

## Lecture 6: Probability and Uncertainty: the quantum mechanical view of nature

[BBC TV Film Leader]

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### Probability and Uncertainty: the quantum mechanical view of nature

In the beginning of the history of experimental observation, or any other kind of observation on scientific things, it's intuition—which is really based just on experience with everyday objects—that suggests reasonable explanations for things.

But as we try to widen, and make more consistent, our description of what we see, as it gets wider and wider and we see a greater range of phenomena, the explanations become what we call laws, instead of simple explanations. But the one important odd characteristic is that they often seem to become more and more unreasonable, and more and more intuitively far from obvious.

To take an example is the relativity theory, in which, for instance, the proposition is that if you think that two things occur at the same time, that's just a subjective opinion: someone else could conclude that those two events, one was before the other, and that simultaneity is merely a subjective impression.

Now, there's no reason why this should be otherwise, really. The things of the direct everyday experience involve large numbers of particles, or involve things moving very slowly, or involve other conditions that are very special and represent, in fact, a very limited experience with nature. It's a small section only that one gets of natural phenomena from a direct experience; it's only through the refined measurements and careful experimentation that we can get a wider vision—and then we see unexpected things.

We see things that are far from what we would guess; we see things that are very far from what we would—could have imagined. And so our imagination is stretched to the utmost—not as in fiction, to imagine things which aren't really there, but our imagination is stretched to the utmost just to comprehend those things which are there—and it's this kind of a situation that I want to talk about tonight. Start, for instance, with the history of light.

At first, light was seen to behave—it would appear to behave—very much like a rain of particles, of "corpuscles," like rain, bullets from a gun, same idea. Then, with further research, it was clear that it was not right, but that light actually behaved like waves, like water waves, for instance. And then in the 20th century, on further research, it appeared that light actually behaved, in many ways, again like particles. In the photoelectric effect you could count these particles—they're called photons now—and so forth.

Again: electrons, when they were first discovered, behaved exactly like particles—bullets—very simple. Further research showed—in electron diffraction experiments, and so on—that they behaved like waves. As time went on, there was a growing confusion in the question of how the things really behaved—waves or particles, particle or waves—everything looked like both.

Now, this growing confusion was resolved in 1925 or 1926 with the advent of the correct equations for quantum mechanics. Now we know how the particles—how the electrons and how light—behave, but what can I call it? I can say they behave like a particle wave, or they behave in [a] "typical quantum mechanical manner"—there isn't any word for it.

If I say they behave like particles, I give the wrong impression; if I say they behave like waves, they behave in their own inimitable way—which, technically, could be called the "quantum mechanical way"—they behave in a way that is like nothing that you have ever seen before! Your experience with things that you have seen before is inadequate, is incomplete. The behavior of things on a very tiny scale is simply different: they do not behave just like particles; they do not behave just like waves.

Atoms do not behave like weights hanging on a spring and oscillating, nor do they behave like miniature representations of the solar system, with little planets going around in orbits, nor does it appear to be somewhat like a cloud or fog of some sort surrounding a nucleus—it behaves like nothing that you've seen before.

Well, there's one simplification: at least electrons behave exactly the same, in this respect, as photons—that is, they're both screwy, but in exactly the same way. How they behave, therefore, takes a great deal of imagination to appreciate, because we are going to describe something which is different than anything you know about.

This—in that respect at least—makes this perhaps the most difficult lecture of the series, in the sense that it's abstract, in the sense that it is not close to experience—and I cannot avoid that. I could give a series of lectures on the character of physical law, and to leave out from this series the description of the actual behavior of particles on a small scale, I would certainly not be doing the job, because this thing is completely characteristic of all of the particles of nature, and is a universal character; it is, if you want to hear about the character of physical law, essential to talk about this particular aspect.

So it will be difficult. But the difficulty, really, is psychological, and exists in the perpetual torment that results from your saying to yourself, "But how can it be like that!"—which really is a reflection of an uncontrolled, but I say utterly vain desire, to see it in terms of some analogy with something familiar.

I will not describe it in terms of an analogy with something familiar; I will simply describe it.

There was a time when the newspaper said that only twelve men understood the theory of relativity. I don't believe there ever was such a time: there might have been of time

when only one man did, because he's the only guy who caught on, before he wrote his paper. But after people read the paper, a lot of people kind of understood the theory of relativity in some way or other, but more than twelve.

On the other hand, I think I can safely say that nobody understands quantum mechanics! Now, if you appreciate this, and don't take the lecture too seriously that you really have to understand, in terms of some model, what I'm going to describe, and just relax and enjoy it, I'm going to tell you what nature behaves like, and if you will simply admit that maybe she does behave like this, you will find her a delightful, entrancing thing.

