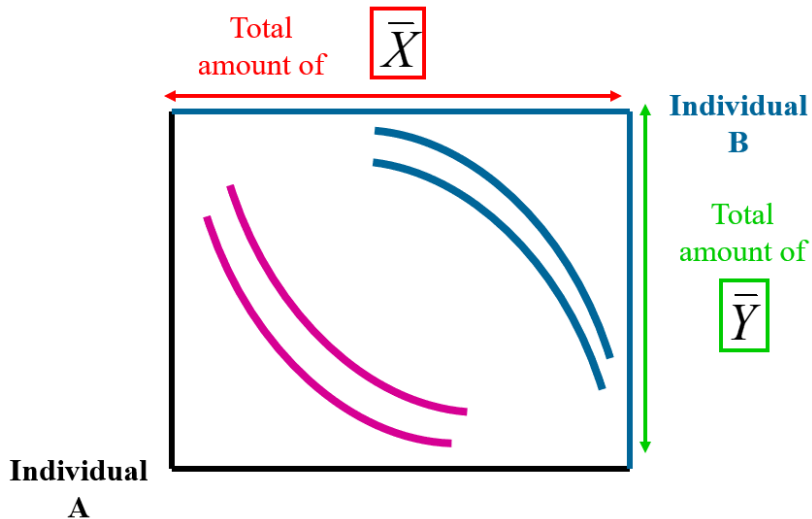# Microeconomic Theory: TA Session

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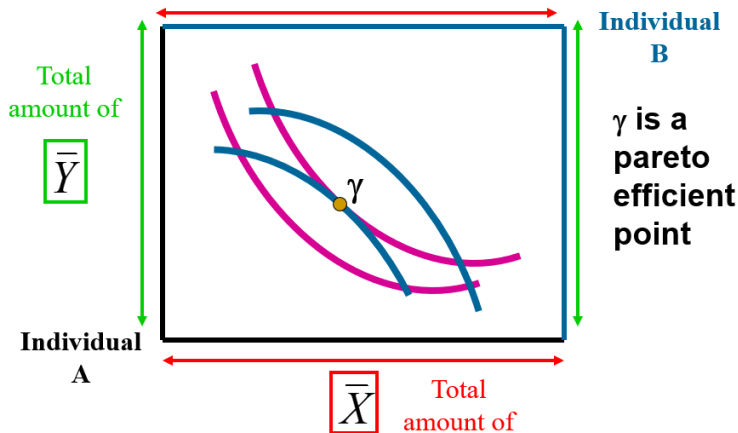Johns Hopkins University

November 14th, 2025
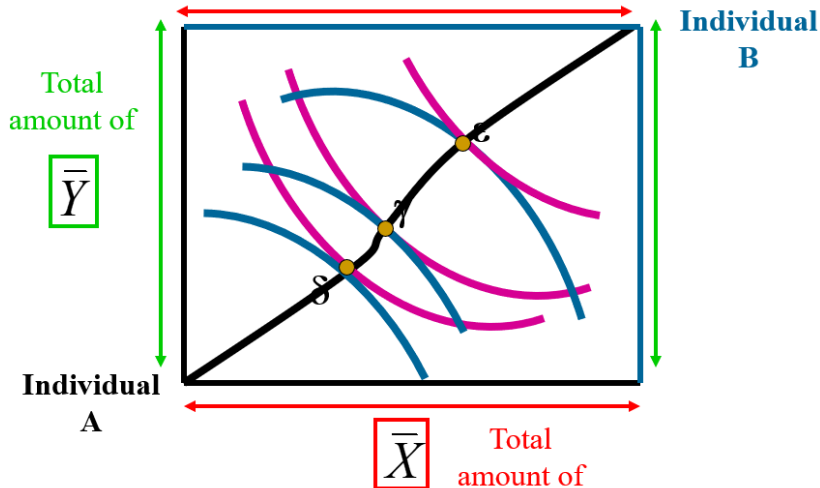
Edgeworth box

Pareto efficient allocation

A **Pareto improvement** is a change that makes at least one person better off without making anyone else worse off

An allocation is **Pareto efficient** if it is impossible to make someone better off without making anyone else worse off

Contract curve

Pareto efficient allocation - exercise

There are two consumers A and B, and two goods x and y. The total available amount of both goods is 4. The utility of consumer A is $U_A = x_A^{3/4} y_A^{1/4}$; the utility of consumer B is $U_B = x_B^{1/4} y_B^{3/4}$.

1. Suppose A consumes $(x_A, y_A) = (4, 0)$ and B consumes $(x_B, y_B) = (0, 4)$. Is this Pareto efficient?

2. Suppose A consumes (4,4) and B consumes (0,0). Is this Pareto efficient?

3. Find a Pareto efficient allocation that is in the interior (i.e. not at the corner of the Edgeworth box).

4. Derive the contract curve in terms of x_A and y_A.

Pareto efficient allocation - answer

Answer:

1. No. With this allocation, both consumers have a utility of 0. Any other interior allocation (e.g. A consumes (2,2) and B consumes (2,2)) is a Pareto improvement upon this.

2. Yes. Any other allocation will make consumer A worse off.

3. $MRS_A = \frac{\partial U_A / \partial x_A}{\partial U_A / \partial y_A} = \frac{3y_A}{x_A}$; $MRS_B = \frac{\partial U_B / \partial x_B}{\partial U_B / \partial y_B} = \frac{y_B}{3x_B}$.

