Microeconomic Theory: TA Session

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Homework 1, Q1

Professor Barbara likes his coffee sweet. He states that he strictly prefers cup-of-coffee A to cup-of-coffee B if A has more than $1/4$ teaspoon more sugar than B. If two cups of coffee have a sugar difference of $1/4$ teaspoon or less he cannot taste the difference between them and is indifferent between them.

1. Are his stated weak preferences complete?
2. Are his states weak preferences transitive?

Answer:

1. Yes. Any two cups of coffee are comparable.
2. No. Counter-example: three cups of coffee: A has 0.5 teaspoon of sugar, B has 0.7 teaspoon, and C has 0.9 teaspoon. We have $A \precsim B$ and $B \precsim C$, but $C \succsim A$. Preferences are not transitive.

Homework 1, Q2

Consider the following utility functions over mangos and nectarines, $(m, n) \geq (0, 0)$:

1. $u_i(m, n) = 2m + 3n$
2. $u_{ii}(m, n) = 16m^2 + 24mn + 9n^2$
3. $u_{iii}(m, n) = 8m + 12n - 24$

Two of these utility functions represent the same preferences. (a) Which two? Explain. (b) For the other utility function, find a comparison (i.e. (m_1, n_1) vs (m_2, n_2)) on which the two preferences disagree.

Answer:
(a) $u_{ii}(m, n) = (4m + 3n)^2$ is not a strictly increasing transformation of $u_i(m, n) = 2m + 3n$, whereas $u_{iii}(m, n) = 4(2m + 3n) - 24$ is a strictly increasing transformation of $u_i(m, n)$. So, the first and third utility functions represent the same preferences.
(b) The two preferences disagree on $(m_1, n_1) = (1, 0)$ and $(m_2, n_2) = (0, 1)$.

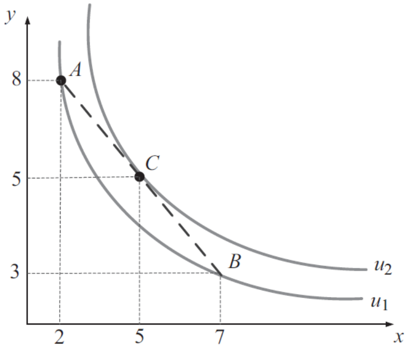
Marginal rate of substitution

Marginal rate of substitution: $MRS_{xy} = \frac{MU_x}{MU_y}$

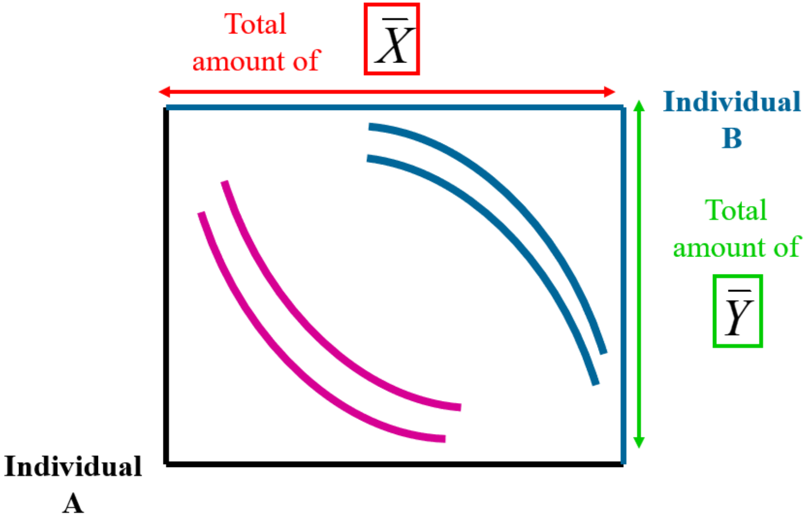
Graphically, MRS is the absolute value of the slope of the indifference curve.