So that's the way to look at the lectures—not to try to understand. (Well, you have to understand the English, of course.) But, in any sense, in terms of something else, don't keep saying to yourself, if you can possibly avoid it, "But how could it be like that!" Of course, you'll get down the drain; you'll get down into a blind alley which nobody has yet escaped—nobody knows how it can be like that.

So then just let me describe to you the behavior of electrons, or a photon, in their typical quantum mechanical way. Now, the way I'm going to do this is by a mixture of analogy and contrast. If I made a pure analogy we would fail, so it must be analogy and contrast to things that you're familiar with.

And so I make it, by analogy and contrast, first to the behavior of particles, for which I will use bullets, and second, to the behavior of waves, for which I will use, say, water waves, or sound waves.

So we begin. First, to discuss in a particular—what I'm going to do is, I'm going to invent a particular experiment, and first tell how it would behave, what the situation would be, in that experiment using particles, [then] what you would expect to happen if waves were involved, and then what happens when there are actually electrons or photons in the system.

I will just take this one experiment which has been designed to contain all of the mystery of quantum mechanics to put you up against the paradoxes and mysteries and peculiarities of nature 100 percent. Any other situation in quantum mechanics, it turns out, can always be explained afterwards by saying, "You remember the case of the experiment with the two holes? It's the same thing."

So I'm going to tell you about the experiment with the two holes, which is the general mystery: it contains, it does contain, the general mystery; I am avoiding nothing. I am baring nature in her most elegant and difficult form.

So you start with bullets. All the experiments are going to be in the same general design, so I'll draw it this way: suppose that we have some source of bullets, which just represents the source, which we call "the source"—and is, in fact, in the case of bullets, a machine gun. Then we have a plate in front here, with a hole in it for the bullets to come out of, and this plate, in the case of bullets, is armor plate. Then a long distance from here we have another plate, which I'm drawing only a short distance, because I

haven't got room on the blackboard for everything, but this distance is supposed to be much longer in proportion to the width.

Please expand that; that's a small point, and it has two holes in it—that's the famous "two—hole" business.

I'm going to talk a lot about these holes, so I'll talk about this hole as Number 1 Hole, and the other hole as Number 2. And I'm only drawing it in two dimensions. You can, if you wish, imagine these as round holes in three dimensions, but just say it's a cross section.

And then again, a long distance away (but we'll draw it relatively short distance, because of the limitations of this blackboard), we have another screen here, which is just a backstop of some sort, into and on which we can put in various places what I will call a detector—I'm going to mark that "detector"—which, in the case of the bullets, is a box of sand into which the bullets will be caught, and we can count them—that's the detector for bullets.

I don't want to have to redraw the experiment each time so I'll label everything in this way, and then we'll be able to catch on to situations for different cases. Also, I'm going to do experiments in which I count how many bullets come into this detector, or box of sand, when the box is here, or here, or here, or here; and to describe that, I'll measure the distance of the box from somewhere out here, and call that  $X$ —and I talk about what happens when we change  $X$ : it means only you move the doggone thing up and down. All right.

Now first, I would like to make a few modifications from real bullets into idealizations: the first is, the machine gun is very shaky and wobbly, and that the bullets go in various directions, not just exactly straight on and bounce back, and they can ricochet off the edges of the slips—the slits, rather—the holes in these armor plates.

Let's say, for instance, that the bullets have all the same speed or energy, if you want (but that's not very important), but the most important idealization in which it differs from real bullets is: I want these bullets to be absolutely indestructible, so that what we find in the box is not pieces of lead of some bullet that broke in half; we get the whole bullet, please. So imagine indestructible bullets, or hard bullets and soft armor plate, or something.

Now, the first thing that we will notice about bullets is that the things that arrive come in lumps: when the energy comes, it's all in one bullet—full of bang. If you count the bullets, there's 1, 2, 3, 4 bullets—the things come in lumps; they're equal in size, we suppose, in this case. When a thing comes into the box, it's either all in the box or it's not in the box—it comes in lumps.

More: if I put up two boxes here, I never get two bullets in the boxes at the same time. (Well—if the gun isn't going off too fast and I have enough time between—slow down the gun so they go off very slowly, bing, bing—bing, bing—then put the two things here, and look very quickly in the two boxes, you'll never get two bullets at the time in the two

boxes, because a bullet is a single identifiable lump.) I call that characteristic of the object that it comes in lumps.

The first thing about bullets is that they come in lumps. Now, what I'm going to measure is how many bullets arrive here on the average in a long period of time. Say you wait an hour, and you count how many bullets are in the can-in the sand-and average that.

