COS30019 – Final Assessment

Answering Instructions:

Please do not use a red pen/type in red.

There are 5 problems.

Total marks on paper: 90 + 8 bonus marks
The maximum mark you can get for the final assessment is 90 (100%). However, if you lose marks in some questions and you get the bonus marks, the bonus marks will be added to your total of the final assessment.

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Problem 1 – Propositional Logic (18 marks)

About the insurance policy the intelligent agent is reviewing, the agent knows the following:

- *IF the policy holder has made no insurance claims AND* **(***the policy holder is a new customer OR the policy holder holds multiple policies with the company***)**, *THEN the policy holder is eligible for a discount.*
- *The policy holder is NOT eligible for a discount.*
- *The policy holder has made no insurance claims.*

Represent the above knowledge base in propositional logic using the following vocabulary:

(5 marks)

NC for *The policy holder has made no insurance claims*,
NEW for *The policy holder is a new customer*,
MP for *The policy holder holds multiple policies with the company*,
DC for *The policy holder is eligible for a discount*.

Using a truth table, please answer the following questions:

a. How many models does the knowledge base have? (5 marks)
b. Is the policy holder a new customer? (4 marks)
c. Does policy holder hold multiple policies with the company? (4 marks)

For questions (b) and (c), your answer has to be **Yes** or **No** or **Don't know**. For instance, if you answer **Yes** to question **1.b.**, you'll have to demonstrate that the knowledge base entails '*The policy holder is a new customer*'; if you answer **No** to question **1.b.**, you'll have to demonstrate that the knowledge base entails '*The policy holder is NOT a new customer*.' Clearly indicate which **rows** of the truth table support your answer.

The knowledge base is:

$$\mathrm{NC} \land (\mathrm{NEW} \lor \mathrm{MP}) \Rightarrow \mathrm{DC}$$
$$\neg \mathrm{DC}$$
$$\mathrm{NC}$$

Truth table:

NC	NEW	MP	DC	$\mathbf{NC} \land (\mathbf{NEW} \lor \mathbf{MP}) \Rightarrow \mathbf{DC}$	$\neg\mathbf{DC}$	KB
T	T	T	T	T	F	F
T	T	T	F	F	T	F
T	T	F	T	T	F	F
T	F	T	T	T	F	F
F	T	T	T	T	F	F
T	T	F	F	F	T	F
T	F	F	T	T	F	F
F	F	T	T	T	F	F
T	F	T	F	F	T	F
F	T	F	T	T	F	F
F	T	T	F	T	T	F
T	F	F	F	T	T	T
F	F	F	T	T	F	F

F	T	F	F	T	T	F
F	F	T	F	T	T	F
F	F	F	F	T	T	F

A. The knowledge base is true in only one model (blue row), so its number of models is 1.

B. **No**, the policy holder is **not** a new customer because the knowledge base does not entail NEW. The KB is TRUE in only one model (highlighted in blue) but in this model NEW is FALSE.

C. **No**, the the policy holder does **not** hold multiple policies with the company because the knowledge base does not entail MP. The KB is TRUE in only one model (highlighted in blue) but in this model MP is FALSE.

Problem 2 – Propositional Logic (16 marks)

Decide whether each of the following sentences is **valid**, **unsatisfiable**, or **neither**. Verify your decisions using truth table. Clearly indicate which **rows** of the table support your answer.

a. $(\neg A \vee B) \wedge (A \wedge \neg B)$
b. $(A \wedge \neg B) \Rightarrow A$
c. $A \vee (B \Rightarrow B)$
d. $(A \Rightarrow \neg A) \wedge B$

Hint: You can use just one truth table for all four sentences.

(4x4=16 marks)

Truth table:

A	**B**	$(\neg A \vee B) \wedge (A \wedge \neg B)$	$(A \wedge \neg B) \Rightarrow A$	$A \vee (B \Rightarrow B)$	$(A \Rightarrow \neg A) \wedge B$
T	T	F	T	T	F
T	F	F	T	T	F
F	T	F	T	T	T
F	F	F	T	T	F

Sentence (a) is **unsatisfiable**.
Senetences (b) and (c) are **valid**.
Sentence (d) is **neither**.

Problem 3 – First-Order Logic (16 marks)

Represent the following statements in first-order logic, using the following vocabulary:

Student(x):	x **is a student**
Phone(y):	y **is a phone**
Game(z):	z **is a game**
Has(u,v):	u **has** v
Plays(u,v):	u **plays** v

1. Every student has some phones.
2. There is a student who has all games or plays some games.
3. Every student has all games but does not play some games.
4. There is a student who has exactly one phone.

