Microeconomic Theory: TA Session

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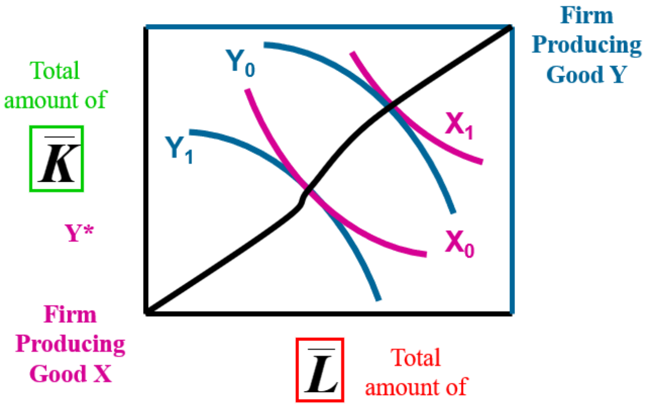
Production Edgeworth Box

Two factors of production L and K; two goods X and Y

The curves are isoquants; the slope of a curve is MRTS



Pareto efficiency is achieved when the two firms' isoquants are tangential to each other: $MRTS_X = MRTS_Y$

Production Edgeworth Box - Exercise

There are two sectors producing goods X and Y: $X = L_X^{1/4} K_X^{3/4}$, $Y = L_Y^{2/3} K_Y^{1/3}$, with $L_X + L_Y = K_X + K_Y = 10$.

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2. Find any Pareto efficient allocation in this market
3. Is $(L_X, K_X) = (8, 2)$ Pareto efficient?

Nice properties of Cobb-Douglas function

With a Cobb-Douglas production function $X = L^{\alpha}K^{1-\alpha}$, a firm will optimally allocate a proportion α of the production cost to L, and allocate a proportion $(1-\alpha)$ to K.

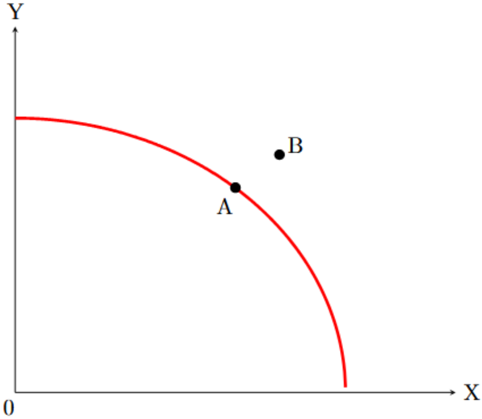
Proof:
Denote the per-unit cost of L as w, and the per-unit cost of K as r. Cost minimization is to minimize $wL + rK$ s.t. $L^{\alpha}K^{1-\alpha} = X$.
The Lagrangian: $\mathcal{L} = wL + rK - \lambda(L^{\alpha}K^{1-\alpha} - X)$
$\frac{\partial \mathcal{L}}{\partial L} = w - \lambda\alpha L^{\alpha-1}K^{1-\alpha}=0$
$\frac{\partial \mathcal{L}}{\partial K} = r - \lambda(1-\alpha)L^{\alpha}K^{-\alpha} = 0$
Dividing the two above conditions, we get: $\frac{w}{r} = \frac{\alpha L^{\alpha-1}K^{1-\alpha}}{(1-\alpha)L^{\alpha}K^{-\alpha}} = \frac{\alpha}{1-\alpha} \cdot \frac{K}{L}$.
Rearranging, we get $\frac{wL}{\alpha} = \frac{Kr}{1-\alpha}$.

Nice properties of the Cobb-Douglas function

Show that, with a Cobb-Douglas utility function $U = x^{\alpha}y^{1-\alpha}$, a consumer will optimally allocate a proportion α of his budget to x, and a proportion $(1-\alpha)$ to y.

Production Possibilities Frontier

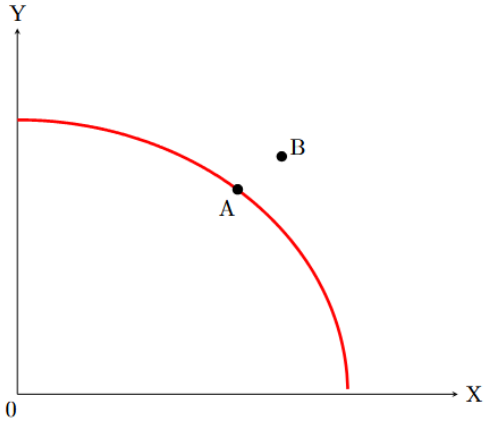
Each point on the production possibility frontier is Pareto efficient.



Production Possibilities Frontier

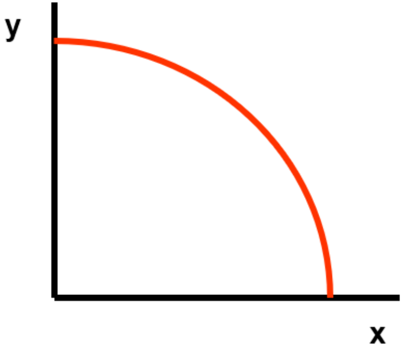
Each point on the production possibility frontier is Pareto efficient.

Proof: Suppose a point A on the PPF is not Pareto efficient. Then, there exists a Pareto efficient production B to the northeast of A. But then B would fall outside of the PPF — it is impossible to produce B!



Production Possibilities Frontier

The slope of the PPF is called the **marginal rate of product transformation (MRT)**: how many units of production of Y you'll need to give up in order to produce an extra unit of X



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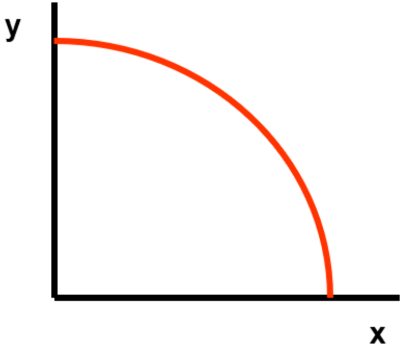
MRT equals the ratio of marginal costs: $MRT = \frac{MC_X}{MC_Y}$



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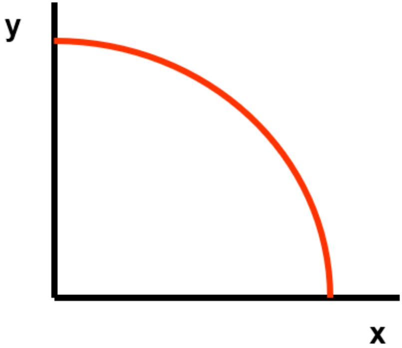
- Intuition: when we move along the PPF, we're essentially moving a bit of both inputs (i.e. moving a bit of production cost) from the production of Y to the production of X.



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$MRT = P_X/P_Y$

- This comes from perfect competition. We have shown previously that $MRT = MC_X/MC_Y$. Under perfect competition, $P_X = MC_X$ and $P_Y = MC_Y$, so $MRT = P_X/P_Y$

The welfare theorems

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Second fundamental theorem of welfare economics: Any Pareto efficient allocation can be obtained as the outcome of competitive market processes, provided that the economy's initial endowment of resources can be redistributed, via lump sum taxes and subsidies, among agents.