Now, we call that, if you want per—let's say we take a definite time, like, per hour, and say the number of bullets that arrive per hour, and sometimes you could call that what's called the probability of arrival, because it just gives the chance that a bullet going through this thing arrives in this particular box—at least it's proportionate to the chance. One way to measure is to measure the average number of bullets that arrive over a period of time. Now, the number of bullets that arrive in this box here will vary as I vary X: I'm going to make a graph here in which I plot horizontally the number of bullets that I get if I hold this thing here for an hour. I'll get a curve that will probably look more or less like this because when the box is behind one of these holes, it gets a lot of bullets—it gets the ones that went through this hole—and otherwise it gets in the one through this hole, and if it's a little bit out of line it doesn't get as many (they have to bounce a little off the edges of the hole, and so it disappears like this).

This is the number that we get in an hour when both holes are open. I call that by an abbreviation,  $N_{1-2}$ , which merely means the number which arrives when Hole Number 1 and Hole Number 2 are both open. Looks like that, say.

Now, I must insist that the number that we're plotting here doesn't come in lumps; it can have any size it wants—for example, there can be two and a half bullets in an hour. In spite of the fact that the bullets come in lumps, what I mean by two and a half bullets in an hour is, that if you run a long time, like 10 hours, you get 25 bullets, so it's on the average two and a half bullets. The  $N$  can have any size; it doesn't have to be in lumps because it's an average.

I'm sure you're all familiar with the joke about the fact that the average family in the United States seems to have two and a half children. It doesn't mean that there's a half a child in any family whatever; the children come in lumps. But nevertheless, when you take the average number per family, it can be any number whatsoever.

And so this number  $N$ , which is the number that will arrive in this container per hour on the average, need not be an integer—it can be a tenth, which would mean under those circumstances that you'd have to wait on the average 10 hours, more or less, per bullet. What we measure, then, is the probability of arrival, which is a technical measure, the probability of arrival. It is a technical term, really, for the average number that arrive in a given length of time.

Now, finally, if we go to analyze this curve  $N_{1-2}$ , we can interpret it very nicely: we can interpret it as a sum of two curves, which I will draw here. (That's why I need the blackboard where I've got several cases, so I draw two curves here.) One which would represent what I'll call  $N_{1-1}$ . The number which would come if Hole Number 2 is closed

by another piece of armor plate in front, and so they all come through Number 1.  $N-2$  would be the number that come through Hole Number 2 alone. So  $N-1$  is the number that come through Hole Number 1 alone, and  $N-2$  is the number that come through Hole Number 2 alone, those numbers being determined by closing the respective holes. Then we discover a very important law, which is that the number that arrive with both holes open is the number that arrive by coming through Number 1 Hole plus the number that come through Number 2 Hole. This proposition—the fact that all you have to do is add these two together—I'd call "nice," or "no interference." That is, what you get from the two holes open is the same as you'd get by simply adding each hole separately. That's for bullets.

Done—we're done with bullets.

All right. I begin again, this time with water waves. Here is standing some kind of a big mass of stuff which is being shaken up and down. This is a long line of barges, or jetties, with a gap in the water inbetween. (Perhaps it's better to do it with ripples than it is to do it with big ocean waves; it sounds more sensible.) I wiggle my finger up and down here, and I have a little piece of wood here, and ripples start out here, and then I've arranged in a tank to put boards in the way here so that I have these two holes, and then I have this so-called detector.

Then what I do with the detector—what the detector detects is how much the water is jiggling: for instance, I put a cork in the water and measure how it moves up and down. What I'm going to measure, in fact, is the energy of the agitation of the cork, which is exactly proportional to the energy carried by the waves.

Also, I forgot to say that this jiggling is made very regular and perfect, so that the waves are all of the same spacing from one another, and then I'll describe what we get under those circumstances.

For that, I first remark—well, let's see: first we can measure the energy of the cork. But then another thing is important for light, or for water waves—for waves, water waves—is that the thing that we're measuring can have any size at all: we're measuring the intensity of the waves, or the energy in the cork; if the waves are very quiet, if the fellow over here is only jiggling a little bit, then there will be very little motion to the cork—no matter how much it is, it's proportional. So I can have any size. It doesn't come in lumps, it's not all there or nothing.

What we're going to measure is the intensity of the waves, which to be precise, if you want, is the energy generated by the waves at a point. Now, what happens if we measure the intensity, which I'll draw on a third curve here, which I'll call "I"—to remind you it's an intensity, and not a number of particles of any kind, and I-1-2 when both holes are open—is a curve that looks something like this: another interesting, complicated—looking curve, which ought to be symmetrical.

(I didn't do too badly, actually.)

A very complicated—looking curve. That is, if we put the [detector] in different places, we get a very, very different intensity which varies very rapidly in a peculiar manner. You're probably all familiar with the reason for that. The reason is that the ripples, as they come out of here, have crests and troughs spreading from here, and they have crests and troughs spreading from here.

Now, if we're at a place which, say, is exactly even between these two things so that the two waves arrive at the same time, the crests will come on top of each other and there'll be plenty of jiggling, which is the exact opposite of this curve—so I'll have to put there should be another bump. You have a lot of jiggling right in dead center.

