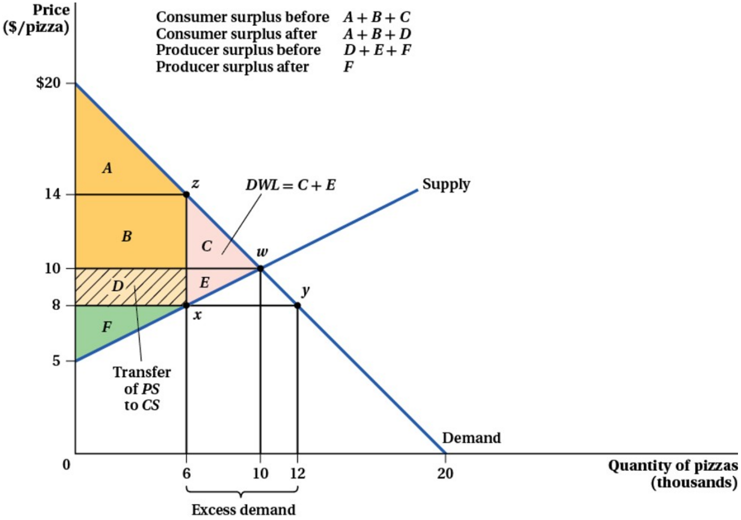
1. $MRS = \frac{\partial U/\partial x}{\partial U/\partial y} = \frac{y+4}{x}$. $\frac{P_x}{P_y} = 1$. At optimum, $MRS = \frac{P_x}{P_y}$ $\Rightarrow \frac{y+4}{x} = 1 \Rightarrow x = y + 4$. Plugging this into the budget constraint: $x + y = (y + 4) + y = 44 \Rightarrow x_{old} = 24, y_{old} = 20$
2. With the price rise, the budget constraint becomes $2x + y = 44$, and $\frac{P_x}{P_y} = 2$. We have $MRS = \frac{P_x}{P_y} \Rightarrow \frac{y+4}{x} = 2 \Rightarrow y = 2x - 4$. Plugging this into the budget constraint:
$2x + y = 2x + (2x - 4) = 44 \Rightarrow x_{new} = 12, y_{new} = 20$
3. The Slutsky method changes the budget constraint so that, at the new prices, the consumer can just afford the old bundle. So the amount of money the consumer has is
$2x_{old} + y_{old} = 2 \times 24 + 20 = 68$. The Slutsky budget constraint is $2x + y = 68$. We use the $MRS = \frac{P_x}{P_y}$ relationship for the new prices (from Question 2) and plug it into the Slutsky budget constraint:
$2x + y = 2x + (2x - 4) = 68 \Rightarrow x_{slustky} = 18, y_{slutsky} = 32$.
Substitution effects:
$SE_x = x_{slutsky} - x_{old} = -6$, $SE_y = y_{slutsky} - y_{old} = 12$
Income effects:
$IE_x = x_{new} - x_{slutsky} = -6$, $IE_y = y_{new} - y_{slutsky} = -12$

Consumer and producer surplus



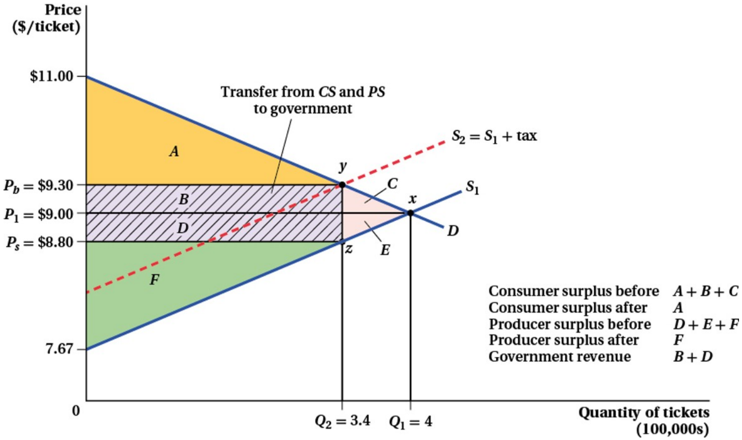
Price ceiling

Imposing a price ceiling of \$8:



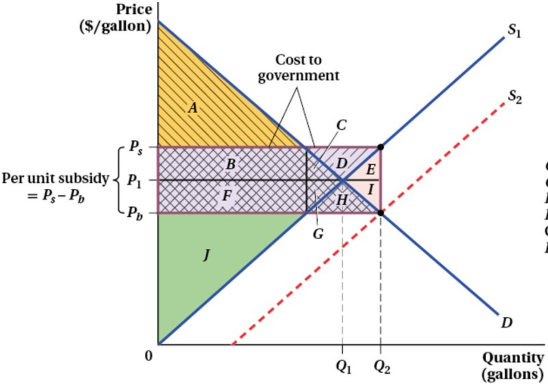
Tax

Imposing a tax of \$0.5 per unit:



Subsidy

Giving a subsidy of $P_s - P_b$ per unit:



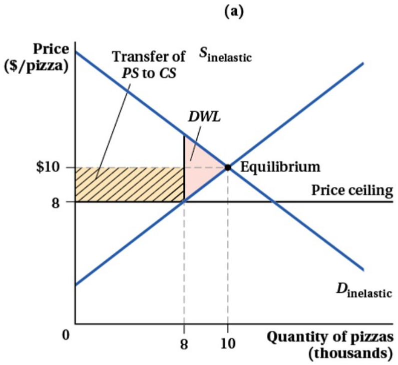
$$
\begin{aligned}
CS_{\text{before}} &= A + B + C \\
CS_{\text{after}} &= A + B + C + F + G + H \\
PS_{\text{before}} &= F + G + J \\
PS_{\text{after}} &= F + G + J + B + C + D \\
\text{Cost} &= B + C + D + E + F + G + H + I \\
DWL &= E + I
\end{aligned}
$$

Consumer and producer surplus

1. How does the size of deadweight loss from a fixed level of per-unit tax relate with elasticity? Give both intuitive and graphical explanations.
2. Which side shoulders a larger burden from a per-unit tax?
3. The supply curve is $P = \frac{1}{2}Q$, and the demand curve is $P = -2Q + 100$. Suppose the government imposes a per-unit tax of \$10. Calculate the buyers' and sellers' tax burden per unit.

Consumer and producer surplus - answer

1. The size of the deadweight loss increases with elasticity. The intuition is that, when elasticity is high, equilibrium quantity will decrease very much when taxes are imposed. Since the new equilibrium strays far from the original, optimal equilibrium, deadweight loss will be high.



Consumer and producer surplus - answer

2. The side with a smaller elasticity.
3. The equilibrium without taxes is $Q = 40, P = 20$. With a \$20 tax, the new equilibrium is $Q = 32, P = 36$. Buyer's tax burden is $36 - 20 = \$16$, and seller's tax burden is \$4.