(4+4+4+4 = 16 marks)

1. $\forall x(\text{Student}(x) \Rightarrow \exists y(\text{Phone}(y) \land \text{Has}(x,y)))$
2. $\exists x(\text{Student}(x) \land (\forall y(\text{Game}(y) \Rightarrow \text{Has}(x,y)) \lor \exists z(\text{Game}(z) \land \text{Plays}(x,z))))$
3. $\forall x(\text{Student}(x) \Rightarrow \forall y(\text{Game}(y) \Rightarrow \text{Has}(x,y)) \land \exists z(\text{Game}(z) \land \neg\text{Plays}(x,z)))$
4. $\exists x(\text{Student}(x) \land \exists y\left(\text{Phone}(y) \land \text{Has}(x,y) \land \forall z(\text{Phone}(z) \land (z \neq y) \Rightarrow \neg\text{Has}(x,z))\right))$

Problem 4 – AI Planning (25 marks)

Our agent is a robot with two hands: *Hand1* and *Hand2*. The robot's task is to tidy up the room by putting rubbish into the *Bin* and putting things at their right places. Initially, the robot is at the *Door*, the *Rubbish* and the *Toy* are at the *Table* and both hands of the robot are free. The right place for the *Toy* is at the *Shelf*. The actions available to the robot include *Go* from one place to another, and *Grasp* or *Ungrasp* an object. Grasping results in holding the object using a free hand if the robot and object are at the same place. One effect of grasping is that the free hand that the robot uses to grasp the object will no longer be free after grasping the object.

1. Write down the initial state description and the agent's goals. (7 marks)
2. Write down STRIPS-style definitions of the three actions. (9 marks)
3. Write down a consistent partial-order plan (POP) with no open preconditions for this problem. (9 marks)

(*Hint*: You may consider using the following template for question 3 of this problem:
Actions={ Start, Go(Door, Table), …}
Orderings={ Start < Go(Door, Table), …}
Links={
Start - RobotAt(Door)-> Go(Door, Table), … }

Open preconditions={…}

And complete it with your answer.

You may also want to include a hand-drawn diagram showing the partial-order plan POP if you wish.)

1. Initial state:

   $\mathrm{At(Robot, Door)} \wedge \mathrm{At(Rubbish, Table)} \wedge \mathrm{At(Toy, Table)} \wedge$
   $\mathrm{Free(Hand1)} \wedge \mathrm{Free(Hand2)} \wedge$
   $\mathrm{Hand(Hand1)} \wedge \mathrm{Hand(Hand2)} \wedge$
   $\mathrm{Place(Door)} \wedge \mathrm{Place(Table)} \wedge \mathrm{Place(Bin)} \wedge \mathrm{Place(Shelf)} \wedge$
   $\mathrm{CanGrasp(Rubbish)} \wedge \mathrm{CanGrasp(Bin)}$

   Goal state:
   $\mathrm{At(Rubbish, Bin)} \wedge \mathrm{At(Toy, Shelf)}$

2. Actions:

   $\mathrm{Action(Go(a, b),}$
   $\quad \mathrm{PRECOND: At(Robot, a)} \wedge \mathrm{Place(a)} \wedge \mathrm{Place(b),}$
   $\quad \mathrm{EFFECT: \neg At(Robot, a)} \wedge \mathrm{At(Robot, b))}$

   $\mathrm{Action(Grasp(h, o, l),}$
   $\quad \mathrm{PRECOND: Free(h)} \wedge \mathrm{At(Robot, l)} \wedge \mathrm{At(o, l)} \wedge \mathrm{Hand(h)} \wedge \mathrm{CanGrasp(o)} \wedge \mathrm{Place(l),}$
   $\quad \mathrm{EFFECT: \neg Free(h)} \wedge \mathrm{InHand(o, h)} \wedge \mathrm{\neg At(o, l))}$

   $\mathrm{Action(Ungrasp(h, o, l),}$
   $\quad \mathrm{PRECOND: InHand(o, h)} \wedge \mathrm{At(Robot, l)} \wedge \mathrm{Hand(h)} \wedge \mathrm{CanGrasp(o)} \wedge \mathrm{Place(l),}$
   $\quad \mathrm{EFFECT: Free(h)} \wedge \mathrm{\neg InHand(o, h)} \wedge \mathrm{At(o, l))}$

3. POP plan:

$Actions = \{$
$Start,$
$Go(Door, Table),$
$Grasp(Hand1, Rubbish, Table),$
$Grasp(Hand2, Toy, Table),$
$Go(Table, Bin),$
$Ungrasp(Hand1, Rubbish, Bin),$
$Go(Bin, Shelf),$
$Ungrasp(Hand2, Toy, Shelf),$
$Finish$
$\}$