On the other hand, if I were to move to some point here, since I'm further from Hole 2 than Hole 1, it takes a little longer for the waves to come from 2 than from 1. When 1 has a crest arriving, the crest hasn't quite reached there yet from 2—in fact, it's a trough from 2—so that water tries to move up, and it tries to move down, from the influences of the waves coming from the two holes, and the net result is it doesn't move at all, or practically not at all, and so we have these low bumps at that place. And then if you move still further over, you get enough delay that when a crest is here, this other crest is one whole wave behind, so it's a crest that—two crests are coming on top of each other, but not the same crest, so to speak.

The fourth crest from here, and the fifth crest from there on top, so you get a big one again, then a small one, a big one, a small one, depending upon the way the crests and troughs interfere, as we say.

The word "interference," again, is used in science in a funny way, because we'll have what we call "constructive interference": when they both interfere here, it makes it stronger. Well, they call it interference anyway, but the very important thing is that  $I-1-2$  is not the same as  $I-1$  plus  $I-2$ , and we say it shows interference—yes, interference; that's a funny term, we use it—constructive and destructive interference.

I didn't mention what  $I-1$  and  $I-2$  look like, but we can find out by closing this, for instance, to find  $I-1$ . The intensity that you get here, if the hole is closed, is simply the waves from one hole, for which there's no interference, and that's this curve.  $N-1$  is the same as  $I-1$ , and the same way, otherwise,  $I-2$ —and this curve is quite different than the sum of these two.

As a matter of fact, the mathematics of this curve is rather an interesting one. What is true is this: that the height of the water, when both holes are open, is equal to the height that you would get from Number 1 open, plus the height that you would get from Number 2 open.

Thus, if it's a trough, the height from 2 is negative, and cancels out the height from 1. So you can represent it by talking about the height of the water, but it turns out that the intensity, in any case (for instance, when both holes are open) is not the same as the height, but it's proportional to the square of the height, and it's because of this fact, that we're dealing with the squares, that we get these very interesting curves.

All right.

Now we erase the machinery and start over; this time we start with electrons. We have a filament here, tungsten plate, holes in the tungsten plate, and for a detector any electrical system which is sufficiently sensitive to pick up the charge of an electron arriving with whatever energy the source has.

Or, if you would prefer, we could use photons: this is a black paper with a hole in it, two holes in another sheet of black paper (paper isn't very good, because the fibers don't make a sharp hole, so use something better), and here, for a detector, a photomultiplier that can detect the individual photons arriving.

Now, what happens with either case—and I'll discuss only the electron case; the other case is exactly the same, the case with photons—is this: first, that what we receive in this electrical detector, with a sufficiently powerful amplifier behind it, are clicks. Click, click-click-click, and so on, with the source here—lumps, absolutely lumps: when the click comes it's a certain size, and the size is the same.

If you turn the source weaker, the clicks come further apart, but it's the same size click. If you turn it up, they go click- click-click-click-click, and it jams the amplifier—you have to turn it down enough that there aren't too many clicks for the machinery that you're using to detect.

Next, if you were to put up another detector here, and listen to both of them, you'd never get two clicks at the same time (at least if the source is weak enough) because of the precision with which you measure the same time: if you cut down the intensity of the source so they come few and far between, they never come a click in both detectors. That means that the thing which is coming, comes in lumps: it has a definite size, and it only comes to one place at a time.

All right, so for electrons—or for photons, we'll just use electrons—it comes in lumps. Therefore, what we can do is the same thing as we did with the bullets: we measure how many come; we measure the probability of arrival. What we do is we hold the detector in a certain place. (Actually, if we wanted to, although it's expensive, we could put detectors all over at the same time, and make the whole curve simultaneously.) But let's suppose we put it at a certain place, and we measure at the end of an hour how many electrons came, and we average it. (By the way, if I put detectors all along the back here, when one comes it comes into one, but not [into] others: it's just one goes off, then the other goes off, and this goes off, and that one goes off, and so on—just like with bullets.)

We measure, then, the probability of arrival of the electrons. And what do we get? The number of electrons that arrive—the same kind of an N-1-2 as before. This is what we get for N-1-2. N-1-2, this is what we get with both holes open!

That's the phenomena of nature: that she produces the curve which is the same as you would get from an interference of waves.



But she produces a curve for what? Not for the energy in a way but for the probability of arrival of one of these lumps. The mathematics is simple. You change  $I$  to  $N$ , and you have to change  $H$  to something else, which is new, and you call it something, because it's not the height of anything, but it in order [to express this] curve as a simple mathematical form, there's an  $A$ , which can be represented as an  $A-1$  plus an  $A-2$ , which we call a probability amplitude (because we don't know what it means), which, to arrive from Hole 1, plus the probability amplitude to arrive from Hole 2, and you add the two together to get the total probability amplitude to arrive, and square it.

