Microeconomic Theory: TA Session

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Reminders

Midterm exam 1 is next Tuesday, Sep 30th, same time and place as the lecture

No TA Section Next Friday, Oct 3rd

Uncertainty

Suppose you have \$10,000. There is a 10% probability that a disaster happens and you lose \$9,000. You end up with

- \$1,000 with 10% probability
- \$10,000 with 90% probability

This risk can be reduced with insurance. Suppose you can buy x dollars of insurance at a premium γx. That is to say, you pay γx today. If no disaster happens, the insurance company does not pay you back; if disaster happens, you're paid x in compensation. Now you'll end up with

- $(1000 + x) - \gamma x$ with 10% probability
- $10000 - \gamma x$ with 90% probability

Expected utility

For utility under uncertainty, we use the **expected utility function**

Suppose there are n possible scenarios with probability $p_1, p_2, ..., p_n$ respectively (note that $p_1 + p_2 + ... + p_n = 1$), and the consumption under each scenario is $c_1, c_2, ..., c_n$. The utility function (with no uncertainty) is $u(c)$.

Then, the expected utility is:

$$p_1 u(c_1) + p_2 u(c_2) + ... + p_n u(c_n)$$

Risk aversion

We know intuitively that:

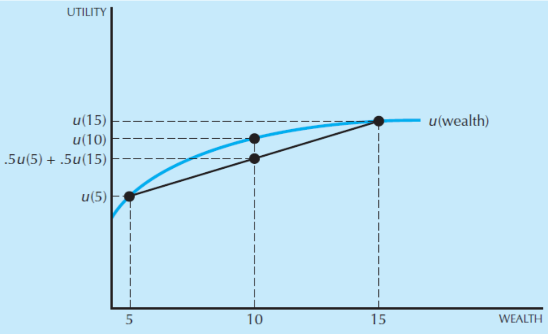
1. Marginal utility is (usually) diminishing
2. (Most) people are risk averse

In the following, we show that these two are actually linked together! From (1) we can derive (2)

Risk aversion

Marginal utility is $u'(c)$. Diminishing marginal utility, then, means $u''(c) < 0$. (We say that the function $u(c)$ is **concave** if $u''(c) < 0$.)

Suppose $u''(c) < 0$. For any $p_1, p_2 > 0$ and $p_1 + p_2 = 1$, we have $p_1 u(c_1) + p_2 u(c_2) < u(p_1 c_1 + p_2 c_2)$



This person is **risk averse**: given the same average outcome, he prefers outcomes with low uncertainty to outcomes with high uncertainty

Diminishing marginal utility leads to risk aversion

Uncertainty - exercise

Jessica have \$10,000. There is a 10% probability that a disaster happens and she loses \$9,000. Suppose you can buy x dollars of insurance at a premium γx.

1. What is the "fair value" of γ that makes the insurance company earn zero profit?
2. Suppose Jessica's utility of wealth is $u(w) = \ln w$. Derive the optimum amount of insurance that she buys.
3. Suppose we don't know the exact form of $u(w)$; all we know is that Jessica is risk averse. Does your answer to Question (2) change?
4. Now suppose that, instead of the fair value, the insurance company wants to make a profit and charges $\gamma = 0.2$. Does your answer to Question (2) change?

Uncertainty - answer

Answer:

1. The insurance company's profit is $\gamma x - 0.1x - 0.9 \times 0 = 0 \;\Rightarrow\; \gamma = 0.1$
2. Suppose she buys x dollars of insurance. Her expected utility is $EU = 0.1\ln(1000 + 0.9x) + 0.9\ln(10000 - 0.1x)$. Taking derivative with regard to x: $\frac{dEU}{dx} = \frac{0.09}{1000+0.9x} - \frac{0.09}{10000-0.1x} = 0$ (using the chain rule: if $h(x) = f(g(x))$, then $h'(x) = f'(g'(x))g'(x)$.) Rearranging, we get $1000 + 0.9x = 10000 - 0.1x \;\Rightarrow\; x = 9000$. She'll be fully insured.
3. Her expected utility is $EU = 0.1u(1000 + 0.9x) + 0.9u(10000 - 0.1x)$. Taking derivative: $0.09u'(1000 + 0.9x) - 0.09u'(10000 - 0.1x) = 0 \;\Rightarrow$ $u'(1000 + 0.9x) = u'(10000 - 0.1x)$. Since $u''(w) < 0$, it must be that $u(1000 + 0.9x) = u(10000 - 0.1x) \;\Rightarrow 1000 + 0.9x = 10000 - 0.1x \;\Rightarrow$ $x = 9000$. It turns out that a risk-averse individual always prefers to be fully insured, regardless of the utility function form.
4. Now, $EU = 0.1\ln(1000 + 0.8x) + 0.9\ln(10000 - 0.2x)$. Taking derivative: $\frac{dEU}{dx} = \frac{0.08}{1000+0.8x} - \frac{0.18}{10000-0.2x} = 0 \;\Rightarrow\; 0.08(10000 - 0.2x) = 0.18(1000 + 0.8x) \;\Rightarrow\; x = 3875$. If the insurance premium is higher than the fair value, the individual will prefer to be partially insured.