$Orderings = \{$
$Start < Go(Door, Table),$
$Go(Door, Table) < Grasp(Hand1, Rubbish, Table),$
$Go(Door, Table) < Grasp(Hand2, Toy, Table),$
$Grasp(Hand1, Rubbish, Table) < Go(Table, Bin),$
$Grasp(Hand2, Toy, Table) < Go(Table, Bin),$
$Go(Table, Bin) < Ungrasp(Hand1, Rubbish, Bin),$
$Ungrasp(Hand1, Rubbish, Bin) < Go(Bin, Shelf),$
$Go(Bin, Shelf) < Ungrasp(Hand2, Toy, Shelf),$
$Ungrasp(Hand1, Rubbish, Bin) < Finish,$
$Ungrasp(Hand2, Toy, Shelf) < Finish$
$\}$

$Links = \{$
$Start - [At(Robot, Door), Place(Door), Place(Table)] \rightarrow Go(Door, Table),$

$Go(Door, Table) - [At(Robot, Table)] \rightarrow Grasp(Hand1, Rubbish, Table),$
$Start - [Free(Hand1), At(Rubbish, Table), Hand(Hand1), CanGrasp(Rubbish), Place(Table)]$
$\rightarrow Grasp(Hand1, Rubbish, Table),$

$Go(Door, Table) - [At(Robot, Table)] \rightarrow Grasp(Hand2, Toy, Table),$
$Start - [Free(Hand2), At(Toy, Table), Hand(Hand2), CanGrasp(Toy), Place(Table)]$
$\rightarrow Grasp(Hand2, Toy, Table),$

$Start - [Place(Table), Place(Bin)] \rightarrow Go(Table, Bin),$
$Go(Door, Table) - [At(Robot, Table)] \rightarrow Go(Table, Bin),$

$Start - [Hand(Hand1), CanGrasp(Rubbish), Place(Bin)] \rightarrow Ungrasp(Hand1, Rubbish, Bin),$
$Go(Table, Bin) - [At(Robot, Bin)] \rightarrow Ungrasp(Hand1, Rubbish, Bin),$
$Grasp(Hand1, Rubbish, Table) - [InHand(Rubbish, Hand1)] \rightarrow Ungrasp(Hand1, Rubbish, Bin),$

$Go(Table, Bin) - [At(Robot, Bin)] \rightarrow Go(Bin, Shelf),$
$Start - [Place(Bin), Place(Shelf)] \rightarrow Go(Bin, Shelf),$

$\text{Start} - [\text{Hand}(\text{Hand2}), \text{CanGrasp}(\text{Toy}), \text{Place}(\text{Shelf})] \rightarrow \text{Ungrasp}(\text{Hand2}, \text{Toy}, \text{Shelf}),$
$\text{Go}(\text{Bin}, \text{Shelf}) - [\text{At}(\text{Robot}, \text{Shelf})] \rightarrow \text{Ungrasp}(\text{Hand2}, \text{Toy}, \text{Shelf}),$
$\text{Grasp}(\text{Hand2}, \text{Toy}, \text{Table}) - [\text{InHand}(\text{Toy}, \text{Hand2})] \rightarrow \text{Ungrasp}(\text{Hand2}, \text{Toy}, \text{Shelf}),$

$\text{Ungrasp}(\text{Hand1}, \text{Rubbish}, \text{Bin}) - [\text{At}(\text{Rubbish}, \text{Bin})] \rightarrow \text{Finish},$
$\text{Ungrasp}(\text{Hand2}, \text{Toy}, \text{Shelf}) - [\text{At}(\text{Toy}, \text{Shelf})] \rightarrow \text{Finish}$
}

$\text{OpenPreconditions} = \{ \}$

Problem 5 – Uncertain reasoning (15 marks + 8 bonus marks)

Mr James Bond takes his car to the mechanic for regular servicing. The mechanic runs a test on the car transmission. The test would return one of two values: **TF** or **NF**. If the test returns **TF**, it indicates that the transmission has a major issue and needs to be replaced. If the test returns **NF**, it indicates that the tests finds no issues and the transmission does not need to be replaced. The accuracy of the test is as follows: The probability of the test returning **TF** when the car transmission is actually faulty is 0.99, and the probability of the test returning **NF** when the car transmission is NOT faulty is 0.97. After running the test on Mr James Bond's car, the mechanic told him that the test returns **TF**. According to the manufacturer of Mr James Bond's car, at the age of his car, only 1 in 500 cars would have a faulty transmission that needs replacement.