It's a direct imitation of what happens with the waves, because we've got to get the same curve out so we use the same mathematics.

Let's find out if I'd better check on one point, though, about the interference: I forgot to say what happens if we close one of the holes. Let's try to analyze this interesting curve, which now, for electrons (I erase all the stuff with the light, so everything with light is erased, and now we're talking about electrons; this curve isn't important in our case) this is the number which arrives.

Now, we would like to analyze this curve. We try this: we say, "Maybe it comes—we can analyze this—by thinking that the electrons come through this hole or through the other"—so we can close one hole and measure how many come through Hole Number 1, and we get that curve; or, we can close this hole, and measure how many come through Hole Number 2, and we get that curve. These two added together is not this, and so this is not the same as  $N-1$  plus  $N-2$ —it does show interference; it shows interference.

And in fact the mathematics is given by this funny formula that the probability of arrival is the square of an amplitude, which itself is the sum of two pieces. Now, nobody-a question is, how can that come about, that when they go through Hole 1, they would be distributed this way; when they go through Hole 2, they would be distributed that way; how could it be that when both holes are open, you don't get the sum of the two? Instance: if I hold the detector at this point here, I get practically nothing. If I close one of the holes, I get plenty. If I close the other hole, I get something. If I leave both holes open, I get nothing—if I let them go through both holes, they don't come anymore. Or take the point in the center: you can show that that's higher than the sum that it was in the other case, than the sum of these two; I get more here when both holes are open, than I would get with either one of the two closed.

Now you might think that if you were clever enough, you could argue that they have some way of going around through the holes, back and forth, and they do something complicated, or it splits in half and goes through the two holes, and so forth, in order to explain this phenomenon.

Nobody, however, has succeeded to get an explanation of this that's satisfactory, because the mathematics in the end is so very simple—the curve is so very simple.

I will summarize, then, by saying that electrons arrive in lumps, like particles. But the probability of arrival of these lumps is determined like the intensity of waves would be. It is in this sense that the electron behaves, as you might say, sometimes like a particle, and sometimes like a wave; it behaves in these two different ways at the same time—that's all there is to say.

I give a mathematical description to figure out the probability of arrival of electrons on any circumstances, and that would, in principle, be the end of the lecture—except that there are a number of subtleties involved in the fact that nature works this way; there's a number of peculiar things, and I would like to discuss those peculiarities, because they may not be self-evident at this point.

To discuss the subtleties, we begin by discussing a proposition which we would have thought to use since these things are lumps: since what comes is always one complete "p ah" (which I'll call an electron—one complete lump, one complete electron), it's obvious that it's reasonable that either an electron arrives—or goes, let's say—that either an electron goes through Hole Number 1, or it goes through Hole Number 2.

That seems like . . . that seems very obvious that it can't do anything else if it's a lump. I'm going to discuss this proposition, so I have to give it a name; I'll call it Proposition A. Now, we've already discussed a little bit what happens with Proposition A. If it were true that an electron either goes through Hole Number 1 or it goes through Hole Number 2, then the total number which arrive here would have to be analyzable as the sum of two contributions—the total number which arrive here will be the number that come here via Hole 1 plus the number that come via Hole 2—and since this curve cannot easily be analyzed as the sum of two pieces in such a nice manner, and since the experiments which determine how many would have arrived—would have arrived—if only Hole Number 1 were open don't give the result that this number is the sum of these two, it is obvious that we should conclude that this Proposition is false—it is not true, that the electron either comes through Hole Number 1 or Hole Number 2; maybe [the electron] divides itself in half temporarily, and so on.

So Proposition A is false.

That's logic.

Unfortunately (or otherwise), we can test logic by experiment, and so we just have to do—to find out whether it's true or not that the electrons come through Hole 1 and Hole 2 (or maybe they go around through both holes, or they split up, and so on). . . —all we have to do is watch them.

To watch them we need light. So we put back here, behind the holes, a source of light—very intense light. Light is scattered by electrons; it's bounced off electrons—in other words, you can see electrons as they go by if the light's strong enough.

We stand back here, and we look to see whether we see, when the electron is counted here, a flash—or have seen, the moment before the electron is counted here, a flash

behind Hole 1, or a flash behind Hole 2, or maybe a sort of "half flash" in each place at the same time—because we're going to find out, now, how it goes, by looking!

Well, you turn on the light and look. And lo, you discover that you see flashes behind either one hole or the other hole every time you get a count here. Every time there's a count here, you see a flash behind Number 1, or behind Number 2.

What you see is that the electron comes 100 percent complete through Hole 1 or through Hole 2—when you look.

Kind of a paradox.