The production function

The **production function** describes how output is made from different inputs:

$$Q = f(K, L)$$

where Q is the quantity of production, K is capital, L is labor

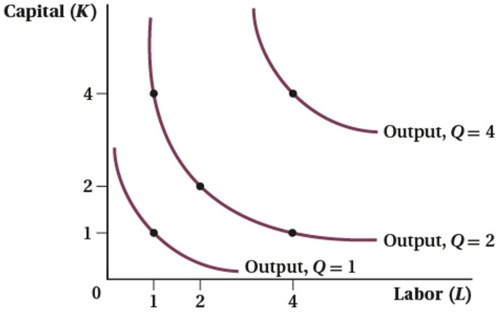
e.g. $Q = K^{0.5}L^{0.5}$

The **Marginal product** is the additional output that a firm can produce when it uses an additional unit of an input (while holding other inputs constant):

$$MP_L = \frac{\partial Q}{\partial L},\ MP_K = \frac{\partial Q}{\partial K}$$

Isoquant

Isoquant is a curve representing all possible combinations of inputs that yields the same amount of output



The **Marginal rate of technical substitution (MRTS)** is the (absolute value of the) slope of the isoquant:

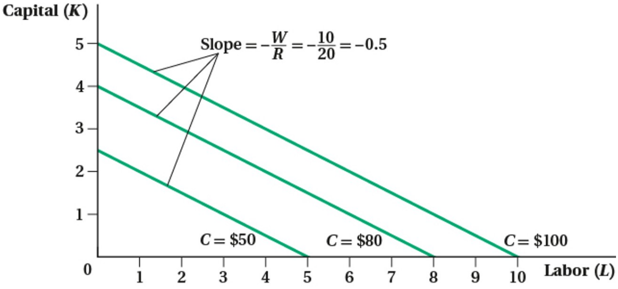
$$MRTS = \frac{MP_L}{MP_K}$$

Isocost

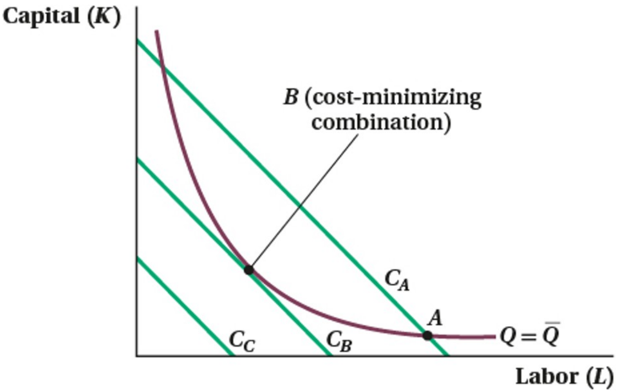
An **isocost line** represents all possible combinations of inputs with the same cost:

$$C = RK + WL$$

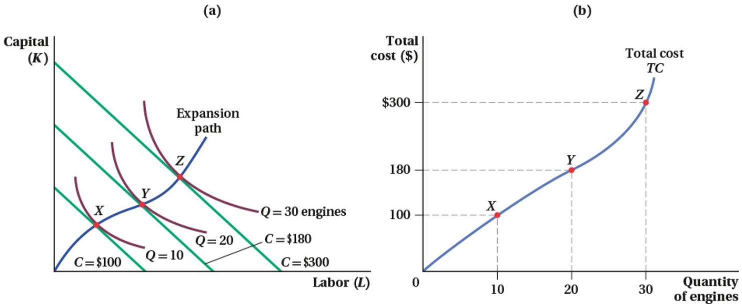
where C is the total cost, R is the cost ("rent") of capital, W is the cost ("wage") of labor



Cost minimization



Expansion path and total cost curve



Production - exercise

Suppose a firm's production function is $Q = K^{0.5}L^{0.5}$, the rental rate of capital is $r = 1$, and the wage for labor is $w = 4$.

1. Find the cost minimizing K and L to produce a quantity of 20.
2. What is the minimized total cost?
3. Derive the expansion path and the (optimal) total cost curve.

Production - answer

Answer:

1. The problem is:
   $\min K + 4L \quad \text{s.t. } K^{0.5}L^{0.5} = 20$
   The Lagrangian is $\mathcal{L} = K + 4L + \lambda(K^{0.5}L^{0.5} - 20)$. At the optimum, its derivative with regard to K, L, λ should all be zero:
   $\frac{\partial \mathcal{L}}{\partial K} = 1 + \lambda L = 0, \ \frac{\partial \mathcal{L}}{\partial L} = 4 + \lambda K = 0, \ \frac{\mathcal{L}}{\partial \lambda} = K^{0.5}L^{0.5} - 20 = 0.$
   Rearranging he first two equations gives us $K = 4L$. Plugging into the third equation, we have $(4L)^{0.5}L^{0.5} = 20 \ \Rightarrow \ K = 40, L = 10$
2. The minimized total cost is $C = K + 4L = 40 + 4 \times 10 = 80$.
3. The expansion path is just $K = 4L$, which we derived in Question (1) with the $\frac{\partial \mathcal{L}}{\partial K} = 0$ and $\frac{\partial \mathcal{L}}{\partial L} = 0$ equations (note that these equations don't depend on the quantity Q). For the total cost curve, we just replace the 20 in Question (1) with Q: $(4L)^{0.5}L^{0.5} = Q \ \Rightarrow \ K = 2Q, L = \frac{Q}{2}$. The total cost curve is $C = K + 4L = 2Q + 4 \times \frac{Q}{2} \ \Rightarrow \ C = 4Q$.