22.520 Numerical Methods for Partial Differential Equations

Term Project Description: Version 1

1 Summary

You will perform a project that involves solving a PDE of your choice using FDM, FVM or FEM. The project should introduce more complexity that the weekly homework, but should also be carefully bounded to represent a reasonable effort for a semester long course. Several examples for projects are provided in this description.

2 Suggested Project Components

The projects should demonstrate a higher level of complexity than the homework assignments in the class. Some of the ways to add complexity to your project could include one, two, or several of the following components:

1. Increasing the geometric complexity of the problem being solved: In this case you may wish to solve the PDE on a complex domain such as over an airfoil, or in cylindrical as opposed to cartesian coordinates.

2. Examining systems of PDE equations: Many physical systems are not simple one PDE, but rather they involve the solution of two PDEs simultaneously. This is the case in fluid mechanics with the conservation of mass and the conservation of momentum. Be careful however, that this complexity can quickly get out of control.

3. Comparing two PDE solution methods: In this project type, you might choose to compare the solution of a PDE using finite differences and finite elements for example, and highlight the differences between the two approaches.

4. Solving non-linear problems: Solving a non-linear PDE or system of PDEs is more complex and can be attempted by those comfortable with numerical methods looking for a challenge.

5. PDE based mesh generation: There are several techniques that use PDEs to generate the meshes that are used later in the solution process. These meshes are usually of high quality and may be useful to somebody else's solution methodology.

6. Comparing different PDE models of the same phenomena: Sometimes different PDEs are used as models for the same general phenomena. For example, Laplace's equation can describe incompressible potential flow, whereas the Euler equations describe compressible inviscid flow. These two models of flow can be compared.

7. Increasing the problem dimensions: For some of you, you may wish to try to solve the problems in class in a higher dimension than done in class. Eg. 1-D hyperbolic equations vs. 2D hyperbolic equations.

8. Exploring different PDE solution techniques: For some of you, FDM, FEM and FVM will not be exciting enough. As such, you may consider exploring a different PDE solution technique such as spectral element method or the discontinuous galerkin method or meshless methods or compact high order finite difference methods. These are all viable topics should you wish to be a little creative and exploratory. Caution, this is not recommended for the faint of heart.

9. Implicit vs. explicit solutions in time: You could explore the differences between explicit and implicit time marching.

These are a few ideas on how to increase the complexity of your project problems. You are encouraged to use online resources to find and define a simple, yet challenging problem.

3 Deliverables

1. **Project proposal (20%)**: You must submit a first project proposal by Monday Feb 19th. This proposal (max 2 pages) should be typed (single spaced) using a computer and will include:

   (a) Summary: A 1 paragraph summary of the problem you wish to solve along with the methods you will use.

   (b) Physical Problem: A 1/2 page description including relevant equations, of the problem you will be solving. The PDE you will solve should be clearly defined, along with any boundary conditions you need to specify.

   (c) Numerical Method: You should write a 1/2 page summary of the numerical approach you will take to solve the problem. You must clearly present the domain over which the equation is to be solved. This should include any pitfalls you expect to encounter.

   (d) Validation: A short, 1-paragraph summary of how you will validate your numerical solution to show you are getting the correct answer.

   (e) Approximate timeline: You should include a brief timeline of the development and testing schedule for your computer code.

   (f) References: You should provide a list of references used in the development of the project proposal.

   Depending on the quality and subject of the proposal, you may be asked to submit a revised proposal. This must be revised and approved before spring break.

2. **Computer code (40%)**: You must submit a working (matlab or similar) computer code that addresses the proposed problem.

3. **Final technical report (40%)**: You must submit a typed technical report including the technical explanation of: (1) The physics of your problem, (2) The mathematics/numerical methods used and (3) The software/code design. The strong *preference* is that this report be typeset in LaTeX. You will be provided the LaTeX template and the description of the final project deliverable after the proposal has been submitted.

The format will be similar to an AIAA journal article, and will be approximately 10-12 pages (absolute max 14 pages) including results.

4 Example Projects

Listed below are some example projects you can investigate for further ideas. You may select one of these topics as a project if you are unable to come up with one of your own.

1. Poisson airfoil mesh generation: In this project you might examine a simplified version of: http://www.grc.nasa.gov/WWW/5810/rvc/grape.htm . The goal would be to research and understand the theory behind this mesh generation technique and then apply it to generate high quality orthonormal structured grids. Ambitious students may also try to solve some PDE using the mesh (potential flow around symmetric airfoils).

2. Unstructured mesh generation: In this project you might examine developing a simplified version of: http://persson.berkeley.edu/distmesh/ . The goal would again be to understand how these meshes are constructed. Ambitious students may develop this grid generator and the FEM/FVM code to solve a particular problem of interest.

3. Finite differences (or finite elements) for the Navier-Stokes equations in Vorticity-Streamfunction ($\omega - \Psi$) form (2-D): This is one of the simpler versions of the Navier Stokes equations. You can solve this particular form on a lid-driven cavity flow quite simply. I would not recommend making life much more complicated than this unless you are confident in your numerical abilities – in which case, you may contemplate mappings to more complicated domains.

4. Finite elements for structural analysis: In this type of project you may solve the 2D FEM problem for a structural deformation of a (simple) object. Ambitious students may consider looking at Distmesh to generate high quality elements.

5. Finite volume solutions exploring the 1-D Euler equations for the shock tube problem: In this problem 1D is likely complicated enough. You can use the 1-D finite volume method to illustrate the propagation of a shock wave and an expansion fan in a compressible fluid.

6. Convection-diffusion equation: This could be any application you are interested in. I would recommend looking at 1-D problems. If you are very ambitious you may wish to look at 2-D problems.

7. Level sets: The Hamilton-Jacobi equation can be solved (in 2D in this class) to find the level sets within a particular domain. These level sets can be used to perform path planning, interface evolution, shadow/light ray analysis, etc. These equations are pretty cool. Anyone I know who has worked on these equations is generally enthralled by them.

Additional project ideas will be appended to this list over time as they are conceived.