Well, let's squeeze nature into some kind of a difficulty here: I'll show you what we're going to do, see . . . We're going to keep the light on; we're going to watch. And you're going to count—we're going to count how many electrons come through. We're going to make two columns. I'll watch the holes very carefully while you, please, count how many are arriving in the detector.

All right, you say, one arrived. I said, I saw that when it went through Hole Number 1. We put here two columns, which is column one for Number 1 Hole and [Column 2 for] Number 2 Hole. Every time you get one, you tell me you got one I have seen it, of course, and I say either Number 1 or 2.

The first one was 1, what's the next one? Number 2. All right. Number 2, Number 2, Number 1, so on.

As we watch the electrons—as I watch the electrons, for every one that you count, I can separate them experimentally into two columns: them that have arrived via Hole 1 and those (I know the English is right; I'm just trying to) that arrived via Hole 2. So the number—the total number that arrived—well, first, what does this column look like when you add it all together for different positions here, which is just the number that have supposed to have come through 1.

I watch behind 1 and what do I see?

I see this curve—that column is distributed this way, just like we thought when we closed Hole 2; it works the same way whether we're looking or not: if we close Hole 2, we get the same distribution—those that arrive [are] as if we were watching. Likewise, in this column that is supposed to have arrived via Hole Number 2, is also the simple curve. Now, look, the total number which arrives has to be the total number—I'm just counting little marks! It has to be the sum of this number plus that number. The total number which arrived absolutely has to be the sum of these two; it has to be distributed this way! I said it was distributed this way; it's distributed this way—it really is, of course; it has to be. It is. It's distributed this way.

If, then, we mark with a prime the results when a light is lit (prime means with a light lit), then we find  $N-1$ -prime is practically the same as  $N-1$  without the light, and  $N-2$ -prime is almost the same as  $N-2$ . But the number that we see when the light is on is not—is equal, is equal—to the number that we see through 1 plus the number that we see through 2.

This is the result that we get when the light is on. In other words, we get a different answer whether I turned on the light or not. If I have the light turned on, this is the distribution which you measure over here. If I turn off the light, this is the distribution that you measure over here. Turn on the light, this is the answer; turn off the light, that's the answer.

See, nature's squeezed out.

Now, we could say then, that the light affects the result. If the light is on, you get a different answer than if the light is off. If you want to, you could say there are light effects, it does affect. In fact, we've found by this experiment we get a difference with the light on and off.

Light affects the behavior of electrons.

If you want to talk about the motion of the electrons through here, which is a little inaccurate, you can: you can say that the light affects the motion, so that those which might have arrived at the maximum are somehow been deviated or kicked by the light and arrived at the minimum instead, thus smoothing the curve to produce this thing.

You see, electrons are very delicate.

Although, when you're looking at a baseball and you shine light on it, it doesn't make any difference, the baseball goes the same way.

Electrons are very flimsy, very delicate, and when you shine a light on, a little tough on the electron, it knocks them about a bit, and instead of doing that, they do this, because you turned the light on so strong—you hit 'em with a hammer; it's not just a delicate thing like when you're looking at a baseball with light.

There, you hit 'em with a hammer.

But you used—you turned up the light too strong: turn it weaker and weaker and weaker until it's very dim, and then use very careful detectors that can see very dim light, and look with the dim light. Now, as the light gets dimmer and dimmer, you can't expect with very, very, very weak light to affect the electron so completely as to change the pattern 100 percent from this pattern to this pattern; as the light gets weaker and weaker and weaker, somehow it should get more and more like no light at all.

How, then, does this turn into that?

Well, it turns out that light is not like a wave of water, but light also comes in particle-like character called photon: as you turn down the intensity of the light, you're not turning down the effect; you're turning down the of photon particle—like things that are coming out of the source. So as I turn down the light, I'm getting fewer and fewer photons. The least I can scatter from an electron is one photon, and if I have too few photons, well, sometimes the electron will get through, and it just happened there wasn't enough light—there was no photon coming by; I didn't see it. A very weak light doesn't mean a small disturbance; it just means a few photons.

What happens is that I have to invent a third column.

You see, you get a click over here. I say, "I saw that one, that was in Number 1 Hole; this was behind Hole Number 2." Then another comes: "Sorry, I didn't see that"—there wasn't enough light to give a photon at that time, so there must be a third column under "didn't see." When the light is very strong there are very few in there. When the light is very weak, most of them end in there. That there are three columns, this one, this one, and, sometimes, in here.

You can guess what happens.

The ones I do see are distributed this way; the ones I didn't see are distributed that way, and as I turn the light weaker and weaker, well, I see less and less of them—with greater and greater fraction are not seen. The actual curve, in any case, is a mixture of this and this, and as the light gets weaker so that fewer and fewer are seen, [it] gets more and more like that, in a continuous fashion. In this case, if the electrons are not seen, and nothing bounced off the light under those circumstances, you get this complicated pattern for those electrons which were not seen—the ones in the column "didn't see" are exactly distributed in this complicated way, and the other two columns are in these two ways here.