- In the graph below, MRS at point C is 1

Convexity $\Leftrightarrow$ diminishing MRS



Edgeworth box



Pareto efficiency

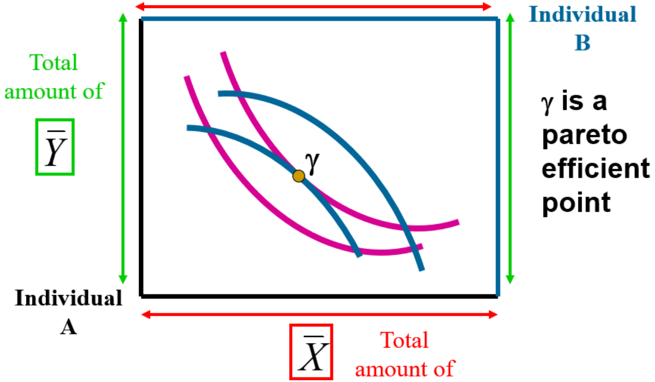
A **Pareto improvement** is a change that makes at least one person better off without making anyone else worse off

An allocation is **Pareto efficient** if it is impossible to make someone better off without making anyone else worse off

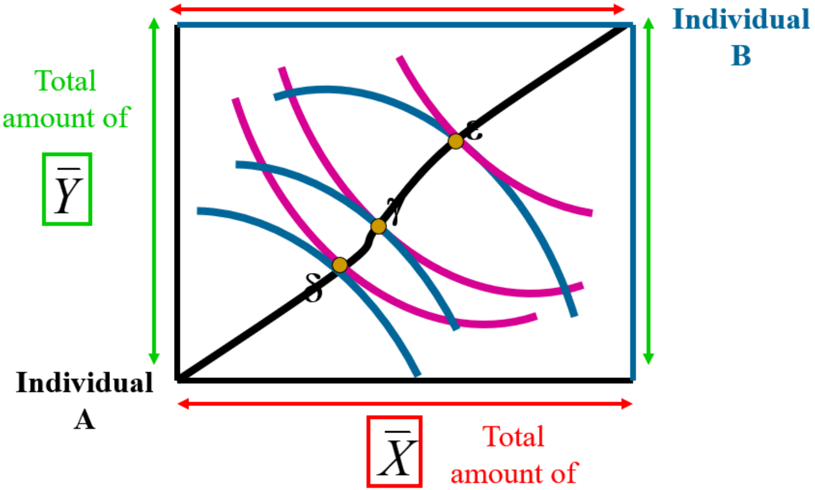
Pareto efficiency

If an allocation is not on the edges, indifference curves of the two individuals must be **tangent** at the point of a Pareto efficient allocation: $MRS_A = MRS_B$

- Otherwise we can always move "inward" to find a point which lies above both consumers' original indifference curves



Contract curve



Pareto efficiency - exercise

There are two consumers A and B, and two goods x and y. The total available amount of both goods is 4. The utility of consumer A is $U_A = x_A^{3/4} y_A^{1/4}$; the utility of consumer B is $U_B = x_B^{1/4} y_B^{3/4}$.

1. Suppose A consumes $(x_A, y_A) = (4, 0)$ and B consumes $(x_B, y_B) = (0, 4)$. Is this Pareto efficient?

   Answer: No. With this allocation, both consumers have a utility of 0. Any other interior allocation (e.g. A consumes (2,2) and B consumes (2,2)) is a Pareto improvement upon this.

2. Suppose A consumes (4,4) and B consumes (0,0). Is this Pareto efficient?

   Answer: Yes. Any other allocation will make consumer A worse off.

Pareto efficiency - exercise

There are two consumers A and B, and two goods x and y. The total available amount of both goods is 4. The utility of consumer A is $U_A = x_A^{3/4} y_A^{1/4}$; the utility of consumer B is $U_B = x_B^{1/4} y_B^{3/4}$.

3. Find a Pareto efficient allocation that is in the interior (i.e. not at the corner of the Edgeworth box).

   Answer: $MRS_A = \frac{\partial U_A/\partial x_A}{\partial U_A/\partial y_A} = \frac{3y_A}{x_A}$; $MRS_B = \frac{\partial U_B/\partial x_B}{\partial U_B/\partial y_B} = \frac{y_B}{3x_B}$.
   With a Pareto efficient allocation, it must be that $MRS_A = MRS_B$ $\Rightarrow \frac{3y_A}{x_A} = \frac{y_B}{3x_B}$. A convenient solution is $(x_A, y_A) = (3, 1)$, and $(x_B, y_B) = (1, 3)$. Another example of a solution would be $(x_A, y_A) = (2, \frac{2}{5})$, and $(x_B, y_B) = (2, \frac{18}{5})$.

