Microeconomic Theory: TA Session

Pinda Wang

Johns Hopkins University

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Reminder

Midterm Exam 2 is next Tuesday, November 4th

No TA session next Friday, November 7th

Game Theory: Nash Equilibrium

Nash equilibrium: an equilibrium in which players each choose their best strategy given the strategies that all other players have chosen

- i.e. nobody has an incentive to (unilaterally) deviate from their equilibrium choice

Pure-strategy Nash equilibrium: prisoners' dilemma

The police arrested two criminals, A and B, who are suspected of committing a major crime, and **questions them separately**

- If one of them confesses and the other remain silent, the confessed one goes free, whereas the silent one gets 10 years
- If both of them confess, they both get 5 years
- If none of them confess, they both get 2 years

		B	
		Confess	Silent
A	Confess	-5, -5	0, -10
	Silent	-10, 0	-2, -2

Pure-strategy Nash equilibrium: prisoners' dilemma

		B	
		Confess	Silent
A	Confess	-5, -5	0, -10
	Silent	-10, 0	-2, -2

Nash equilibrium: (Confess, Confess)

- This is not optimal: welfare is maximized at (Silent, Silent)

Mixed-strategy Nash equilibrium

Consider the following game:

		Player B	
		L	R
Player A	U	2, 1	0, 0
	D	0, 0	3, 2

There are two pure-strategy Nash equilibria: (U, L) and (D, R)

Mixed-strategy Nash equilibrium:

- Each player randomizes between his actions
- Given the other player's randomization strategy, the player must be **indifferent** between his own actions
  - Otherwise, if a player prefers action x over y, he will not randomize: he'll choose x with probability 1

Mixed-strategy Nash equilibrium

		Player B	
		L	R
Player A	U	2, 1	0, 0
	D	0, 0	3, 2

Suppose that Player A chooses U with probability p and D with probability $1-p$; Player B chooses L with probability q and R with probability $1-q$

Given the strategy of Player B, Player A is indifferent between U and D:

$$2p + 0 \cdot (1-p) = 0 \cdot p + 3(1-p)$$
$$\Rightarrow \ p = \frac{3}{5}$$

Mixed-strategy Nash equilibrium

		Player B	
		L	R
Player A	U	2, 1	0, 0
	D	0, 0	3, 2

Suppose that Player A chooses U with probability p and D with probability $1-p$; Player B chooses L with probability q and R with probability $1-q$

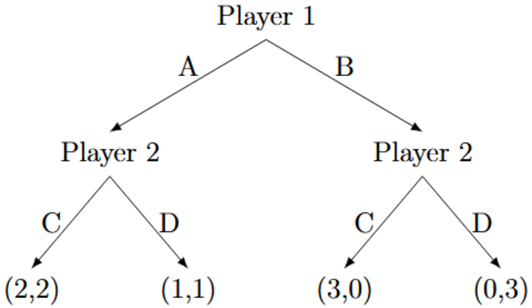
Given the strategy of Player A, Player B is indifferent between L and R:

$$1 \cdot q + 0 \cdot (1-q) = 0 \cdot q + 2(1-q)$$
$$\Rightarrow \quad q = \frac{2}{3}$$

Mixed-strategy Nash equilibrium: $((\frac{3}{5}, \frac{2}{5}),\ (\frac{2}{3}, \frac{1}{3}))$

Sequential games

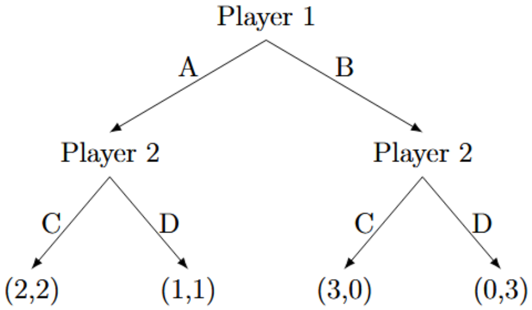
Consider the following sequential game. In this game, player 1 acts first by choosing between A and B. Then, after observing Player 1's action, Player 2 chooses between C and D.



In a sequential game, a strategy needs to be a **contingent plan**

- e.g. "Choose C if Player 1 chose A, choose D if Player 1 chose B"

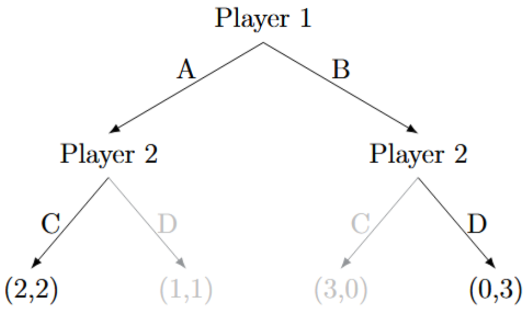
Sequential games: subgame perfect equilibrium



To solve for the equilibrium of sequential games, we use **backward induction**: working from the end of the game tree back towards the top