   With a Pareto efficient allocation, it must be that $MRS_A = MRS_B$ $\Rightarrow \frac{3y_A}{x_A} = \frac{y_B}{3x_B}$. A convenient solution is $(x_A, y_A) = (3, 1)$, and $(x_B, y_B) = (1, 3)$. Another example of a solution would be $(x_A, y_A) = (2, \frac{2}{5})$, and $(x_B, y_B) = (2, \frac{18}{5})$.

4. From (3), we have $\frac{3y_A}{x_A} = \frac{y_B}{3x_B}$. We also know that $x_B = 4 - x_A$, $y_B = 4 - y_A$. Plugging these into the MRS equation, we have $\frac{3y_A}{x_A} = \frac{4 - y_A}{3(4 - x_A)}$ $\Rightarrow y_A = \frac{x_A}{9 - 2x_A}$, $0 \leq x_A \leq 4$, $0 \leq y_A \leq 4$.

Equilibrium of an exchange economy
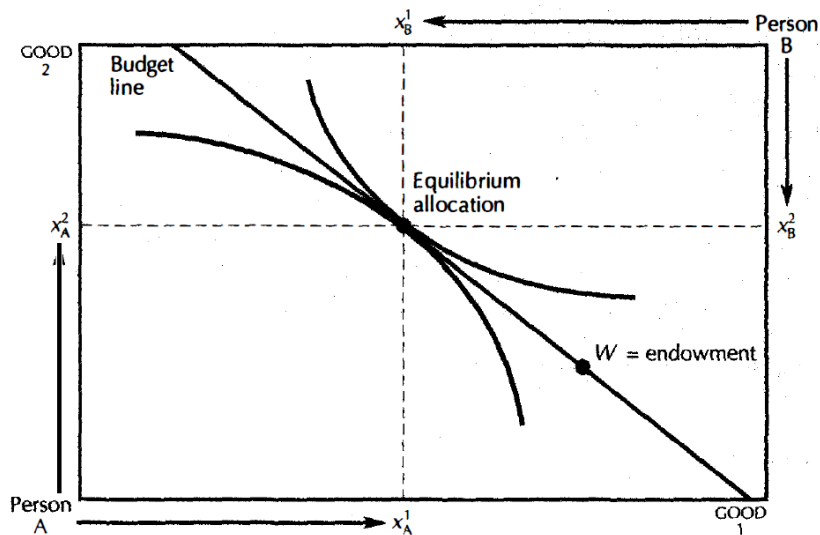
Equilibrium of an exchange economy

Suppose there are two consumers A and B, and two goods x and y.

Endowment of consumer A is ω_x^A and ω_y^A. Endowment of consumer B is ω_x^B and ω_y^B.

Equilibrium quantities are x_A, x_B, y_A, y_B

Price of good x is p_x; we normalize the price of good y to 1.

1. Budget constraint of consumer A: $p_x x_A + y_A = p_x \omega_x^A + \omega_y^A$

2. Budget constraint of consumer B: $p_x x_B + y_B = p_x \omega_x^B + \omega_y^B$

3. Total available amount of good x: $x_A + x_B = \omega_x^A + \omega_x^B$

4. Total available amount of good y: $y_A + y_B = \omega_y^A + \omega_y^B$

5. $MRS = \frac{p_x}{p_y}: \frac{\partial U_A/\partial x_A}{\partial U_A/\partial y_A} = p_x = \frac{\partial U_B/\partial x_B}{\partial U_B/\partial y_B}$

We can solve for the equilibrium using the above equations.

In an exchange economy, there are two consumers A and B, and two goods x and y. The endowment of consumer A is $(\omega_x^A, \omega_y^A) = (3, 1)$; the endowment of consumer B is $(\omega_x^B, \omega_y^B) = (1, 3)$. The utility of consumer A is $U_A = x_A^{1/2} y_A^{1/2}$; the utility of consumer B is $U_B = x_B^{1/4} y_B^{3/4}$.

Solve for the equilibrium. That is to say, solve for x_A, x_B, y_A, y_B, p_x.

Equilibrium of an exchange economy - answer

Answer:

1. Budget constraint of consumer A: $p_x x_A + y_A = 3p_x + 1$
2. Budget constraint of consumer B: $p_x x_B + y_B = p_x + 3$
3. Total available amount of good x: $x_A + x_B = 4$
4. Total available amount of good y: $y_A + y_B = 4$
5. $MRS = \frac{p_x}{p_y}$: $\frac{\partial U_A/\partial x_A}{\partial U_A/\partial y_A} = \frac{y_A}{x_A} = p_x = \frac{y_B}{3x_B} = \frac{\partial U_B/\partial x_B}{\partial U_B/\partial y_B}$

For consumer A, we combine (1) and (5) to get $p_x x_A + y_A = 2y_A = 3p_x + 1$
$\Rightarrow$ $y_A = \frac{3p_x+1}{2}$, $x_A = \frac{3p_x+1}{2p_x}$.

For consumer B, we combine (2) and (5) to get $p_x x_B + y_B = \frac{4y_B}{3} = p_x + 3$
$\Rightarrow$ $y_B = \frac{3(p_x+3)}{4}$, $x_B = \frac{p_x+3}{4p_x}$.

Use (4): $y_A + y_B = \frac{3p_x+1}{2} + \frac{3(p_x+3)}{4} = 4$ $\Rightarrow$ $p_x = \frac{5}{9}$.

Plugging p_x into the x_A, y_A, x_B, y_B that we just derived, we have:
$(x_A, y_A) = (\frac{12}{5}, \frac{4}{3})$, $(x_B, y_B) = (\frac{8}{5}, \frac{8}{3})$