Microeconomic Theory: TA Session

Pinda Wang

Johns Hopkins University

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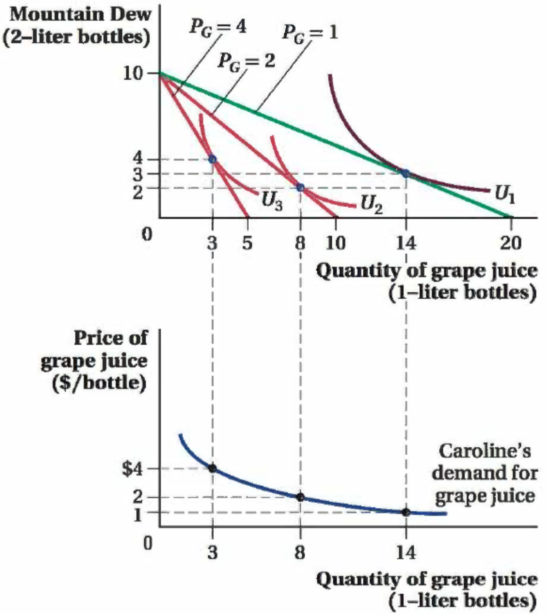
Assignment 1

Reminder: Assignment 1 is due on Sep 11th on Canvas

Deriving the demand curve



Deriving the demand curve

1. John's utility function for goods x and y is $U = xy$. He has \$20 dollars, and each unit of y costs \$2. Derive his demand curve for good x.

Answer: Let the price of x be p. Then this is essentially the same question as Q4 of Week 1's slides, but with a budget constraint of $px + 2y = 20$. With $y/x = MRS = p_x/p_y = p/2$, the quantity of good x is $x = \frac{10}{p}$. So the demand curve for x is $p = \frac{10}{x}$.

Deriving the demand curve

2. Now suppose his utility function is $U = x + y$. Derive his demand curve for good x.

Answer: In this question we have to deal with "corner solutions". Again let the price of x be p. For $p > 2$, indifference curves are linear and are steeper than the budget constraint (see figure on next page for an example with $p = 4$; indifference curves are black and budget constraint red). The highest indiff curve that intersects with the budget constraint does so at $(x, y) = (0, 10)$, so the demand for x is 0.

When $p = 2$, indifference curve overlaps with budget constraint, and demand for x can be anywhere between 0 and 10.

When $p < 2$, highest indifference curve will intersect with budget constraint at $(x, y) = (\frac{20}{p}, 0)$, so the demand for x is $\frac{20}{p}$.

The demand curve is also shown in the next page.

Deriving the demand curve

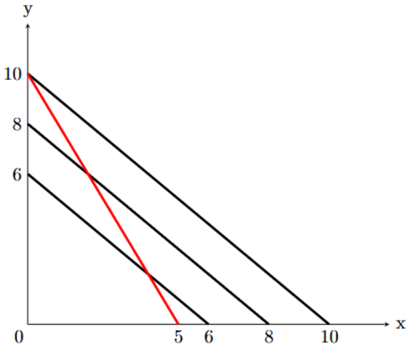


Figure: $p = 4$

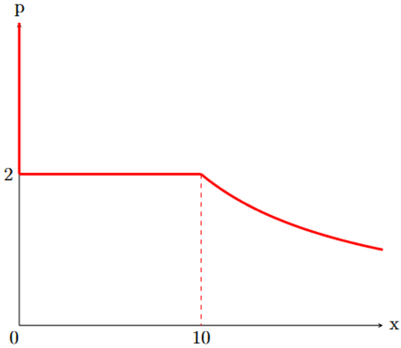


Figure: Demand curve

Income and substitution effects

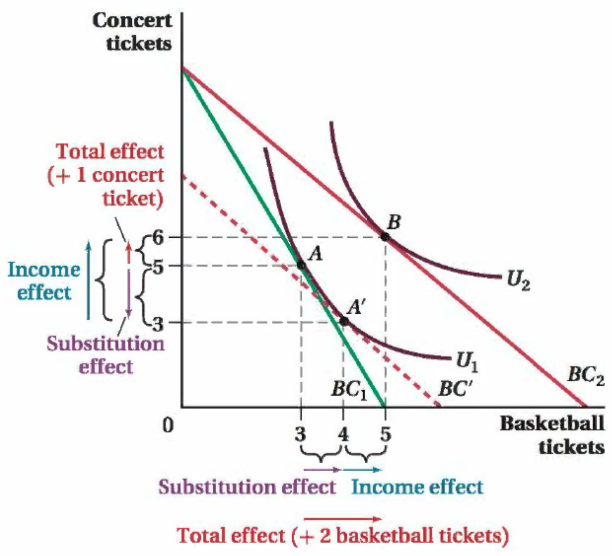
How does consumer respond to price changes?

- **Substitution effect:** the changes in quantities demanded due to the change in relative prices (while maintaining utility level constant)
- **Income effect:** the changes in quantities demanded due to the change in purchasing power

Total effect = Substitution effect + income effect

Income and substitution effects

Consider a fall in the price of basketball tickets:



Income and substitution effects - exercise

Jack's utility function for goods x and y is $U = 2xy + 10x$. He has \$100 dollars. Each unit of x costs \$5, and each unit of y costs \$4.

1. Calculate his optimal consumption bundle.
2. Now the price of good y falls to \$1. Calculate his optimal consumption bundle.
3. Calculate the income and substitution effects of both goods resulting from this change.

Income and substitution effects - answer

Answer:

1. With $U = 2xy + 10x$, we have $MRS = \frac{\partial U/\partial x}{\partial U/\partial y} = \frac{2y+10}{2x} = \frac{y+5}{x}$. THe price ratio is $\frac{p_x}{p_y} = \frac{5}{4}$. At the optimal consumption bundle,
   $MRS = \frac{p_x}{p_y} \Rightarrow \frac{y+5}{x} = \frac{5}{4} \Rightarrow y = \frac{5}{4}x - 5.$
   Plugging this into the budget constraint:
   $5x + 4y = 5x + 4(\frac{5}{4}x - 5) = 10x - 20 = 100.$ We have the optimal consumption bundle $x_{old} = 12,\ y_{old} = 10.$
2. $MRS = \frac{p_x}{p_y} \Rightarrow \frac{y+5}{x} = 5 \Rightarrow y = 5x - 5.$ Plugging into the budget constraint, we have: $5x + y = 5x + (5x - 5) = 10x - 5 = 100.$ The optimal consumption bundle is $x_{new} = 10.5,\ y_{new} = 47.5.$

Income and substitution effects - answer

Answer:

3. We first calculate the optimal consumption bundle under the old utility level and the new prices. The old utility level is $U_{old} = 2xy + 10x = 2 \times 12 \times 10 + 10 \times 10 = 360$. For the $MRS = \frac{p_x}{p_y}$ relationship under the new prices, we have $\frac{y+5}{x} = 5 \;\Rightarrow\; y = 5x - 5$. Plugging into the utility function, we have $2x(5x-5) + 10x = 360 \;\Rightarrow\; x_{interim} = 6,\; y_{interim} = 25$.

   The substitution effect for x is $x_{interim} - x_{old} = -6$, the income effect for x is $x_{new} - x_{interim} = 4.5$.

   The substitution effect for y is $y_{interim} - y_{old} = 15$, the income effect for y is $y_{new} - y_{interim} = 22.5$.