Now, you say, "I got another way to measure which hole it goes through." I'm sorry, I haven't got enough time to discuss a large number of different inventions that you might have, to find out which hole the electron went through, and what happens in each case. It always turns out, however, that it's impossible to arrange the light in any way, so that you can tell through which hole the thing is going, without disturbing the pattern of arrival of the electrons from this form to this form, without destroying the interference. Not only light, but anything else—you use neutrinos—you use anything, there's [a] principle that it's impossible to do it.

You can, if you want, invent a way to tell which hole the electron's going through. Then it turns out it's going through one or the other. If you try to make that instrument so that at the same time it doesn't disturb the motion of the electrons, then what happens is you get back—you can't tell anymore which one it goes through, and you get this; if you can tell, you get this. Heisenberg noticed, when he discovered the laws of quantum mechanics, that the new laws of nature that he discovered could only be consistent if there was some basic limitation to our experimental abilities that had not been previously recognized.

In other words, you can't experimentally be as delicate as you wish—and he proposed his uncertainty principle, which, stated in terms of our experiment, is the following (he stated it in another way, but they're exactly equivalent; you can get from one to the other, but unfortunately, I haven't time to explain that)—in our experiment, his uncertainty principle would be stated in this manner: "it is impossible to design any apparatus whatsoever to determine which hole the electron passes (if one succeeds in determining which hole the electron passes—through which hole the elec—which can

determine through which hole the electron passes) that will not at the same time disturb the electron enough to destroy the interference pattern."

No one has found a way around this, and I'm sure you're all itching with inventions as to methods of detecting which hole the electron went through, but if each one of them is analyzed carefully, you'll find out there's something the matter with it— that if, without disturbing the electron, you think you can do it, but it turns out there's always something the matter—and you can account for the difference in the patterns due to the disturbance of the instruments used to determine through which hole the electron comes.

Now this, therefore, is a basic characteristic of nature and tells us something about everything. If a new particle is found tomorrow, the kaon (actually it's already been found—something, give it a name)—let's say a kaon, and I use kaons to interact with electrons to determine which hole the electron is going through, I already know ahead of time (I hope) enough about the behavior of the kaon to say that it cannot be of such a type that I could tell through which hole the electron would go, without at the same time producing a disturbance on the electron to change the pattern from here to here.

The uncertainty principle is used as a general principle to guess ahead at many of the characteristics of unknown objects, Think how? limited in their character. Well, then, let's go back.

What about this Proposition A? Does it go either through one hole or the other, or not? Well, physicists have a convention, a way of avoiding the pitfalls which exist, and they make their game, their rules of thinking, as follows. That if you have an apparatus which is capable of telling which hole the electron goes through (and you can have such apparati), then one can say that it either goes through one hole or the other.

It does, when you look; it always is going through one hole or the other—when you look. But when you do not try to determine, or you have no disturbance—no apparatus there to determine through which hole a thing goes— under those circumstances, you cannot say that it either goes through one hole or the other. You could always say it, provided you stop thinking immediately and don't make any deduction from it; we prefer not to say it, rather than to stop thinking at the moment.

In other words, when we don't look, we can't say through which hole it's going, but if you try to look to see, you find that it always goes through one hole or the other.

Still, to conclude that it goes either through one hole or the other when you're not looking is to produce an error in prediction. That is the logical tightrope on which we have to walk if we wish to interpret nature. This proposition that I'm talking about is more general. It's not just for two holes; it's a general proposition—[it] reads something like this: that the probability of any event in an ideal experiment (that just means when everything is specified as well as it can be), the probability of an event is the square of something, which I call "A" here, is called the probability amplitude—and when an event can occur in several alternative ways, the probability amplitude—this "A" number—is

the sum of the A's for each of the various alternatives; and finally, if an experiment is performed which is capable of determining which alternative is taken, the probability of the event is the sum of the probabilities for each alternative—that is, you lose the interference. Now, the question is, how does it really work?

By what machinery is actually producing this thing?

Well, nobody knows any machinery. Nobody can give you a deeper explanation of this phenomenon than I have given—that is, a description of it.

They can give you a wider explanation, in the sense that they could do more examples to show how it's impossible to tell which hole it goes through, and at the same time not destroy the interference pattern. They can give a wider class of experiments than just the two—slit interference experiment, and so on, but they're all—it's just repeating the same thing to drive it in.

It's not any deeper, it's only wider: the mathematics can be made more precise; you can mention that they're complex numbers instead of real numbers, and a couple of other minor points, which have nothing to do with the main idea.

The deep mystery is what I describe, and no one can go any deeper today—but only wider.