- If Player 1 has already chosen A: Player 2 should choose C
- If Player 1 has already chosen B: Player 2 should choose D
- Player 2's strategy is (C,D): "Choose C if Player 1 chose A, and choose D if Player 1 chose B"

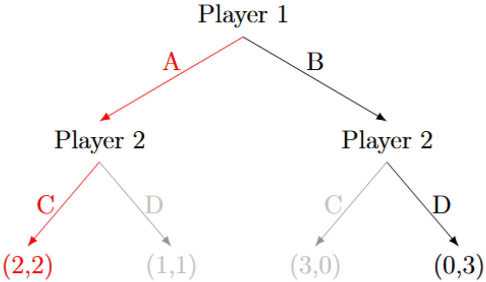
Sequential games: subgame perfect equilibrium



Now that we know what Player 2's best strategy is, we go back to Player 1

- If Player 1 chooses A, he'll get a payoff of 2
- If Player 1 chooses B, he'll get a payoff of 0
- Player 1's strategy is A

Sequential games: subgame perfect equilibrium



We now have the **subgame perfect equilibrium (SPE)**: Player 1's strategy is A, and Player 2's strategy is (C,D)

- In a sequential game, a strategy profile is a SPE if it's a Nash equilibrium for every subgame (including the original game)

Finitely repeated games

Suppose two players play the following prisoners' dilemma game repeatedly for 5 times. What is the subgame perfect equilibrium?

		Player B	
		Confess	Silent
Player A	Confess	-4, -4	0, -5
	Silent	-5, 0	-3, -3

Using backward induction:

- In the 5th and last game, both players will choose to confess, because this is the last game
- In the 4th game, both players know that they'll choose confess in the 5th game, so they'll both choose to confess
- ...
- In the 1st game, by the same logic, both choose to confess

The SPE: everyone chooses confess in every stage of the game

- The players cannot credibly enforce any other outcome, because they know they'll confess in the last period

Infinitely repeated games

What if the two players play the same game for an infinite number of times?

		Player B	
		Confess	Silent
Player A	Confess	-4, -4	0, -5
	Silent	-5, 0	-3, -3

We can no longer use backward induction, because there is no "last period"

Can the two players credibly enforce silence?

Infinitely repeated games

		Player B	
		Confess	Silent
Player A	Confess	-4, -4	0, -5
	Silent	-5, 0	-3, -3

It turns out that the cooperative outcome (i.e. both choose silence) is possible as long as players are sufficiently patient

Suppose, in the eyes of the players, \$1 in the next period is worth \$$d$ this period

- In economic jargon, this d is called the discount factor
- If $d = 0$, the players are infinitely impatient: they don't care about the future at all
- If $d = 1$, the players are infinitely patient: they value the future just as much as the present

Infinitely repeated games

		Player B	
		Confess	Silent
Player A	Confess	-4, -4	0, -5
	Silent	-5, 0	-3, -3

Players can use a **grim trigger** strategy to enforce cooperation:

- Choose silence if no one has chosen to confess before
- Choose to confess forever after if anyone has confessed before

Infinitely repeated games

		Player B	
		Confess	Silent
Player A	Confess	-4, -4	0, -5
	Silent	-5, 0	-3, -3

Suppose Player B uses the grim trigger strategy.

Player A's payoff from always silence: $-3 - 3d - 3d^2 - 3d^3 - \ldots = -\frac{3}{1-d}$

If Player A defects to confession in the 1st period: he takes more advantage in the 1st period, at the cost of suffering from the (Confess, Confess) outcome for every period thereafter. His payoff is $0 - 4d - 4d^2 - 4d^3 - \ldots = -\frac{4d}{1-d}$

For the grim trigger strategy to work, Player A need to prefer "always silent" over "confess in 1st period":

$$-\frac{3}{1-d} \geq -\frac{4d}{1-d} \;\Rightarrow\; d \geq \frac{3}{4}$$

Infinitely repeated games

		Player B	
		Confess	Silent
Player A	Confess	-4, -4	0, -5
	Silent	-5, 0	-3, -3

With the equilibrium above, the payoff of each player is $-\frac{3}{1-d}$.

We can do even better than this! Instead of enforcing (Silent, Silent) each period, we can enforce (Confess, Silent) and (Silent, Confess) alternately:

- Play (Confess, Silent) in every odd period
- Play (Silent, Confess) in every even period
- If anyone deviated from the above plan, play (Confess, Confess) forever after

This can also be sustained if d is large enough

Infinitely repeated games

		Player B	
		Confess	Silent
Player A	Confess	-4, -4	0, -5
	Silent	-5, 0	-3, -3

For Player A, the payoff of this strategy is
$-5 - 0d - 5d^2 - 0d^3 - \ldots = -\frac{5}{1-d^2}$

For Player B, the payoff of this strategy is
$0 - 5d - 0d^2 - 5d^3 - \ldots = -\frac{5d}{1-d^2}$

As long as $-\frac{5}{1-d^2} > -\frac{3}{1-d} \Rightarrow d > \frac{2}{3}$, this "alternating" grim trigger strategy makes both players better off than the original (Silent, Silent) grim trigger strategy.

