The Proof of Complex Analysis

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Preliminaries to Complex Analysis

1 Complex number and the complex plane

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1.2 Convergence

Theorem 1.1. $\mathbb{C}$*, the complex numbers, is complete.*

Proof. For a Cauchy sequence of complex numbers $\{z_n\}$, then

$$|z_n - z_m| \to 0 \quad \text{as} \quad n, m \to \infty.$$

In other words, given $\epsilon > 0$ there exists an integer $N > 0$ so that $|z_n - z_m| < \epsilon$ whenever $n, m > N$. If assuming $z_n = x_n + iy_n, z_m = x_m + iy_m$, so we can get

$$|z_n - z_m| = \sqrt{(x_n - x_m)^2 + (y_n - y_m)^2}.$$

According to Cauchy's convergence theorem: every Cauchy sequence of real numbers converges to a real number. So we can get the Cauchy's convergence theorem of complex numbers. □

Theorem 1.2. *The set* $\Omega \subset \mathbb{C}$ *is compact if and only if every sequence* $\{z_n\} \subset \Omega$ *has a subsequence that converges to a point in* Ω*.*

Proof. □

Theorem 1.3. *A set* Ω *is compact if and only if every open covering of* Ω *has a finite subcovering.*

Proof. □

Proposition 1.4. *if* Ω_1

Proof. □

2 Functions on the complex plane

2.1 Continuous functions

Theorem 2.1. *A continuous function on a compact set* Ω *is bounded and attains a maximum and minimum on* Ω*.*

Chapter 2

Cauchy's Theorem and Its Applications

1 Goursat's theorem

Theorem 1.1. *If Ω is an open set in $\mathbb{C}$, and $T \subset \Omega$*