Now, I mentioned probabilities in this calculation. What we're calculating here, this curve, is the probability of arrival of an electron. The question is, is there any way to determine where it really arrives? We are not averse to using the theory of probability—that is, calculating odds when a situation's very complicated: you throw up a die, and it spins so many times in the air with the various resistance, and atoms, and all this complicated business that we're perfectly willing to allow that we don't know enough details—and so we calculate the odds that the thing'll come this way or that way. But here, what we're proposing, is it not, is that there be probability all the way back at the fundamental laws, that in the fundamental laws of physics there are odds. For example, suppose that I have an experiment so set up that with the light out, I get this interference situation, and know that. Then I say that with the light on, I can't predict through which hole it will go. I only know that each time I look it'll be one hole or the other, but there is no way to predict ahead of time through which hole it goes. The future, in other words, is unpredictable: it is impossible to predict in any way, from any information ahead of time, through which hole the thing will go—or which hole it will be seen behind. That means that physics has kind of given up, if the original purpose was (and everybody thought it was) to know enough that in given a situation you could predict what's going to happen next, that given the circumstances you can predict what happens. Here are the circumstances: source, strong light source; tell me, behind which hole will I see the electron?

You say, "Well, the reason you can't tell through which hole you're going to see the electron is, it's determined by some very complicated things back here: if I knew enough about that electron—it has internal wheels, internal gears, and so forth—and that this is

what determines through which hole it goes. It's 50/50 probability because, like a die, it's set sort of at random—and if I were to have studied it carefully enough, your physics is incomplete: if you get a complete enough physics, then you'll be able to predict through which hole it goes." That's the "hidden variable" theory, so called.

Well, that's not possible.

It is not due to a lack of detailed knowledge that we cannot make a prediction, because I said that if I didn't turn on the light, I should get this interference pattern. If I have a circumstance in which I get that interference pattern, then it is impossible to analyze it in terms of saying, it goes through here or here, because that curve is so simple, mathematically—a different thing than the contribution of this and this as probabilities.

So if it were possible for you to have determined through which hole it was going to go if I had the light on, the fact that I had the light on hasn't got anything to do with it!

Whatever gears there are back here that you observe, which permitted you to tell me whether it was going to go through 1 or 2, you could have observed if I had the light off. And therefore you could have told me with the light off which hole—each time an electron goes—which hole it's going to go through. But if you can do this, then that curve would have to be represented as the sum of those that go through there and those that go through there—and it ain't.

Therefore, it's impossible to have information ahead of time as to which hole it's going to go through when the light is out—or when the light is on, or out—in a circumstance where the experiment is set up that can produce this interference pattern.

It is not a lack of unknown gears—a lack of internal complications—that makes nature have probability in it; it seems to be in some sense intrinsic. Someone has said it this way: "nature herself doesn't know which way the electron is going to go."

A philosopher once said (a pompous one): "it is necessary for the very existence of science that the same conditions always produce the same result."

Well, they don't: if you set up electrons in any way—I mean, you set up the circumstance here, in the same conditions every time, and you cannot predict behind which hole you'll see the electron. They don't—and yet the science goes on in spite of him. Although, the same conditions don't produce the same results, that makes us unhappy, that we can't predict exactly what'll happen.

Incidentally, you can make a circumstance which is very dangerous and serious and man must know and you still can't predict. For instance, we could cook up (I don't know if we'd want to do that or not), but we could cook up a scheme by which we arrange photo cells so that if you see the electron—one electron's going to go through—if we see it behind Hole Number 1, we set off the atomic bomb, and start World War III. If we see it behind Hole 2, we have—just make peace feelers, and delay the war a little longer.

Then the thing is, that the future of man would then be dependent upon something which no amount of science can predict. The world is—the future is unpredictable. What



is necessary for the very existence of science, and what the characteristics of nature are, are not to be determined by pompous preconditions. They're to be determined—they are determined always by the material with which we work, by nature herself: we look and we see what we're going to find—what we find. We cannot say ahead of time, successfully, what it's going to look like. The most reasonable possibilities turn out often not to be the situation. What is necessary for the very existence of science is just the ability to experiment, the honesty in reporting results—the results must be reported without somebody saying what they'd like the results to have had been—and finally, an important thing, is the intelligence to interpret the results, but important point about this intelligence is: that it must not—it should not—be sure ahead of time about what must be.

Now, it can be prejudiced, and say, "That's very unlikely—I don't like that." Prejudice is different than absolute certainty. I don't mean absolute prejudice, just bias—but not strict bias, not complete prejudice. As long as you're biased, it doesn't make any difference, because if the fact is true, there will be a perpetual accumulation of experiments to perpetually annoy you, until they cannot be disregarded any longer.

[They] only can be disregarded if you're absolutely sure ahead of time of some precondition that science has to have. In fact, it is necessary for the very existence of science that minds exist, which do not allow that nature must satisfy some preconceived conditions like those of our philosophers.

Thank you.