Folk Theorem: If players are patient enough, any "reasonable" outcome can be sustained as a SPE in an infinitely repeated game

Oligopoly: Cournot competition

Cournot competition: firms compete in quantity; price is determined by the total output of the firms

Suppose there are N identical firms in the market. Each firm incurs constant marginal cost of $MC = 2$ and no fixed costs. The market demand curve is $P = 10 - Q$. The firms engage in Cournot competition.

1. Calculate the equilibrium output of each firm q_i, total output of all firms Q, the market price P, each firm's profit π_i, and total profit of all firms Π.
2. As N increases to infinity, how does Q, p, π_i, Π change?

Cournot competition - answer

Answer:

1. Firms are identical, so we can assume that the equilibrium is symmetric. Denote q_{-i} as the total output of all firms except firm i (this notation is very commonly used in game theory). Then, the demand curve that firm i faces is $P = (10 - q_{-i}) - q_i$. Then, $MR = (10 - q_{-i}) - 2q_i$.

   Profit maximization means $MR = MC: \ (10 - q_{-i}) - 2q_i = 2$ $\Rightarrow \ q_i = \frac{8-q_{-i}}{2}$. This is true for all i from 1 to N, so all firms have the same q_i, and $q_{-i} = (N-1)q_i$. So, $q_i = \frac{8-(N-1)q_i}{2}$ $\Rightarrow \ q_i = \frac{8}{N+1}$.

   Total output $Q = \frac{8N}{N+1}$, price $P = 10 - \frac{8N}{N+1}$, each firm's profit $\pi_i = (10 - \frac{8N}{N+1} - 2)\frac{8}{N+1} = (\frac{8}{N+1})^2$. Total profit $\Pi = N(\frac{8}{N+1})^2$.

2. As N increases to infinity, $Q = 8, P = 2, \pi_i = 0, \Pi = 0$. The equilibrium approaches the prefect competition outcome as N increases.

Bertrand competition

Bertrand competition: firms compete in price. The firm with the lowest price captures the entire market demand. If there is a tie in the lowest price, the tied firms divide the market share equally.

In class we have seen that, if firms have identical marginal cost and prices are continuous, the Nash equilibrium in Bertrand competition is that every firm charges the marginal cost — same equilibrium as perfect competition

Bertrand competition

Bertrand competition with different marginal costs. Suppose two firms engage in Bertrand competition. Firm A has a constant marginal cost of 5. Firm B has a constant marginal cost of 10.

1. Is there a Nash Equilibrium? If so, find all Nash equilibrium. If not, explain.

Bertrand competition - answer

Answer:
There is no Nash equilibrium! We prove this by looking at different cases:

1. $p_1 \neq p_2$. In this case, the firm with the lower price has an incentive to increase its price by just a little bit, so that his new price is still the lowest. Now he still captures the entire market, but he earns more profit from the higher price. So this cannot be an equilibrium.
2. $p_1 = p_2 < 10$. In this case, firm B captures half of the market and earns negative profit. So he has an incentive to raise price, capture 0% of the market, and earn zero profit. This cannot be an equilibrium.
3. $p_1 = p_2 = 10$. In this case, firm A has an incentive to lower his price just a bit, say to \$9.9999, so that he captures the entire market (instead of half) and earns more profit. This cannot be an equilibrium.
4. $p_1 = p_2 > 10$. In this case, either firm has an incentive to lower his price just a bit, so that he captures the entire market and earns more profit. This cannot be an equilibrium.

We have now ruled out all possible scenarios. There is no Nash equilibrium.

Bertrand competition

Bertrand competition with non-continuous prices. Now suppose prices can only change in the increment of one cent — you can set the price at \$2.45 or \$2.46, but not \$2.455.
Two firms engage in Bertrand competition, and both have a constant marginal cost of \$5. Suppose there is only one consumer in the market: he buys from the firm with the lowest price. If prices are tied, he buys from each firm with equal probability.

1. Is there a Nash equilibrium? If so, find all Nash equilibrium. If not, explain.

Bertrand competition - answer

Answer:
There are three Nash equilibria:

1. (5,5) is still a Nash equilibrium, just like in the original Bertrand problem where continuous pricing is allowed.
2. (5.01, 5.01) is a Nash equilibrium. Each firm's expected profit is \$0.005 $(= 0.01 \times 50\%)$. The only way for a firm to undercut is to lower price to \$5, but then this firm will earn zero profit, so no one has an incentive to deviate.
3. (5.02, 5.02) is also a Nash equilibrium. Each firm's expected profit is \$0.01$(= 0.02 \times 50\%)$. If a firm undercuts by lowering price to \$5.01, he captures the entire market but still earns a profit of \$0.01 $(= 0.01 \times 100\%)$. So no one has an incentive to deviate.

So the Nash equilibria are (5,5), (5.01, 5.01), and (5.02, 5.02).