Please use the following vocabulary when answering the following questions **using Bayes' rules**:
F – The car transmission is actually faulty
TF – The test returns the value **TF** indicating that the transmission has major issues

1. What is the probability that Mr James Bond's car transmission is faulty?

(15 marks)

2. **(Bonus question)** After further investigation, we also know that Mr James Bond has a very aggressive driving style that is really damaging to the car transmission and the car manufacturer informs that with Mr James Bond's driving style, 1 in 10 cars would have a faulty transmission that needs replacement. The mechanic then informs that the cost of replacing the transmission is $4,000. If the car transmission does have a major issue and it is not replaced then it will break during driving causing the entire engine to be broken which will cost $12,000. If Mr James Bond does not replace the transmission of his car now, what is the ***expected cost*** for him? If Mr James Bond is rational, would he replace the car transmission now?

(8 bonus marks)

1. The probability of the test returning TF when the transmission is actually faulty is 0.99:

$$\mathrm{P(TF|F)} = 0.99$$

The probability of the test returning NF (or NOT returning TF) when the transmission is not faulty is 0.97:

$$\mathrm{P(\neg TF|\neg F)} = 0.97$$

Only 1 in 500 cars would have a faulty transmission that needs replacement:

$$\mathrm{P(F)} = \frac{1}{500} = 0.002$$

We need to find $\mathrm{P(F|TF)}$.

Using Bayes' theorem:

$$\mathrm{P(TF|F)} = \frac{\mathrm{P(F|TF)} \times \mathrm{P(TF)}}{\mathrm{P(F)}}$$
$$\Leftrightarrow \mathrm{P(TF)} \times \mathrm{P(F|TF)} = \mathrm{P(TF|F)} \times \mathrm{P(F)} = 0.99 \times 0.002 = 0.00198 \ (1)$$

$$\mathrm{P(TF|\neg F)} = \frac{\mathrm{P(\neg F|TF)} \times \mathrm{P(TF)}}{\mathrm{P(\neg F)}}$$
$$\Leftrightarrow \mathrm{P(TF)} \times \mathrm{P(\neg F|TF)} = \mathrm{P(TF|\neg F)} \times \mathrm{P(\neg F)} = (1 - 0.97) \times (1 - 0.002) = 0.02994 \ (2)$$

Adding (1) and (2), we obtain:

$$\begin{aligned}
&\mathrm{P(TF)} \times \mathrm{P(F|TF)} + \mathrm{P(TF)} \times \mathrm{P(\neg F|TF)} \\
&= \mathrm{P(TF)} \times \big(\mathrm{P(F|TF)} + \mathrm{P(\neg F|TF)}\big) \\
&= \mathrm{P(TF)} \times 1 \\
&= \mathrm{P(TF)} \\
&= 0.00198 + 0.02994 \\
&= 0.03192
\end{aligned}$$

Using Bayes' theorem again:

$$\mathrm{P(F|TF)} = \frac{\mathrm{P(TF|F)} \times \mathrm{P(F)}}{\mathrm{P(TF)}} = \frac{0.99 \times 0.002}{0.03192} \approx 6.203\%$$

So, the probability that Mr James Bond's car transmission is faulty is around 6.203%.

2. Further investigation has elevated the probability P(F) to $\frac{1}{10}$ or 0.1.

   Reusing the formulae (1) and (2) above, we can recalculate $\mathrm{P(TF)}$ with the new value of P(F):

$$\begin{array}{rl}
 & \mathrm{P(TF)} \times \mathrm{P(F|TF)} = \mathrm{P(TF|F)} \times \mathrm{P(F)} = 0.99 \times 0.1 = 0.099 \\
+ & \mathrm{P(TF)} \times \mathrm{P(\neg F|TF)} = \mathrm{P(TF|\neg F)} \times \mathrm{P(\neg F)} = (1 - 0.97) \times (1 - 0.1) = 0.027 \\
\hline
 & \mathrm{P(TF)} = 0.126
\end{array}$$

   The probability of the transmission having a major issue is:

$$\mathrm{P(F|TF)} = \frac{\mathrm{P(TF|F)} \times \mathrm{P(F)}}{\mathrm{P(TF)}} = \frac{0.99 \times 0.1}{0.126} \approx 78.57\%$$

   If Mr. Bond does not replace his transmission now, the probability that his transmission breaks down and costs him \$12000 is 78.57%. The probability that the transmission is fine and costs him \$0 is $100\% - 78.57\% = 21.43\%$.

   The expected cost for Mr. Bond is:

$$12000 \times 78.57\% + 0 \times 21.43\% \approx 9428.4 \text{ dollars}$$

   If Mr. Bond is rational, he would replace his transmission now because the probability that it is indeed faulty is higher.