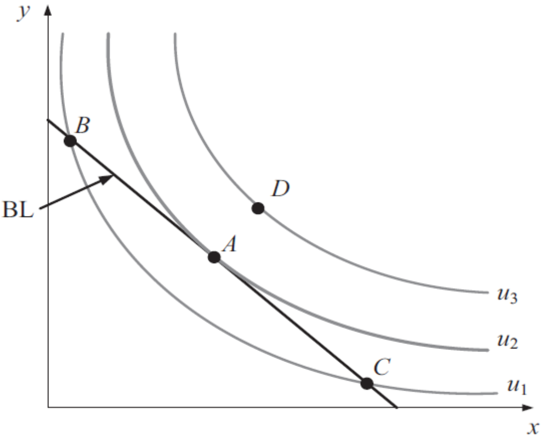
4. Derive the contract curve in terms of x_A and y_A.

   Answer: From (3), we have $\frac{3y_A}{x_A} = \frac{y_B}{3x_B}$. We also know that $x_B = 4 - x_A$, $y_B = 4 - y_A$. Plugging these into the MRS equation, we have $\frac{3y_A}{x_A} = \frac{4-y_A}{3(4-x_A)} \Rightarrow y_A = \frac{x_A}{9-2x_A}$, $0 \le x_A \le 4$, $0 \le y_A \le 4$.

Utility maximization

Given a budget constraint, the consumer's utility is maximized when the indifference curve just touches the budget constraint

- In math, $MRS_{xy} = \frac{MU_x}{MU_y} = \frac{p_x}{p_y}$



Utility maximization - exercise

1. John has the following utility function for apples (x) and bananas (y): $U = 3x^{0.5}y^{0.5}$. He has 20 dollars. One apple costs \$4, and one banana costs \$2. Solve for John's optimal bundle.

Solution method 1: Using $MRS_{xy} = \frac{MU_x}{MU_y} = \frac{p_x}{p_y}$:
$\frac{1.5x^{-0.5}y^{0.5}}{1.5x^{0.5}y^{-0.5}} = \frac{4}{2} \Rightarrow \frac{y}{x} = 2$. Plugging into the budget constraint:
$4x + 2y = 4x + 2 \cdot 2x = 20 \Rightarrow x = \frac{5}{2},\ y = 5$

Utility maximization - Lagrangian method

(See "Appendix" of Class 3 slides)

We are maximizing a function (the utility function) subject to a constraint (BC). This kind of "constrained optimization problem" can be solved with the Lagrangian method.

Set up a Lagrangian function:
$\mathcal{L}(x, y; \lambda) = u(x, y) + \lambda(I - p_x x - p_y y)$

The utility maximization problem can be solved by the following three equations:

$$\frac{\partial \mathcal{L}}{\partial x} = 0$$
$$\frac{\partial \mathcal{L}}{\partial y} = 0$$
$$\frac{\partial \mathcal{L}}{\partial \lambda} = I - p_x x - p_y y = 0$$

Utility maximization - Lagrangian method

Solution method 2:
Lagrangian: $\mathcal{L} = 3x^{0.5}y^{0.5} + \lambda(20 - 4x - 2y)$
Taking derivatives:

$$\frac{\partial \mathcal{L}}{\partial x} = 1.5x^{-0.5}y^{0.5} - 4\lambda = 0 \tag{1}$$

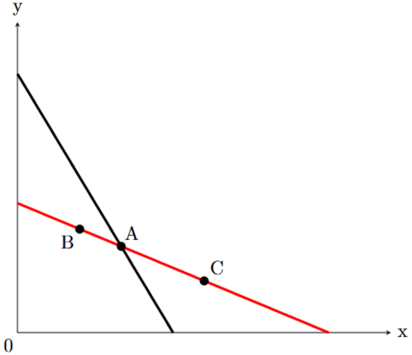
$$\frac{\partial \mathcal{L}}{\partial y} = 1.5x^{0.5}y^{-0.5} - 2\lambda = 0 \tag{2}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 20 - 4x - 2y = 0 \tag{3}$$

Dividing (1) by (2), we have $\frac{y}{x} = 2$. Plugging into (3), we have the exact same thing as in solution method 1: $x = \frac{5}{2}$, $y = 5$.

Revealed preference

Suppose that the consumer's utility is maximized at point A when his budget constraint is the black line. Now his budget constraint shifts to the red line. Which point(s) among A, B, and C can be his new utility maximization bundle?



Answer: A and C. From the consumer's choice under the black budget line, we know he prefers A over B. Under the new red budget line, A and B are still feasible, so he cannot possibly have B as the utility maximization bundle.